LATTICE

DESIGNS

Lecture 6
January 2014

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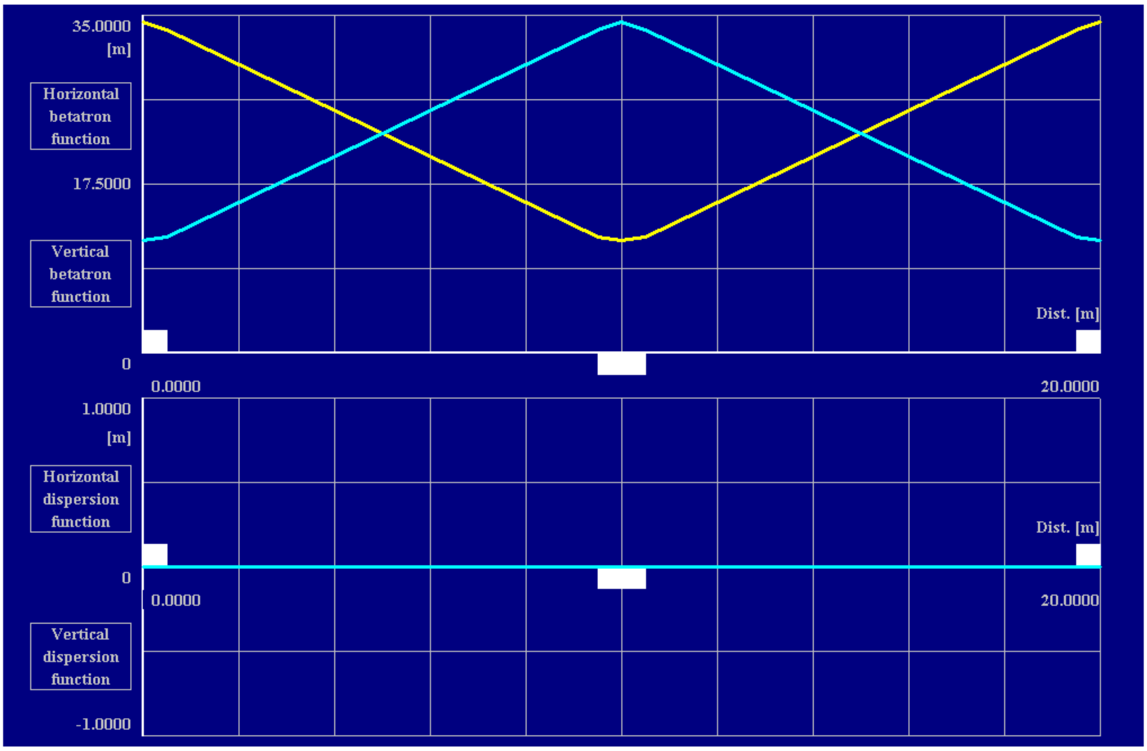
Introduction

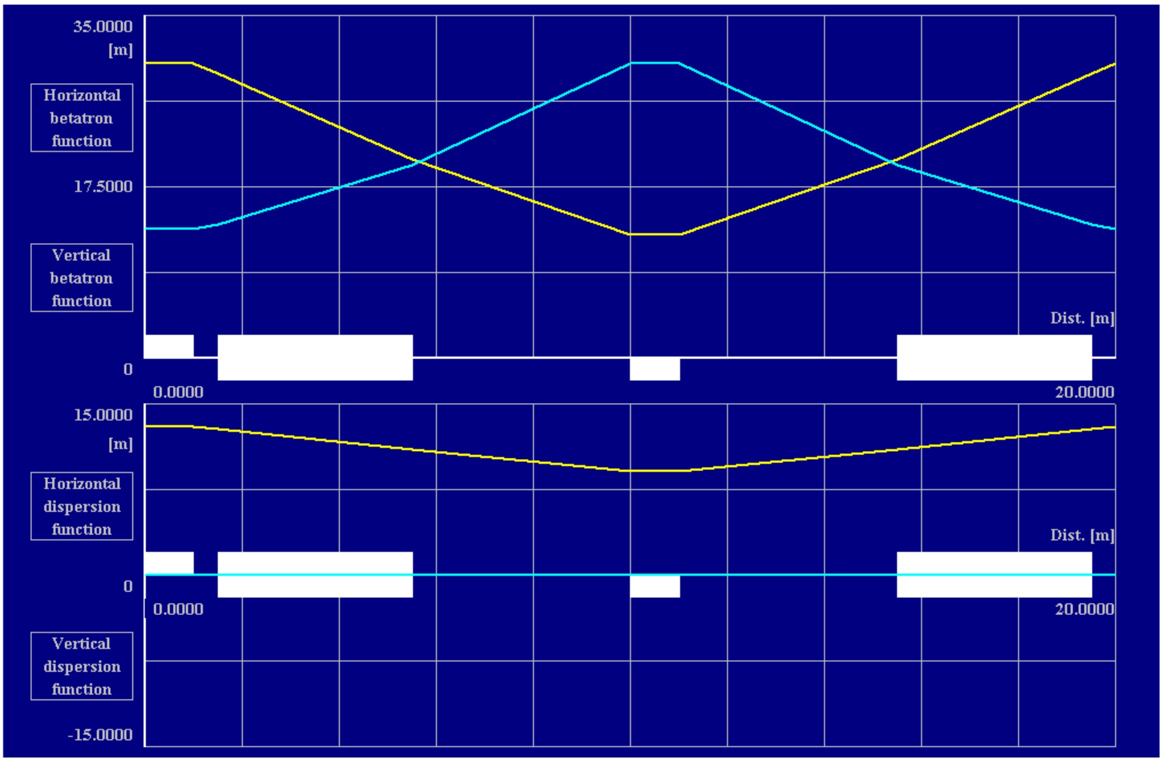
- **So far we have:**
  - **Introduced a local curvilinear co-ordinate system that follows the reference or central orbit.**
  - **Described the behaviour of the beam with respect to the reference or central orbit using a 'hard-edge' model for the lattice elements.**
  - **Derived the transverse motion equations in the local curvilinear co-ordinate system for both magnetic and electrostatic elements.**
  - **Expressed the solutions in terms of matrices.**
  - **Used the matrices to track ions though a lattice.**
  - **Introduced the Twiss parameterisation which is ubiquitous to lattice design.**
  - **Treated the longitudinal plane in the same way as the transverse plane.**
- **We will now look at lattice designs:**

FODO cell

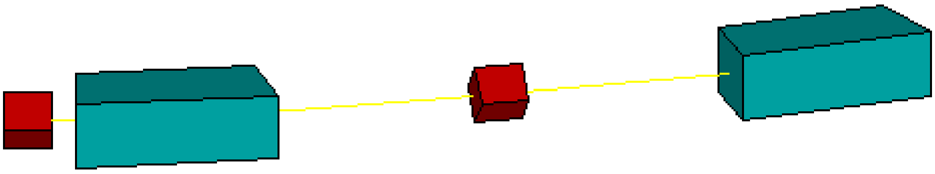


- **The basic FODO cell is the best known and studied cell in lattice optics.**
- **The usual choices for phase advances are 45°, 60° and 90°. The 60° cell has the best all-round characteristics and is close to the minimum beam sizes obtained at ~76°.**
- **Note an 'F quadrupole' is denoted by a box above the axis and a 'D' by a box below the axis. Dipoles are denoted by a box extending above and below.**
- **In the above example: $\Delta\mu$=60°, k_F = -0.1035, k_D = 0.1035 and L_{cell} = 20 m.**

