Lecture 2: Magnetization, cables and ac losses

Magnetization

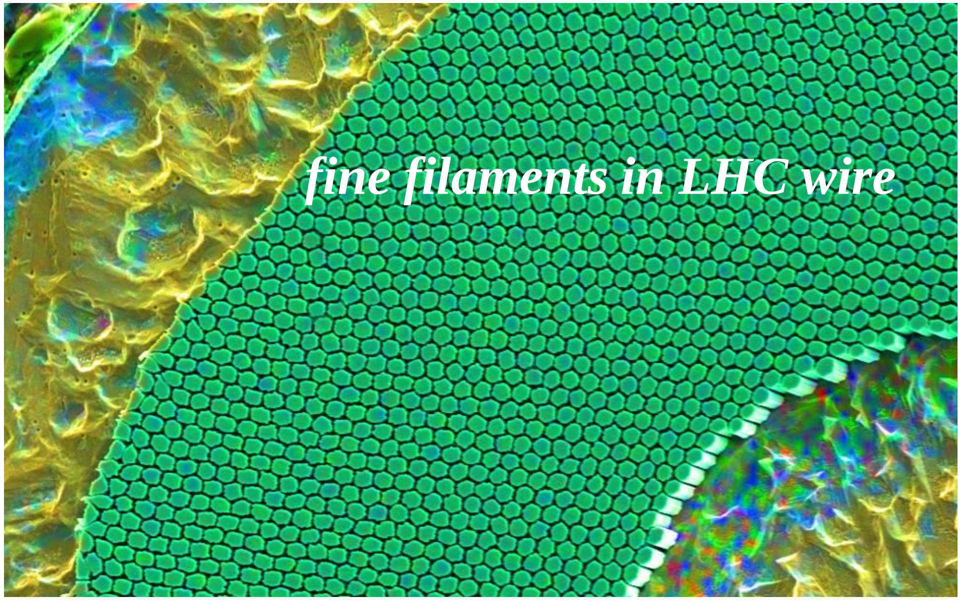
- superconductors in changing fields
- magnetization of wires & filaments
- coupling between filaments

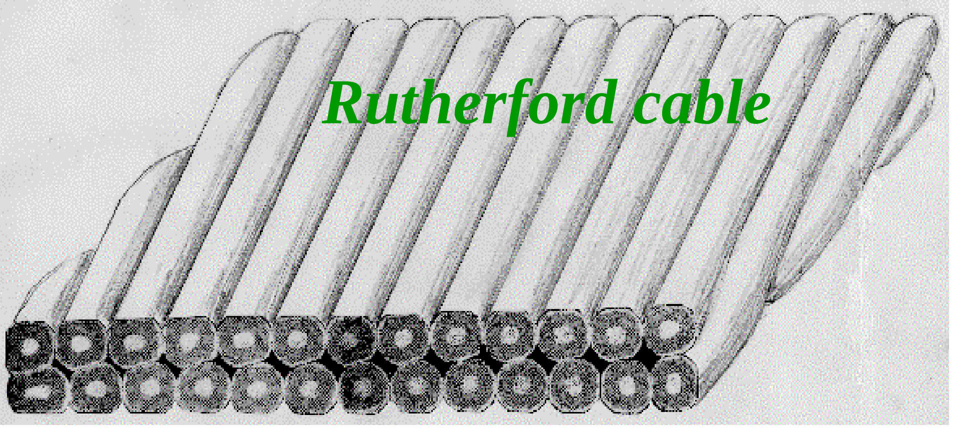
Cables

- why cables?
- coupling in cables
- effect on field error in magnets

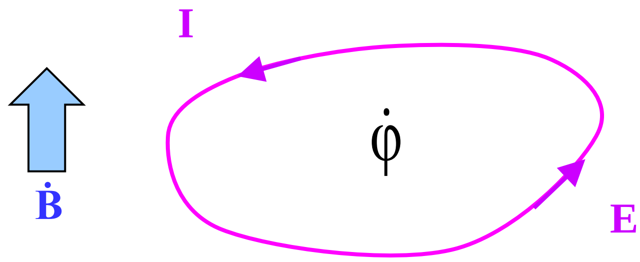
AC losses

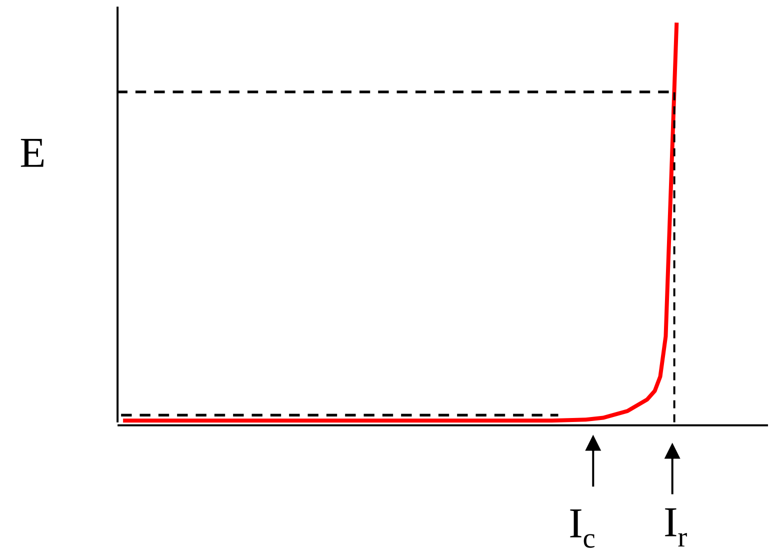
- general expression
- losses within filaments
- losses from coupling





Superconductors in changing magnetic fields





Faraday's Law of EM Induction

$$\oint E dl = \int \dot{B} dA = \dot{\phi}$$

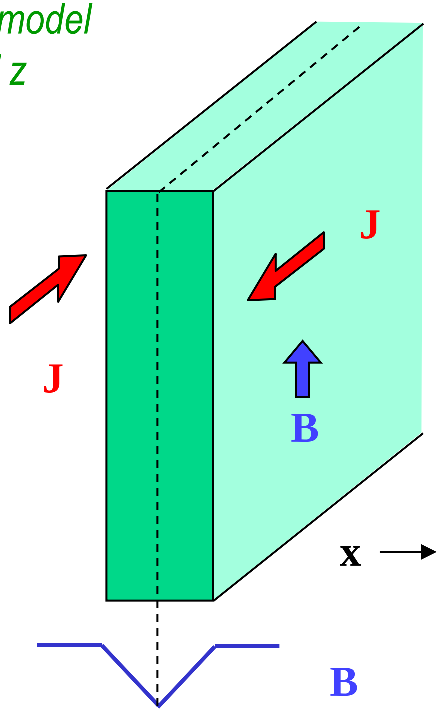
- changing field
  - → changing flux linked by loop
  - → electric field E in superconductor
  - → current I_r flows around the loop
- change stops
  - → electric field goes to zero
  - → superconductor current falls back to I_c (not zero)
  - → current circulates for ever ***persistent current***

changing magnetic fields
on superconductors
→ electric field
→ resistance
→ power dissipation

Persistent screening currents

- ***screening currents*** are in addition to the ***transport current***, which comes from the power supply
- like eddy currents but, because no resistance, they don't decay

simplified slab model
infinite in y and z



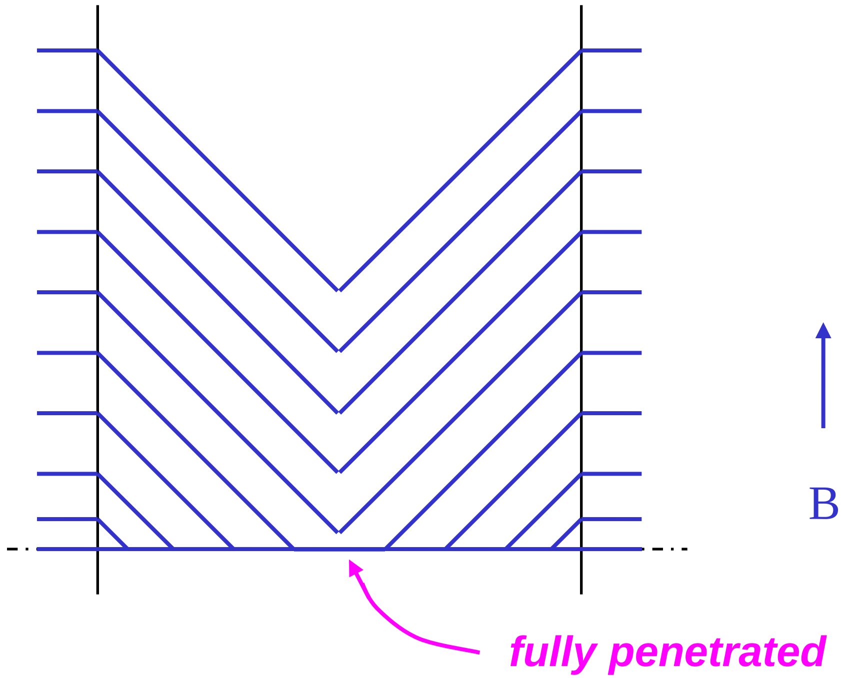
- ***dB/dt*** induces an electric field ***E*** which drives the screening current up to current density J_r
- ***dB/dt*** stops and current falls back to J_c
- so in the steady state we have persistent $J = +J_c$ or $J = -J_c$ or $J = 0$ nothing else
- known as the ***critical state model*** or ***Bean London model***
- in the 1 dim infinite slab geometry, Maxwell's equation says

$$\frac{\partial B_y}{\partial x} = -\mu_o J_z = \mu_o J_c$$

- so a uniform J_c means a constant field gradient inside the superconductor

The flux penetration process

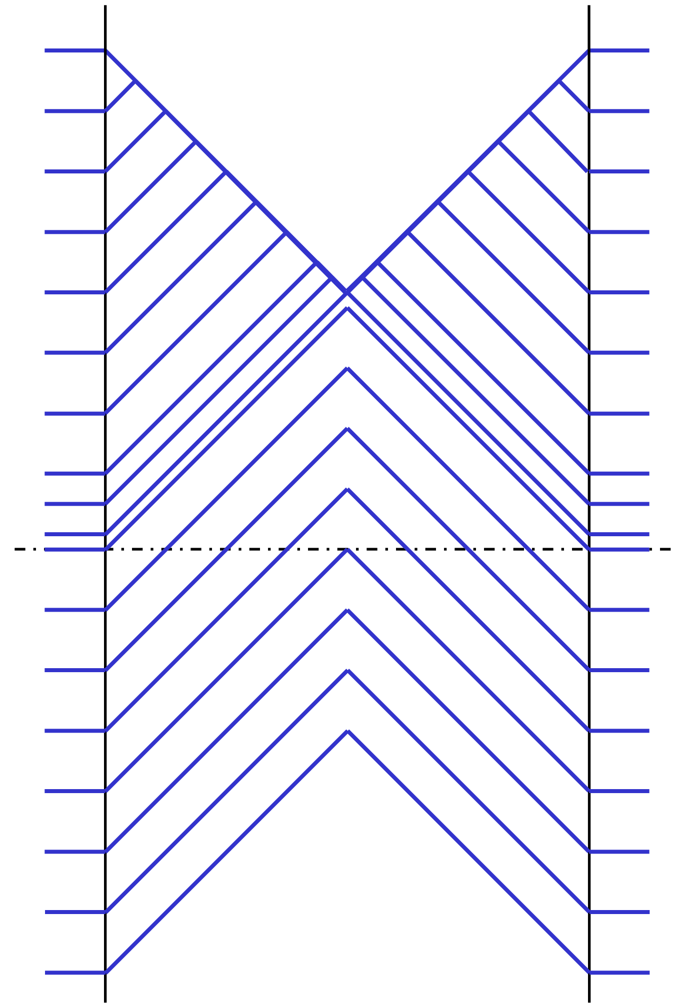
plot field profile across the slab



field increasing from zero

Bean London critical state model

- **current density everywhere is $\pm J_c$ or zero**
- **change comes in from the outer surface**



field decreasing through zero

Magnetization of the Superconductor

When viewed from outside the sample, the persistent currents produce a magnetic moment.

