Joint Universities Accelerator School

JUAS 2014

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Normal-conducting accelerator magnets

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CERN

Scope of the lectures

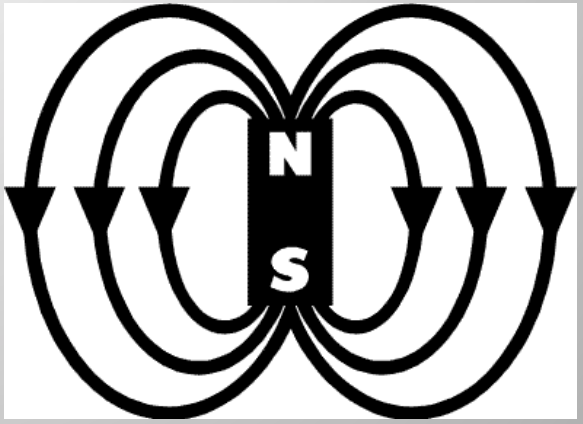
Overview of electro-magnetic technology as used in particle accelerators considering *normal-conducting, iron-dominated* electro-magnets (generally restricted to direct current situations)

Main goal is to:

- create a fundmental understanding in accelerator magnet technology
- provide a guide book with practical instructions how to start with the design of a standard accelerator magnet
- focus on applied and practical design aspects using 'real' examples
- introduce finite element codes for practical magnet design
- present an outlook into as aspects related to magnet prodcution, testing and measurements

Not covered:

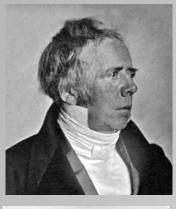
- permanent magnet technology
- super-conducting technology

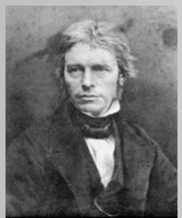
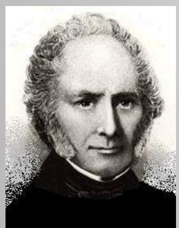
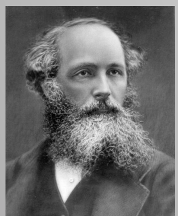


Lecture 1: Basic principles

- A bit of history…
- Why do we need magnets?
- Magnet technologies
- Basic principles and concepts
- Magnet types and applications

A bit of history…



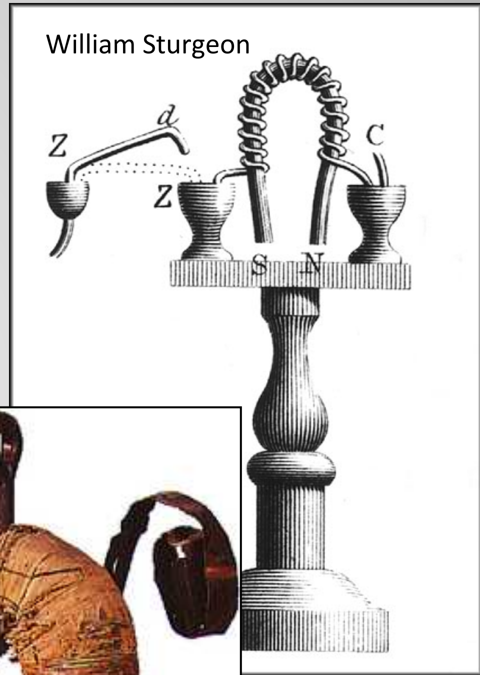
1820: Hans Christian Oersted (1777-1851) finds that electric current affects a compass needle

1820: Andre Marie Ampere (1775-1836) in Paris finds that wires carrying current produce forces on each other

1820: Michael Faraday (1791-1867) at Royal Society in London develops the idea of electric fields and studies the effect of currents on magnets and magnets inducing electric currents

1825: British electrician, William Sturgeon (1783-1850) invented the first electromagnet

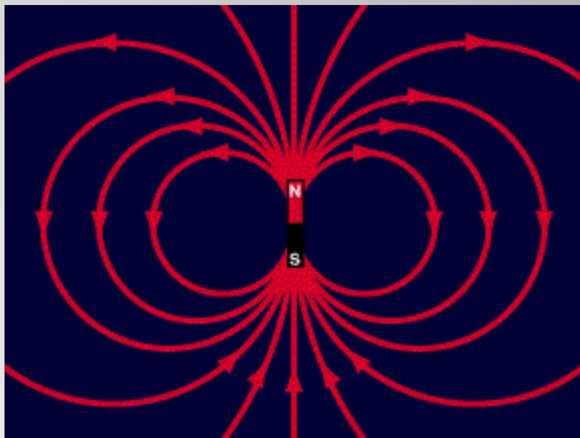
1860: James Clerk Maxwell (1831-1879), a Scottish physicist and mathematician, puts the theory of electromagnetism on mathematical basis



William Sturgeon


Joseph Henry

Magnetic units

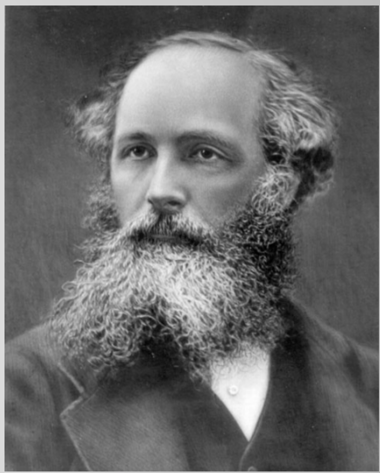


IEEE defines the following units:

- Magnetic field:
  - H (vector) [A/m]
  - the magnetizing force produced by electric currents
- Electromotive force:
  - e.m.f. or U [V or (kg m^2)/(A s^3)]
  - here: voltage generated by a time varying magnetic field
- Magnetic flux density or magnetic induction:
  - B (vector) [T or kg/(A s^2)]
  - the density of magnetic flux driven through a medium by the magnetic field
  - Note: induction is frequently referred to as "Magnetic Field“
  - H, B and μ relates by: $B = \mu H$
- Permeability:
  - $\mu = \mu_0 \mu_r$
  - permeability of free space $\mu_0 = 4 \pi 10^{-7}$ [V s/A m]
  - relative permeability μ_r (dimensionless): $\mu_{air} = 1$; $\mu_{iron} > 1000$ (not saturated)
- Magnetic flux:
  - ϕ [Wb or (kg m^2)/(A s^2)]
  - surface integral of the flux density component perpendicular trough a surface

Maxwell's equations

In 1873, Maxwell published "Treatise on Electricity and Magnetism" in which he summarized the discoveries of Coulomb, Oersted, Ampere, Faraday, et. al. in four mathematical equations:



Gauss' law for electricity:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \oint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Gauss' law of flux conservation:

$$\boxed{\nabla \cdot \vec{B} = 0} \qquad \oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

Faraday's law of induction:

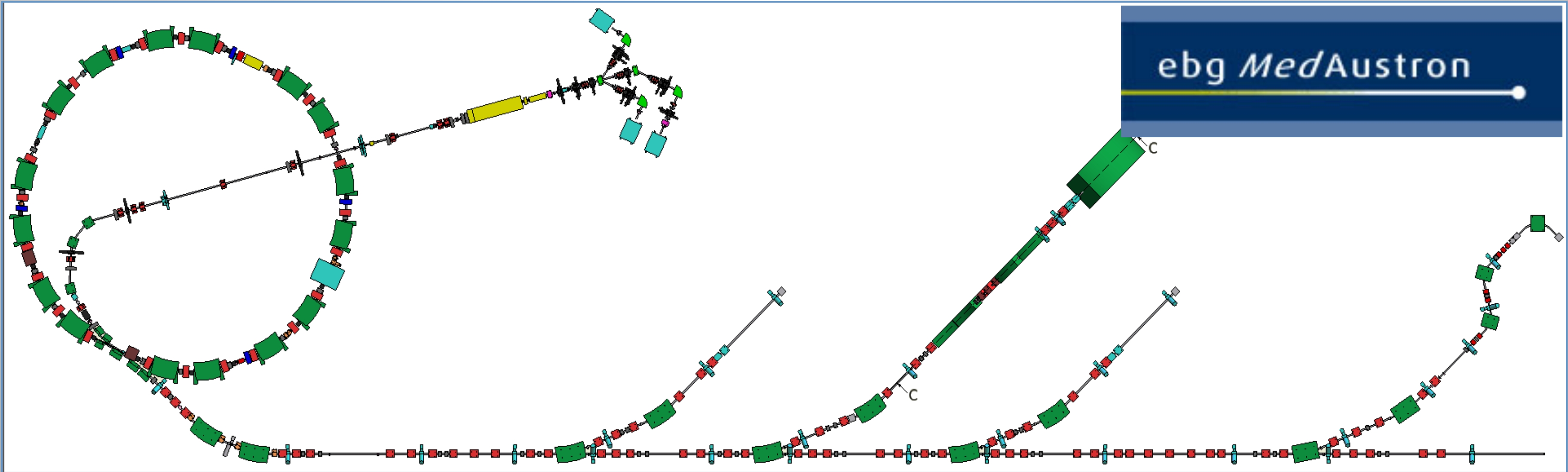
$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \qquad \oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A}$$

Ampere's circuital law:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \oint_{\partial A} \vec{B} \cdot d\vec{s} = \int_A \mu_0 \vec{J} \cdot d\vec{A} + \frac{d}{dt}\int_A \mu_0 \varepsilon_0 \vec{E} \cdot d\vec{A}$$

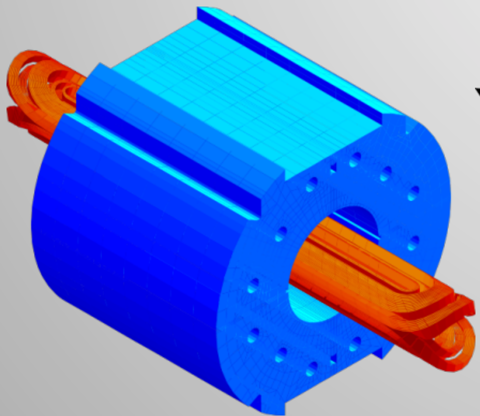
Why do we need magnets?

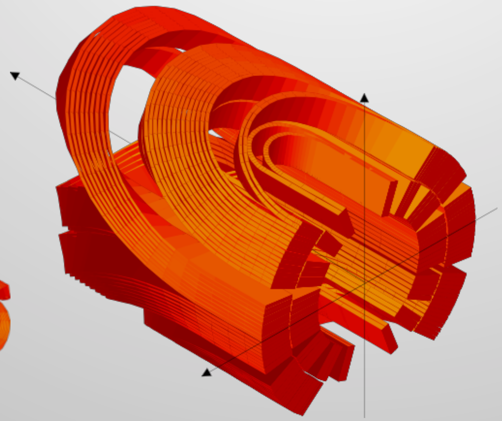
- Interaction with the beam
  - guide the beam to keep it on the orbit
  - focus and shape the beam
- Lorentz's force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
  - for relativistic particles this effect is equivalent if $\vec{E} = c\vec{B}$
  - if $B = 1$ T then $E = 3 \cdot 10^8$ V/m

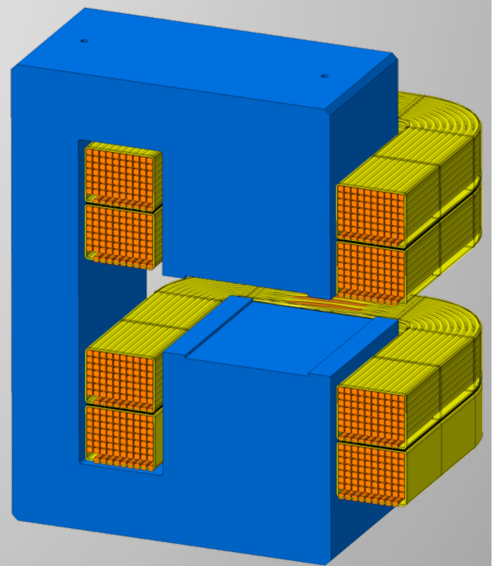


Magnet technologies

- Magnets
  - Electro-magnets
    - Superconducting
      - **Coil dominated** B < 9 T
      - Iron dominated B < 2 T
    - Normal-conducting
      - Coil dominated B < 1 T
      - **Iron dominated** B < 2 T
  - Permanent magnets

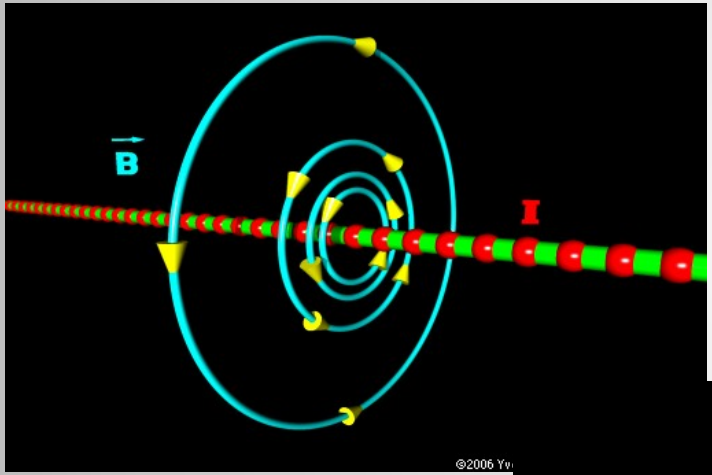






How does a magnet work?