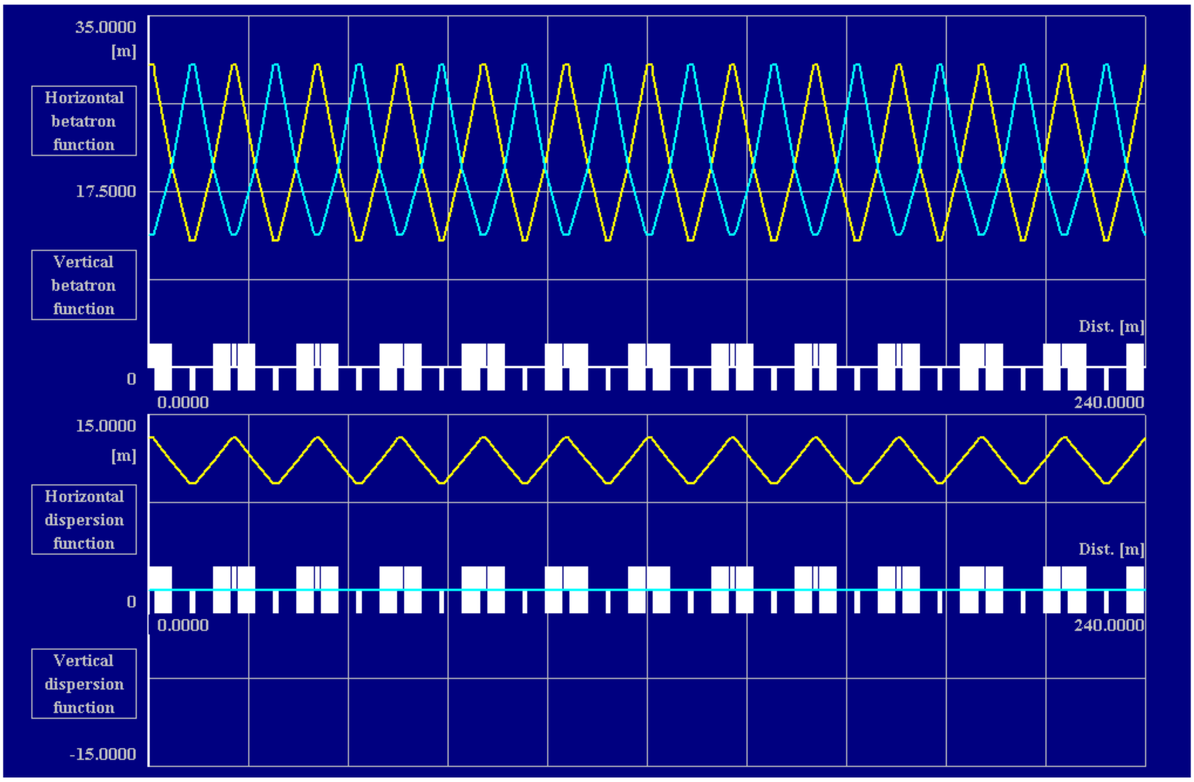
FODO with dipoles



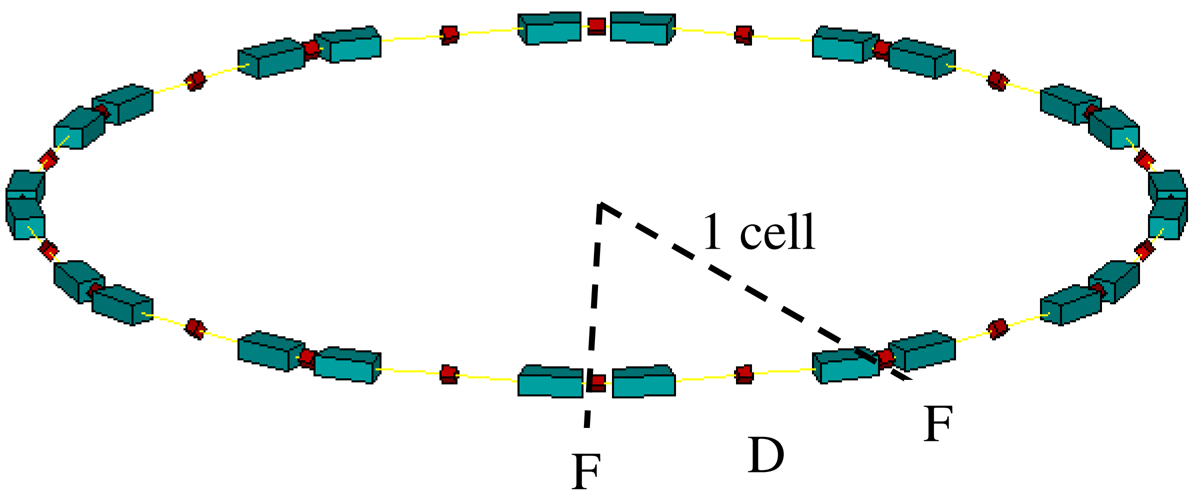
- **The addition of dipoles changes the focusing slightly and introduces dispersion.**
- **In the above example: $\Delta\mu_x = 60°$, $\Delta\mu_z = 60°$, $k_F = -0.0722$, $k_D = 0.0915$, $\theta_H = 0.2618$ rad and $L_{cell} = 20$ m.**



A regular ring using a FODO

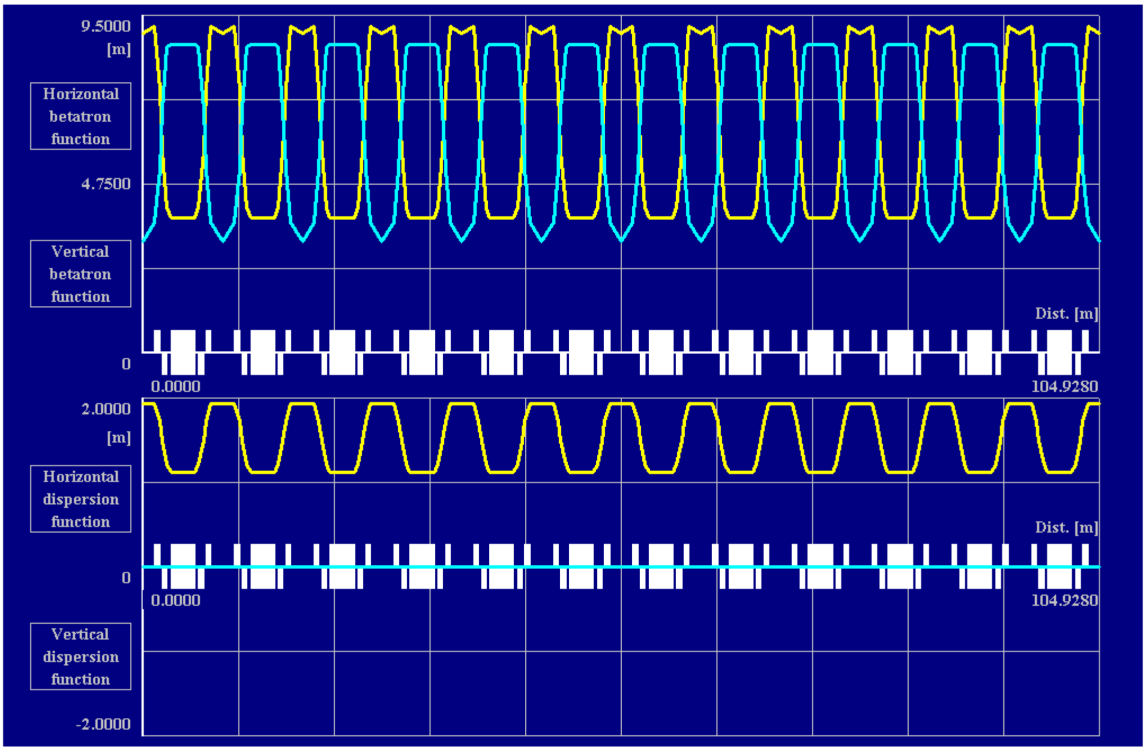


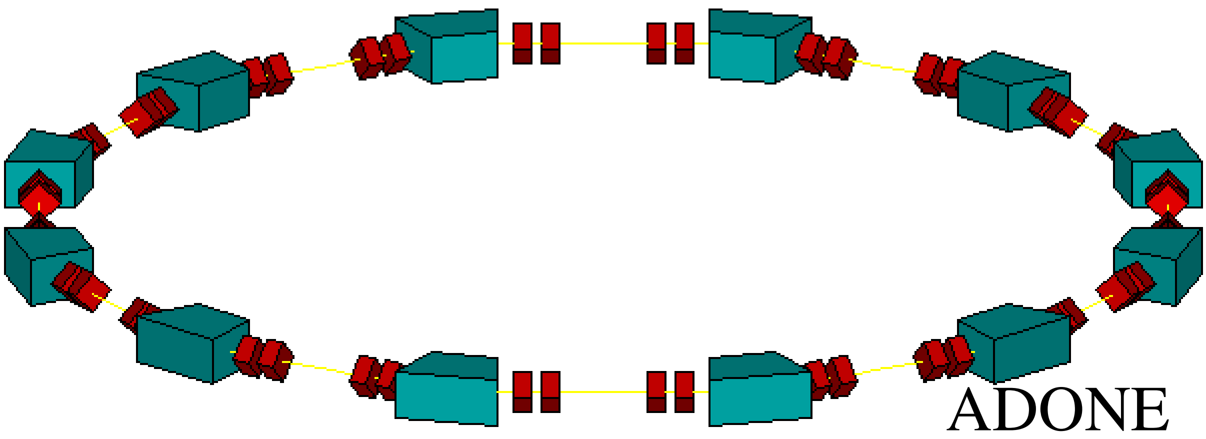
- **Using the same cell we can make a ring, BUT the drift spaces tend to be too short for extraction and injection.**
- **Note that dipoles sit around β_z minimum to save power.**



A ring using a split FODO

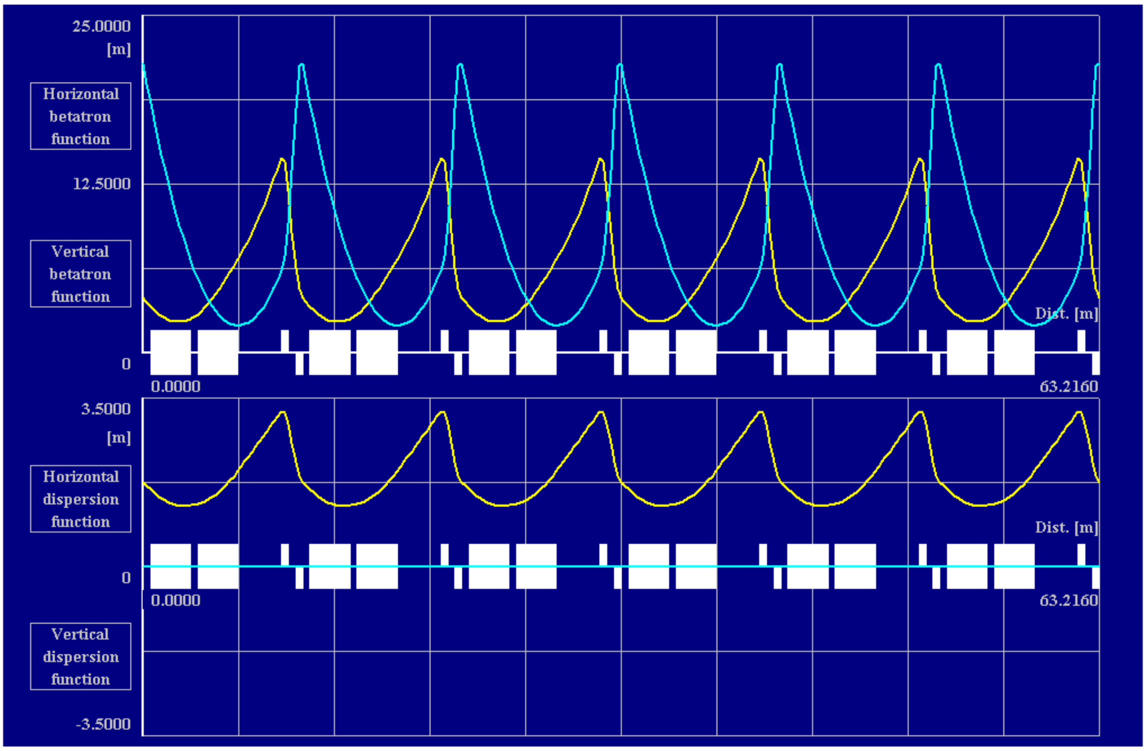
- Here the F and D quads are split into 2 units. Between the 'split' quads, the betatron amplitude functions are quasi constant.
- Unlike the previous lattice, the dipoles sit around β_y max. because the requirements of a light source take precedence over the aperture and cost of the magnets.