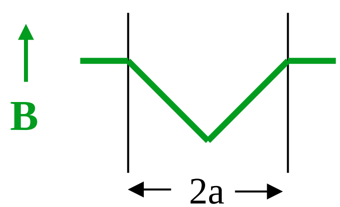
Problem for accelerators because it spoils the precise field shape

We can define a magnetization (magnetic moment per unit volume)

$$M = \sum_V \frac{I.A}{V}$$

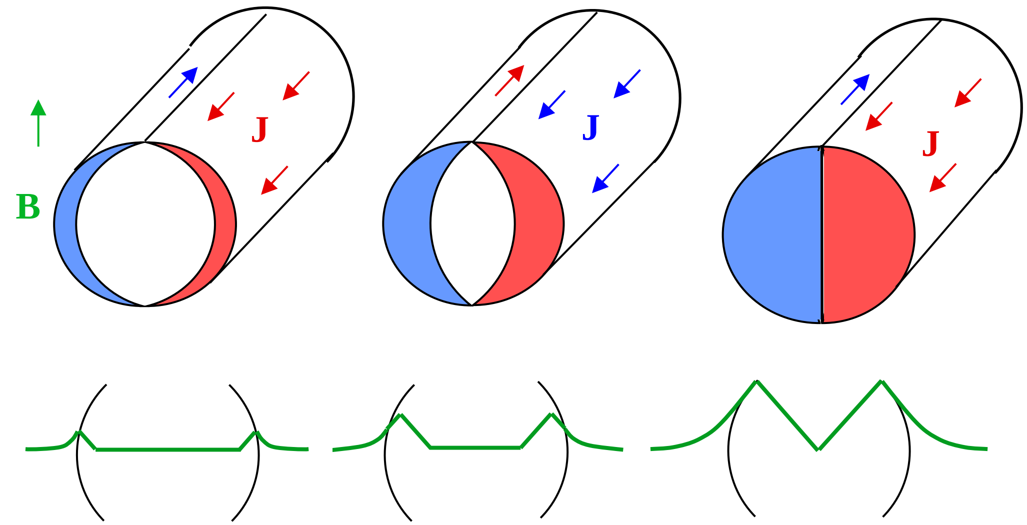
NB units of H

for a fully penetrated **slab**



$$M_s = \frac{1}{a}\int_0^a J_c x dx = \frac{J_c a}{2}$$

for **cylindrical** filaments the inner current boundary is roughly elliptical (controversial)



when fully penetrated, the magnetization is

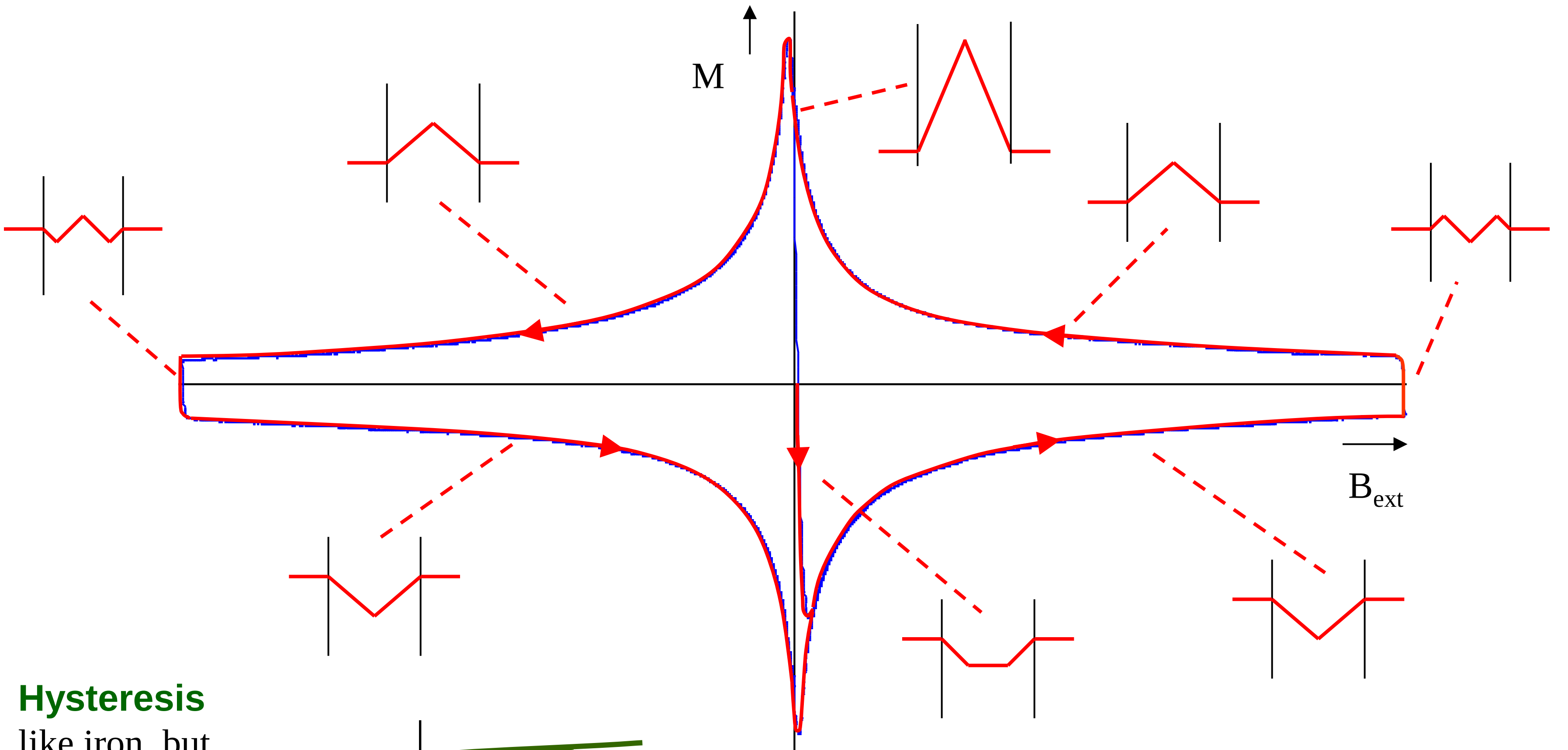
$$M_s = \frac{4}{3\pi} J_c a = \frac{2}{3\pi} J_c d_f$$

where a, d_f = filament radius, diameter

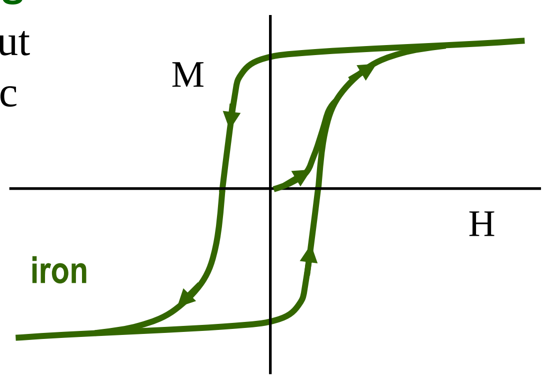
Note: M is here defined per unit volume of NbTi filament

to reduce *M* need small *d* - fine filaments

Magnetization of NbTi



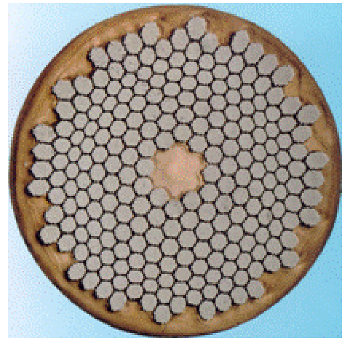
Hysteresis
like iron, but
diamagnetic

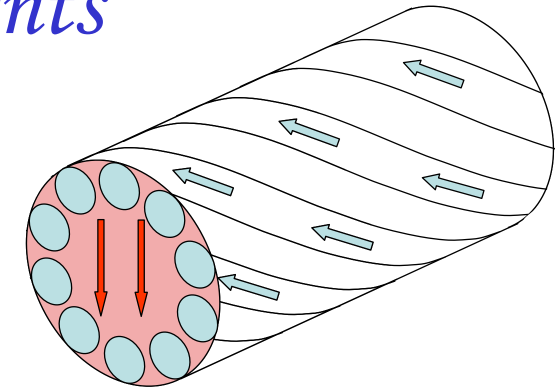


Magnetization is important because
it produces field errors and ac losses

Coupling between filaments

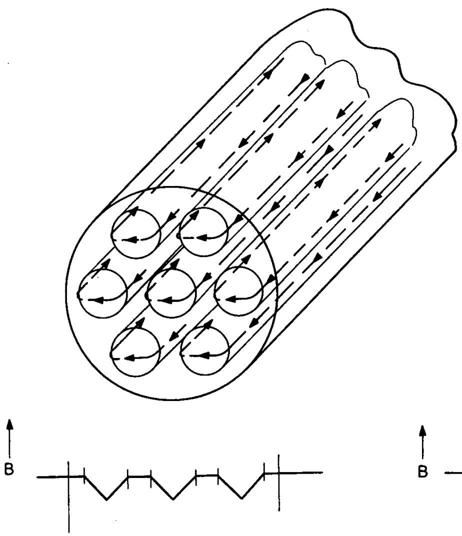
recap $M_s = \frac{2}{3\pi} J_c d_f$

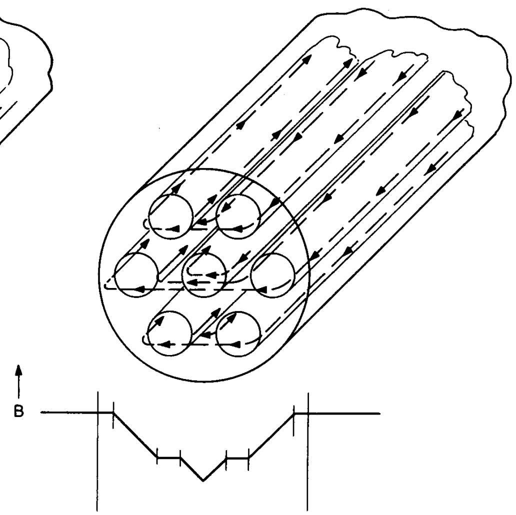




- reduce M by making fine filaments
- for ease of handling, filaments are embedded in a copper matrix

- coupling currents flow along the filaments and across the matrix
- reduce them by twisting the wire
- they behave like eddy currents and produce an additional magnetization





$$M_e = \frac{dB}{dt} \frac{1}{\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

$$M_e = \frac{2}{\mu_o} \frac{dB}{dt} \tau \quad \text{where} \quad \tau = \frac{\mu_o}{2\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

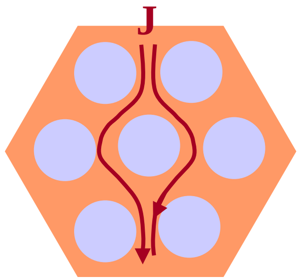
- but in changing fields, the filaments are magnetically coupled
- screening currents go up the left filaments and return down the right

per unit volume of **wire**

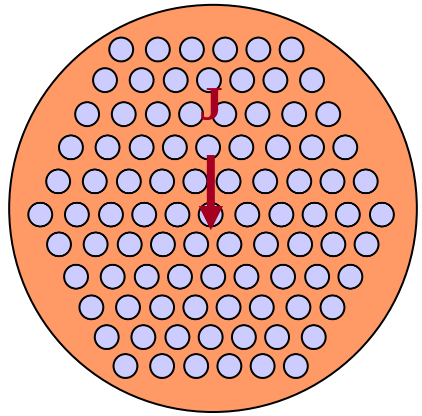
ρ_t = resistivity across matrix, p_w = wire twist pitch

Transverse resistivity across the matrix

Poor contact to filaments

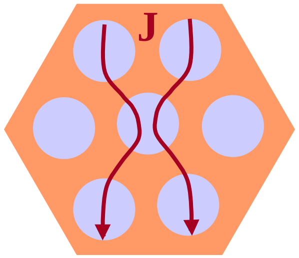


$$\rho_t = \rho_{Cu}\frac{1+\lambda_{sw}}{1-\lambda_{sw}}$$



where λ_{sw} is the fraction of superconductor in the wire cross section (after J Carr)

Good contact to filaments

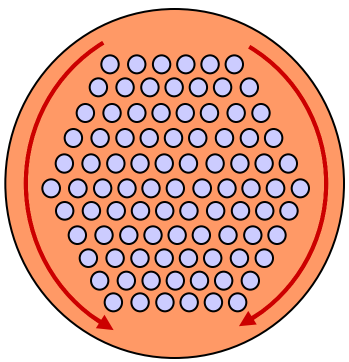


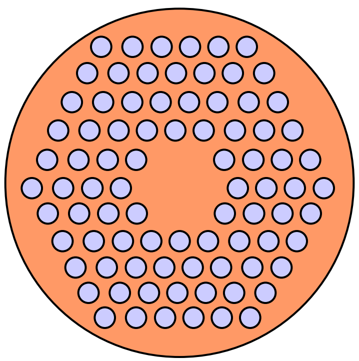
$$\rho_t = \rho_{Cu}\frac{1-\lambda_{sw}}{1+\lambda_{sw}}$$

