# Hadron-Resonance Gas at Freeze-out: Reminder on Importance of Repulsive Interactions


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**V.B., M. Gazdzicki, M.I. Gorenstein PRC 2013**

# Motivation

- **Resonances introduced to the ideal Hadron Gas take into account attractive interactions**

- **Nucleon-nucleon potential includes both parts – attractive at large and repulsive at small distances**

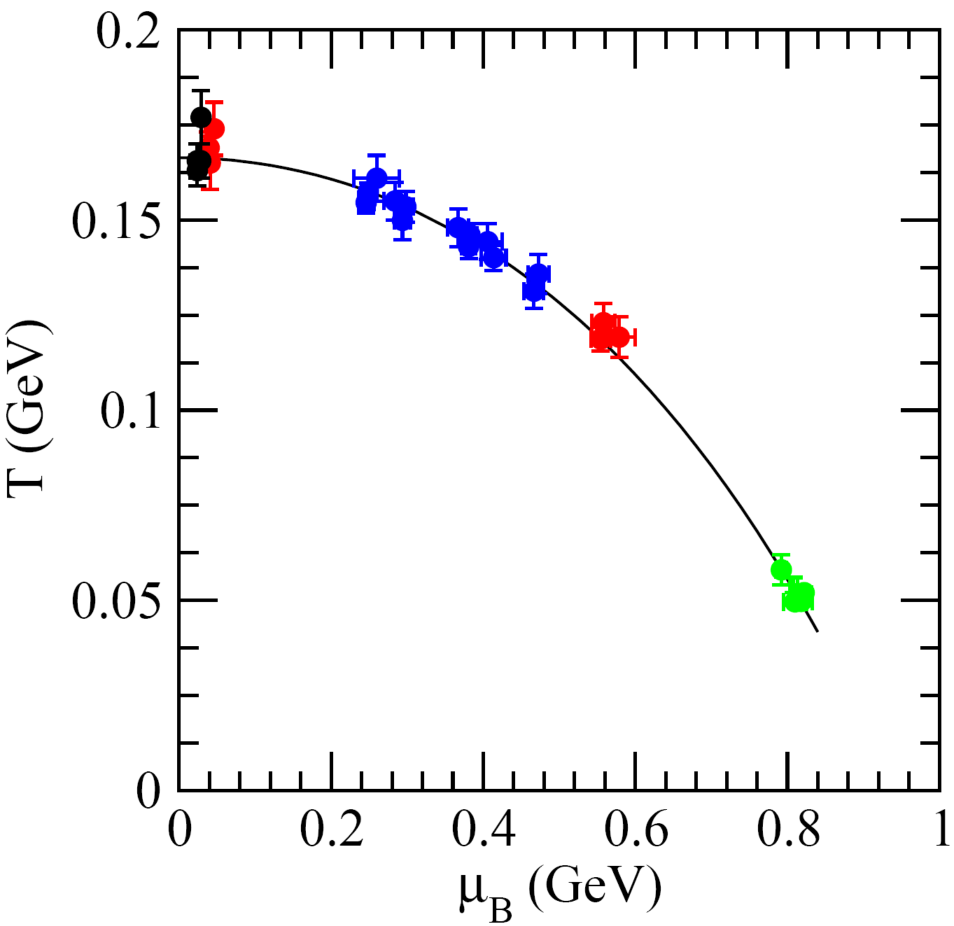
- **Repulsive interactions are accounted for by using van der Waals excluded volume**

- **Point-like hadrons would always become dominant phase at very high energy density due to the large number of different types of hadrons**

- **Just the excluded volume effects ensure a phase transition from a gas of hadrons and resonances to the quark-gluon plasma**

# Hadron Gas of **Point-Like** particles

**Hadron Gas model allows to connect the results of different experiments by the single Freeze-out line $\mathrm{T}(\mu_\mathrm{B})$**




$$\langle N_i \rangle \equiv \sum_{\mathbf{p}} \langle n_{\mathbf{p},i} \rangle \simeq \frac{g_i V}{2\pi^2} \int_0^\infty p^2 dp \, \langle n_{p,i} \rangle \,,$$

$$\langle N^+ \rangle = \sum_{i, q_i > 0} \langle N_i \rangle \,,$$

$$\langle \mathbf{n_{p,i}} \rangle = \frac{\mathbf{1}}{\exp\left[\left(\sqrt{\mathbf{p}^2 + m_i^2} - \mu_\mathbf{i}\right)/\mathbf{T}\right] \pm \mathbf{1}}$$