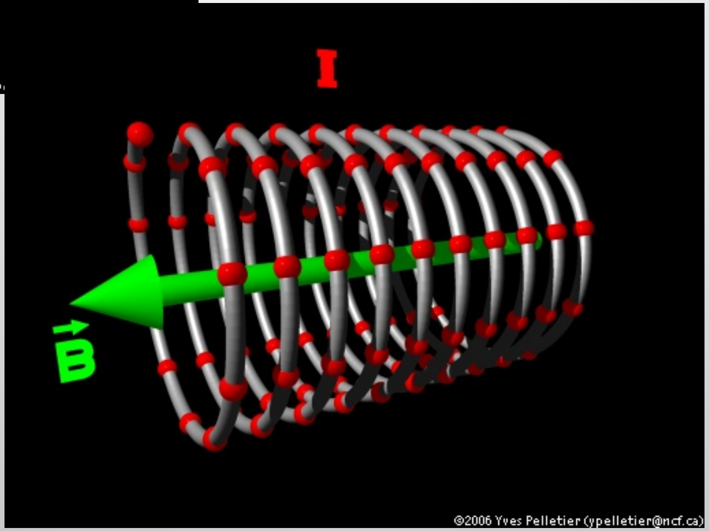
- Permanent magnets provide only constant magnetic fields
- Electro-magnets can provide adjustable magnetic fields



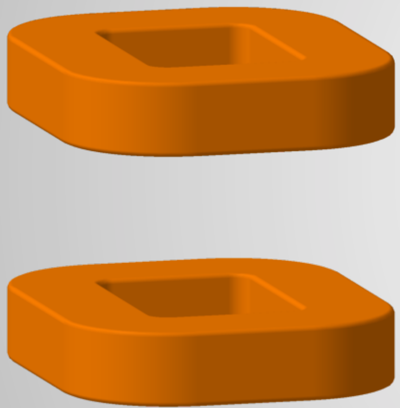
Maxwell & Ampere:

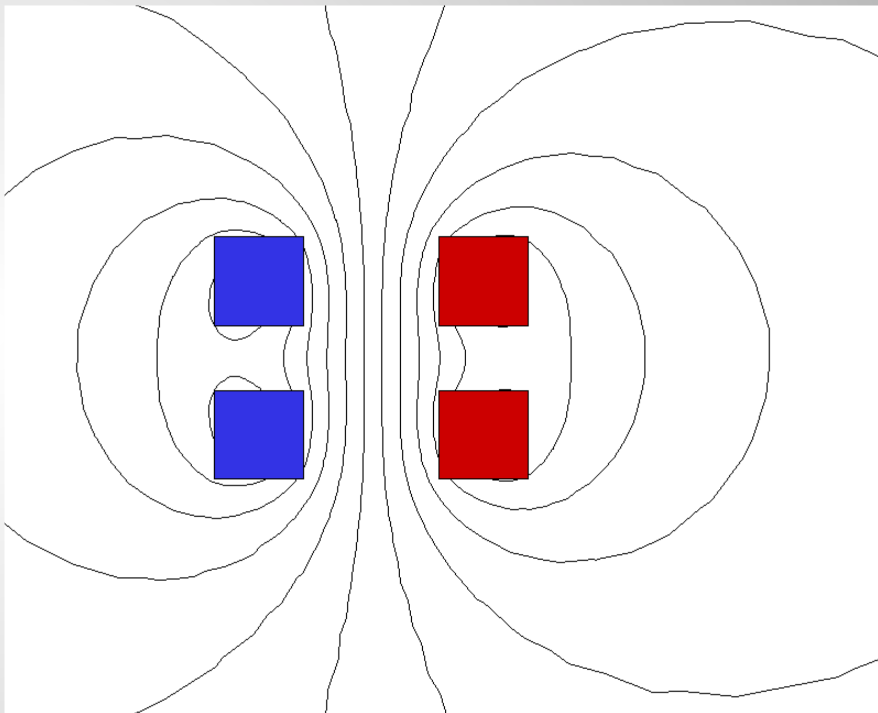
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

„An electrical current is surrounded by a magnetic field“



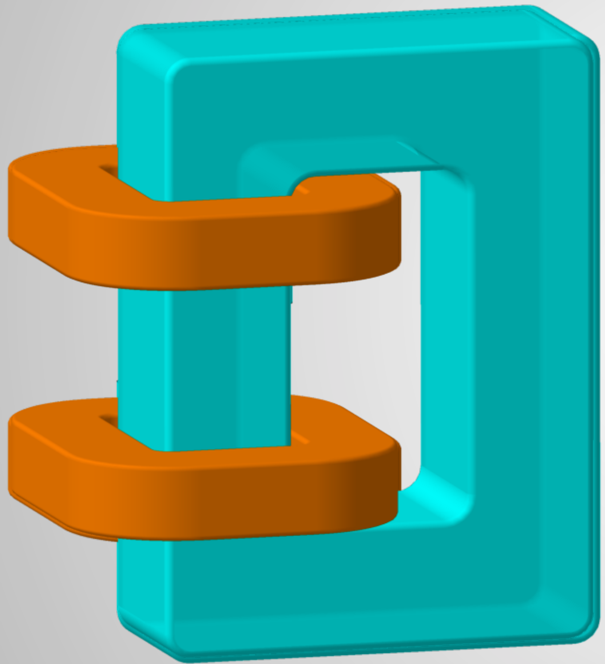
Magnetic circuit

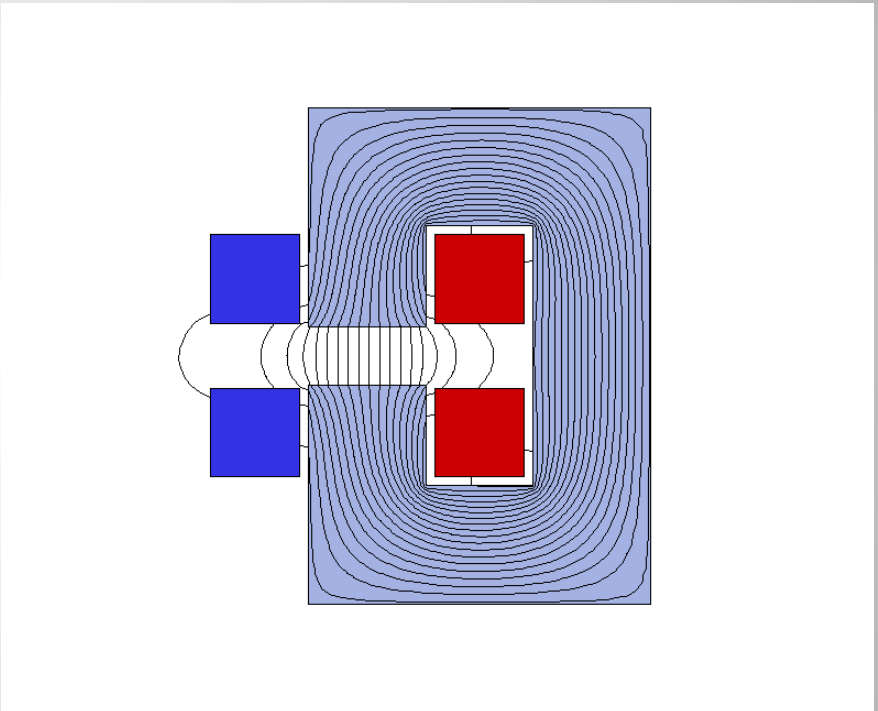




Flux lines represent the magnetic field
Coil colors indicate the current direction

Magnetic circuit





Coils hold the electrical current
Iron holds the magnetic flux

Excitation current in a dipole

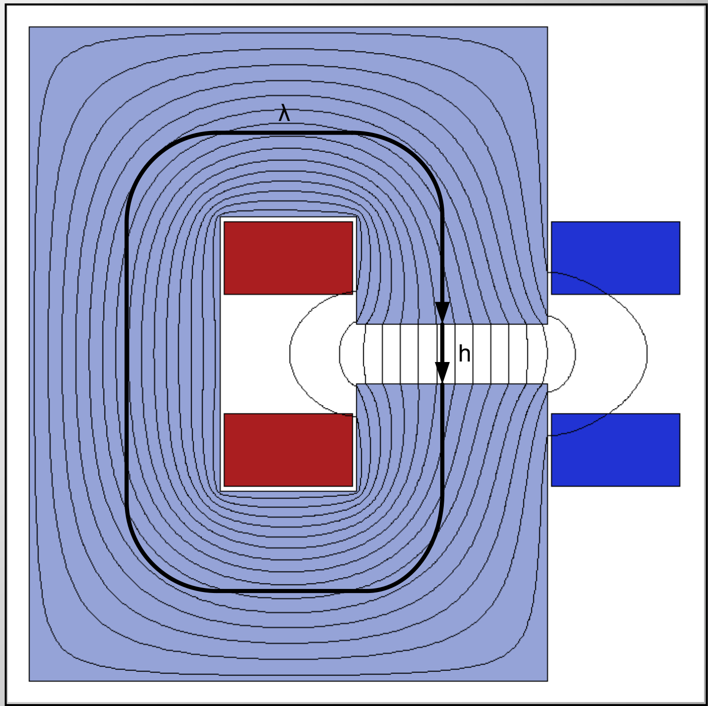
Ampere's law $\oint \vec{H} \cdot d\vec{l} = NI$ and $\vec{B} = \mu \vec{H}$ with $\mu = \mu_0 \mu_r$

leads to $NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = \int_{gap} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{l} + \int_{yoke} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{l} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$

assuming, that B is constant along the path

If the iron is not saturated: $\frac{h}{\mu_{air}} >> \frac{\lambda}{\mu_{iron}}$

then: $\boxed{NI_{(per\ pole)} \approx \frac{Bh}{2\mu_0}}$

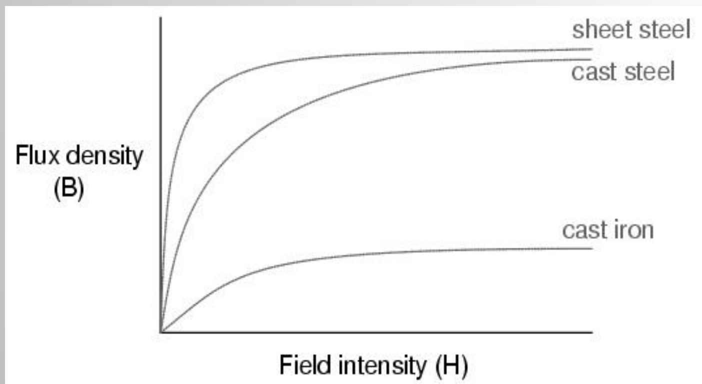


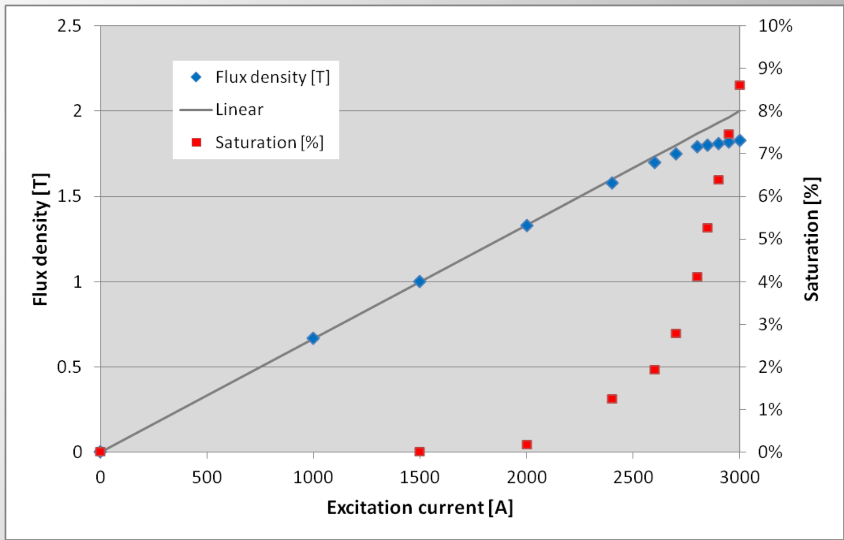
Reluctance and efficiency

Similar to Ohm's law, one can define the 'resistance' of a magnetic circuit, called 'reluctance', as:

- *σ:* conductivity [S/m]
- *NI:* magneto-motive force [A]
- *Φ*: magnetic flux [Wb]
- l_M: flux path length in iron [m]
- A_M: iron cross section perpendicular to flux [m²]

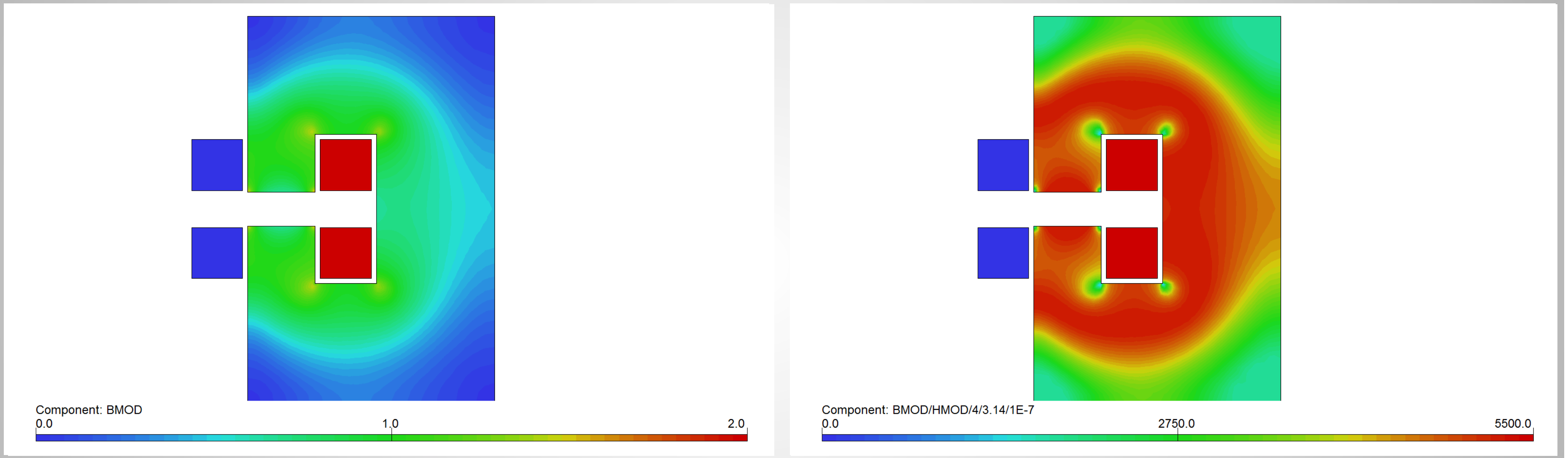
$$R_E = \frac{U}{I} = \frac{l_E}{A_E \sigma} \Rightarrow R_M = \frac{NI}{\Phi} = \frac{l_M}{A_M \mu_r \mu_0}$$





- Increase of B above 1.5 T in iron requires non-proportional increase of H
- Iron saturation (small μ_{iron}) leads to inefficiencies

Iron saturation



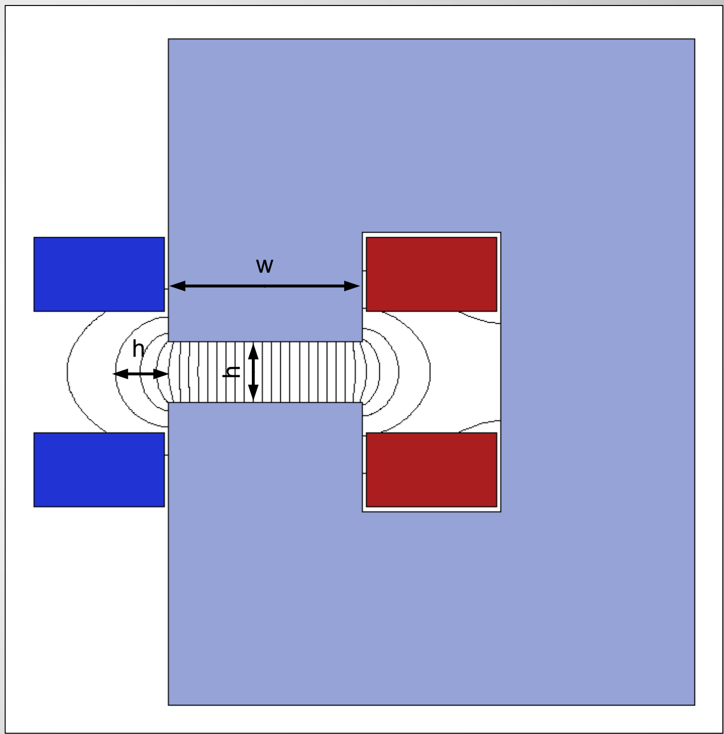
Keep yoke reluctance small by providing sufficient iron cross-section!

Magnetic flux

Flux in the yoke includes the gap flux and stray flux

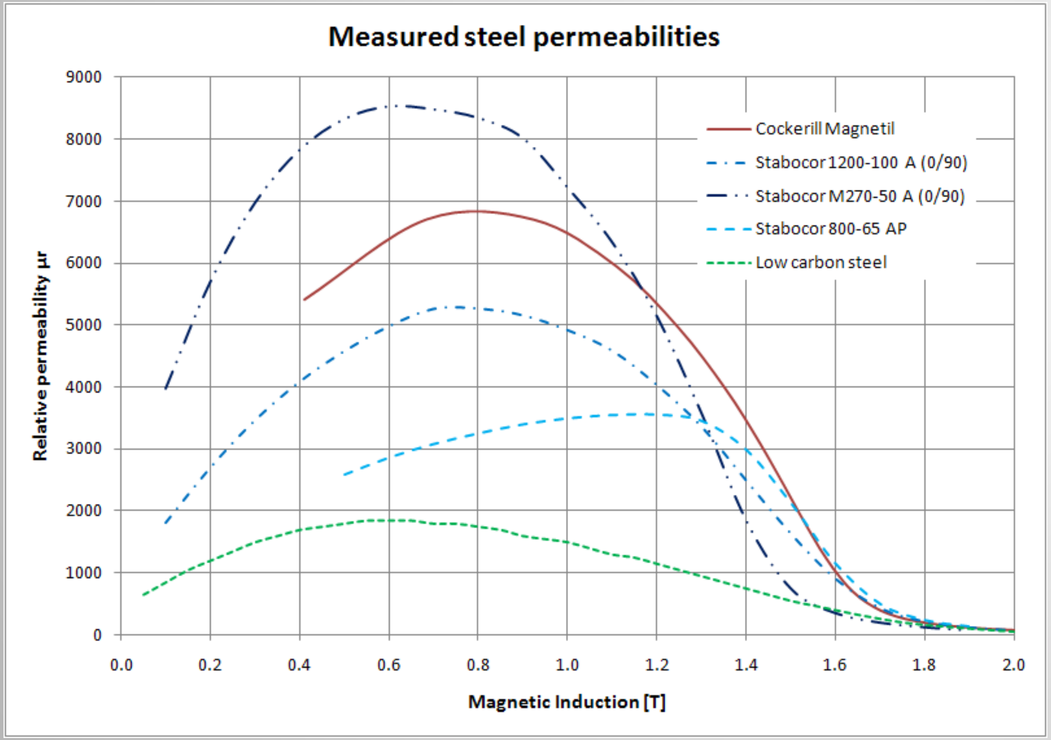
Total flux in the return yoke:

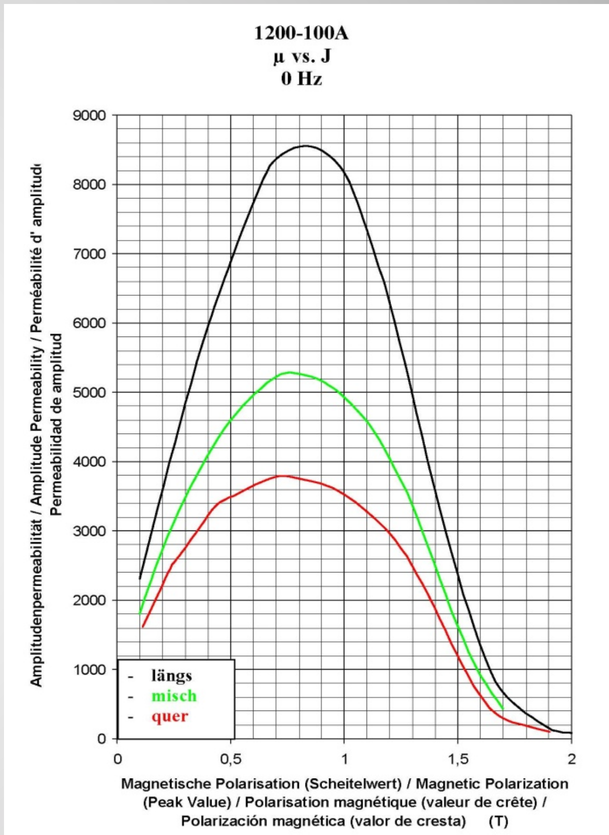
$$\Phi = \int_A B \cdot dA \approx B_{gap}(w + 2h)l_{mag}$$