Some complications

Thick copper jacket

include the copper jacket as a resistance in parallel

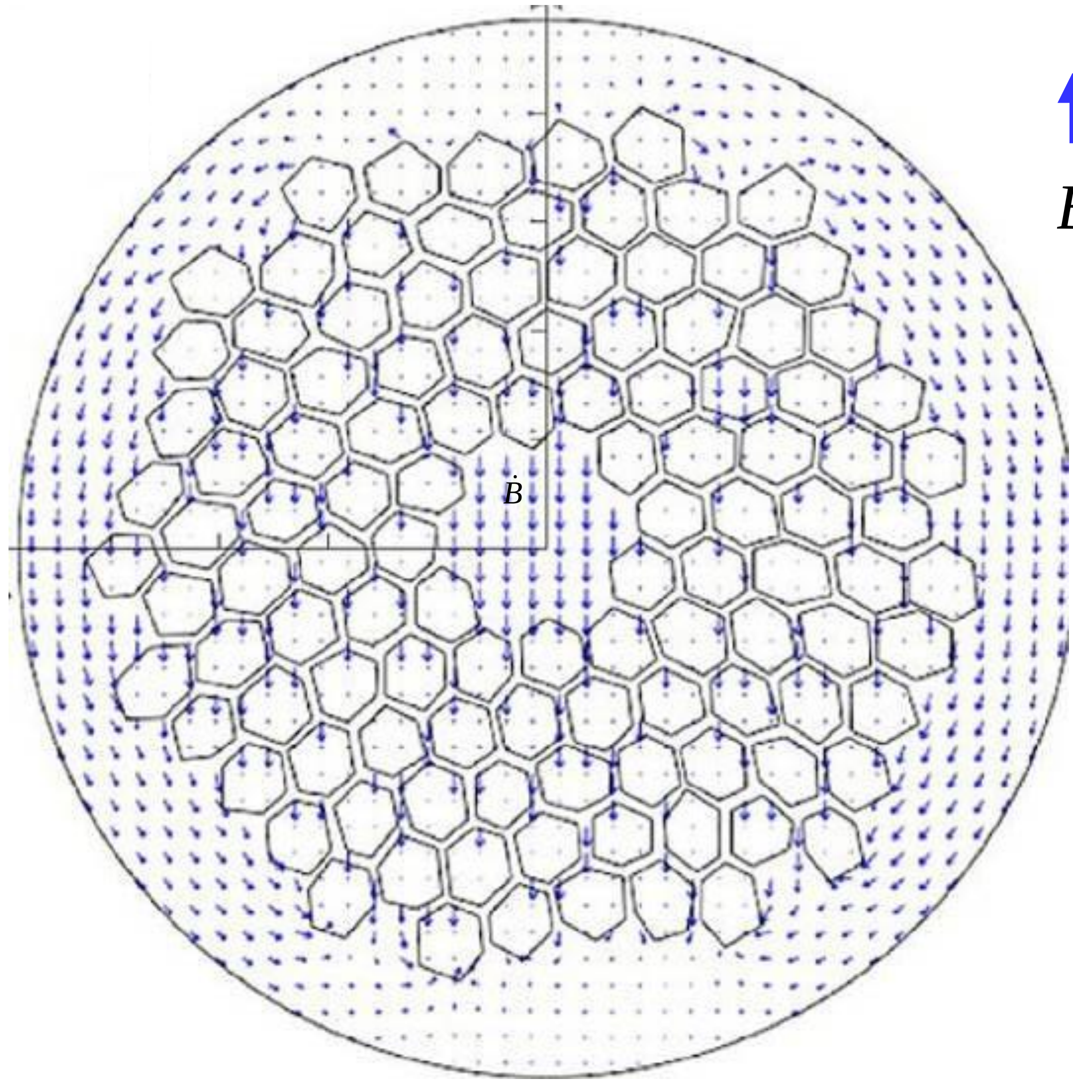




Copper core

resistance in series for part of current path

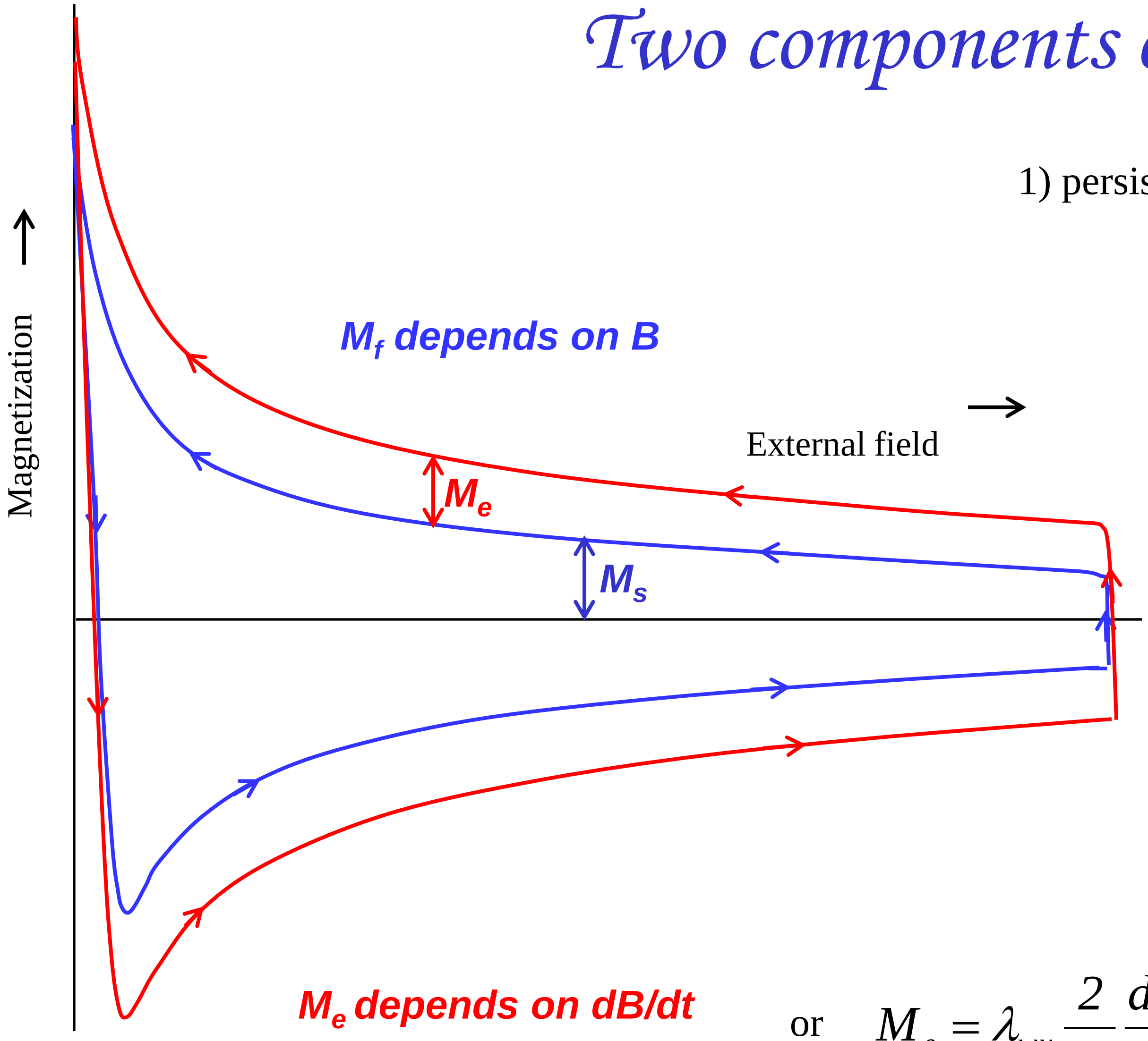
Computation of current flow across matrix



$\dot{B}$

calculated using the COMSOL code by P.Fabbricatore et al JAP, 106, 083905 (2009)

Two components of magnetization



1) persistent current within the filaments

$$M_s = \lambda_{su} \frac{2}{3\pi} J_c(B)\, d_f$$

where λ_{su} = fraction of superconductor in the unit cell

2) eddy current coupling between the filaments

$$M_e = \lambda_{wu} \frac{dB}{dt} \frac{1}{\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

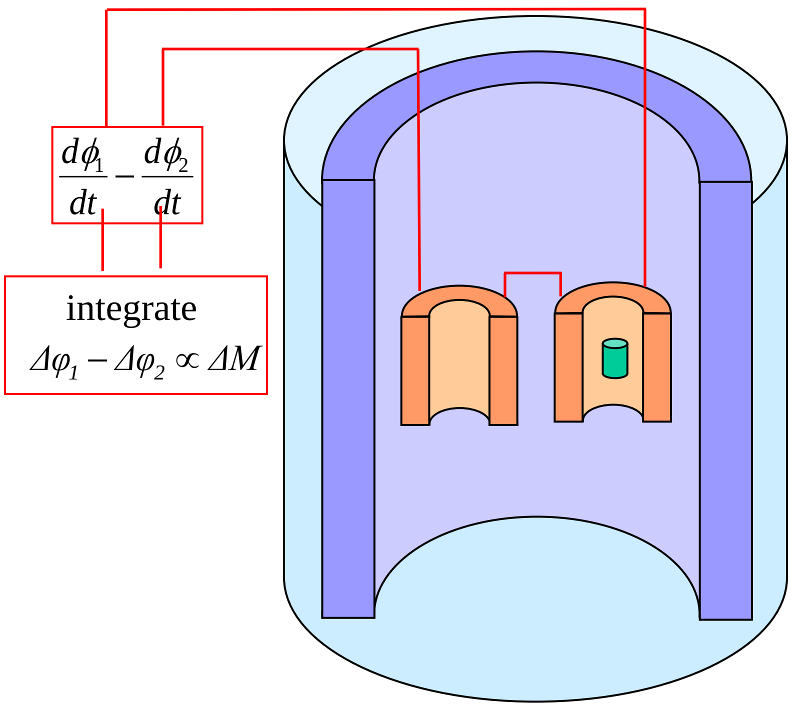
or $$M_e = \lambda_{wu} \frac{2}{\mu_o} \frac{dB}{dt} \tau \quad \text{where} \quad \tau = \frac{\mu_o}{2\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

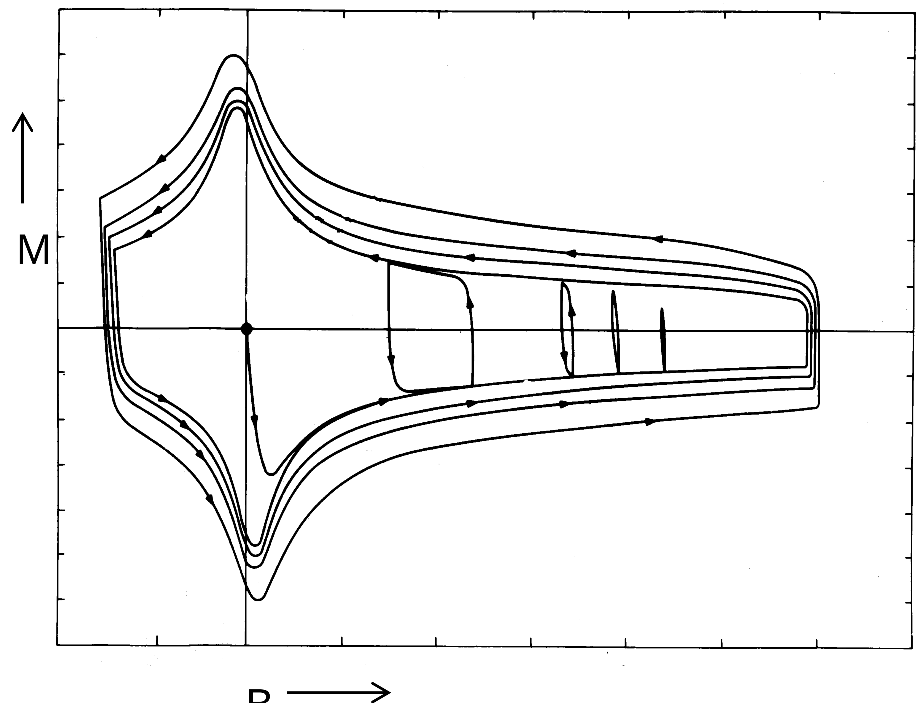
Magnetization is averaged over the ***unit cell***

where λ_{wu} = fraction of wire in the section

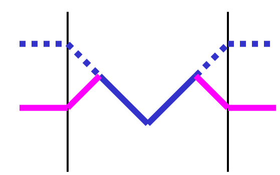
Measurement of magnetization

In field, the superconductor behaves just like a magnetic material. We can plot the magnetization curve using a magnetometer. It shows hysteresis - just like iron only in this case the magnetization is both diamagnetic and paramagnetic.