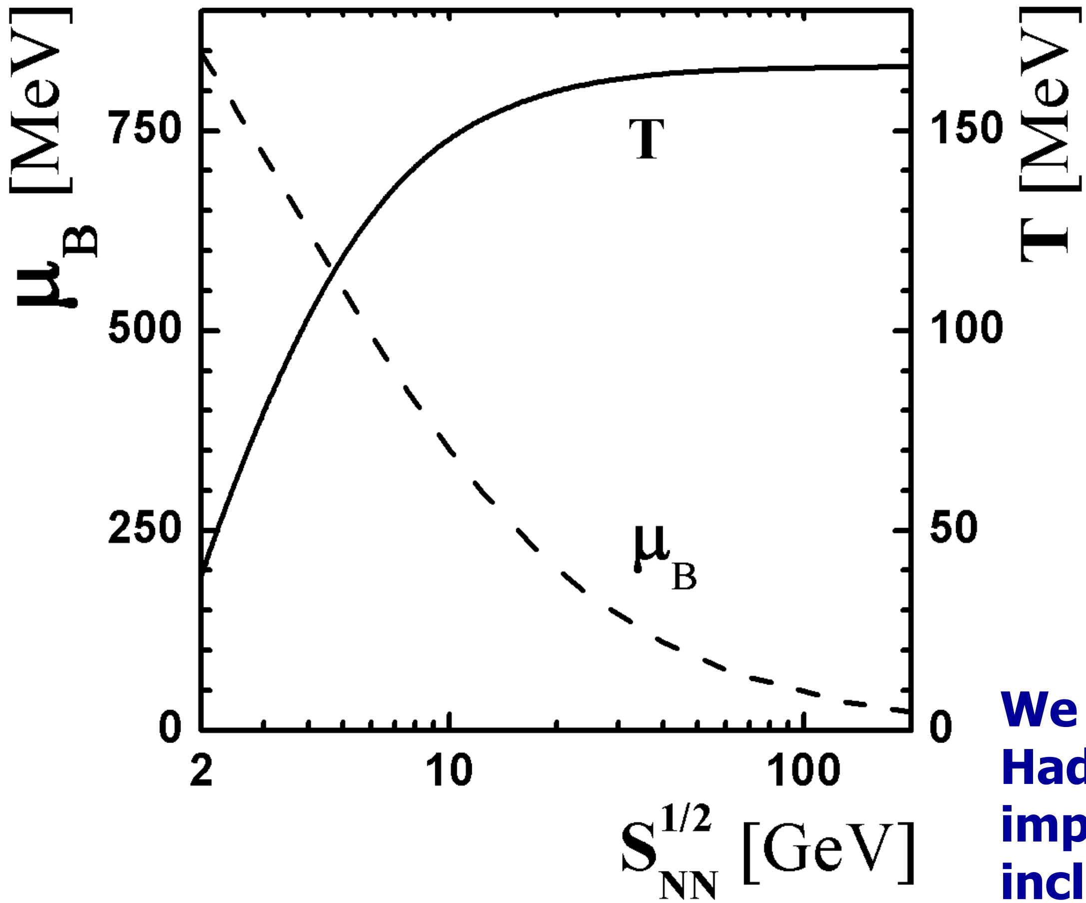
$$\mu_i = q_i \, \mu_Q + b_i \, \mu_B + s_i \, \mu_S,$$

$$Q/B = 0.4 \Rightarrow \mu_Q(T, \mu_B)$$
$$S = 0 \Rightarrow \mu_S(T, \mu_B)$$

$$\mathbf{T} \quad \mu_\mathbf{B} \quad \gamma_\mathbf{S}$$

Cleymans, Oeschler, Redlich, Wheaton, Phys. Rev. C 73, 2006

# Hadron Gas of Point-Like particles




THERMUS: Wheaton, Cleymans
Comp. Phys. 2009 [hep-ph/0407174]