Permeability

Ferro-magnetic materials: high permeability (μ_r >>1), but not constant



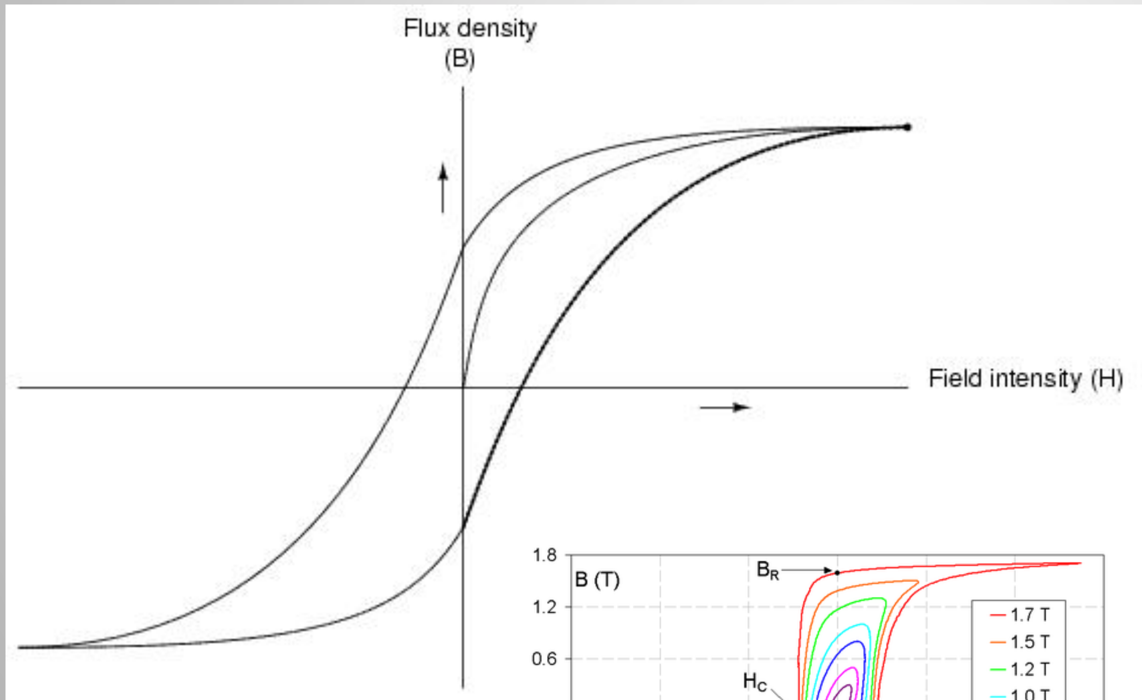


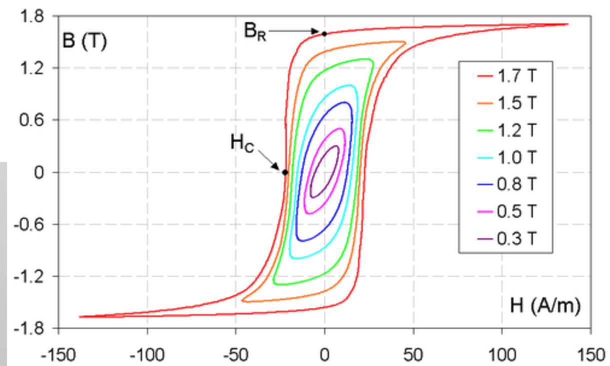
Data source: Thyssen/Germany

Anisotropy in sheet material can be partly cured by final annealing

Steel hysteresis

Flux density *B(H)* as a function of the field strength is different, when increasing and decreasing excitation



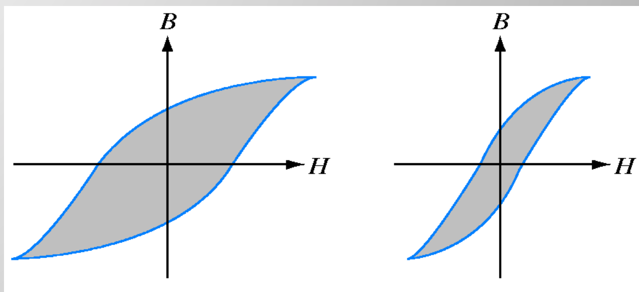


Remanent field (Retentivity):

$H = 0 \rightarrow B = B_r > 0$

Coercivity or coercive force:

$B = 0 \rightarrow H = H_c < 0$



(a) Hard material (b) Soft material

Residual field in a magnet

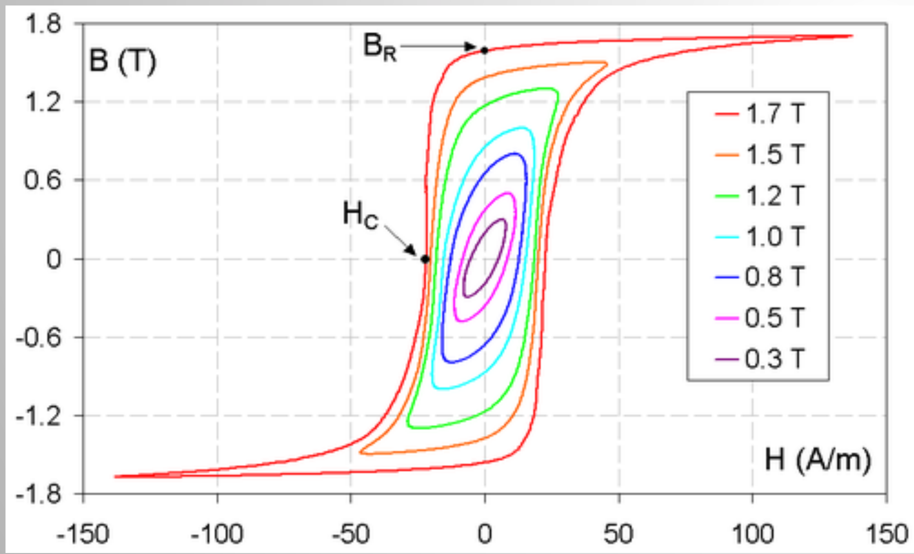
In a continuous ferro-magnetic core (transformer) the residual field is determined by the remanent field B_r

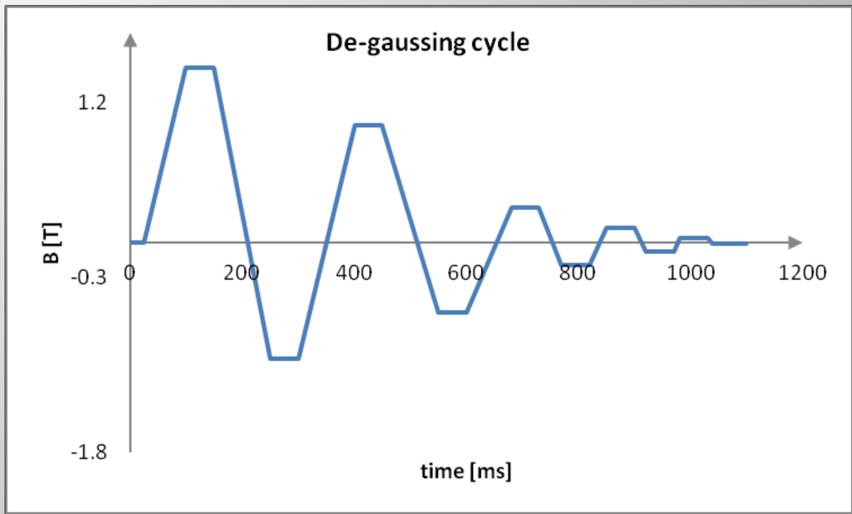
In a magnet core (gap), the residual field is determined by the coercivity H_c

Assuming the coil current *I=0*:

$$\oint \vec{H} \cdot \vec{dl} = \int_{gap} \vec{H}_{gap} \cdot \vec{dl} + \int_{yoke} \vec{H}_c \cdot \vec{dl} = 0$$

$$B_{residual} = -\mu_0 H_C \frac{l}{g}$$



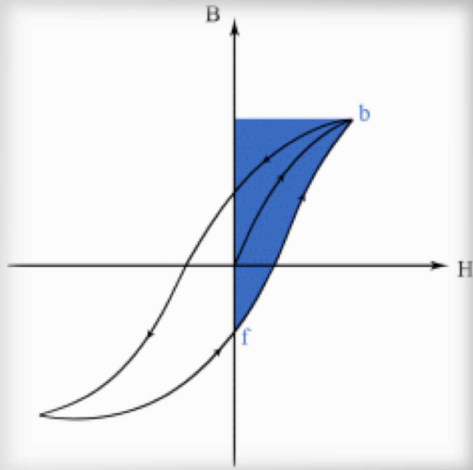


Demagnetization cycle!

Stored energy & Inductance

Stored energy E_S [J, joules] in a magnet depends on (non-uniform) field distribution in the gap, coils, and iron yoke:

$$E_S = \int_V \int_f^b H \cdot dB \cdot dv$$ and in case μ_r is linear: $$E_S = \frac{1}{2}\int_V H \cdot B \cdot dv$$



– difficult to calculate analytically
– usually done by numerical computations
– most of the energy is stored in the air gap

Inductance L [H] of a magnet is given by: $$L = \frac{2E_S}{I^2}$$

– total voltage on a pulsed magnet: $$V_{tot} = RI + L\frac{dI}{dt} = RI + \frac{2E_S}{I^2}\frac{dI}{dt}$$

– low inductance allows fast changes of magnetic field
– inductance depends on the magnetization in the iron

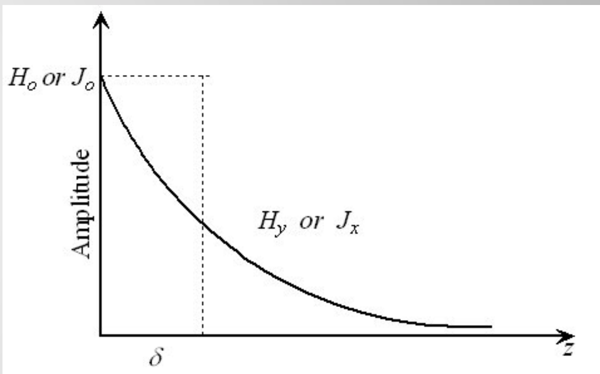
Eddy currents

Faraday's law: varying magnetic field induces an e.m.f. (voltage) $U = -\frac{\partial \Phi}{\partial t}$

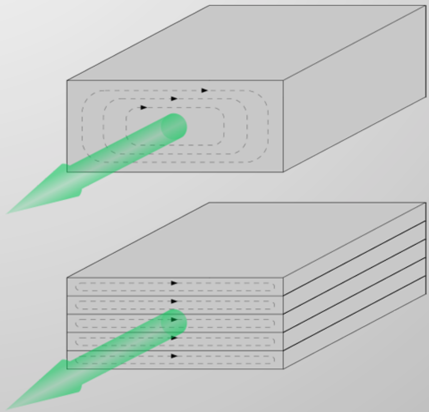
- Circulating (eddy) currents are generated in electrical conducting materials
  - creating a magnetic field opposing the original change in magnetic flux (Lenz's law)
  - opposing to the penetration of the magnetic field (skin effect)
  - producing losses (Joule heating)
  - causing delays to reach stable field value
  - damping high order modes (ripples)

$$H_y(z) = H_0 \cdot e^{-z/\delta} \qquad \delta = \frac{1}{\sqrt{\pi \cdot \mu_0 \cdot \mu_r \cdot f \cdot \sigma}}$$

  - δ: skin depth [m]



- Magnetic circuits are made of insulated laminations to reduce eddy currents,
  - decrease lamination thickness (d < δ/2)
  - increase resistivity
  - decrease permeability
  - decrease frequency ($\partial\Phi/\partial$t)



Losses

Losses in the coils:

Ohmic power loss P_Ω per length unit [W/m] in a coil conductor

$$\frac{P_\Omega}{l} = \frac{\rho}{a_{cond}} I^2$$

- ρ: resistivity [Ωm] (for copper: $1.86 \cdot 10^{-8}$ Ωm @ 40°C)
- a_{cond} : conductor cross-section [m²]

Losses in the iron yoke:

Hysteresis losses: Power loss P_H per mass unit [W/kg] up to 1.5 T using Steinmetz's law

$$\frac{P_H}{m} = \eta \cdot f \cdot B^x$$

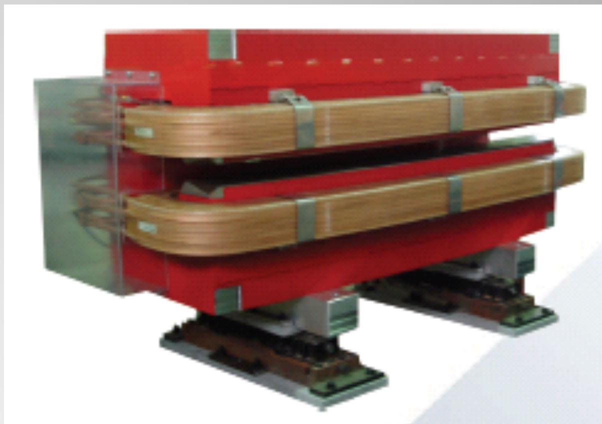
- η: material depending coefficient: $0.01 < \eta < 0.1$; $\eta \approx 0.02$ for silicon steel
- x: Steinmetz exponent: for iron $x = 1.6$
- f: operation frequency [Hz]

