# Transverse Beam Dynamics 

JUAS tutorial 4
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## 1 Exercise: chromaticity in a FODO cell

Consider a ring made of $N_{\text {cell }}$ identical FODO cells with equally spaced quadrupoles. Assume that the two quadrupoles are both of length $l_{q}$, but their strengths may differ.

### 1.1 Calculate the maximum and the minimum betatron function in the FODO cell as a function of the length $l_{q}$, the focal lengths $f_{F}, f_{D}$ and the phase advance $\mu$. (Use the thin lens approximations)

### 1.2 Calculate the natural chromaticities for this machine.

1.3 Show that for short quadrupoles, if $f_{F} \simeq f_{D}$,

$$
\xi_{N} \simeq-\frac{N_{\text {cell }}}{\pi} \tan \frac{\mu}{2}
$$

## 2 Exercise: measurement of Twiss parameters

One of the possible ways to determine experimentally the Twiss parameters at a given point makes use of a so-called quadrupole scan. One can measure the transverse size of the beam in a profile monitor, called Wire Beam Scanner (WBS), located at a distance $L$ downstream a focusing quadrupole, as a function of the normalized gradient in this quadrupole. This allows to compute the emittance of the beam, as well as the $\beta$ and the $\alpha$ functions at the entrance of the quadrupole.

Let's consider a quadrupole $Q$ with a length of $l=20 \mathrm{~cm}$. This quadrupole is installed in an electron transport line where the particle momentum is $300 \mathrm{MeV} / c$. At a distance $L=10 \mathrm{~m}$ from the quadrupole the transverse beam size is measured with a WBS, for various values of the current $I_{Q}$. The maximum value of the quadrupole gradient $G$ is obtained for a current of 100 A, and is $G=1 \mathrm{~T} / \mathrm{m} . G$ is proportional to the current.

Advice: use thin-lens approximation.

### 2.1 How does the normalized focusing strength $K$ vary with $I_{Q}$ ?

2.2 Let $\Sigma_{1}$ and $\Sigma_{2}$ be the $2 \times 2$ matrices with the twiss parameters, $\Sigma=\left(\begin{array}{cc}\beta & -\alpha \\ -\alpha & \gamma\end{array}\right)$, at the quadrupole entrance and at the wire scanner, respectively.

- Give the expression $\Sigma_{2}$ as function of $\alpha_{1}, \beta_{1}$, and $\gamma_{1}$
- Show that $\beta_{2}$ can be written in the form: $\beta_{2}=A_{2}(K l)^{2}+A_{1}(K l)+A_{0}$
- Express $A_{0}, A_{1}$, and $A_{2}$ as a function of $L, \alpha_{1}, \beta_{1}$, and $\gamma_{1}$

Hint for the next questions: show that if you express $\beta_{2}$ as

$$
\beta_{2}=B_{0}+B_{1}\left(K l-B_{2}\right)^{2}
$$

you have:

$$
\begin{aligned}
& B_{0}=A_{0}-A_{1}^{2} / 4 A_{2}^{2}=L^{2} / \beta_{1} \\
& B_{1}=A_{2}=L^{2} \beta_{1} \\
& B_{2}=-A_{1} / A_{2}=1 / L-\alpha_{1} / \beta_{1}
\end{aligned}
$$

2.3 The transverse beam r.m.s. beam size is $\sigma=\sqrt{\epsilon \beta}$, where $\epsilon$ is the transverse emittance. Express $\sigma_{2}$ as a function of $K l$ and find its minimum, $(K l)_{\min }$. Give the expression for $\frac{\mathrm{d} \sigma_{2}}{\mathbf{d}(K l)}$.
2.4 How does $\sigma_{2}$ vary with $K l$ when $\left|K l-(K l)_{\text {min }}\right| \gg 1 / \beta_{1}$ ?
2.5 Deduce the values of $\alpha_{1}, \beta_{1}$, and $\gamma_{1}$ from the measurement $\sigma_{2}$, as a function of the quadrupole current $I_{Q}$.