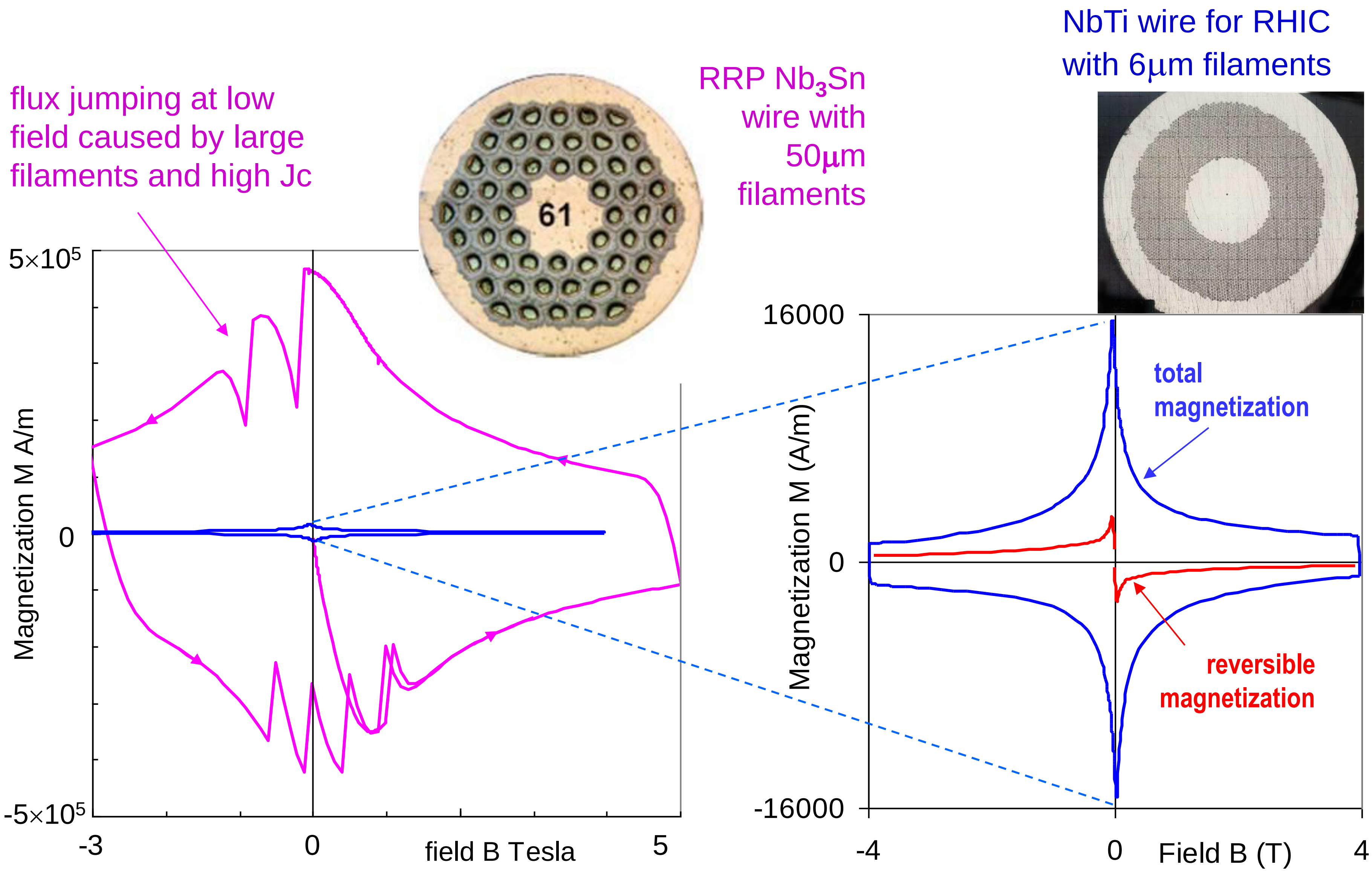


Note the minor loops, where field and therefore screening currents are reversing



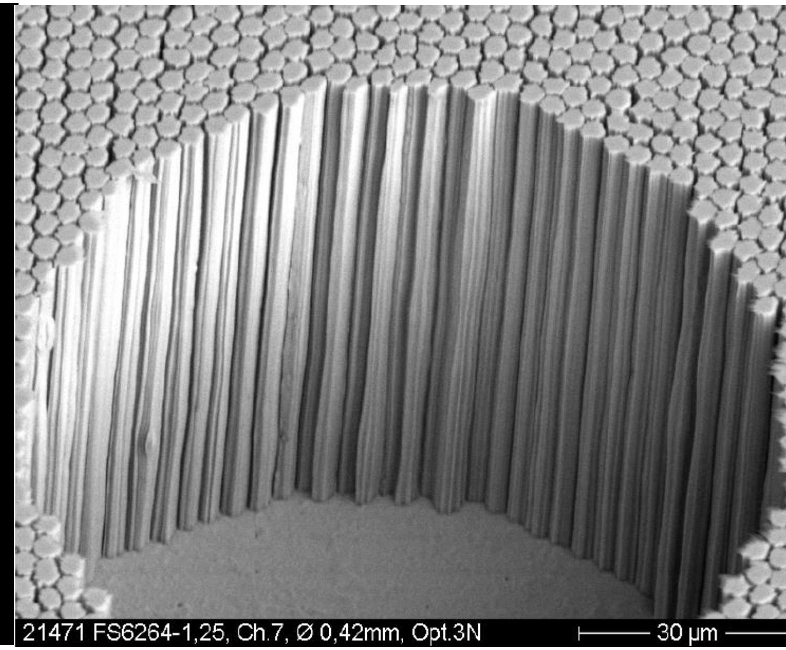
Two balanced search coils connected in series opposition, are placed within the bore of a superconducting solenoid. With a superconducting sample in one coil, the integrator measures ΔM when the solenoid field is swept up and down

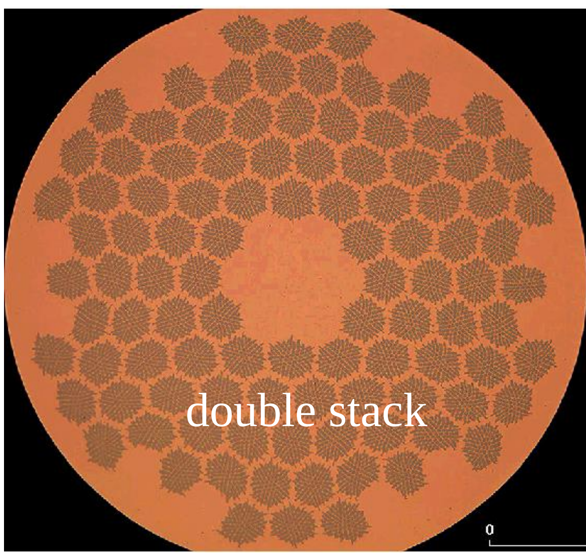
Magnetization measurements

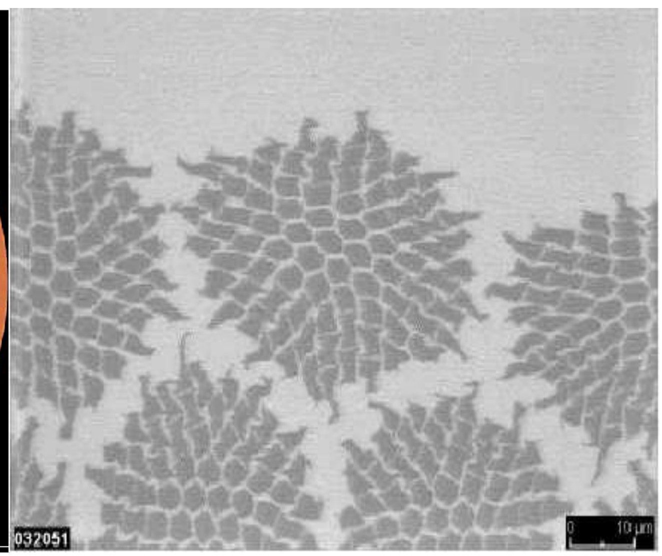


Fine filaments for low magnetization

Accelerator magnets need the finest filaments
- to minimize field errors and ac losses







Typical diameters are in the range 5 - 10µm. Even smaller diameters would give lower magnetization, but at the cost of lower Jc and more difficult production.

Cables - why do we need them?

- for accurate tracking we connect synchrotron magnets in series
- for rise time *t* and operating current *I* , charging voltage is

$$V = \frac{L\,I}{t} = \frac{2\,E}{I\,t}$$

- for a given field and volume, stored energy of a magnet is fixed, regardless of current or inductance

$$E = \frac{1}{2} L I^2 = \frac{B^2}{2\mu_o} vol$$

RHIC E = 40kJ/m, t = 75s, 30 strand cable

cable I = **5kA**, charge voltage per km = **213V**

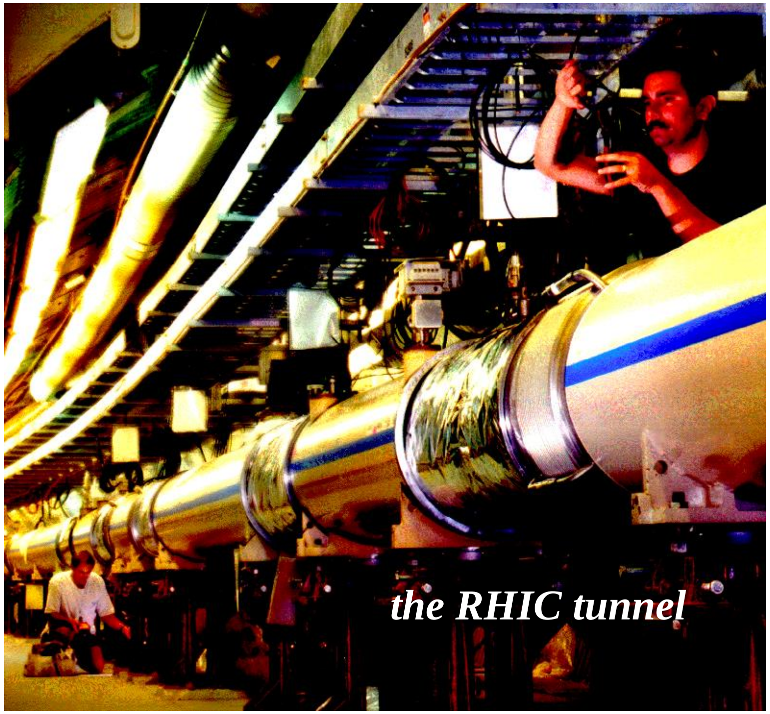
wire I = **167A**, charge voltage per km = **6400V**

FAIR at GSI E = 74kJ/m, t = 4s, 30 strand cable

cable I = **6.8kA**, charge voltage per km = **5.4kV**

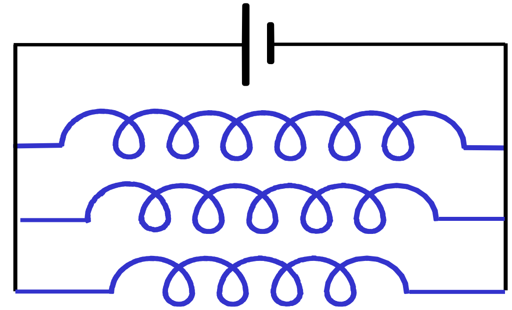
wire I = **227A**, charge voltage per km = **163kV**

- so we need high currents!