**Cleymans et. al.:**

$$\mu_B = \frac{1308}{1000 + 0.273\sqrt{s_{NN}}}$$

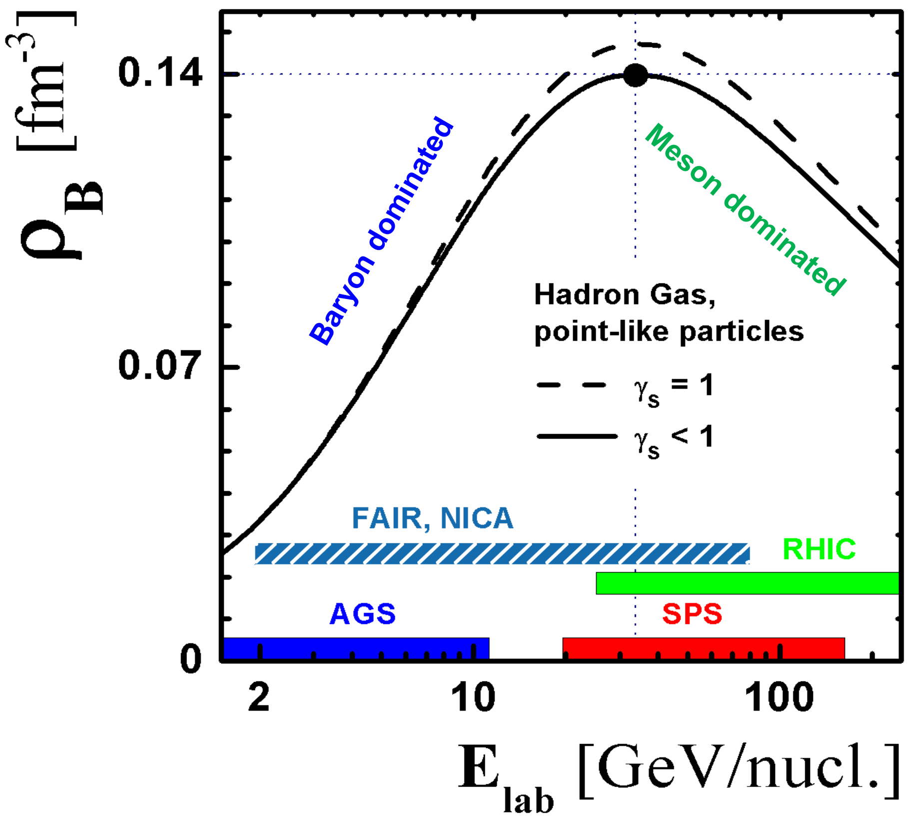
$$T = 166 - 139\mu_B^2 - 53\mu_B^4$$

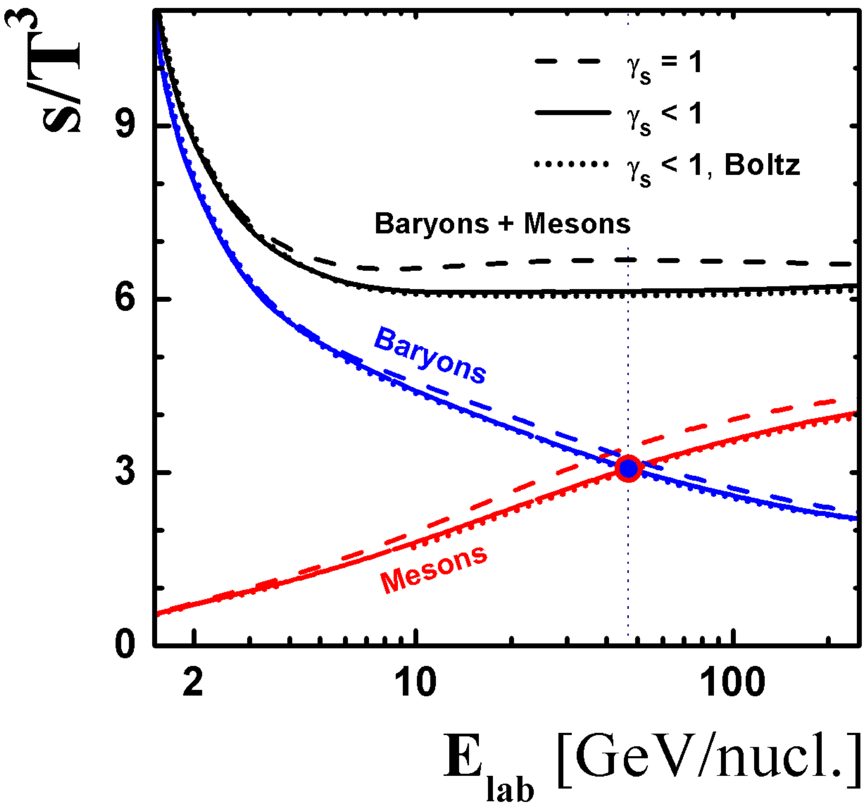
**Becattini, Manninen, Gazdzicki**
**PRC 73, 2006:**

$$\gamma_S = 1 - 0.396 \exp\left(-1.23\frac{T}{\mu_B}\right)$$

**We use this parameterization in Hadron Resonance Gas model implemented in THERMUS that includes all particles and resonances up to $K_4^*(2045)$ meson and $\Omega^-$ baryon, quantum statistics and width of resonances**

# Net-Baryon and Entropy Densities




- Net-baryon density has a maximum at 34A GeV, where NA49 observed the signals of the onset of deconfinement
- With increasing collision energy the baryon-dominated matter changes to meson-dominated matter at 46A GeV

# Excluded Volume HRG

The system volume should be substituted by the available volume $V \rightarrow V - \sum_i v_i N_i$

where $v_i$ is the excluded volume parameter $v_i = 4 \cdot \frac{4}{3} \pi r_i^3$

and $r_i$ is the corresponding hard sphere radius of a particle $i$

In the grand canonical ensemble, this substitution leads to a transcendental equation for the pressure