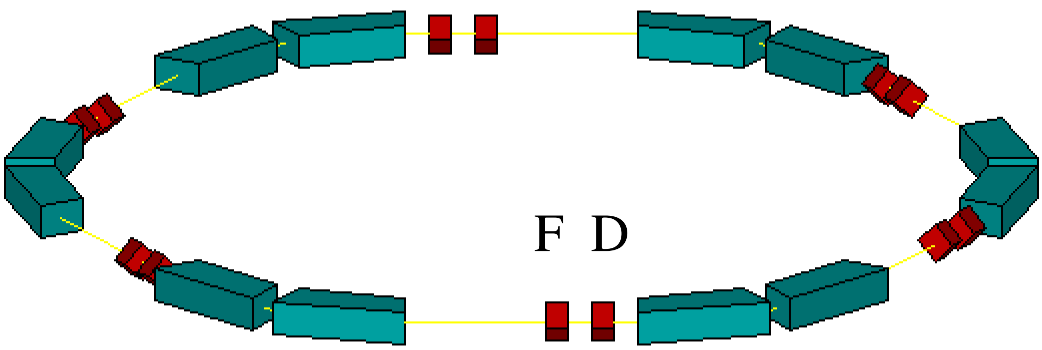




A ring using a doublet

- **Another way to make the space in a FODO more useful is to move the central quadrupole to one side. This effectively creates pairs of quads, or doublets.**
- **Doublets have been very popular, but they do have large peaks and steep asymmetric slopes in the betatron amplitude functions.**





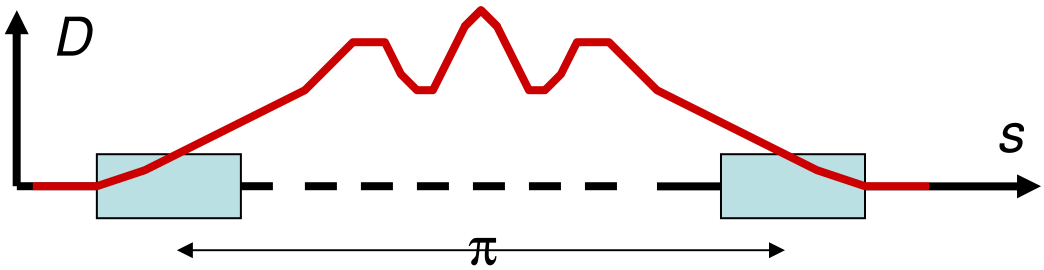
GSI medical ring design

Controlling dispersion

- **All the rings shown so far simply repeat a standard cell n times to reach 2π of bending.**
- **This works for plain accelerators and often leads to an economical solution in which all quadrupoles for example are powered by a single power converter.**
- **In more advanced lattices, we would like to have regions with zero dispersion e.g. for RF cavities. This is done in small rings by closing the dispersion in bumps. For large rings, see later.**
- **To close a dispersion bump one needs a phase advance of 180° to 360° in the plane of bending.**
- **This leads to solutions for rings with two, or three or four or more closed dispersion bumps separated by dispersion-free sections.**
- **Each closed bump forms a 'corner' and the ring looks 'triangular' or 'square' or 'pentagonal' and so on.**

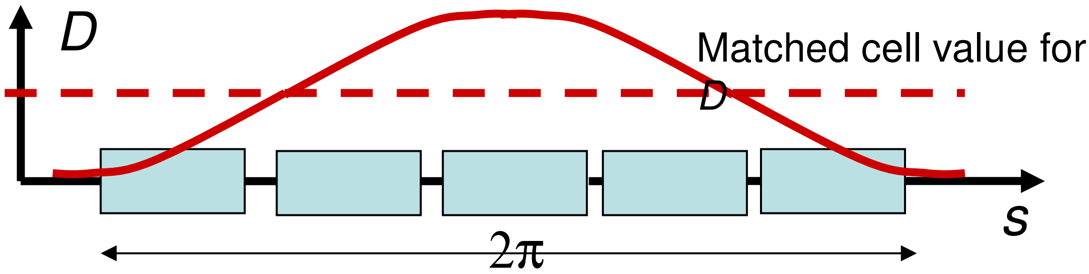
Closing a dispersion bump

- **Case 1. The half-wavelength bump**



Possible where 2 short magnets can provide all of the required bending.

- **Case 2. Uniformly distributed bending**



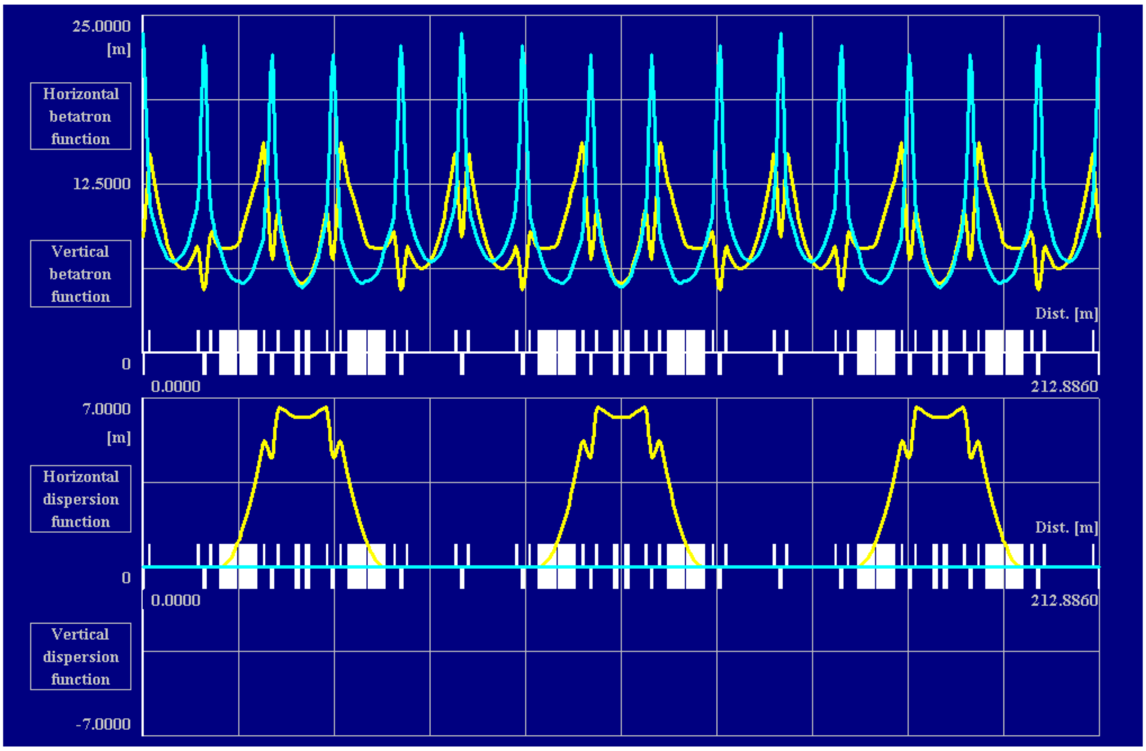
When the bending is uniformly distributed, the dispersion *D* oscillates about the equilibrium value of the matched cell.

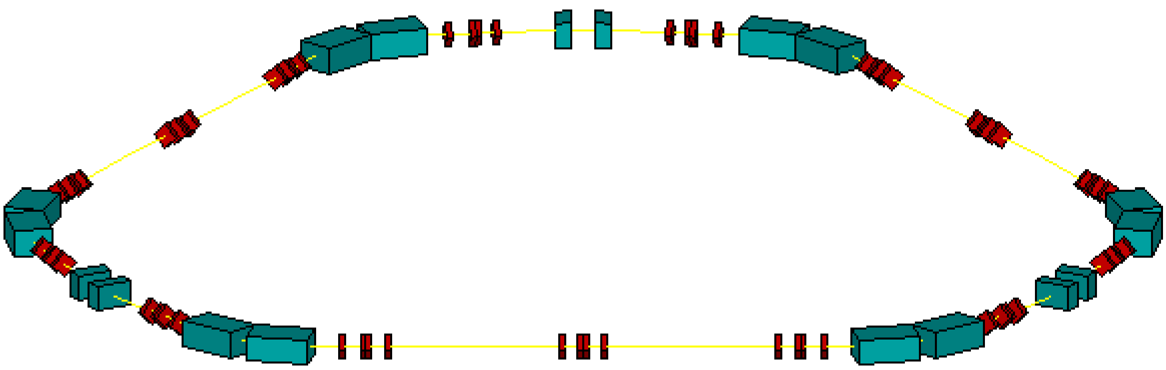
- **Case 3. Hybrid**

Often the lattice of a small ring will be a mixture of the two limiting cases above.

A ring using a triplet

- A triplet is another possible cell for a ring.
- In this case, the large horizontal phase advance at the centre of the triplet is used to make 3 closed dispersion bumps.
- The 'waist' in the vertical betatron amplitude in long straight sections is used for the dipoles. This keeps the aperture requirements and cost down.



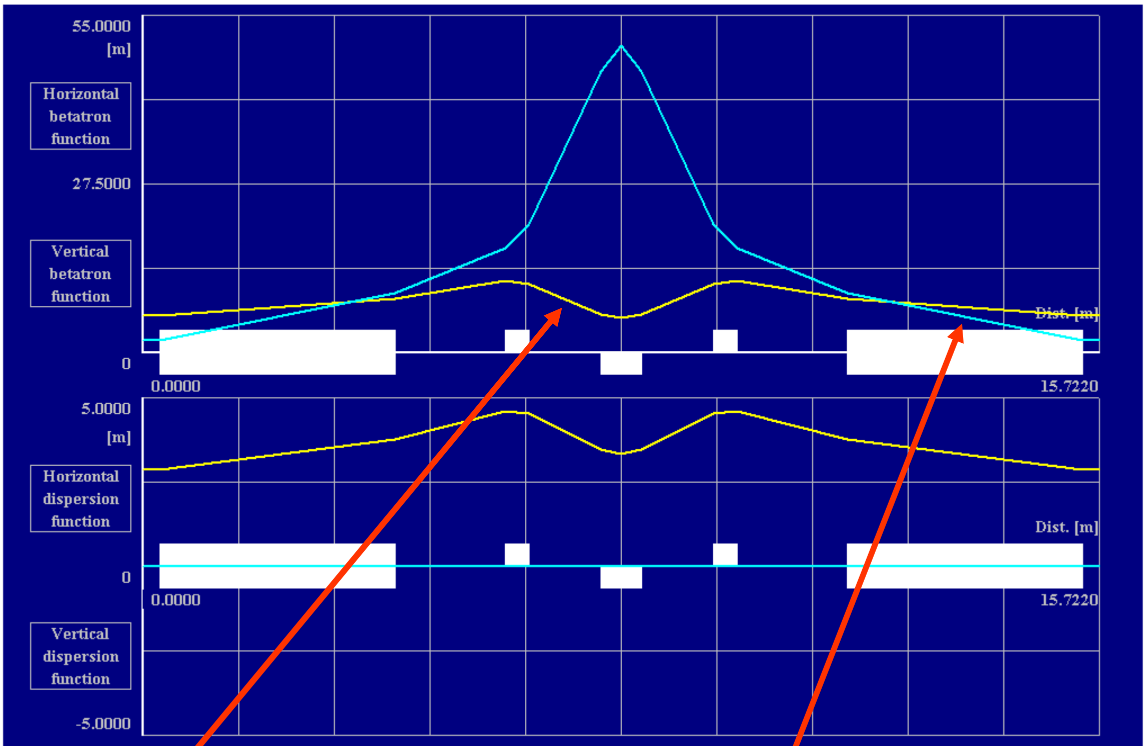


AUSTRON

Characteristics of triplets

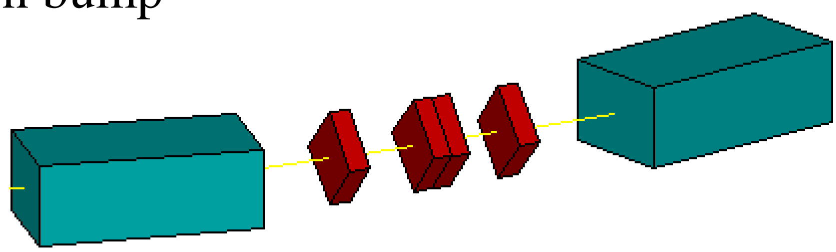
- **Phase advance** $$\mu_{s_1 \rightarrow s_2} = \int_{s_1}^{s_2} \frac{1}{\beta} \, \mathrm{d}s$$