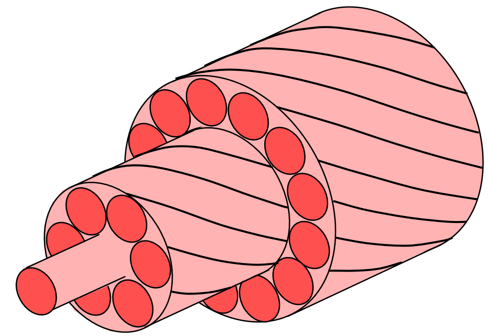


- a single 5µm filament of NbTi in 6T carries 50mA
- a composite wire of fine filaments typically has 5,000 to 10,000 filaments, so it carries 250A to 500A
- for 5 to 10kA, we need 20 to 40 wires in parallel - a cable

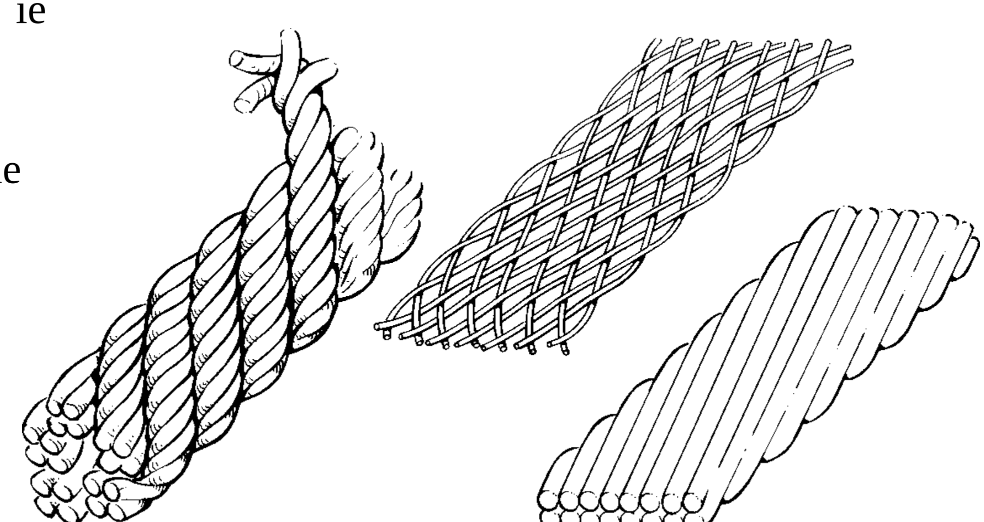
Cable transposition

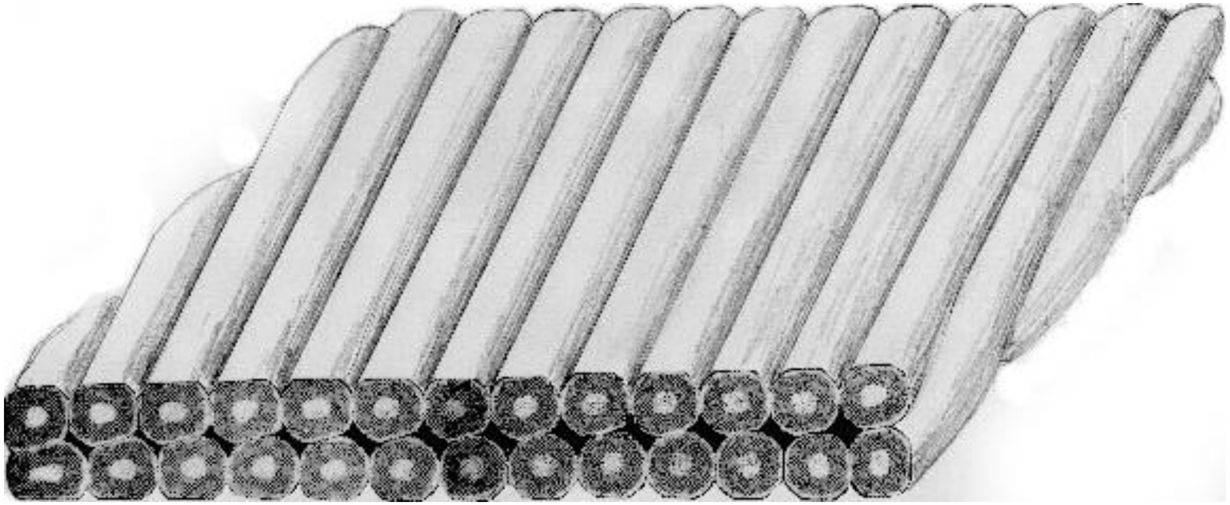


- many wires in parallel - want them all to carry same current zero resistance - so current divides according to inductance
- in a simple twisted cable, wires in the centre have a higher self inductance than those at the outside

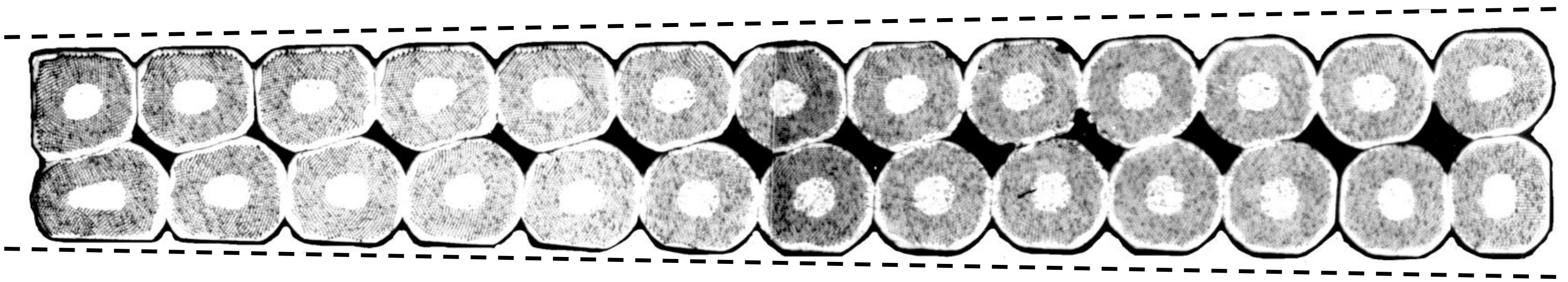


- current fed in from the power supply therefore takes the low inductance path and stays on the outside
- outer wires reach J_c while inner are still empty
- so the wires must be fully **transposed,** ie every wire must change places with every other wire along the length
  inner wires ⇒ outside outer wire ⇒ inside
- three types of fully transposed cable have been tried in accelerators
  - rope
  - braid
  - Rutherford

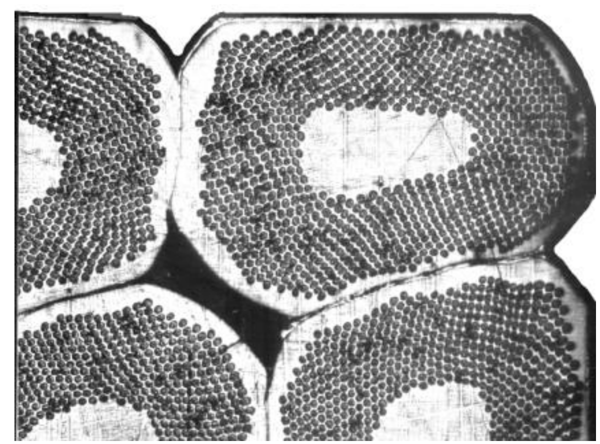




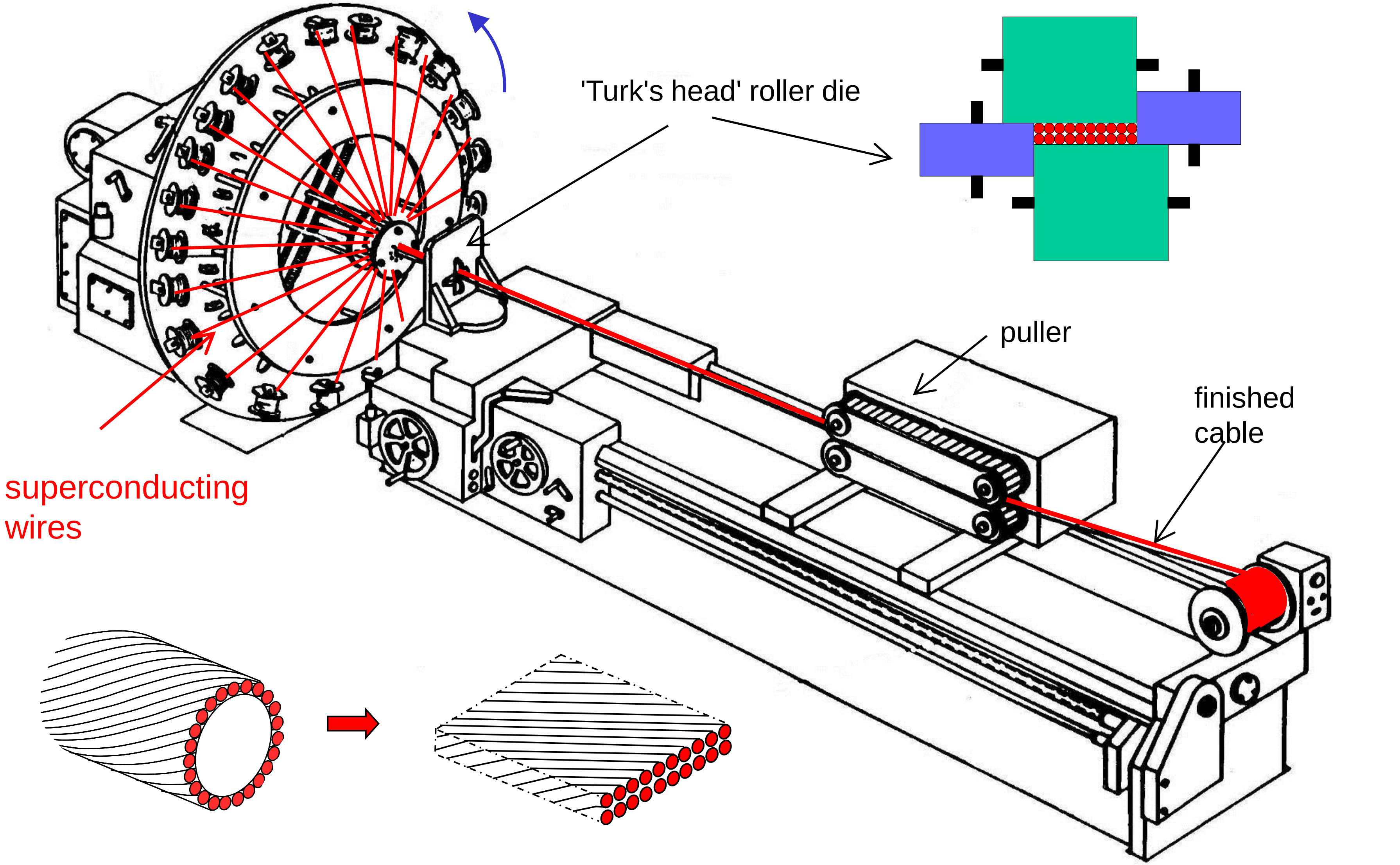
Rutherford cable



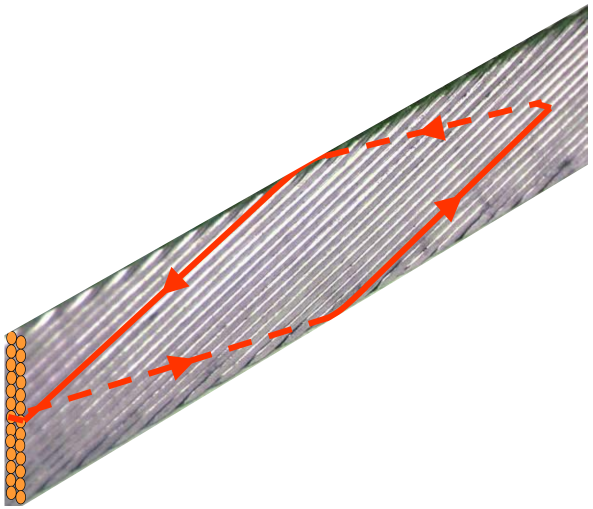
- Rutherford cable succeeded where others failed because it could be compacted to a high density (88 - 94%) without damaging the wires, and rolled to a good dimensional accuracy (~ 10μm).
- Note the 'keystone angle', which enables the cables to be stacked closely round a circular aperture