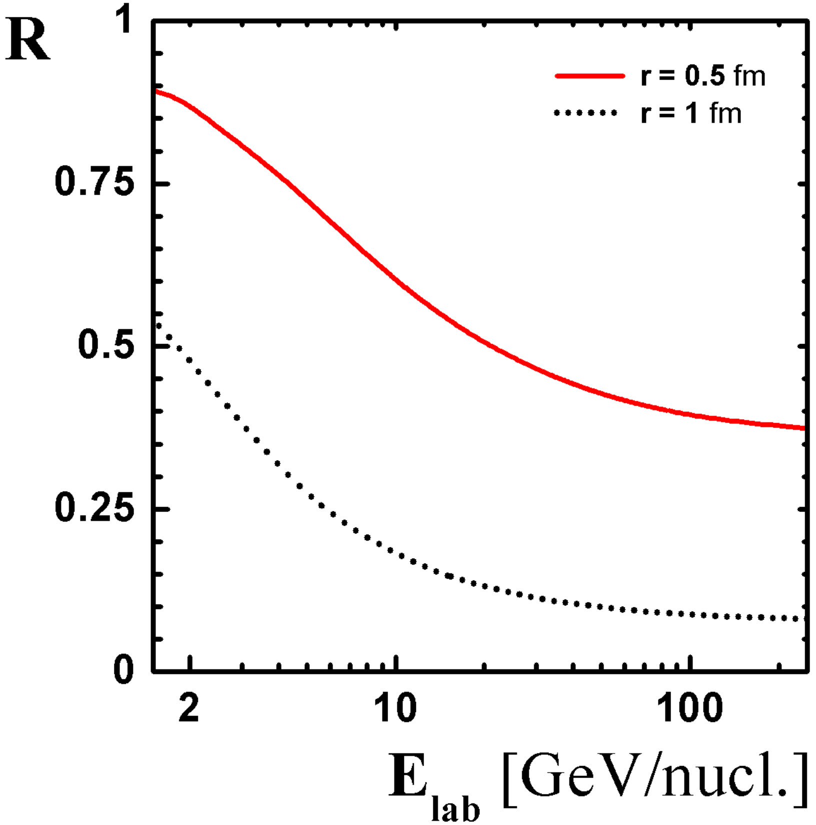
$$p = \sum_i p_i^{id}(T, \widetilde{\mu_i}) \qquad \widetilde{\mu_i} = \mu_i - v_i\, p$$

and modified density

$$n_i = \frac{n_i^{id}(T, \widetilde{\mu_i})}{1 \; + \; \sum_i v_i\, n_i^{id}(T, \widetilde{\mu_i})}$$

Gorenstein, Petrov, Zinovjev, PLB 1981;
Rischke, Gorenstein, Stocker, Greiner, ZPC 1991

# Suppression factor




**If all particles have the same volume then in Boltzmann approximation:**

$$R(T, \mu_B;\ r) = \frac{\exp(-v\, p/T)}{1 + v \sum_j n_j^{id}(T, \tilde{\mu}_j)}$$

**Particle ratios cancel this factor**

**Mean multiplicities absorb it in volume**

**However densities change:**

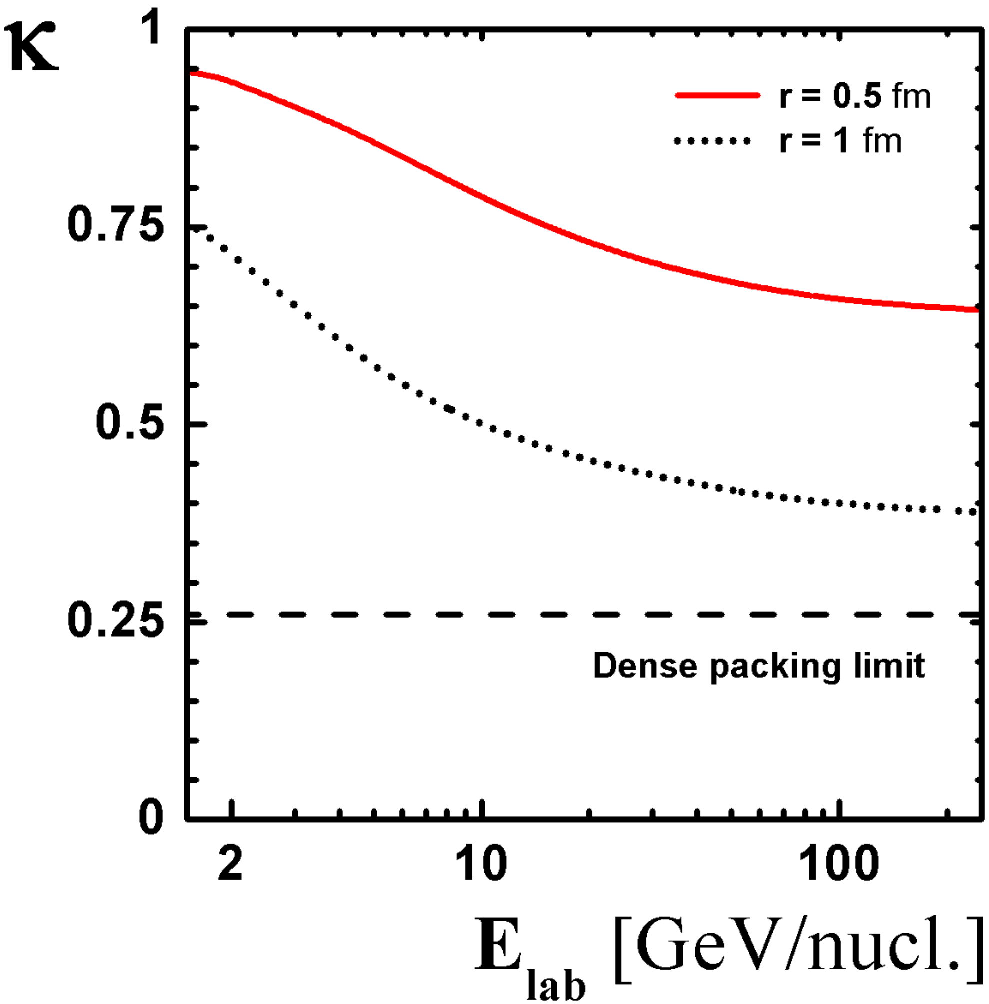
$$\rho_B(T, \mu_B) = R\, \rho_B^{id}(T, \mu_B)$$