- **Thus regions of low β give large phase advances.**



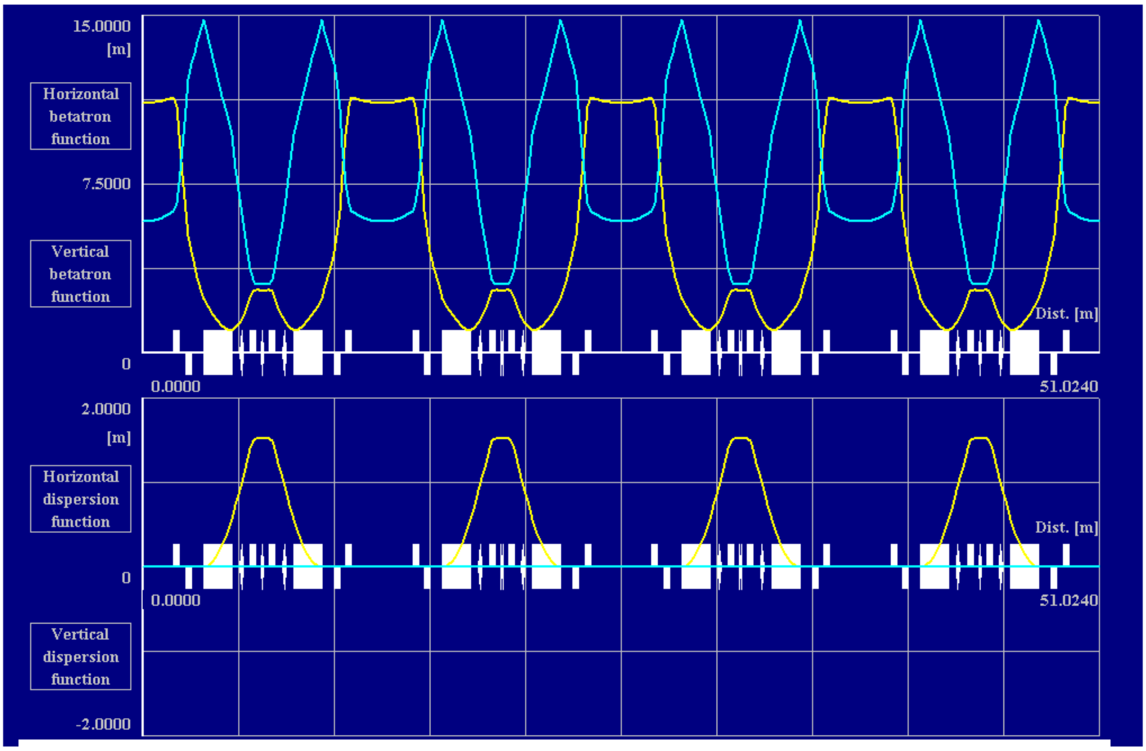
β_x is kept small for large phase advance for closing dispersion bump

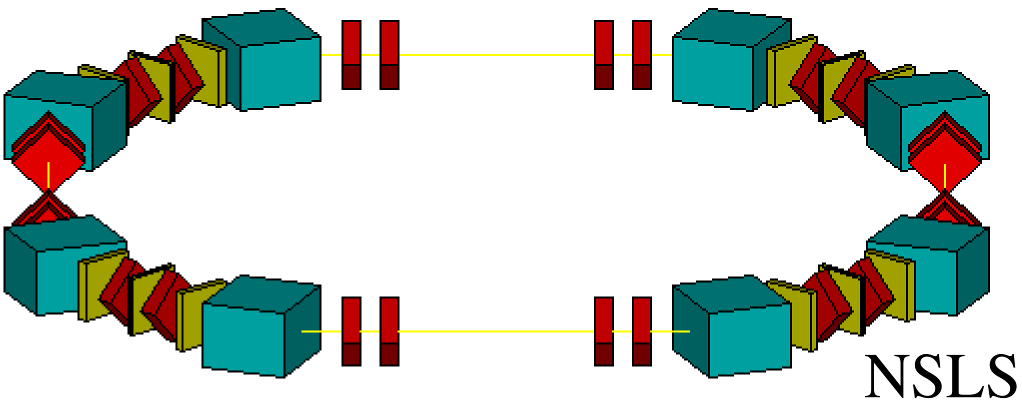
Small β_z in dipole saves money



Light source lattice

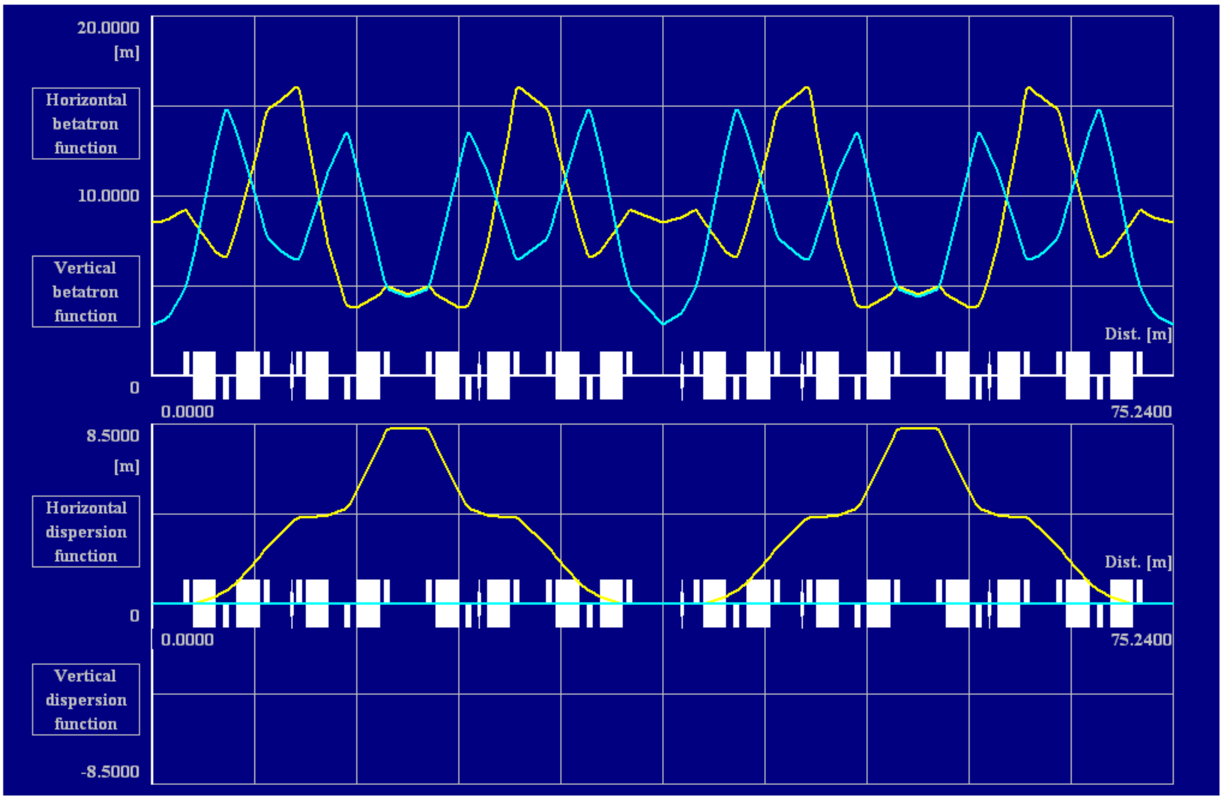
- **Chasman-Greene, double-bend achromat, high-brightness lattice. The aim is to minimise $D_x(s)$ and $\beta_x(s)$ in the dipoles.**
- **Each cell supports a closed dispersion bump. There are 4 bumps making a 'square' ring.**

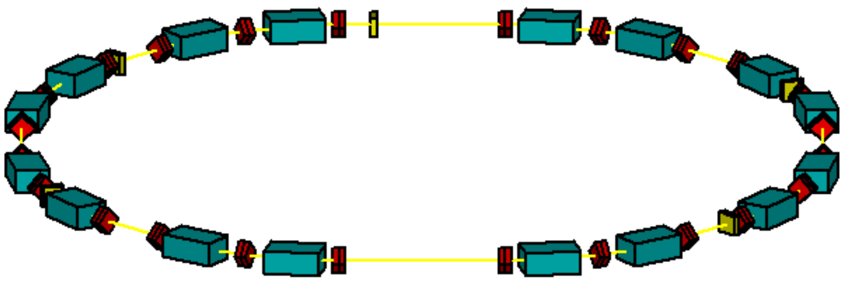




Medical machine lattice

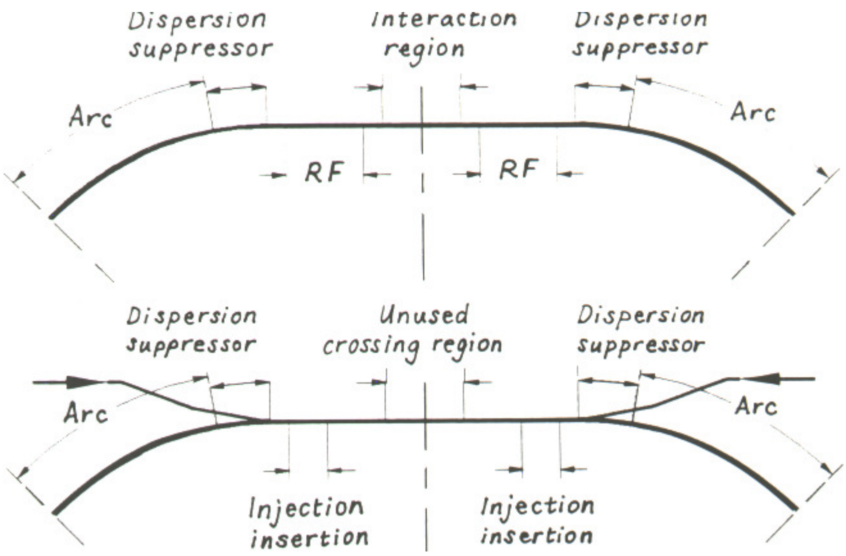
- **The PIMMS medical machine lattice.**
- **This ring has 2 dispersion bumps with distributed bending. Compared to the earlier examples, this creates a ‘rounder’ ring.**