Eddy current losses: Power loss P_E per volume unit [W/m³] if $d_{lam} << \delta$

$$\frac{P_E}{V} = \frac{\pi^2 d_{lam}^2 f^2 B^2}{6\rho}$$

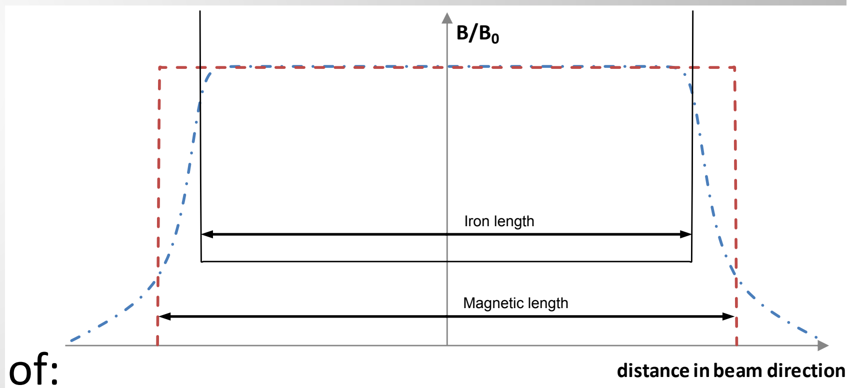
- d_{lam}: lamination thickness [m]

Magnetic length



Coming from ∞, B increases towards the magnet center (stray flux)

Magnetic length: $$l_{mag} = \frac{\int_{-\infty}^{\infty} B(z) \cdot dz}{B_0}$$

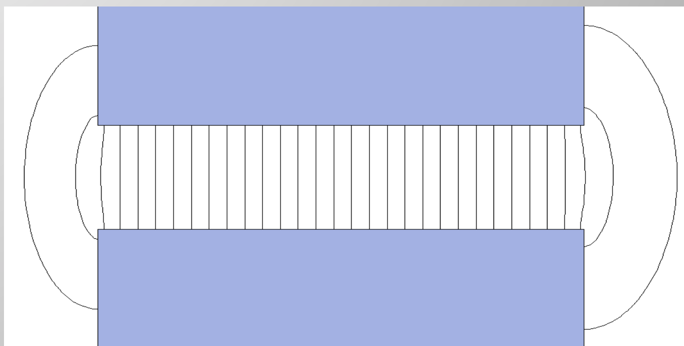


'Magnetic' length > iron length

Approximation for a dipole: $l_{mag} = l_{iron} + 2hk$

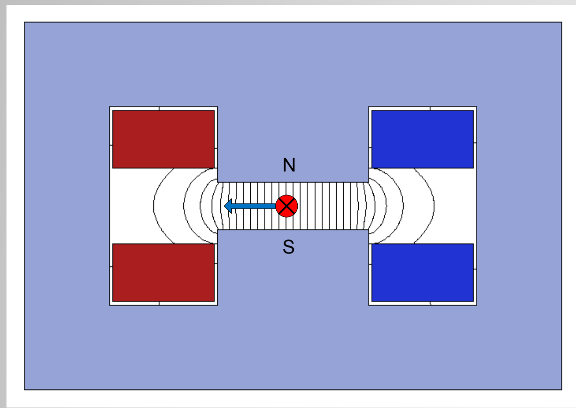
Geometry specific constant *k* gets smaller in case of:

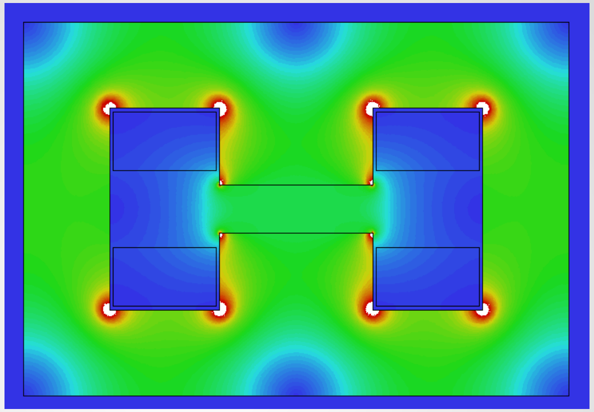
- pole length < gap height
- saturation
- precise determination only by measurements or numerical calculations

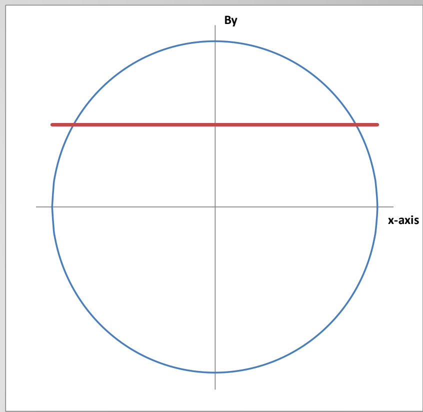


Dipoles

- Purpose: bend or steer the particle beam



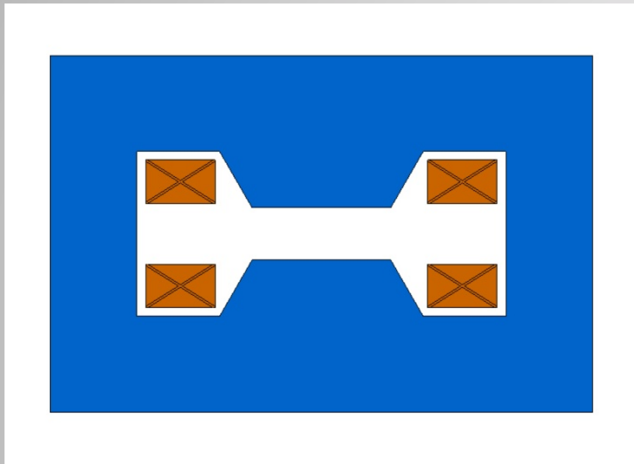


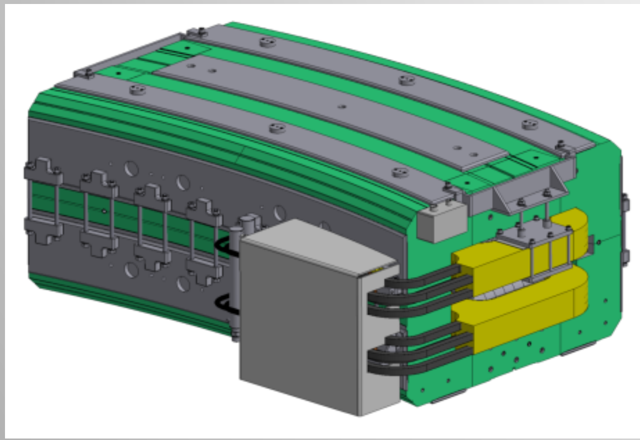


- Equation for normal (non-skew) ideal (infinite) poles: $y = \pm r$
  (r = half gap height)
- Magnetic flux density: $B_x = 0$; $B_y = b_1 = \text{const.}$
- Applications: synchrotrons, transfer lines, spectrometry, beam scanning