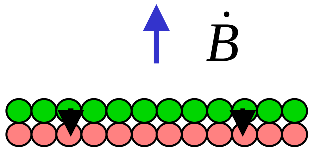
Manufacture of Rutherford cable

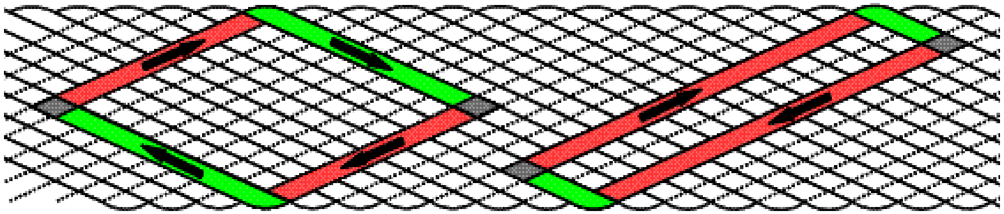


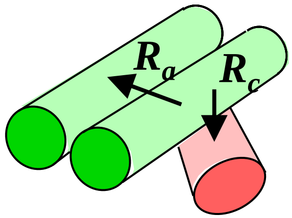
Coupling in Rutherford cables



- Field transverse
  coupling via crossover resistance R_c

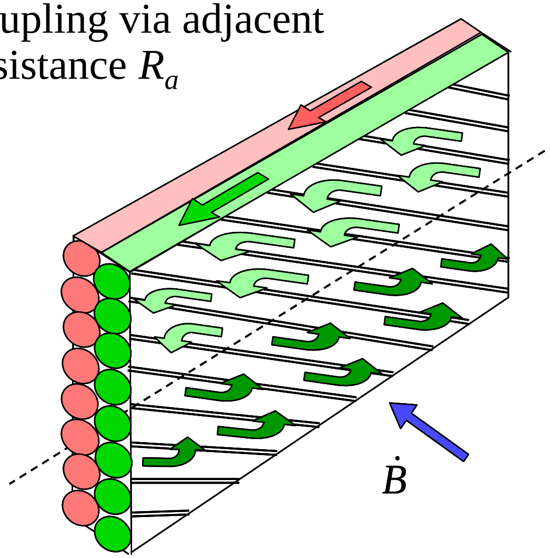




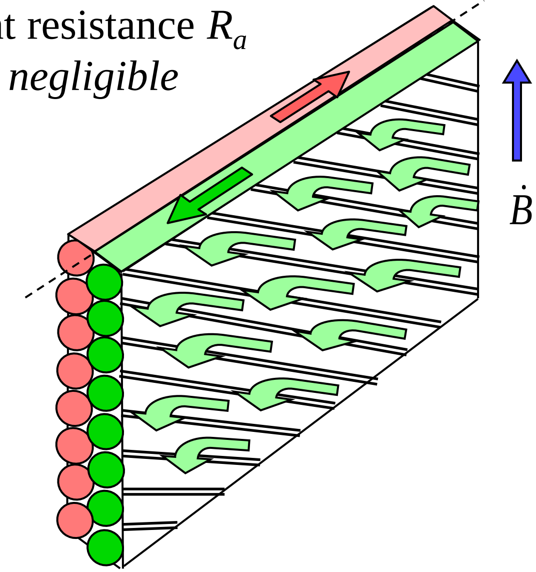


crossover resistance *Rc*
adjacent resistance *Ra*

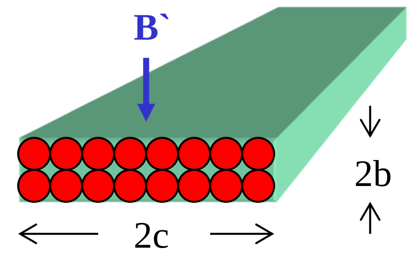
- Field transverse
  coupling via adjacent resistance R_a



- Field parallel coupling via adjacent resistance R_a
  usually negligible



Magnetization from coupling in cables



- Field transverse coupling via crossover resistance R_c

$$M_{tc} = \frac{1}{120}\frac{\dot{B}_t}{R_c}\frac{c}{b}\,p\,N(N-1) = \frac{1}{60}\frac{\dot{B}_t}{\rho_c}\,p^2\frac{c^2}{b^2}$$

where M = magnetization per unit volume of cable*, p= twist pitch, N = number of strands*

R_c R_a = resistance per crossover ρ_c ρ_a = effective resistivity between wire centres

- Field transverse coupling via adjacent resistance R_a

where θ = slope angle of wires $Cos\,\theta$ ~ 1

$$M_{ta} = \frac{1}{6}\frac{\dot{B}_t}{R_a}\,p\,\frac{c}{b} = \frac{1}{48}\frac{\dot{B}_t}{\rho_a}\frac{p^2}{Cos^2\theta}$$

- Field parallel coupling via adjacent resistance R_a

$$M_{pa} = \frac{1}{8}\frac{\dot{B}_p}{R_a}\,p\,\frac{b}{c} = \frac{1}{64}\frac{\dot{B}_p}{\rho_a}\frac{p^2}{\cos^2\theta}\frac{b^2}{c^2}$$

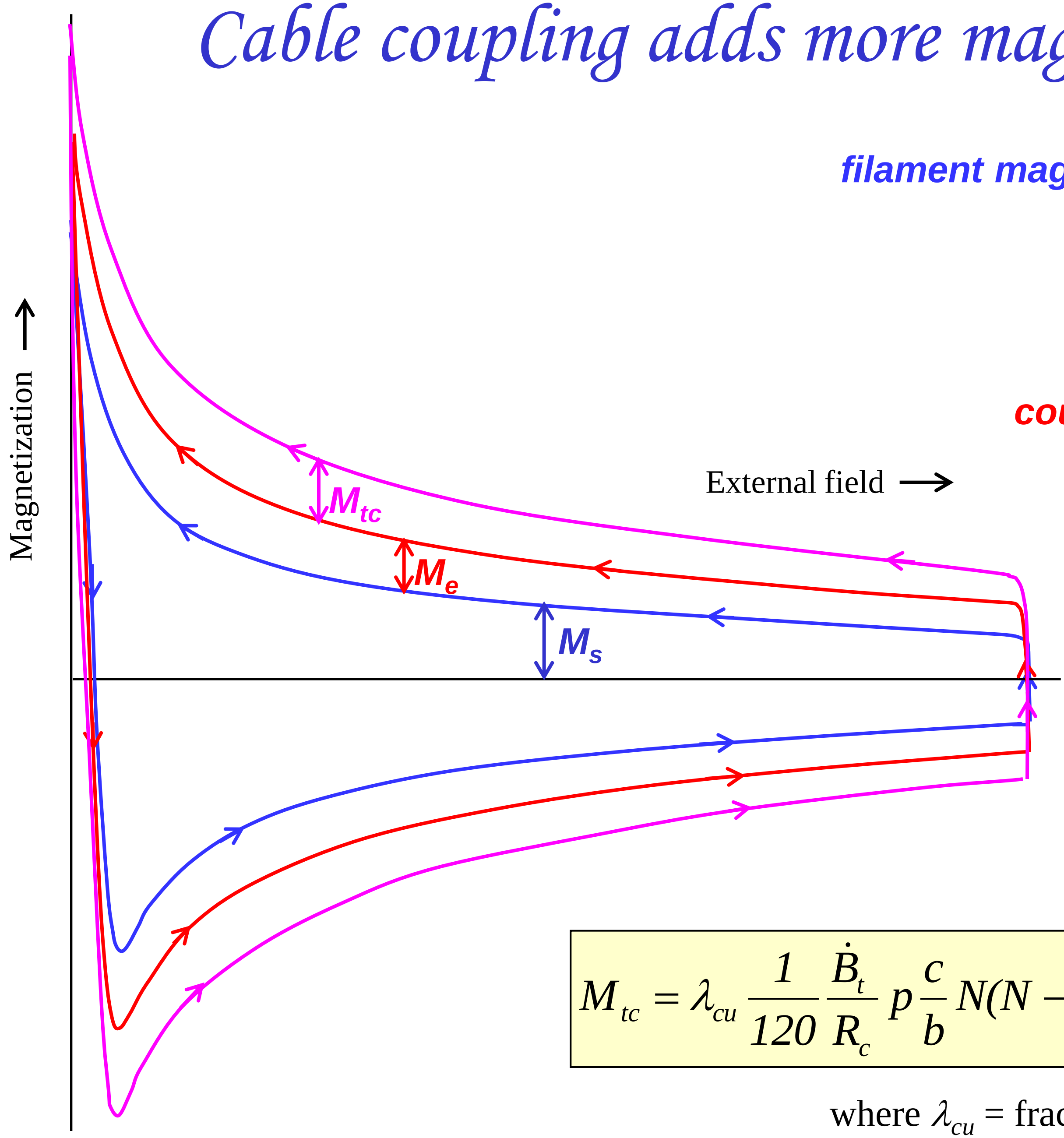
(usually negligible)

- Field transverse ratio crossover/adjacent

$$\frac{M_{tc}}{M_{ta}} = \frac{R_a}{R_c}\frac{N(N-1)}{20} \approx 45\,\frac{R_a}{R_c}$$

So without increasing loss too much can make $\boldsymbol{R_a}$ 50 times less than $\boldsymbol{R_c}$ - anisotropy

Cable coupling adds more magnetization



filament magnetization M_f depends on B

$$M_s = \lambda_{su} \frac{2}{3\pi} J_c(B) d_f$$

coupling between filaments M_e depends on dB/dt

$$M_e = \lambda_{wu} \frac{dB}{dt} \frac{1}{\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

coupling between wires in cable depends on dB/dt

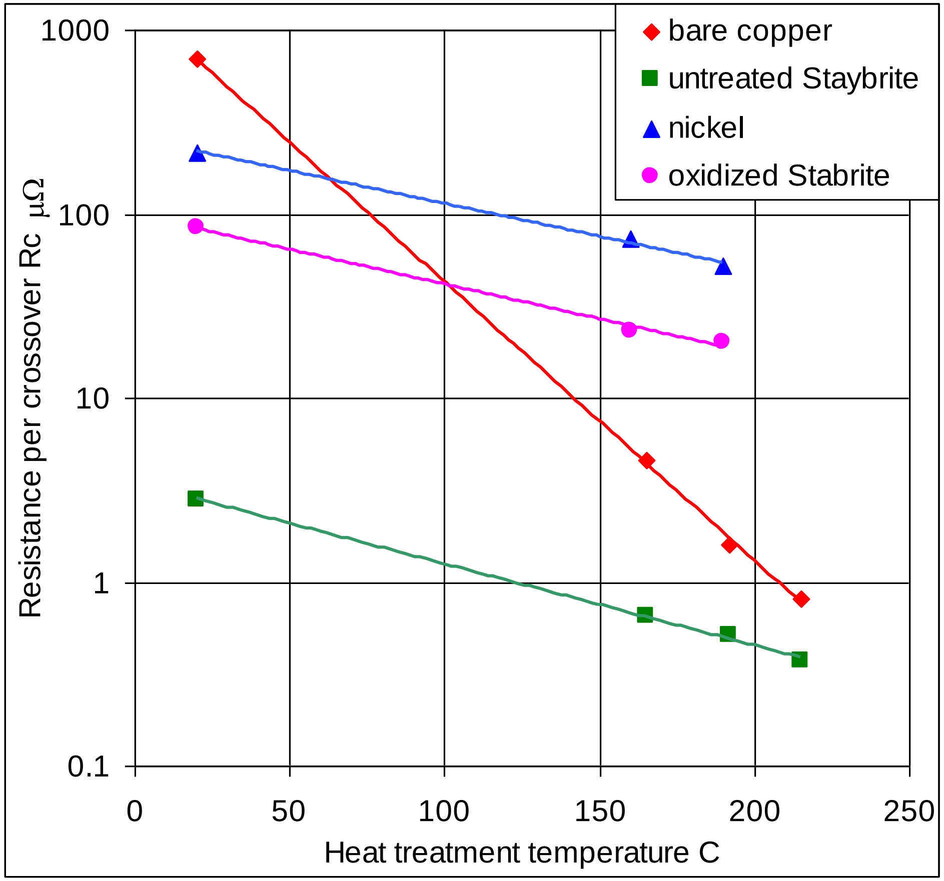
$$\boxed{M_{tc} = \lambda_{cu} \frac{1}{120} \frac{\dot{B}_t}{R_c} p \frac{c}{b} N(N-1)}$$

$$\boxed{M_{ta} = \lambda_{cu} \frac{1}{6} \frac{\dot{B}_t}{R_a} p \frac{c}{b}}$$

where λ_{cu} = fraction of cable in the section

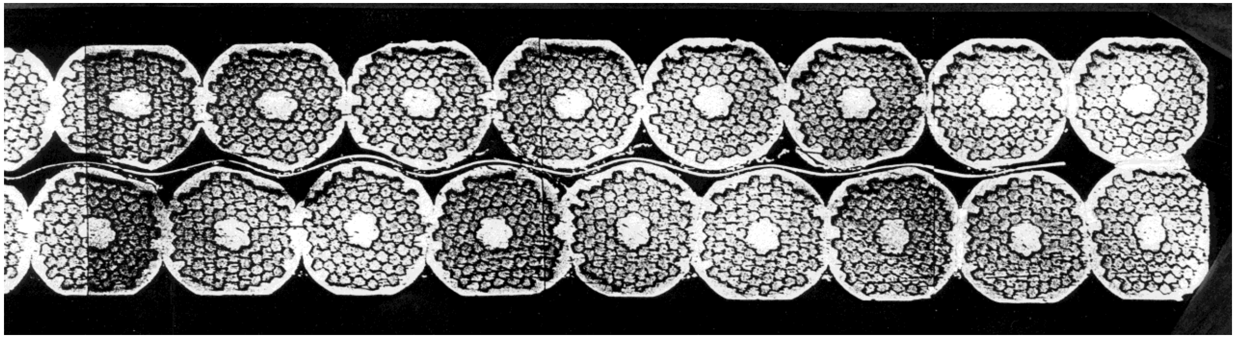
Controlling R_a and R_c

- surface coatings on the wires are used to adjust the contact resistance
- the values obtained are very sensitive to pressure and heat treatments used in coil manufacture (to cure the adhesive between turns)
- *data from David Richter CERN*