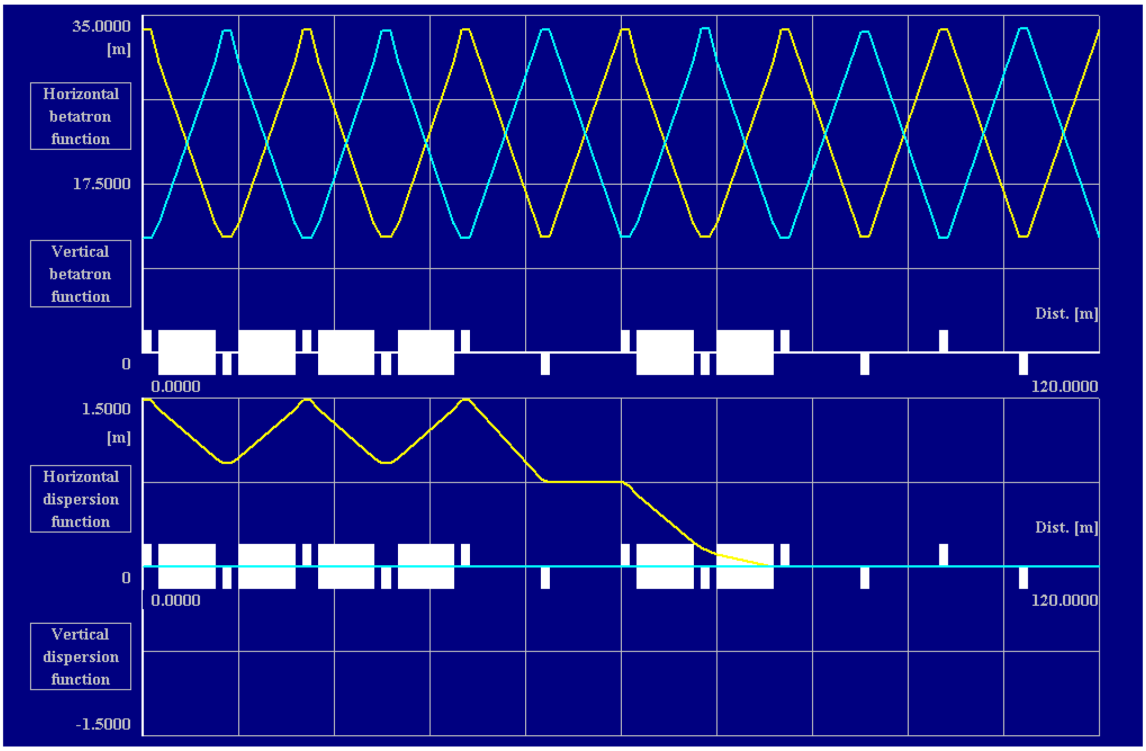
Large rings

- Large rings, such as the LHC, often have a basic FODO cell in the arcs.
- The overall ring has an *n*-fold symmetry containing the *n*-arcs and *n* straight regions in which the physics experiments are mounted.
- Between the arcs and the straight regions there is the so-called *dispersion suppressor* that brings the dispersion function to zero in the straight region in a controlled way. There are several schemes for dispersion suppressors (see next slides).
- The straight regions contain the injection and extraction and the RF cavities, which, in an electron machine like LEP, can occupy hundreds of metres.
- A dispersion-free straight region may also contains a low-β insertion for physics.



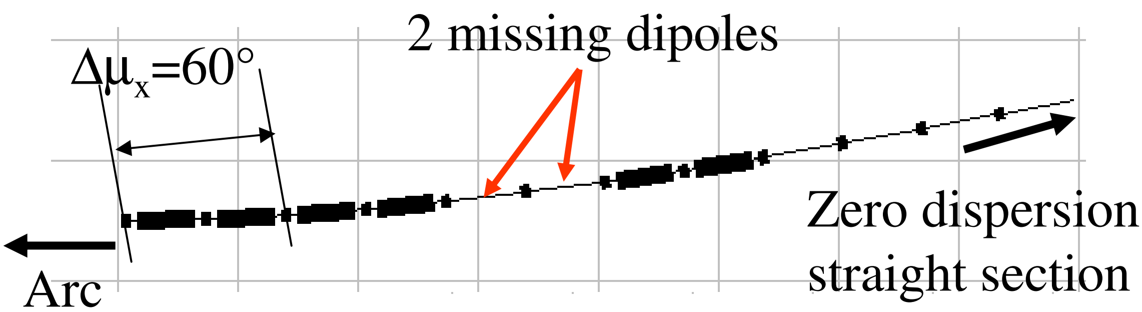
Missing-magnet suppressor

- **Lattice functions of missing-magnet suppressor for a 60° FODO cell. Note how β_x and β_z hardly notice the suppression of D_x.**



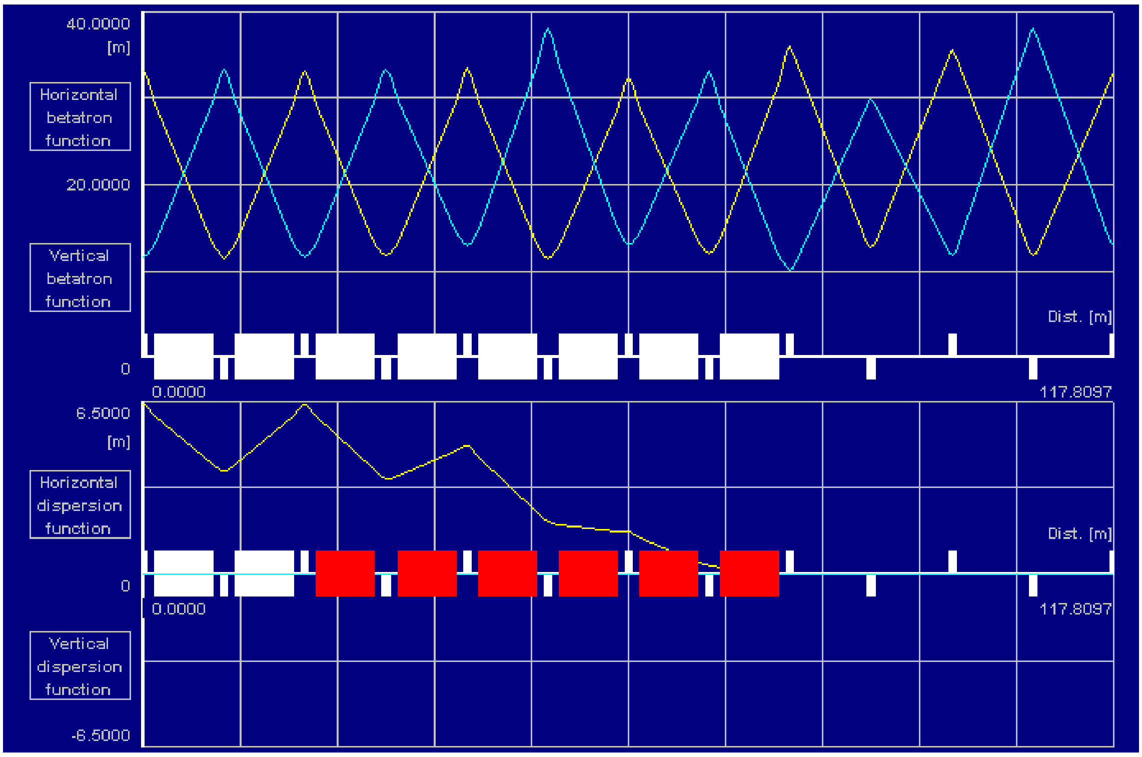
File name: DEMO-004 | LATTICE ELEMENTS (On-Axis) | Date of run: 11/
Treated as: transfer line | Time of run: 8:4
User title: Demonstration FODO cell with 60deg phase advance and rectangular dip.

Unit no. [1]	Name [2]	Type [3]	Length [m] [4]	Hor. Mbend [rad] [5]	Vert. Mbend [rad] [6]	Edge angle-1 [rad] [7]	Edge angle-2 [rad] [8]	k-Mquad [1/m2] [9]
1	FQ	QUADR	1.0000	0.000000	0.000000	0.00000	0.00000	-0.103513
2	ss0	DRIFT	1.0000	0.000000	0.000000	0.00000	0.00000	0.000000
3	Dipole	RBEND	7.0000	0.030000	0.000000	0.01500	0.01500	0.000000
4	ss0	DRIFT	1.0000	0.000000	0.000000	0.00000	0.00000	0.000000
5	DQ	QUADR	1.0000	0.000000	0.000000	0.00000	0.00000	0.103513



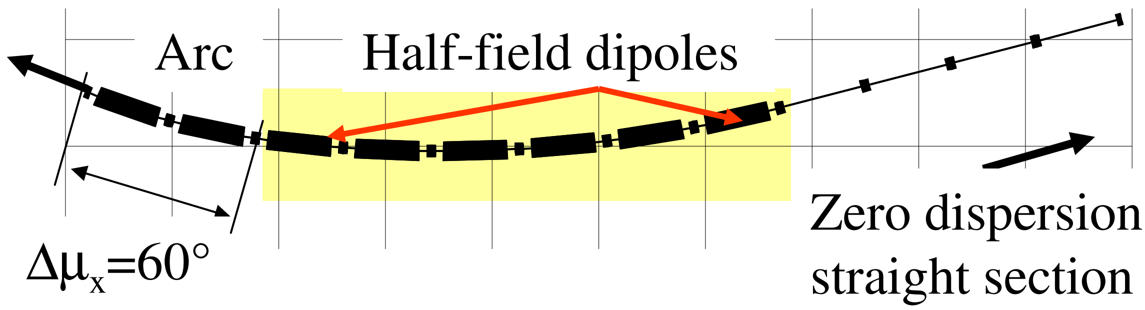
Half-field suppressor

- Lattice functions of the half-field suppressor for a 60° FODO cell. The functions β_x and β_z are slightly perturbed by the suppression of D_x.