$$s(T, \mu_B) = R\, s^{id}(T, \mu_B)$$

$$\varepsilon(T, \mu_B) = R\, \rho_B^{id}(T, \mu_B)$$

**Typical values of hard-core radii considered in the literature are r = 0.3 – 0.8 fm. One may therefore expect a decrease of the density maximum and a shift of its position to a smaller collision energy**

# Available Volume



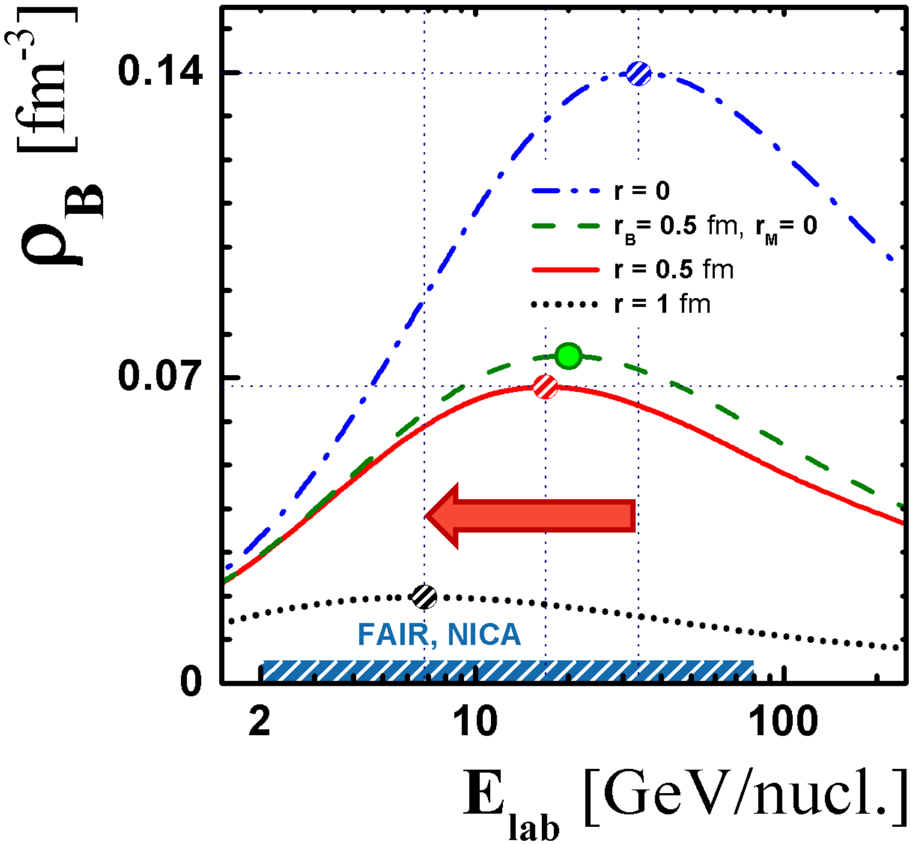

$$\kappa = \frac{V - v\sum_i N_i}{V} = \exp\left(\frac{v\,p}{T}\right) R$$

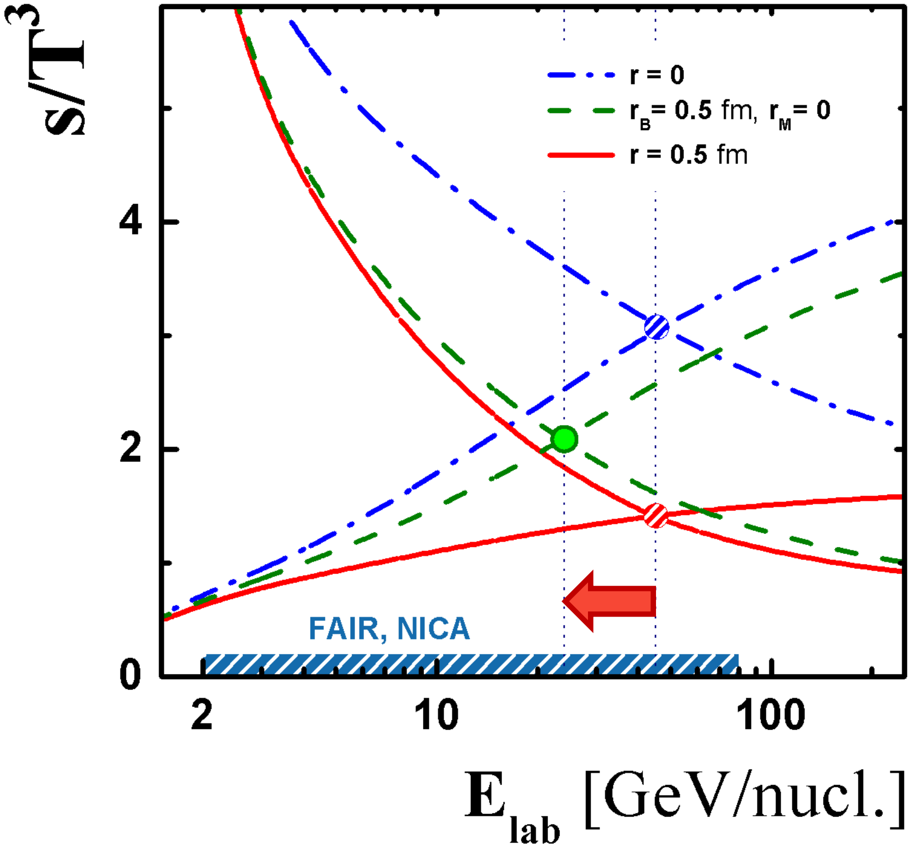
**It is always larger than the dense packing limit for hard spheres.**