Cored Cables

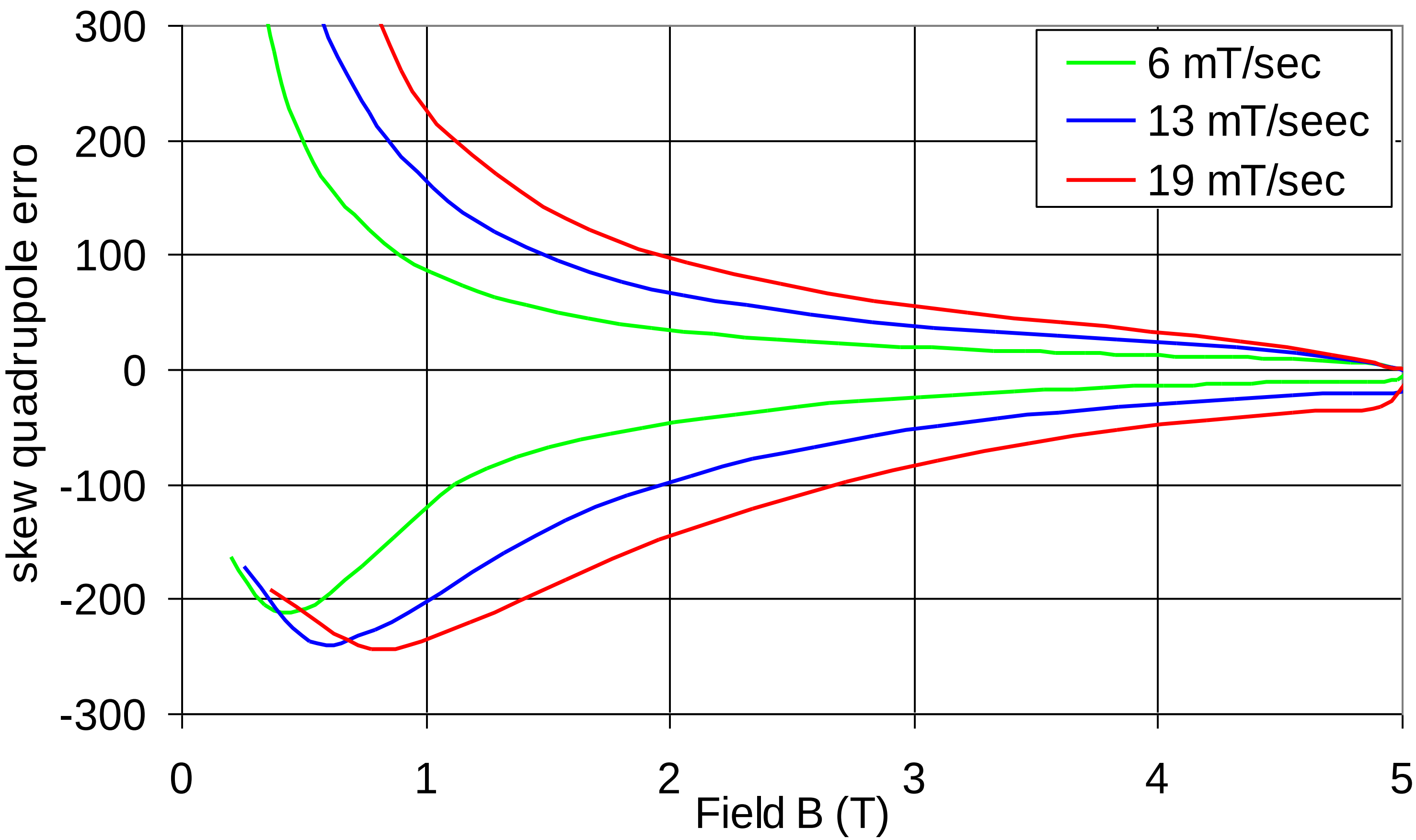
- using a resistive core allows us to increase R_c while keeping R_a the same
- thus we reduce losses but still maintain good current transfer between wires
- not affected by heat treatment



Magnetization and field errors – an extreme case

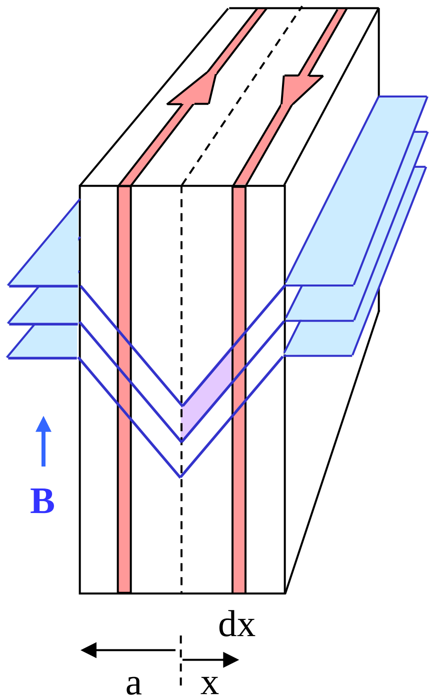
Magnetization is important in accelerators because it produces field error. The effect is worst at injection because

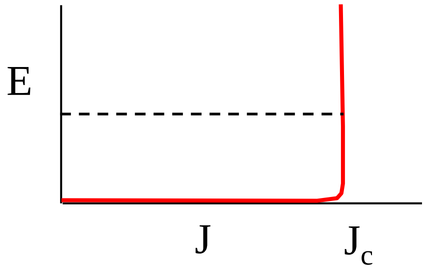
- $\Delta B/B$ is greatest
- magnetization, ie ΔB is greatest at low field



skew quadrupole error in Nb_3Sn dipole which has exceptionally large coupling magnetization (University of Twente)

AC loss power





Faraday's law of induction

$$\oint E dl = \frac{d}{dt}\int_A B dA$$

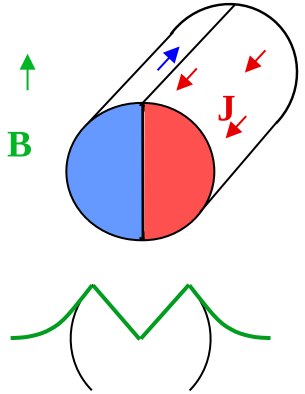
loss power / unit length in slice of width *dx*

$$p(x) = E\,J_c\,dx = \frac{dB}{dt}J_c\,dx$$

$$\boxed{P = \dot{B}M}$$

total loss in slab per unit volume

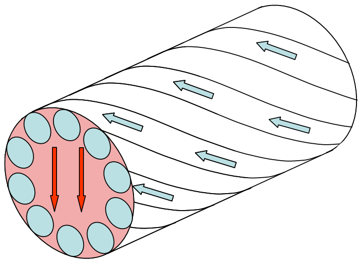
$$P = \frac{1}{a}\int_0^a p(x)dx = \frac{1}{a}\frac{dB}{dt}J_c\int_0^a x\,dx = \dot{B}J_c\frac{a}{2} = \dot{B}M$$



for round wires (not proved here)

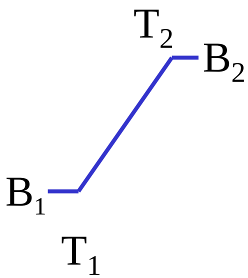
$$P = \dot{B}M = \frac{4}{3\pi}\dot{B}J_c\,a = \frac{2}{3\pi}\dot{B}\,J_c\,d_f$$

also for coupling magnetization



$$P_e = \dot{B}M_e = \dot{B}^2\frac{1}{\rho_t}\left[\frac{p_w}{2\pi}\right]^2 = \frac{\dot{B}^2}{\mu_o}2\tau$$

Hysteresis loss



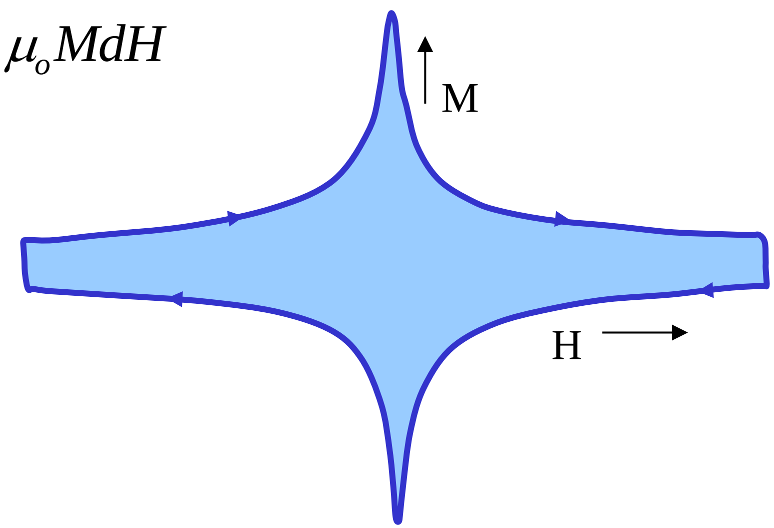
loss over a field ramp $$Q = \int_{T_1}^{T_2} M \frac{dB}{dt} dt = \int_{B_1}^{B_2} M dB$$

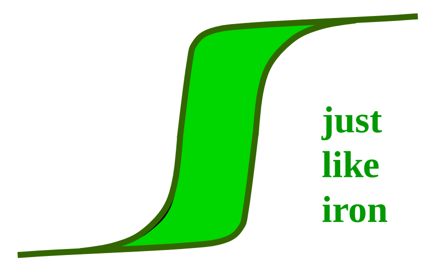
loss per ramp independent of $\dot{B}$

- in general, when the field changes by δB the magnetic field energy changes by $\delta E = H \delta B$ *(see textbooks on electromagnetism)*
- so work done by the field on the material $W = \int \mu_o H dM$
- around a ***closed loop***, this integral must be the energy dissipated in the material

$$Q = \int \mu_o H dM = \int \mu_o M dH$$

hysteresis loss per cycle (not per ramp)





just like iron

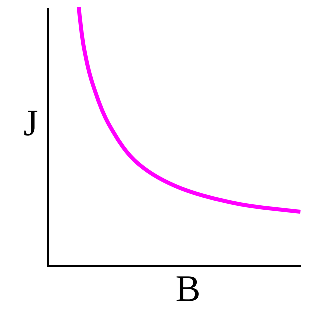
Integrated loss over a ramp

1) Screening currents within filaments

$$Q = \int_{B_1}^{B_2} M(B)dB$$

J_c and M vary with field

Kim Anderson approximation $J_c(B) = \frac{J_o B_o}{(B + B_o)}$



good at low field, less so at high field

round wire $M_s(B) = \frac{2}{3\pi} J_c(B) d_f$

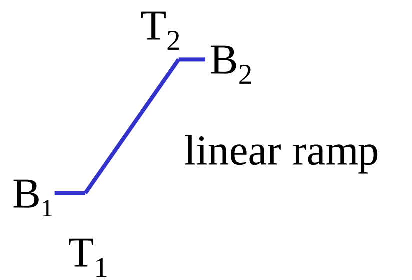
$$Q = \frac{2}{3\pi} \int_{B_1}^{B_2} \frac{J_o B_o}{(B + B_o)} d_f \, dB = \frac{2}{3\pi} d_f \, J_o B_o ln\left\{\frac{B_2 + B_0}{B_1 + B_0}\right\}$$

loss per ramp, independent of $\dot{B}$

2) Coupling currents between filaments

$$M_e = \dot{B} \frac{1}{\rho_t} \left[\frac{p_w}{2\pi}\right]^2$$

$$Q = \int_{B_1}^{B_2} M(\dot{B})dB = \frac{(B_2 - B_1)^2}{(T_2 - T_2)} \frac{1}{\rho_t} \left[\frac{p_w}{2\pi}\right]^2$$