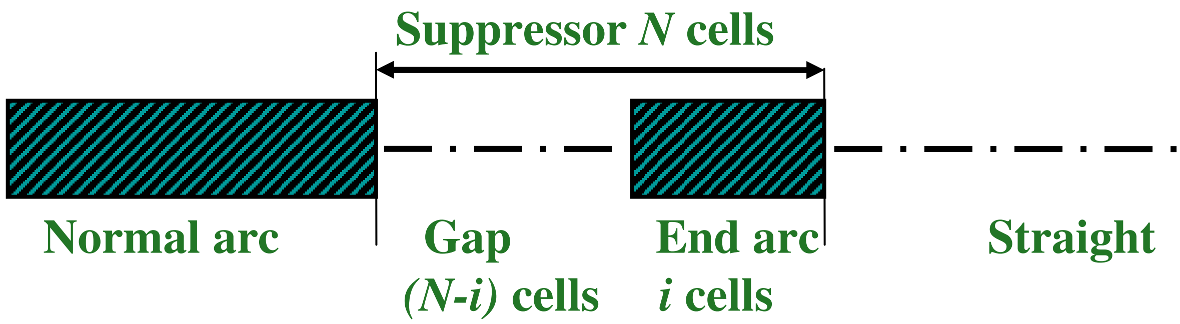


File name : Octant_3 LATTICE ELEMENTS (On-Axis) Date of run: 11/
Treated as: transfer line Time of run: 4:4
User title: Demo disp. suppr. with FODO cell with 60deg phase advance and RBEND.

Unit no.	Name	Type	Length [m]	Hor. Mbend [rad]	Vert. Mbend [rad]	Edge angle-1 [rad]	Edge angle-2 [rad]	k-Mquad [1/m2]
1	FQH1	QUADR	0.4087	0.000000	0.000000	0.00000	0.00000	-0.124042
2	ss0	DRIFT	1.0000	0.000000	0.000000	0.00000	0.00000	0.000000
3	Dipole	SBEND	7.0000	0.130900	0.000000	0.00000	0.00000	0.000000
4	ss0	DRIFT	1.0000	0.000000	0.000000	0.00000	0.00000	0.000000
5	DQH1	QUADR	0.4087	0.000000	0.000000	0.00000	0.00000	0.126769



Dispersion suppressors



- **Missing-magnet suppressors for FODO arcs (Fquad. + Dipole + Dquad. + Dipole):**

N	*Gap*	*i*	$\Delta\mu$	End arc dipole θ
2	1	1	60°	L/ρ
3	1	2	45°	$(L/\rho)/\sqrt{2}$
4	2	2	30°	$(L/\rho)/2$

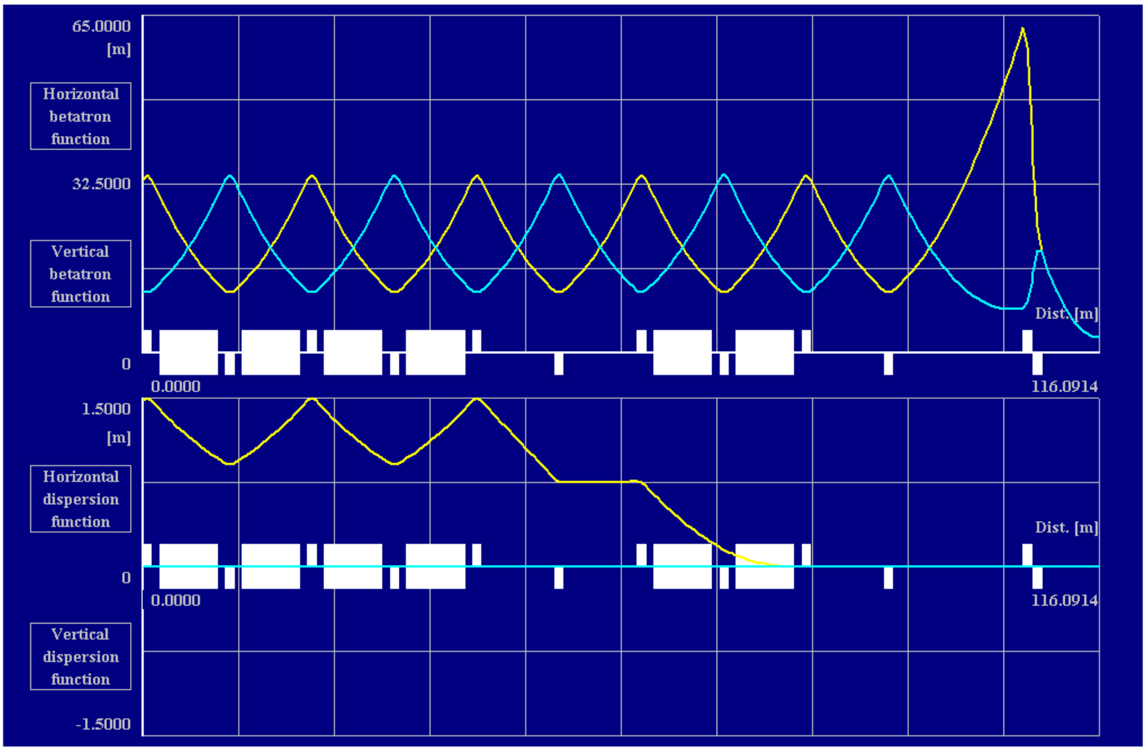
- **Half-field suppressors for FODO arcs (*N* = *i*, <u>no gap</u>)**

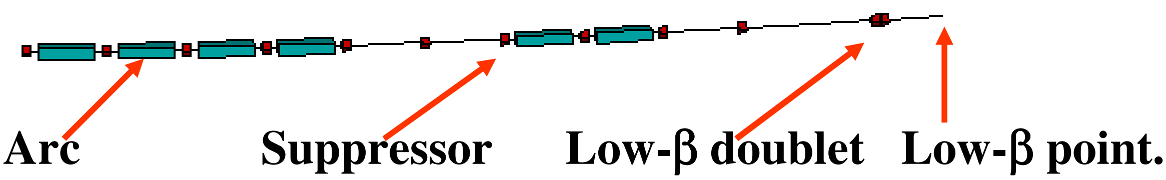
N=i	*Gap*	$\Delta\mu$	End arc dipole θ
2	0	90°	$(L/\rho)/2$
3	0	60°	$(L/\rho)/2$
4	0	45°	$(L/\rho)/2$

[Half field is useful in electron machines as it reduces the synchrotron radiation into the experimental region.]

Low-β insertion

- Frequently it is necessary to make the beam size small in both planes. This requires a so-called *low-β insertion*.
- As an example, a doublet has been added after the dispersion suppressor on slide 16 to bring both betatron amplitudes down to 3 m.
- This case requires some further numerical matching to reduce the peak and separate the doublet quadrupoles a little more.





Numerical matching

- The last example invoked *numerical matching*.
- Although we would like to believe that one can just type in what one wants, push the button and get a good result, it is better to have some strategies.
- Knowledge of some standard modules can be useful.
- The most basic module is the 1:1 module that has the very simple transfer matrix.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- This module will return the input values of x and x' at the exit. Thus any beam distribution will simply be transported unchanged to the exit.
- What does one have to specify in a numerical matching program in terms of β and α to get this matrix?

1:1 and 1:-1 modules

- **A "1 : 1" module returns the entry beam co-ordinates at the exit and the "1:-1" returns the negative values.**
- **Consider the general transfer matrix:**

$$\begin{pmatrix} \left(\dfrac{\beta_2}{\beta_1}\right)^{1/2}(\cos\Delta\mu + \alpha_1 \sin\Delta\mu) & (\beta_1\beta_2)^{1/2}\sin\Delta\mu \\ -(\beta_1\beta_2)^{-1/2}[(1+\alpha_1\alpha_2)\sin\Delta\mu + (\alpha_2-\alpha_1)\cos\Delta\mu] & \left(\dfrac{\beta_1}{\beta_2}\right)^{1/2}(\cos\Delta\mu - \alpha_2\sin\Delta\mu) \end{pmatrix}$$

- **Set $\Delta\mu = 2\pi$, $\beta_2 = \beta_1$ and $\alpha_2 = \alpha_1$ to create the 1:1 matrix.**

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- **Set $\Delta\mu = \pi$, $\beta_2 = \beta_1$ and $\alpha_2 = \alpha_1$ to create the 1:-1 matrix.**

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

- **You can create these matrices in a lattice program with say 4 or 2 FODO cells with 90° phase advance. The module you create would always be 1:1 or 1:-1 and would always return the input beam to the exit accordingly, whatever the input Twiss functions were.**
- **For example, if you had made an arc with a closed dispersion bump and equal input and output Twiss functions, then you could join two of these arcs with 1:1 modules to provide long straight regions.**

Telescope modules

- **Since phase space is conserved, it is clear that when the beam width increases the angular divergence will go down and *vice versa*.**
- **This can be seen in the telescopic modules 1:*n* or 1:-*n*. The matrices are of the form:**

$$\begin{pmatrix} \pm n & 0 \\ 0 & \pm \dfrac{1}{n} \end{pmatrix}$$

- **Matrices of this type scale the excursion *x* by *n* and inversely scale the angular divergence *x'* by 1/*n*. The moduli are still unity so phase space is conserved.**
- **To obtain this type of module put $\Delta\mu = \pi$, or $\Delta\mu = 2\pi$, $\beta_2 = n\,\beta_1$ and $\alpha_2 = \alpha_1$.**

Length scaling of a module

- **From Lecture 4 equation (7), we had**

$$\frac{\mathrm{d}^2\sqrt{\beta}}{\mathrm{d}s^2}+K_\mathrm{y}(s)\sqrt{\beta}=\left(\sqrt{\beta}\right)^{-3}.$$

- **It was stated that this equation is rarely used. Well, here is one case. Let us suppose that you have created an ideal 1:1 or 1:-1 module, but it is too long. How can you shorten it and still have the same transfer matrix?**
- **Rewrite equation (7) with scaling factors,**

$$s=\kappa s,\quad \beta=\tau\beta,\quad K=\lambda K_\mathrm{y}(s)$$

$$\frac{\mathrm{d}^2\sqrt{\tau\beta}}{\kappa^2\mathrm{d}s^2}+\lambda K_\mathrm{y}(s)\sqrt{\tau\beta}=\left(\tau\sqrt{\beta}\right)^{-3}.$$

- **With some re-arrangement,**

$$\frac{\tau^2}{\kappa^2}\frac{\mathrm{d}^2\sqrt{\beta}}{\mathrm{d}s^2}+\lambda\tau^2K_\mathrm{y}(s)\sqrt{\beta}=\left(\sqrt{\beta}\right)^{-3}.$$