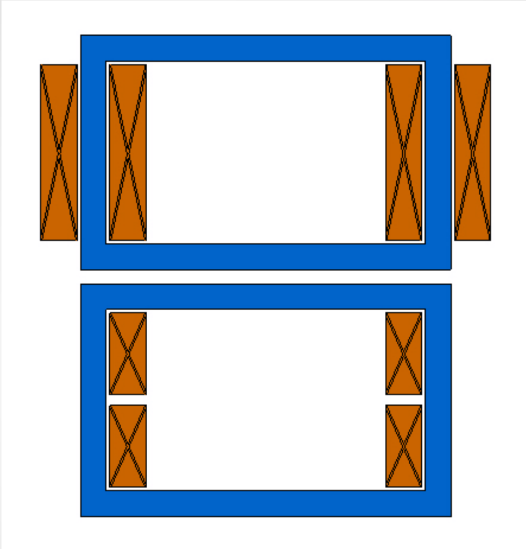
Dipole types

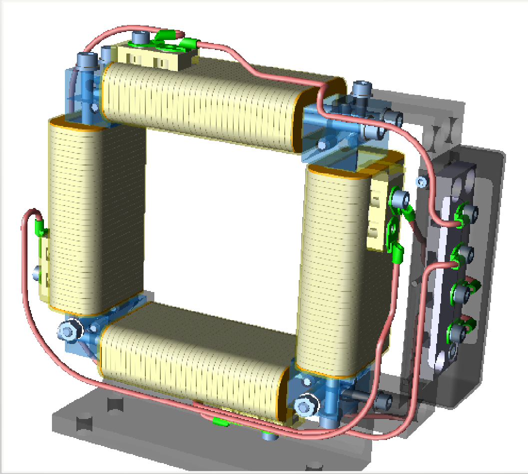
H-Shape



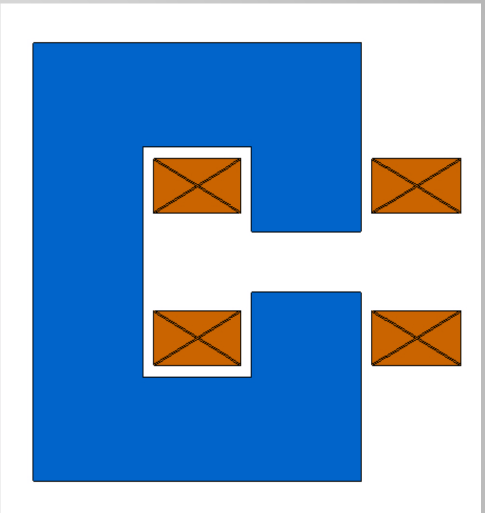


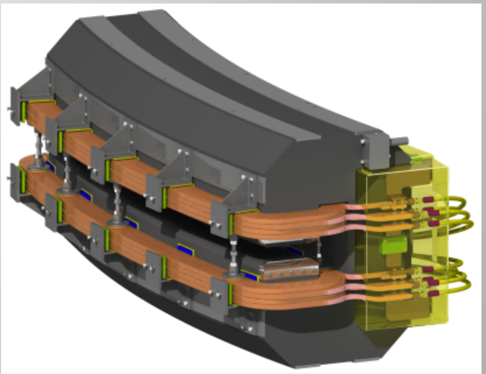
O-Shape





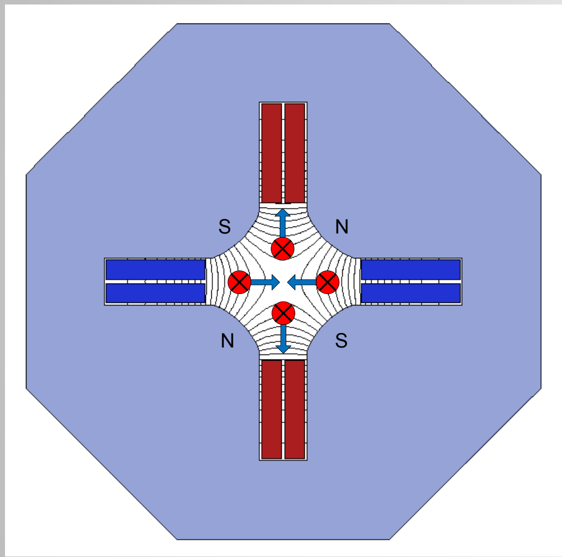
C-Shape

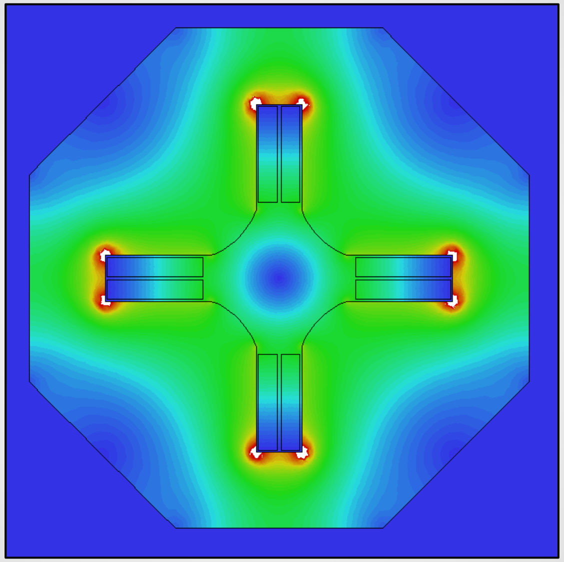


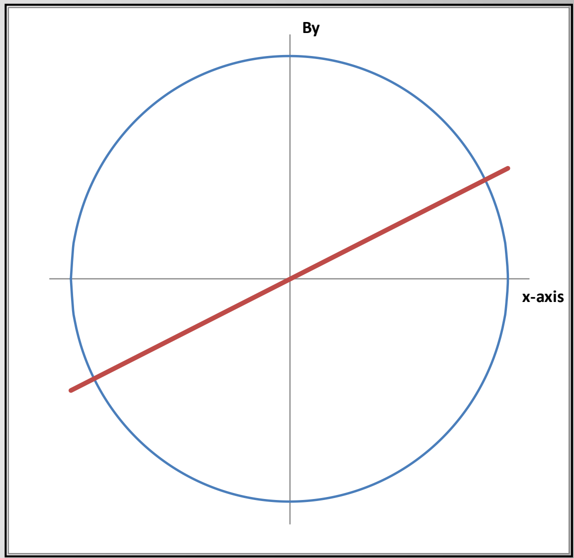


Quadrupoles

- Purpose: focusing the beam (horizontally focused beam is vertically defocused)



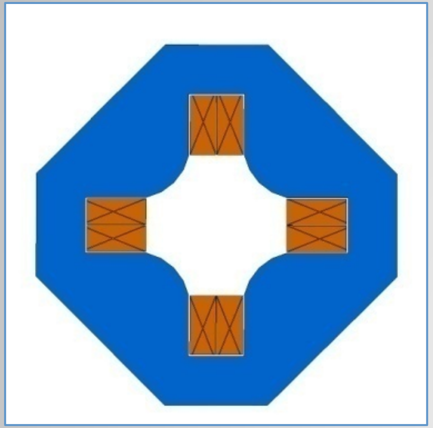




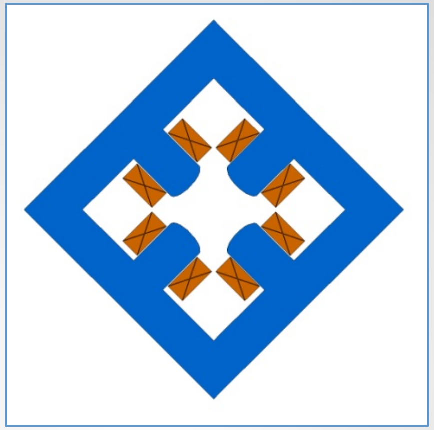
- Equation for normal (non-skew) ideal (infinite) poles: $2xy = \pm r^2$ (r = aperture radius)
- Magnetic flux density: $B_x = b_2 y$; $B_y = b_2 x$

Quadrupole types

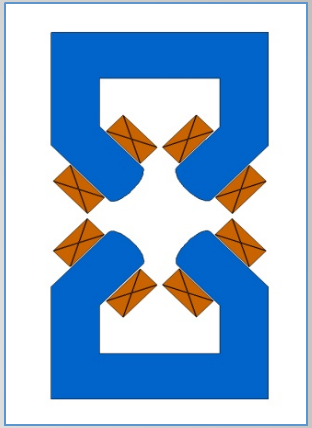
Standard quadrupole

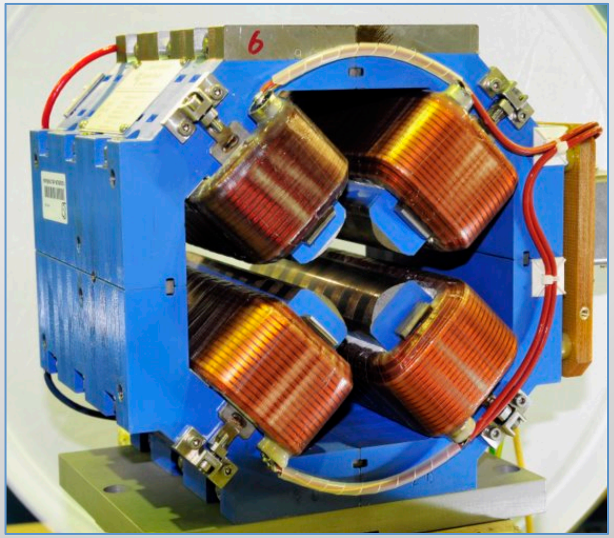


Standard quadrupole



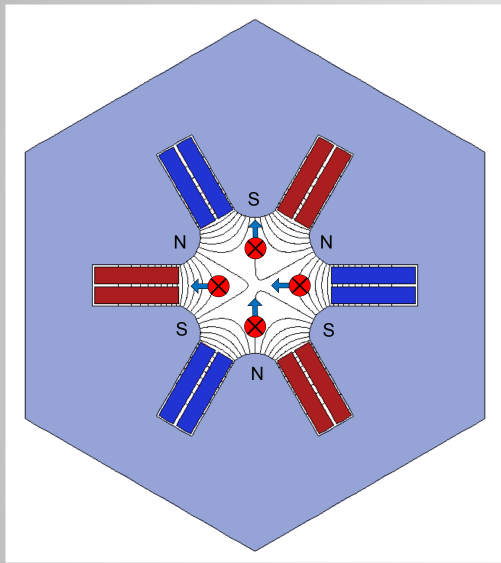
Collins or Figure-of-Eight

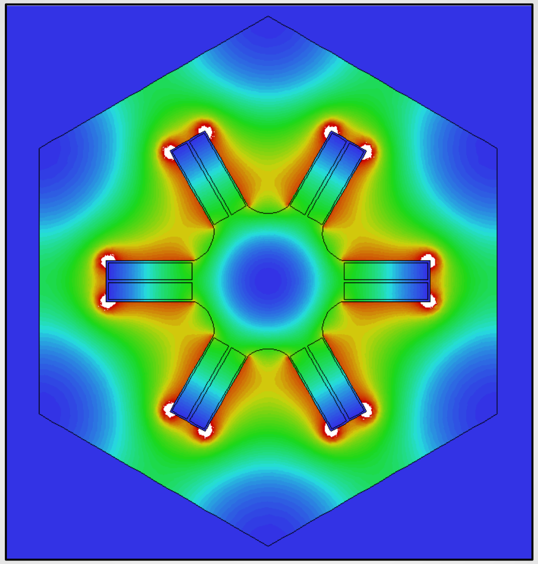


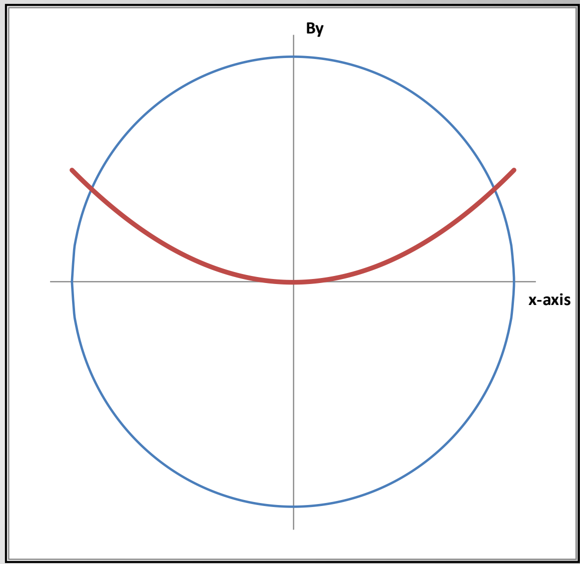


Sextupoles

- Purpose: correct chromatic aberrations of 'off-momentum' particles



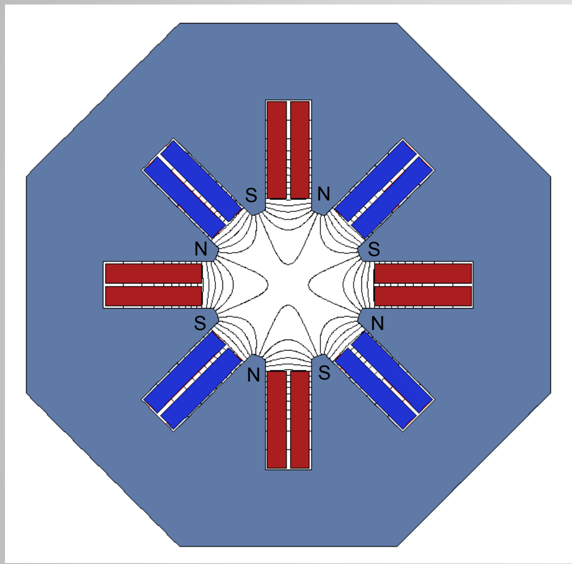


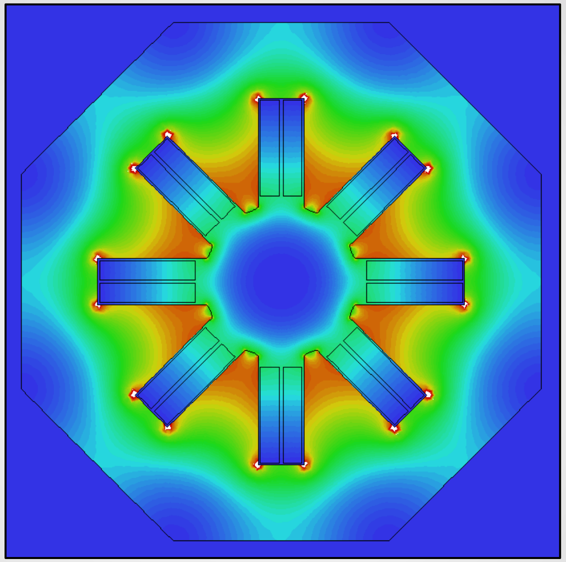


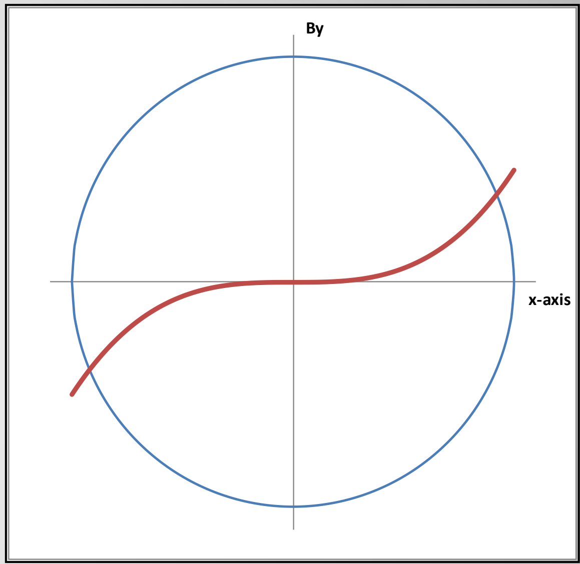
- Equation for normal (non-skew) ideal (infinite) poles: $3x^2y - y^3 = \pm r^3$ (r = aperture radius)
- Magnetic flux density: $B_x = b_3 xy$; $B_y = b_3(x^2 - y^2)/3$