**We also remind that the excluded volume parameter v is assumed to be 4 times larger than the hadron volume**

**This ensures a consistency of the excluded volume approach at all collision energies even for the largest considered radius r = 1 fm**

# Excluded Volume Effects



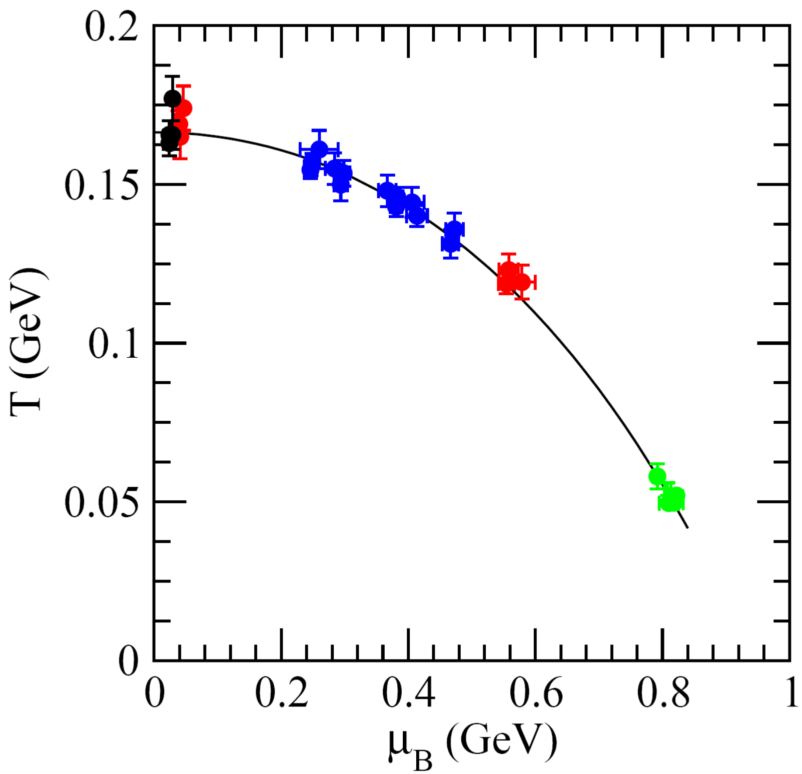

- **Density maximum moves from 34 to 17 and 7 AGeV**
- **Transition point of baryon/meson domination moves from 46 to 23 AGeV**
- **One should refit data and obtain new freeze-out line to conclude about effects arising due to different baryon and meson radiuses**

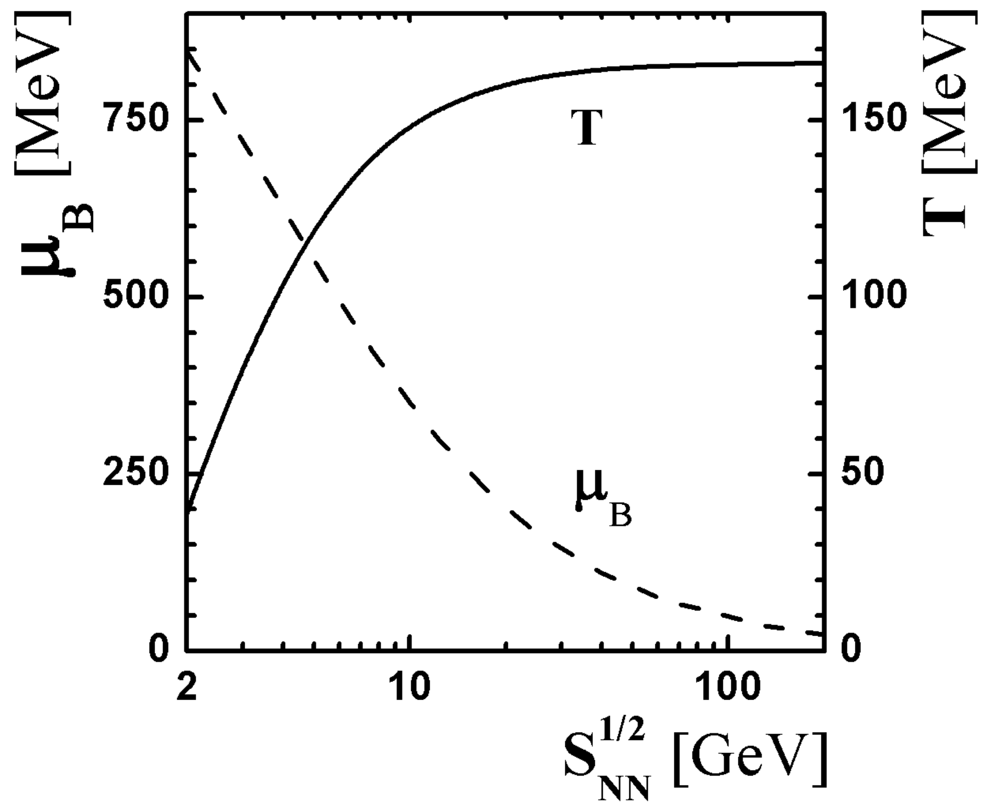
# Conclusions:

- Excluded Volume effects lead to smaller particle number and entropy densities
- The position of the net baryon density maximum moves to lower energies
- The transition point from baryon to meson domination lies at higher energies than density maximum
- The transition point moves to lower energies only for different meson and baryon radiuses in the system
- Further research is needed

# Thank you!

# Freeze-out line for r=0 or $r_B=r_M$



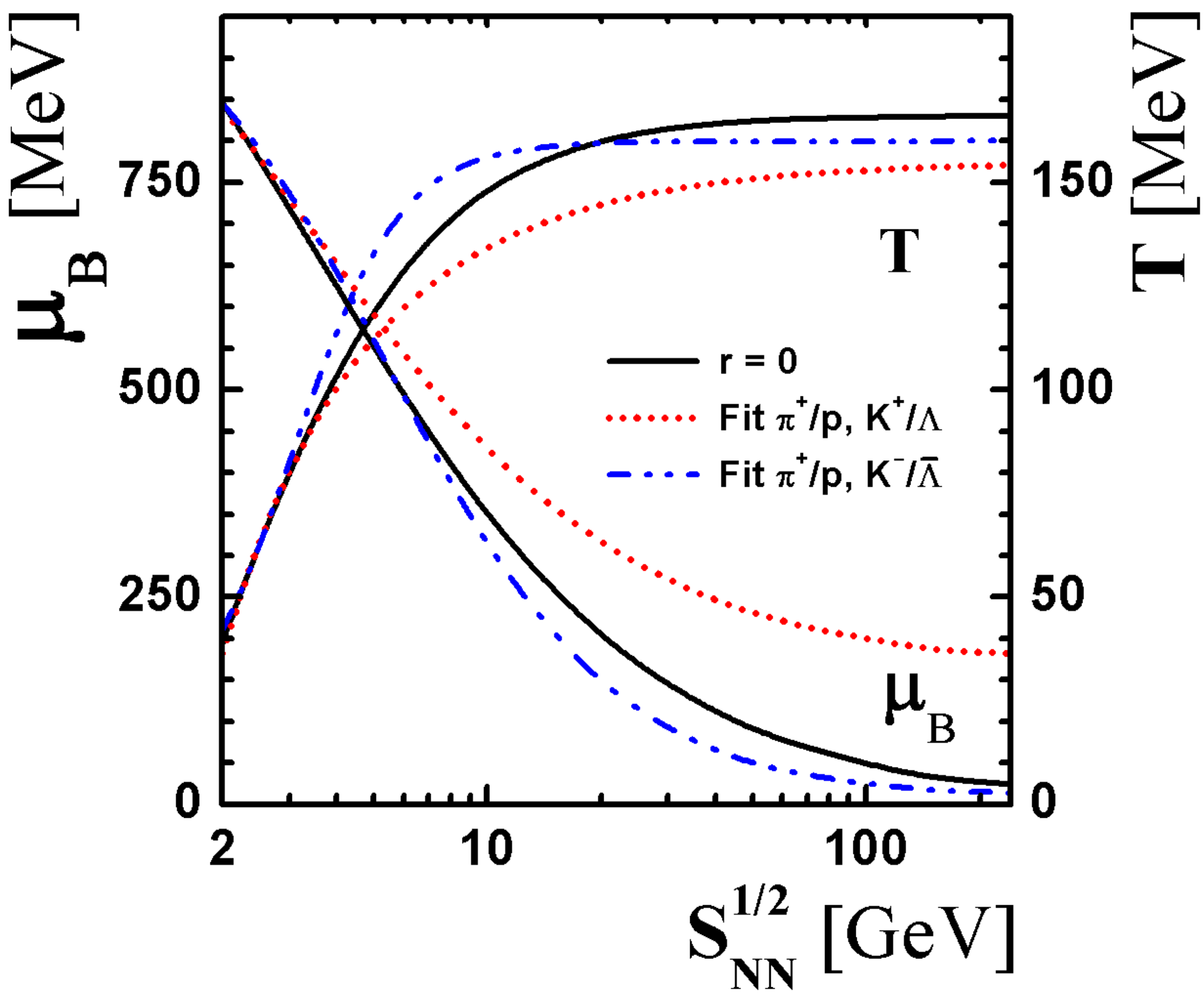




**The r=0 line was obtained as a fit of the $T(\mu_B)$ from different groups.**

**The new fit in a single model (THERMUS) gives a different line.