- **By inspection one sees that the equation is unchanged, if**

$$\kappa^2=\tau^2 \text{ and } \lambda=\tau^{-2}.$$

Try,

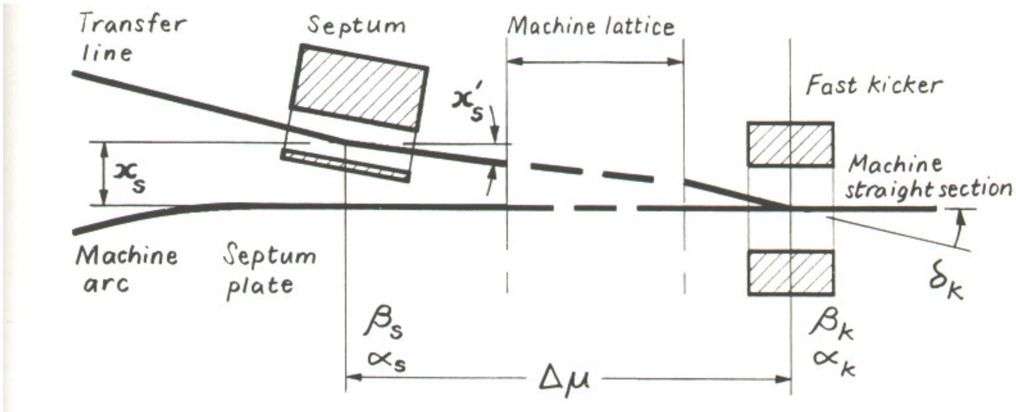
$$\kappa=\tau=0.8 \text{ and } \lambda=1/0.64$$

all β functions and all lengths will be reduced by 20%, and all gradients by 36%, but phase advances and α-functions are unchanged. Thus the 1:1 or 1:-1 module has the same properties as before.

$$\mu=\int\frac{1}{(\tau\beta)}\mathrm{d}|\kappa s|,\qquad \alpha=-\frac{1}{2}\frac{\mathrm{d}(\tau\beta)}{\mathrm{d}(\kappa s)}$$

Single-turn injection/extraction

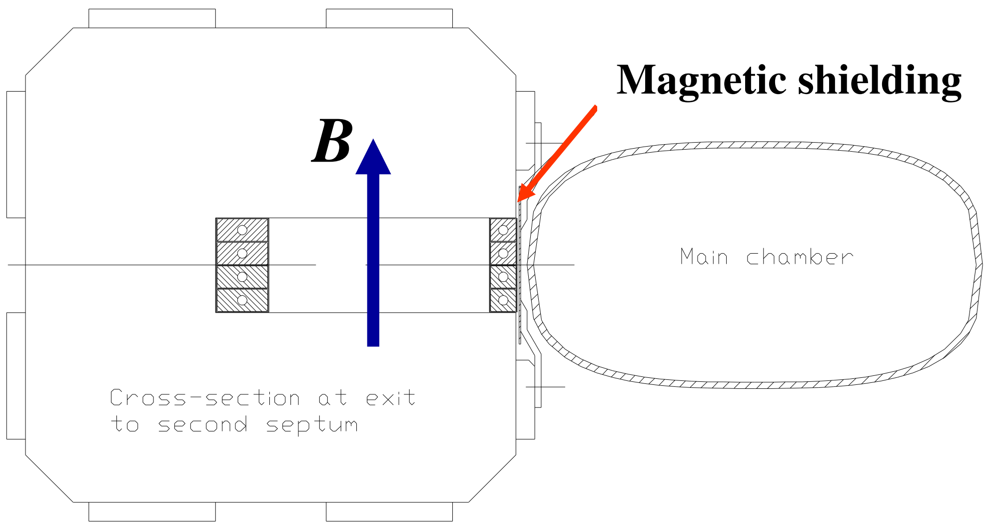
- **A conventional injection/extraction insertion,**



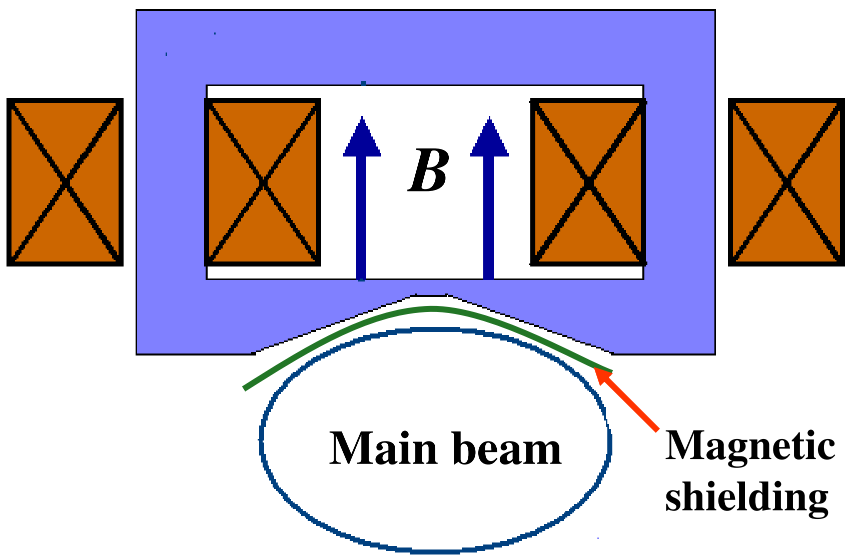
- **The $\Delta\mu$ is ideally 90°.**
- **If there is a quadrupole between the kicker and septum, then it is better to have a defocusing lens to benefit from the outward kick.**
- **It is better to have zero dispersion in order to have a narrow beam.**
- **It is also an advantage to have a large β_x at the kicker.**
- **If the septum bends in the same plane as the kicker then a *current-wall septum* is needed. If the bend is perpendicular to the first kick then a *Lambertson septum* is needed.**