Octupoles

- Purpose: 'Landau' damping

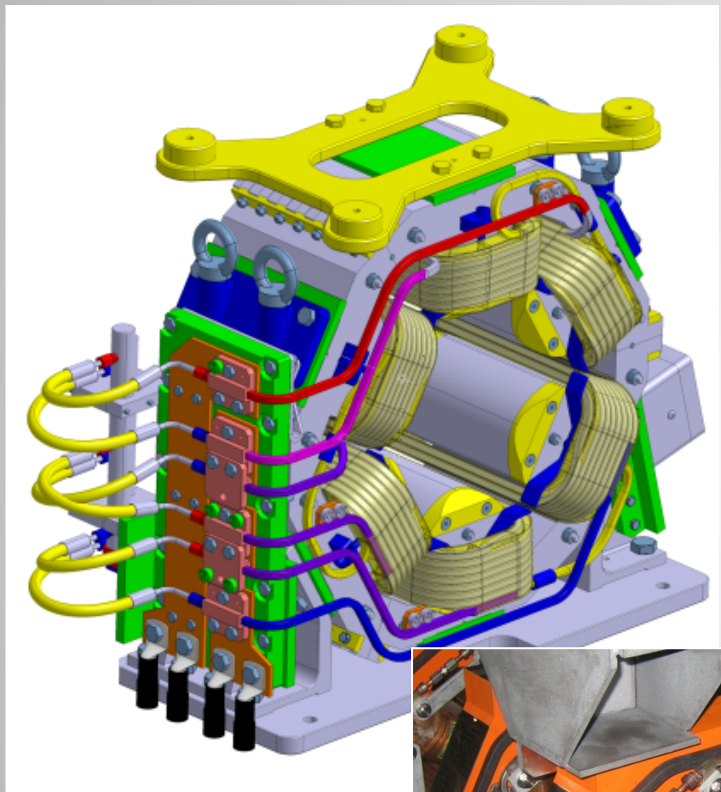


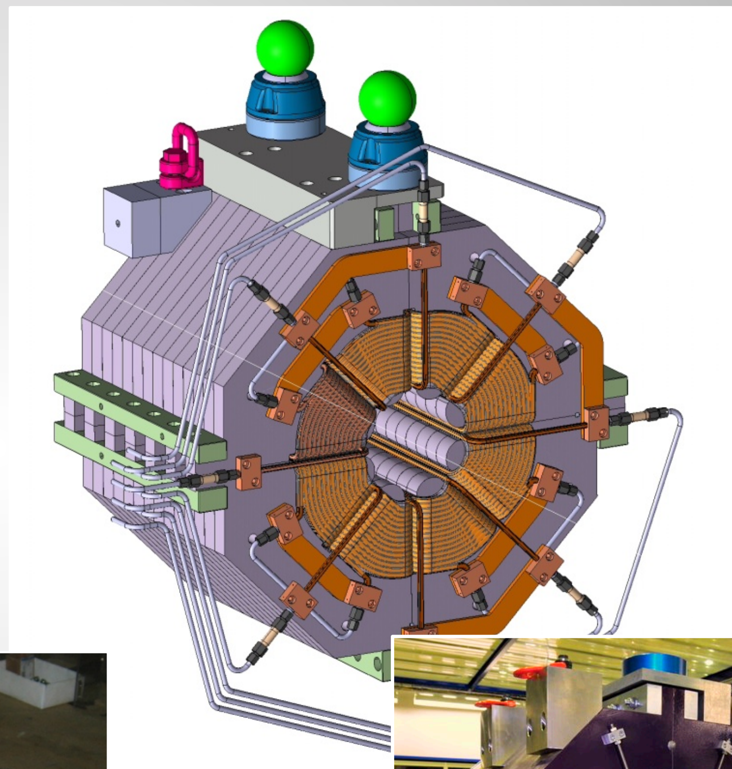


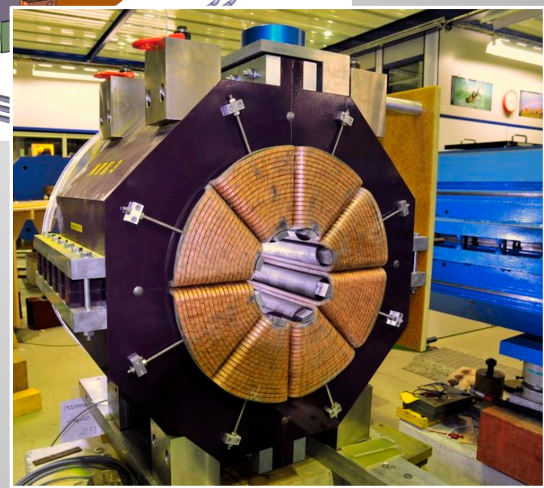


- Equation for normal (non-skew) ideal poles: $4(x^3y - xy^3) = \pm r^4$
  (r = aperture radius)
- Magnetic flux density: $B_x = b_4(3x^2y - y^3)/6$; $B_y = b_4(x^3 - 3xy^2)/6$

Sextupoles & Octupoles

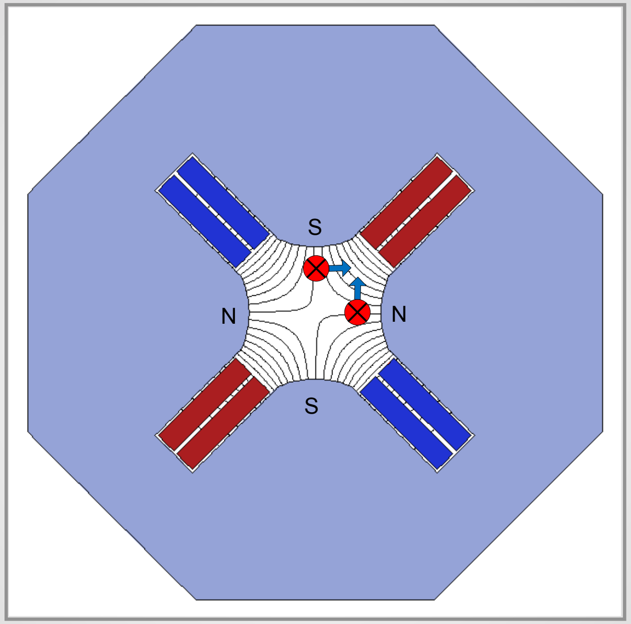






Skew quadrupole

- Purpose: coupling horizontal and vertical betatron oscillations



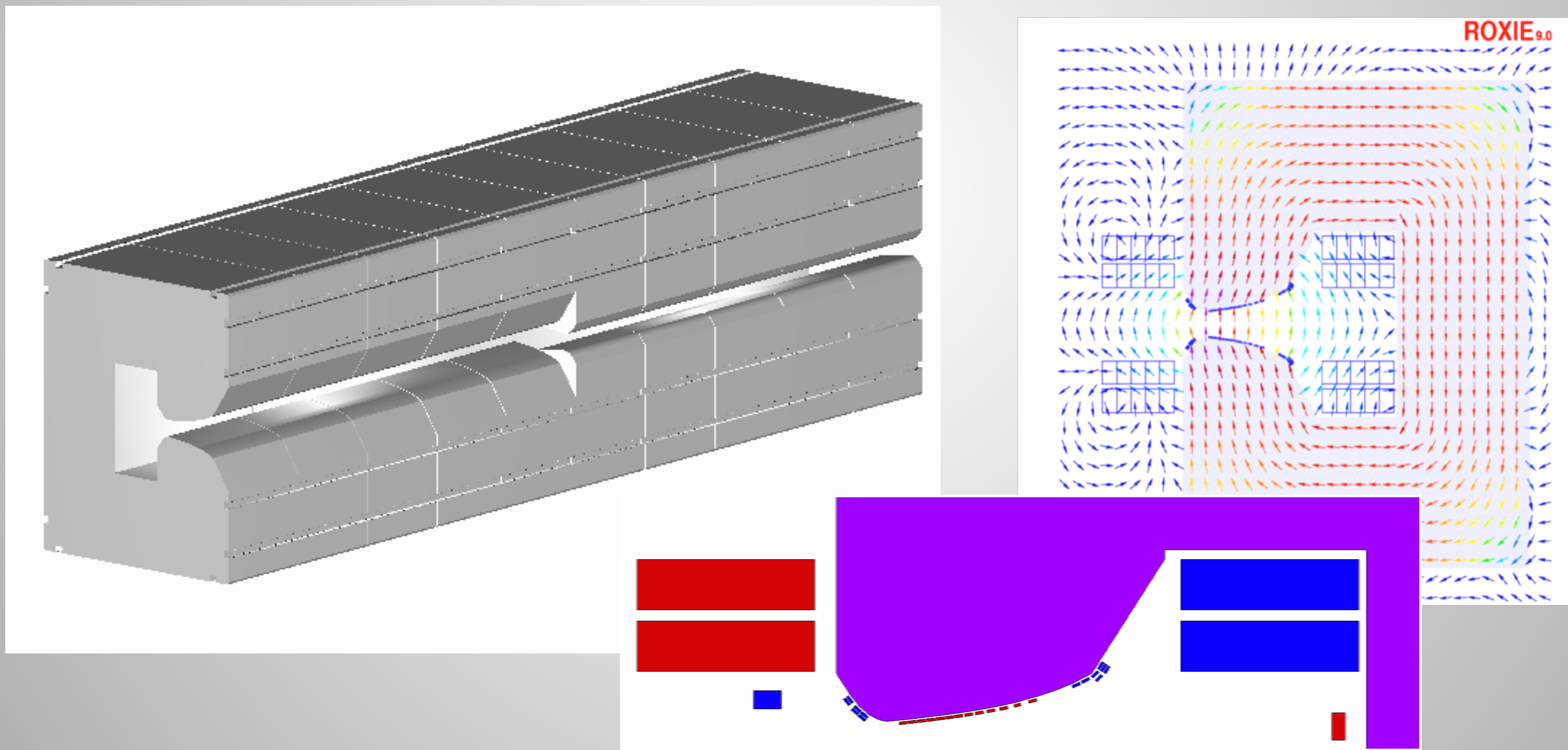
Rotation by π/2n

- Beam that has horizontal displacement (but no vertical) is deflected vertically
- Beam that has vertical displacement (but no horizontal) is deflected horizontally

Combined function magnets

Functions generated by pole shape (sum a scalar potentials):

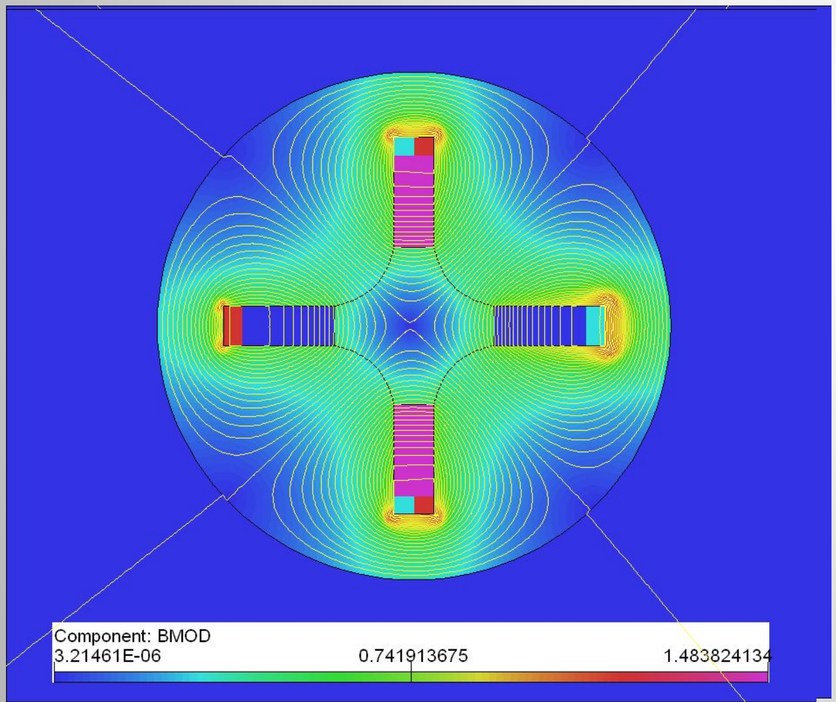
- Amplitudes cannot be varied independently
- Dipole and quadrupole: PS main magnet (PFW, Fo8…)