3) Coupling currents between wires in cable

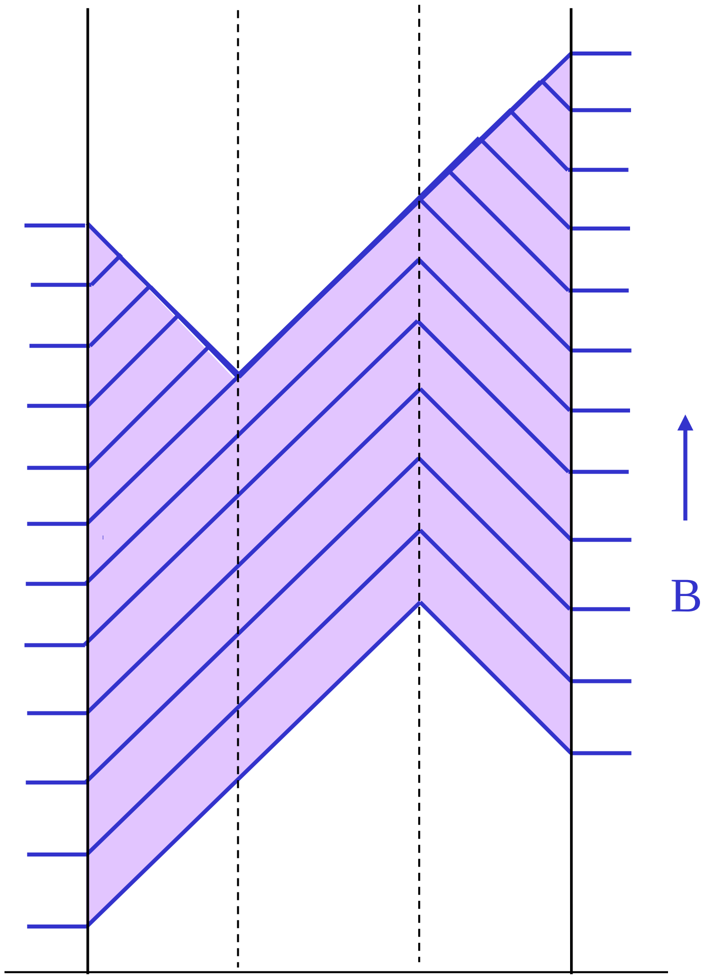
loss per ramp ~ *1/ΔT*

$$M_{tc} = \frac{1}{60} \frac{\dot{B}_t}{\rho_c} p_c^2 \frac{c^2}{b^2}$$

$$Q = \int_{B_1}^{B_2} M_{tc}(\dot{B})dB = \frac{(B_2 - B_1)^2}{(T_2 - T_2)} \frac{1}{\rho_c} \frac{p_c^2}{60} \frac{c^2}{b^2}$$

all per unit volume
- volume of what?

The effect of transport current



plot field profile across the slab

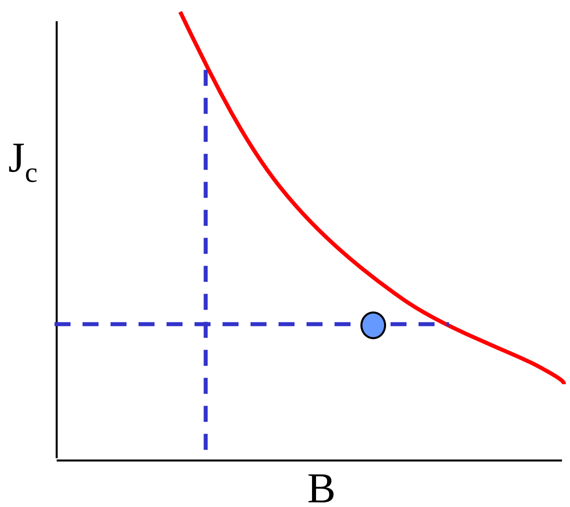
- in magnets there is a transport current, coming from the power supply, in addition to magnetization currents.
- because the transport current 'uses up' some of the available J_c the magnetization is reduced.
- but the loss is increased because the power supply does work and this adds to the work done by external field

total loss is increased by factor $(1+i^2)$ where $i = I_{max} / I_c$

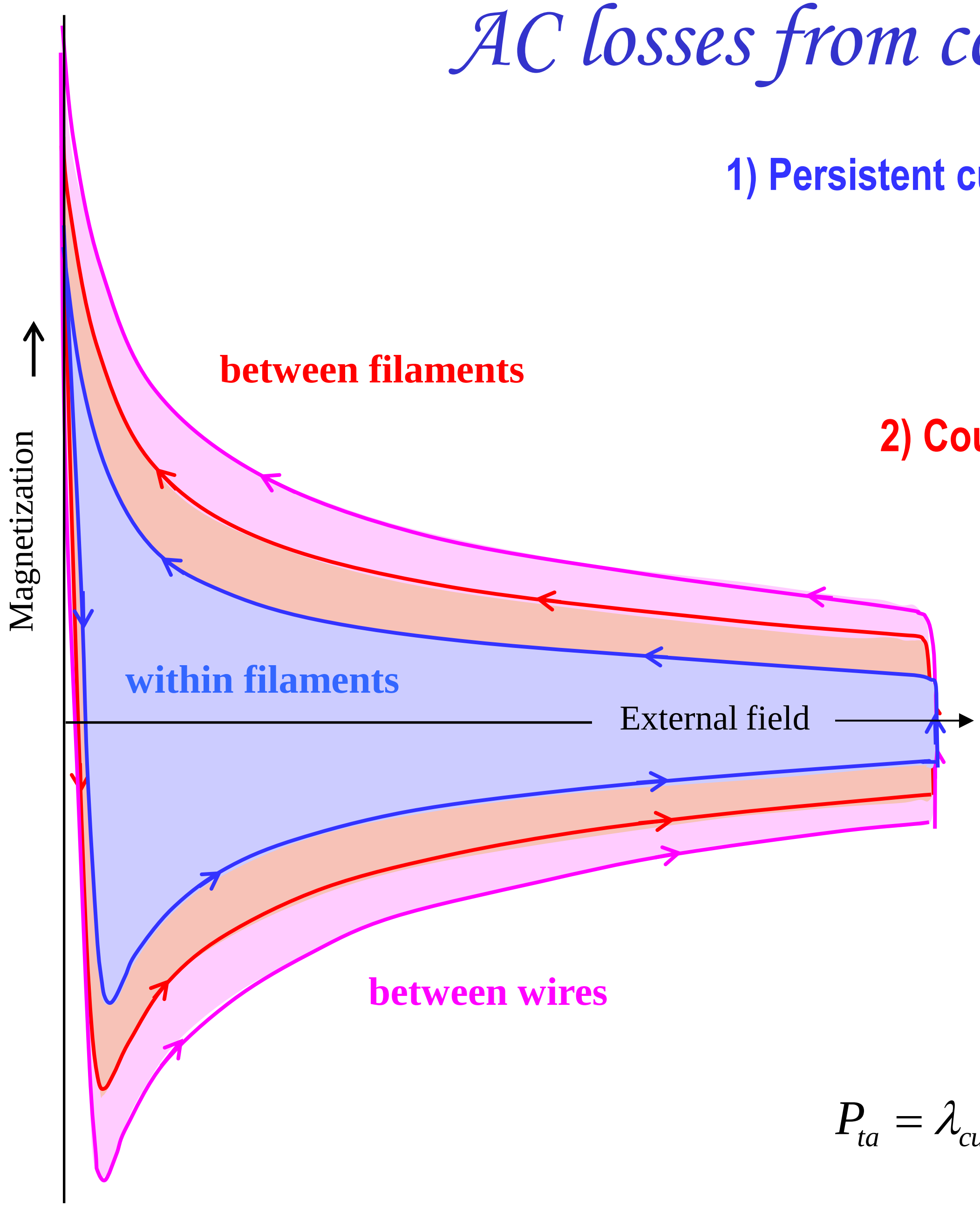
$$E = \frac{2}{3\pi} d_f \, J_o B_o \ln\left\{\frac{B_2 + B_o}{B_1 + B_o}\right\}(1+i^2)$$

usually not such a big factor because

- *design for a margin in Jc*
- *most of magnet is in a field much lower than the peak*



AC losses from coupling



1) Persistent currents within filaments

$$P_s = \frac{2}{3\pi} \lambda_{su} \dot{B}\, J_c\, d_f$$

2) Coupling between filaments within the wire

$$P_e = \lambda_{wu} \dot{B}^2 \frac{1}{\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

3) Coupling between wires in the cable

$$P_{tc} = \lambda_{cu} \frac{1}{120} \frac{\dot{B}_t^2}{R_c} \frac{c}{b} p_c N(N-1)$$

$$P_{ta} = \lambda_{cu} \frac{1}{6} \frac{\dot{B}_t^2}{R_a} p_c \frac{c}{b} \qquad P_{pa} = \lambda_{cu} \frac{1}{8} \frac{\dot{B}_p^2}{R_a} p_c \frac{b}{c}$$

Summary of losses - per unit volume of winding

1) Persistent currents in filaments

power W.m^{-3}

$$P_s = \lambda_{su} M_f \dot{B} = \lambda_{su} \frac{2}{3\pi} J_c(B)\, d_f \dot{B}$$

loss per per ramp J.m-3

$$Q_s = \lambda_{su} \frac{2}{3\pi} d_f J_o B_o \ln\left\{\frac{B_2 + B_o}{B_1 + B_o}\right\}$$

where λ_{su} , λ_{wu} , λ_{cu} = fractions of superconductor, wire and cable in the winding cross section

2) Coupling currents between filaments in the wire

power W.m^{-3}

$$P_e = \lambda_{wu} M_e \dot{B} = \lambda_{wu} \frac{\dot{B}^2}{\rho_t}\left(\frac{p}{2\pi}\right)^2$$

3) Coupling currents between wires in the cable

transverse field crossover resistance power W.m^{-3}

$$P_{tc} = \lambda_{cu} \frac{1}{120} \frac{\dot{B}_t^2}{R_c} p \frac{c}{b} N(N-1)$$

transverse field adjacent resistance power W.m^{-3}

$$P_{ta} = \lambda_{cu} \frac{1}{6} \frac{\dot{B}_t^2}{R_a} p \frac{c}{b}$$

parallel field adjacent resistance power W.m^{-3}

$$P_{pa} = \lambda_{cu} \frac{1}{8} \frac{\dot{B}_p^2}{R_a} p \frac{b}{c}$$

don't forget the filling factors

Concluding remarks

- changing magnetic fields drive superconductor into resistive state ⇒ losses – leave persistent currents
- screening currents produce magnetization (magnetic moment per unit volume)
  ⇒ lots of problems - field errors and ac losses
- in a synchrotron, the field errors from magnetization are worst at injection
- we reduce magnetization by making fine filaments - for practical use embed them in a matrix
- in changing fields, filaments are coupled through the matrix ⇒ increased magnetization
  - reduce it by twisting and by increasing the transverse resistivity of the matrix
- accelerator magnets must run at high current because they are all connected in series
  - combine wires in a cable, it must be fully transposed to ensure equal currents in each wire
- wires in cable must have some resistive contact to allow current sharing
  - in changing fields the wires are coupled via the contact resistance
    - different coupling when the field is parallel and perpendicular to face of cable
      - coupling produces more magnetization ⇒ more field errors
- irreversible magnetization ⇒ ac losses in changing fields
  - coupling between filaments in the wire adds to the loss
    - coupling between wire in the cable adds more

never forget that magnetization and ac loss are defined per unit volume - ***filling factors***