Septa designs

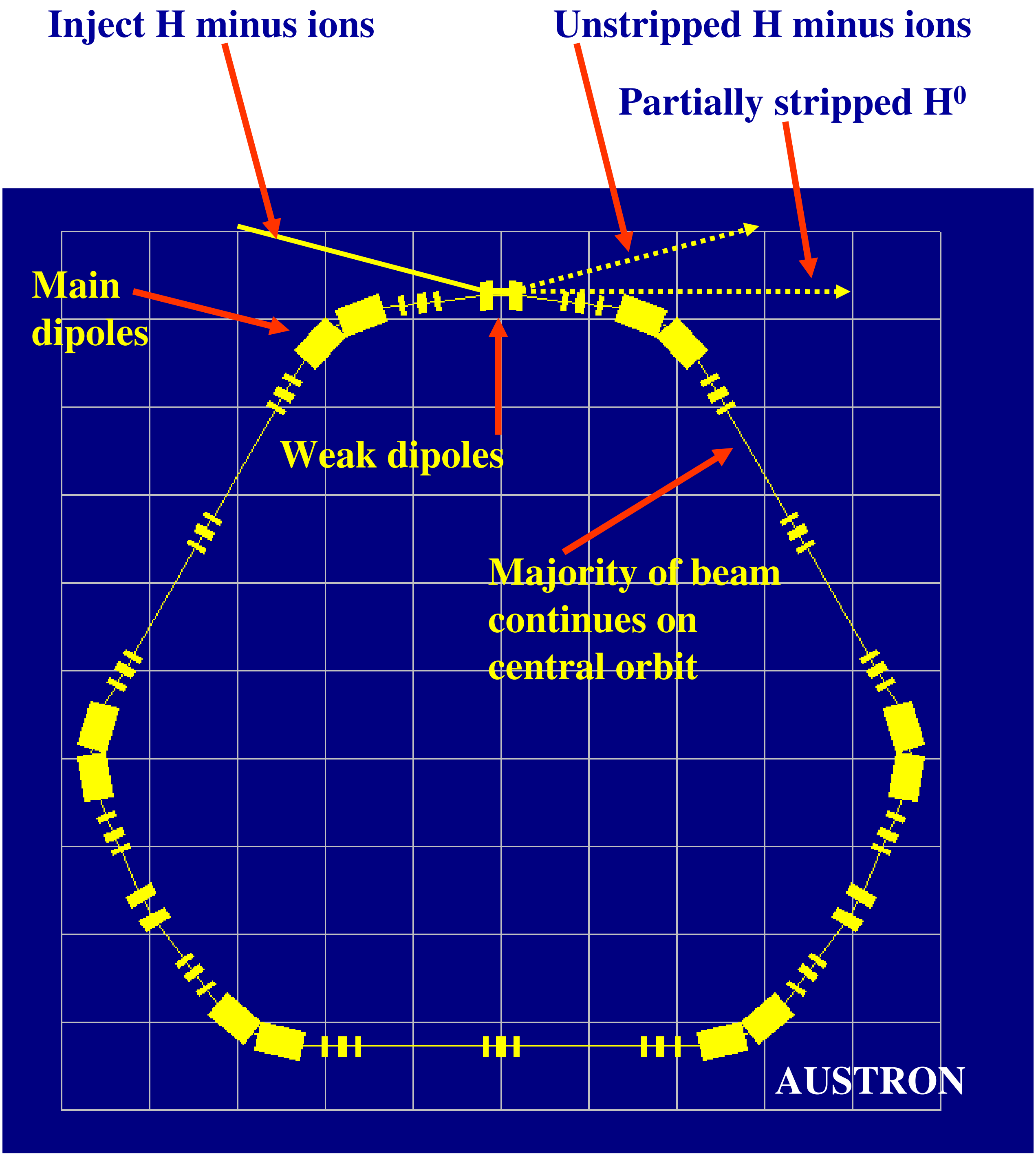
❖ Current-wall septum



❖ Lambertson septum

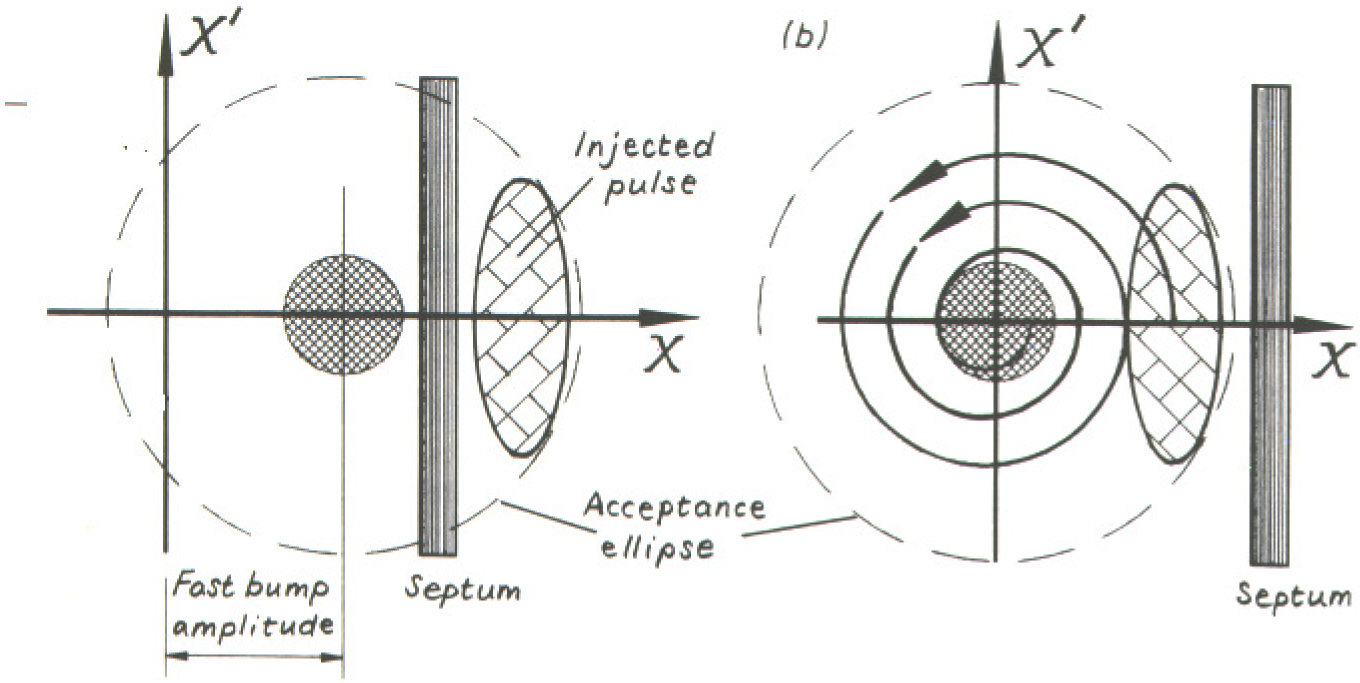


H minus stripping



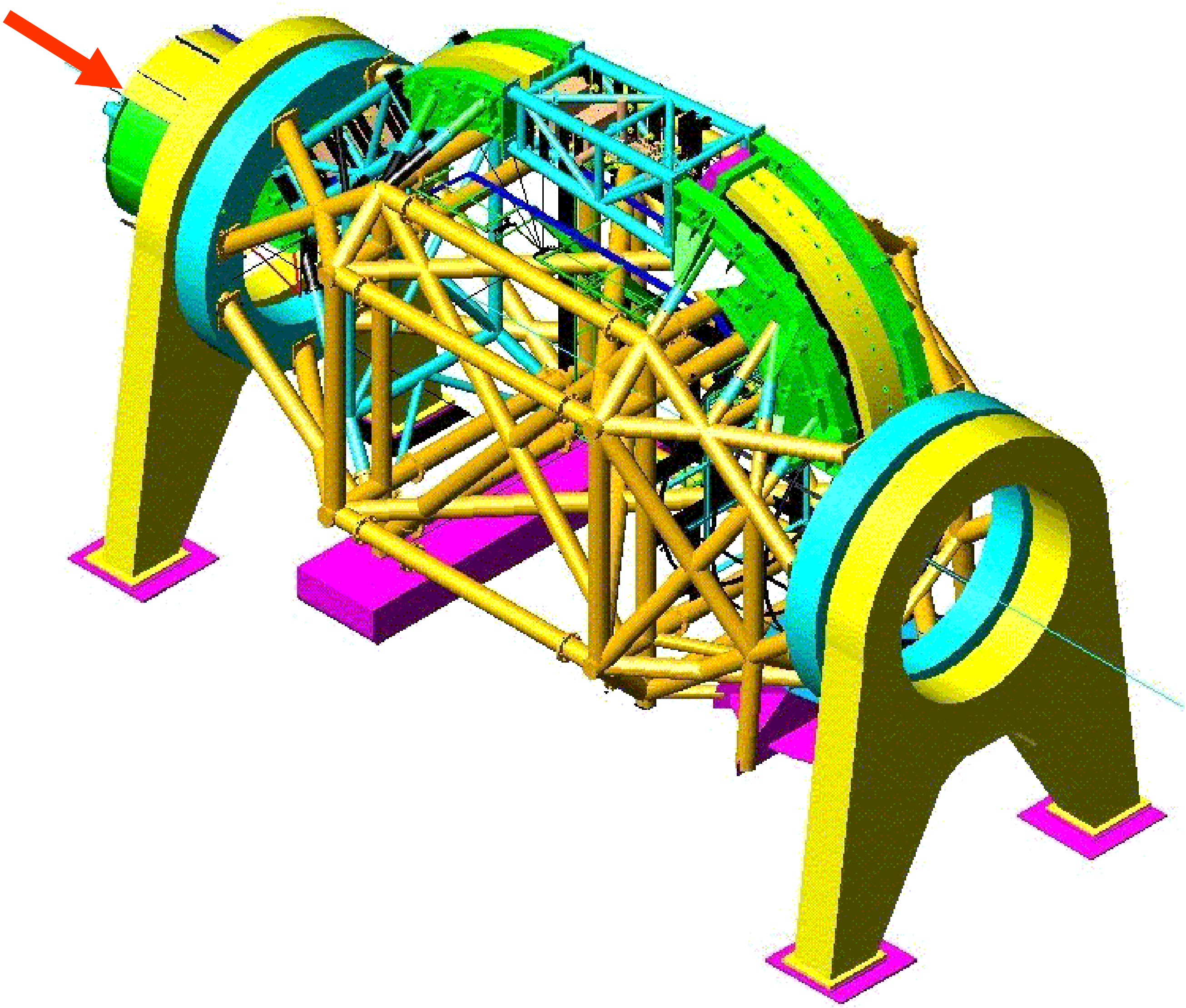
- This injection 'cheats' Liouville, but the beam still suffers some emittance blowup from scattering in the stripping foil.

Injection by radiation damping



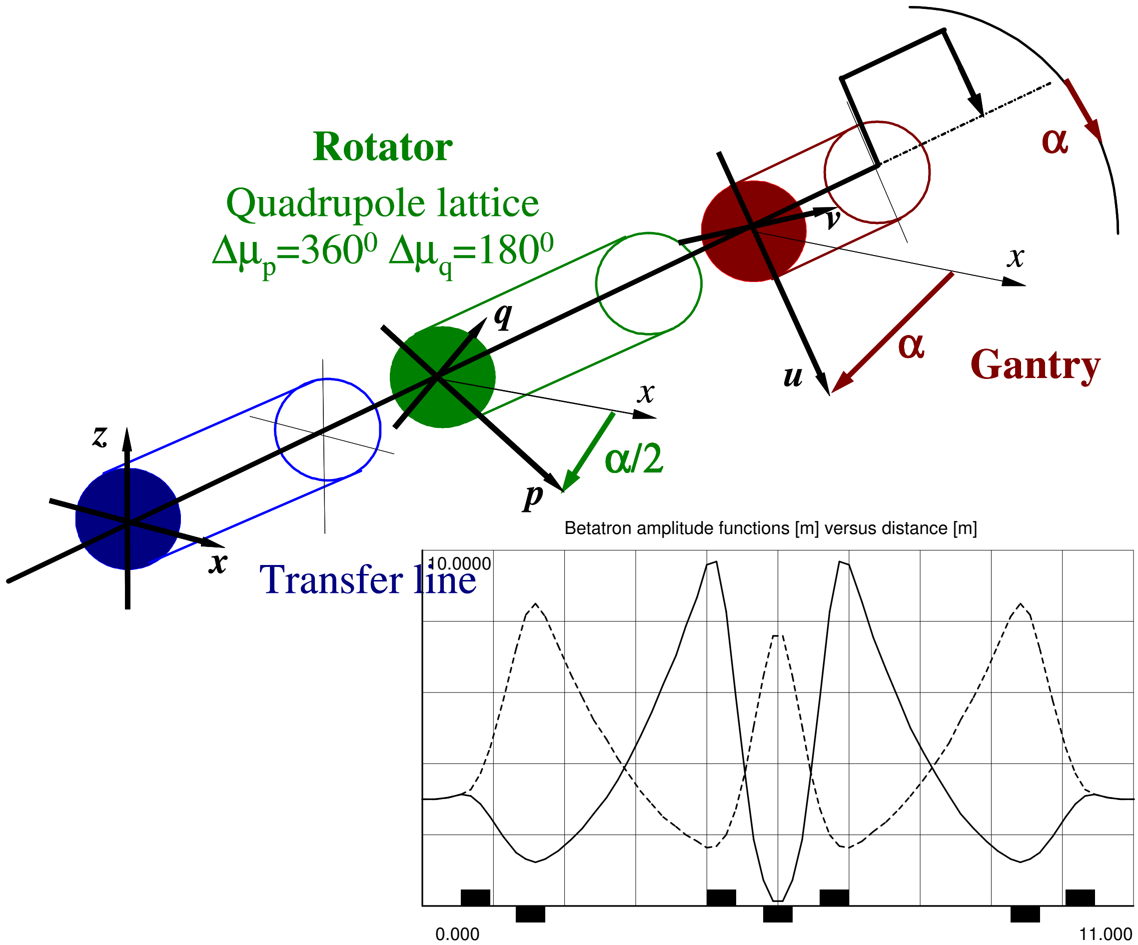
- **Displace central orbit with a fast bump towards the septum.**
- **Inject a pulse.**
- **Collapse bump before injected pulse returns to septum.**
- **Let synchrotron radiation damp newly injected pulse into the core of the beam.**

Medical gantry



- GSI iso-centric gantry.
- Rotates 360° around patient.
- 13 m diameter.
- 25 m length.
- 600 t overall weight.

Rotational optics



$$M_0=\begin{pmatrix}\cos\frac{\alpha}{2} & 0 & \sin\frac{\alpha}{2} & 0\\ 0 & \cos\frac{\alpha}{2} & 0 & \sin\frac{\alpha}{2}\\ -\sin\frac{\alpha}{2} & 0 & \cos\frac{\alpha}{2} & 0\\ 0 & -\sin\frac{\alpha}{2} & 0 & \cos\frac{\alpha}{2}\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}\cos\frac{\alpha}{2} & 0 & \sin\frac{\alpha}{2} & 0\\ 0 & \cos\frac{\alpha}{2} & 0 & \sin\frac{\alpha}{2}\\ -\sin\frac{\alpha}{2} & 0 & \cos\frac{\alpha}{2} & 0\\ 0 & -\sin\frac{\alpha}{2} & 0 & \cos\frac{\alpha}{2}\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$$

- **A *rotator module* (1:1 horizontal, 1:-1 vertical) is mounted between the fixed beam line and the gantry.**
- **The rotator is turned by half the angle of the gantry.**
- **The Twiss and dispersion functions are transferred exactly to the rotated coordinate system of the gantry.**

Summary

- We have looked at the basic FODO cell, which is the best known and studied cell in lattice optics.
- We have also seen the split FODO, the doublet and the triplet cells.
- In order to make simple rings, we have concatenated many cells to give 2π of bending.
- In order to control dispersion, we have looked at closed dispersion bumps for small and medium sized machines and dispersion suppressors for big machines.
- Numerical matching was mentioned with reference to building a low-β insertion.
- Various lattice modules have been described and an analytic method for scaling modules was described.
- Various injection and extraction techniques have been described.
- Finally, the rather exotic topic of rotational optics has been briefly visited.