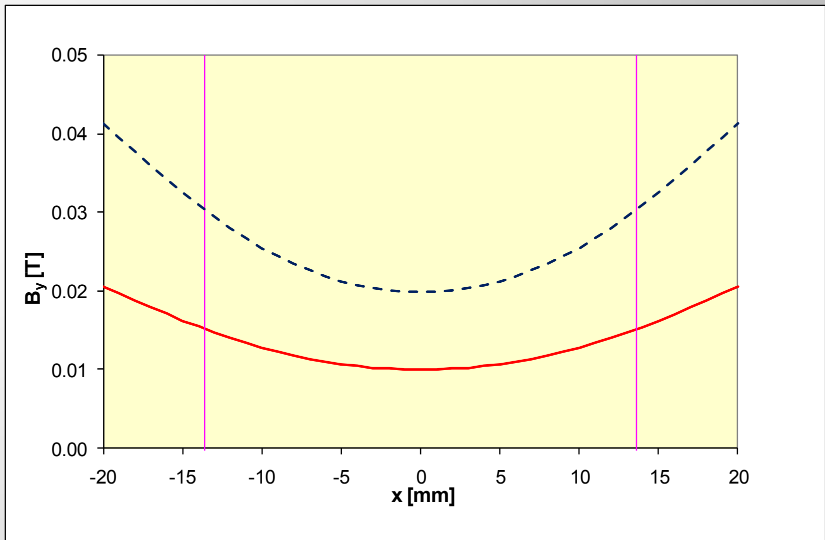


Combined function magnets

Functions generated by individual coils:

Amplitudes can be varied independently

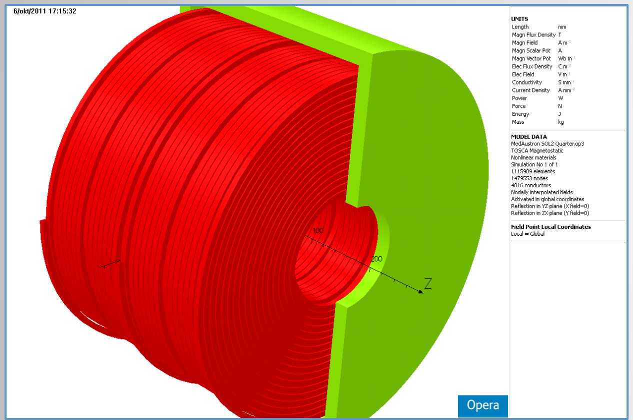


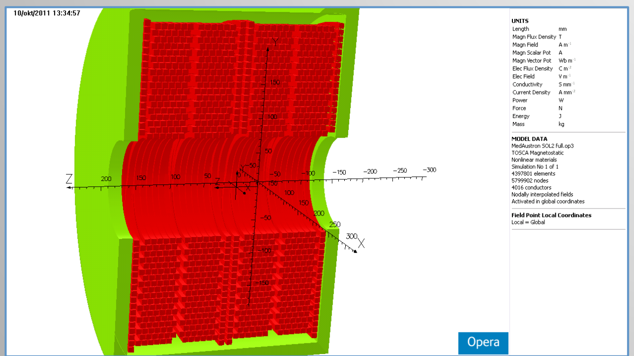


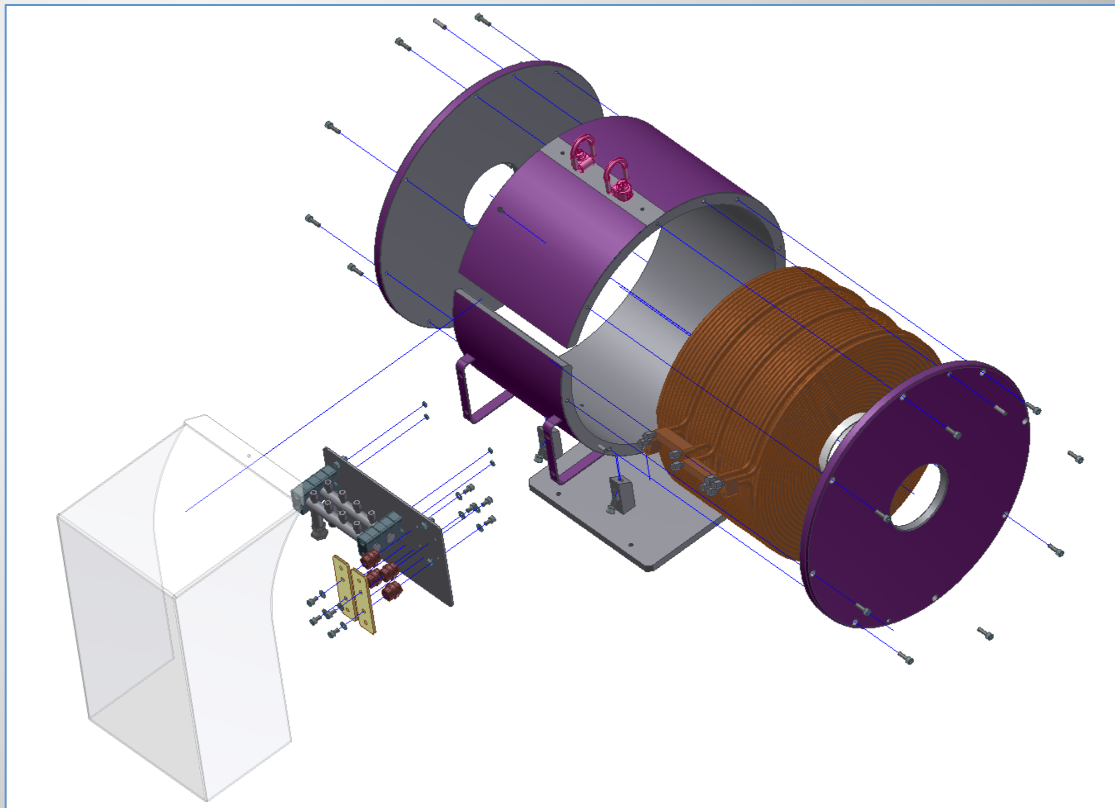
Quadrupole and corrector dipole (strong sextupole component in dipole field)

Solenoids

- Weak focusing, non-linear elements
- Main field component in z-direction, focusing by end fields
- Usually only used in experiments or low-energy beam lines



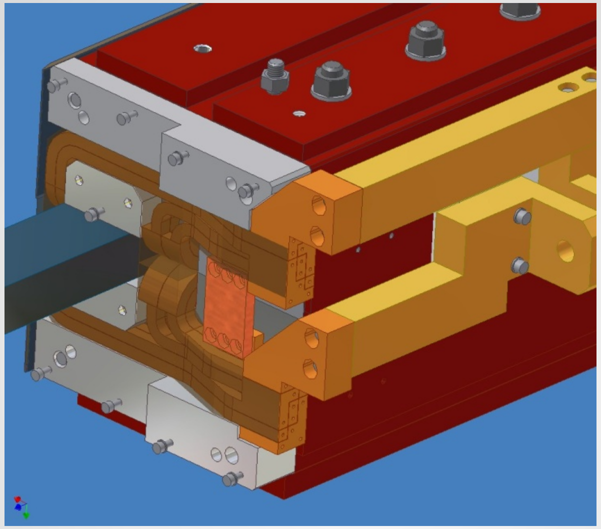




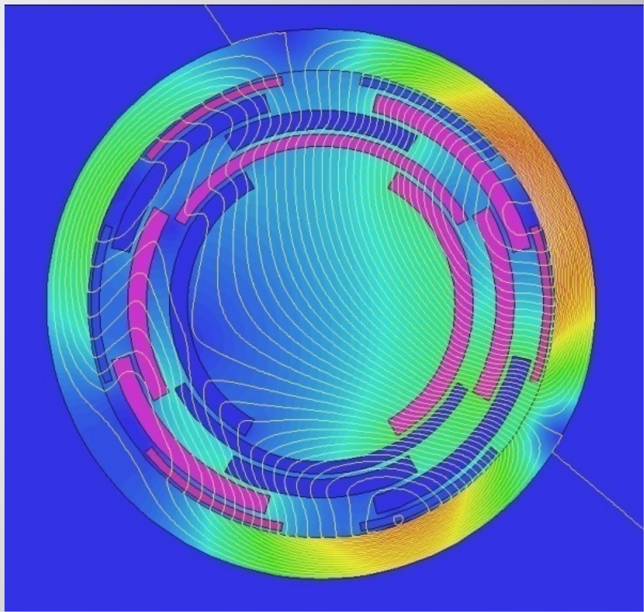
Special magnets

For beam injection and extraction

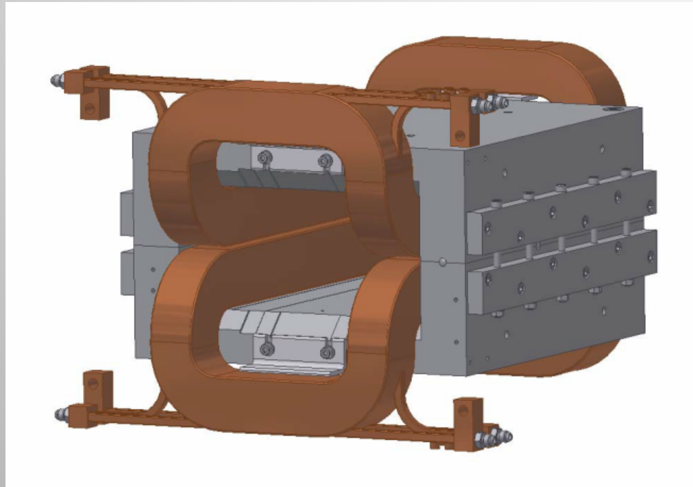
- Septa
- Kicker magnets
- Bumper magnets



Coil-dominated magnets

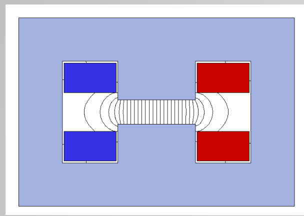
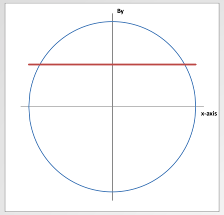
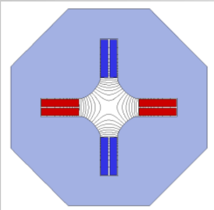
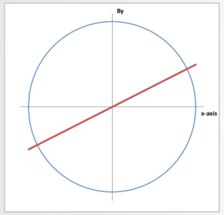
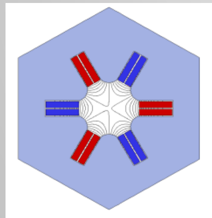
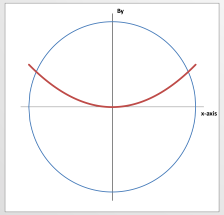
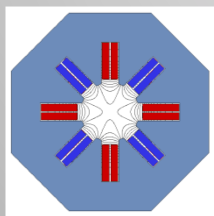
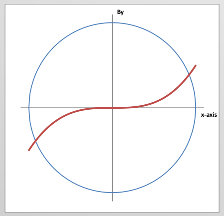


Scanning magnets



JUAS
Joint Universities Accelerator School

Overview

Pole shape	Field distribution	Pole equation	B_x, B_y
		$y = \pm r$	$B_x = 0$ $B_y = b_1 = B_0 = \text{const.}$
		$2xy = \pm r^2$	$B_x = b_2 y$ $B_y = b_2 x$
		$3x^2y - y^3 = \pm r^3$	$B_x = b_3 xy$ $B_y = b_3(x^2 - y^2)/2$
		$4(x^3y - xy^3) = \pm r^4$	$B_x = b_4(3x^2y - y^3)/6$ $B_y = b_4(x^3 - 3xy^2)/6$

Summary

- Magnets are needed to guide and shape particle beams
- Coils carry the electrical current, the iron yoke carries the magnets flux
- Steel properties have a significant influence on the magnet performance
- In case of time-varying fields, eddy currents can appear
- Different magnet types providing different functions