**

**For simplicity we take two ratios obtained from the r=0 freeze-out line and fit them with $r_B$=0.5 fm, $r_M$=0**

# New fit with $r_B$=0.5 fm, $r_M$=0




The new fit gives very different freeze-out lines because

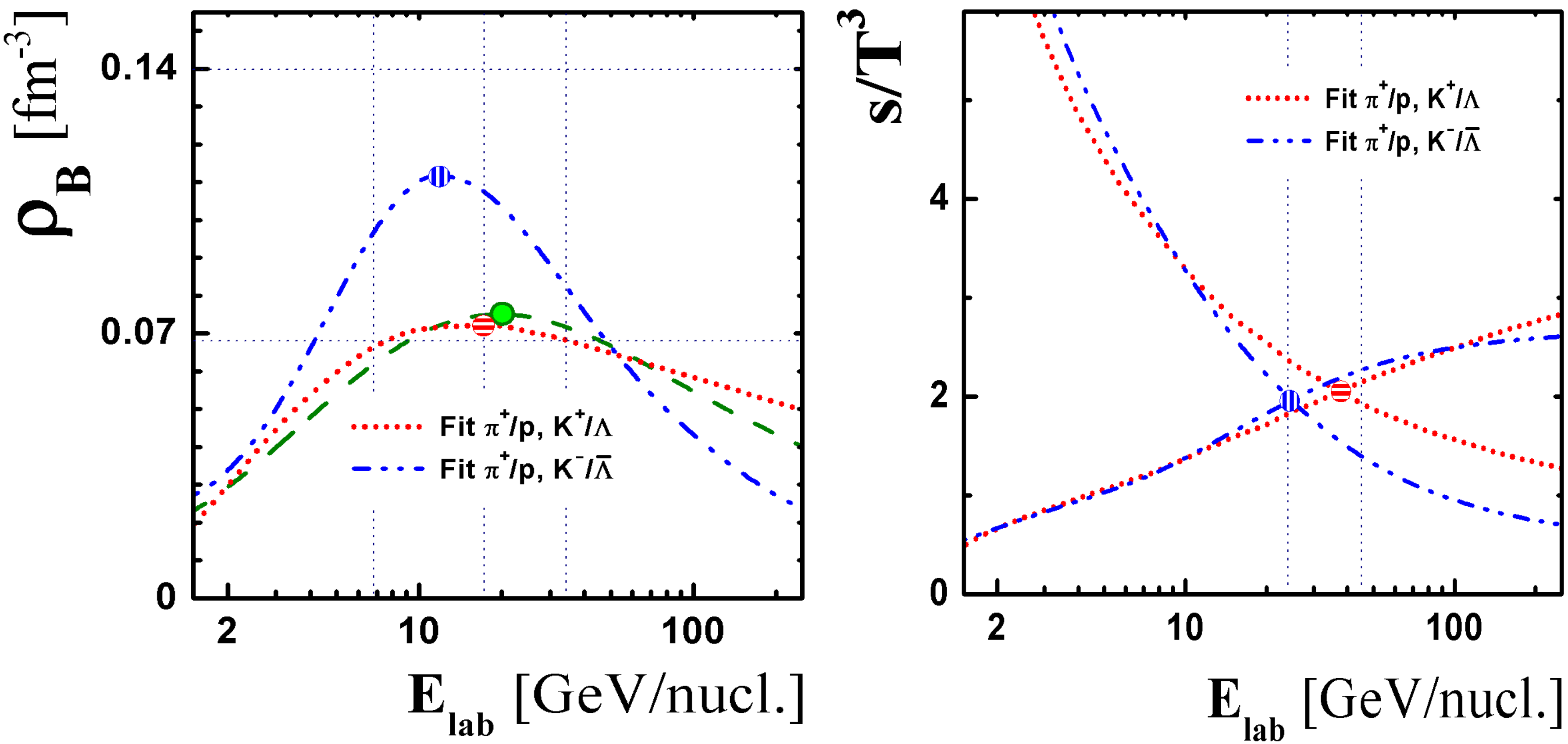
$$\frac{\pi^+}{p},\ \frac{K^+}{\Lambda} \sim e^{-\frac{\mu_B}{T}}$$

$\frac{\mu_B}{T}$ increases

$$\frac{K^-}{\overline{\Lambda}} \sim e^{+\frac{\mu_B}{T}}$$

$\frac{\mu_B}{T}$ decreases

# New fit with $r_B$=0.5 fm, $r_M$=0




**The new fit changes the details and preserves the main features: change in the position of density maximum and baryon/meson transition point.**