



# Joint Universities Accelerator School

## Mini-workshop on Superconducting Magnets

Paolo Ferracin

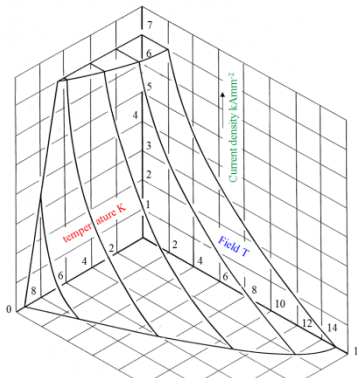
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European Organization for Nuclear Research (CERN)

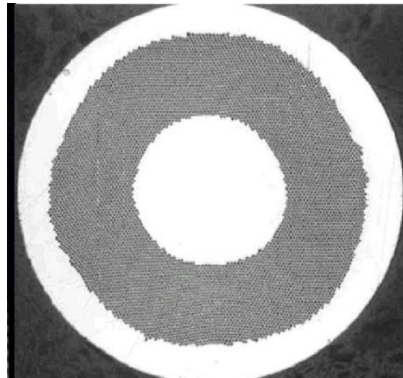
# Mini-workshop introduction

- Goal: outline design of a superconducting magnet
  - Apply the theory explained during lectures to a practical case
  - Solve a case study using analytical formulas, “back of the envelope” calculation, plots, data, etc. provided during the presentations
    - From the superconducting material to the full magnet
  - Understand physics and reasoning behind design options
    - General dimensions, orders of magnitude of different parameters
- Provide a short report of the results (*3 credits*)

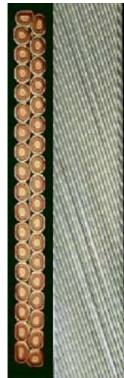
Superconducting material



Superconducting strand



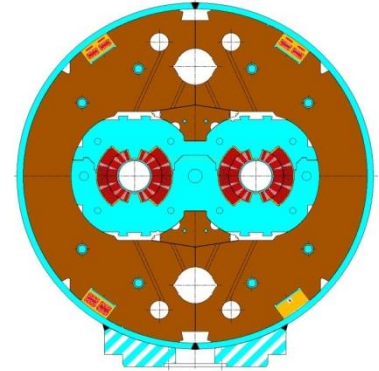
Superconducting cable



Superconducting coil

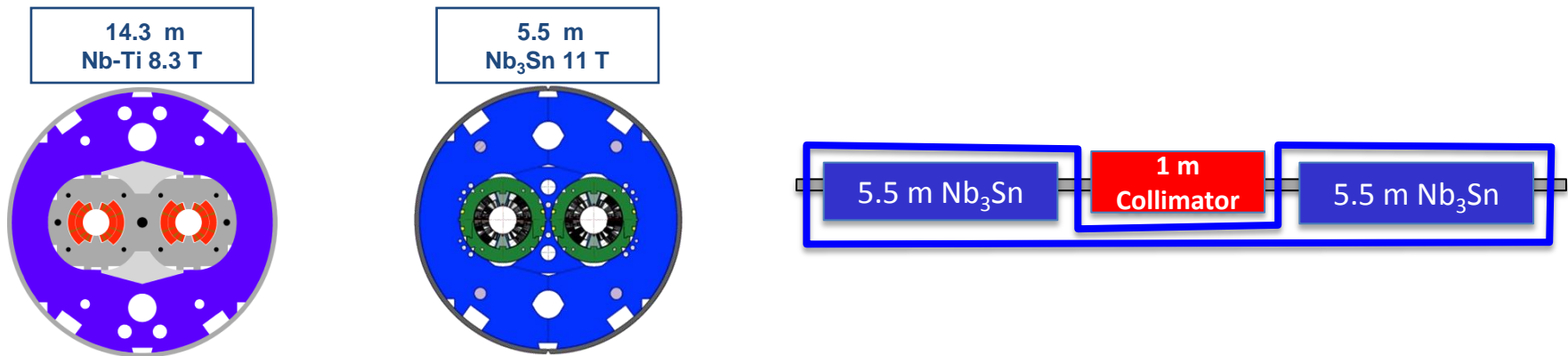


Superconducting magnet

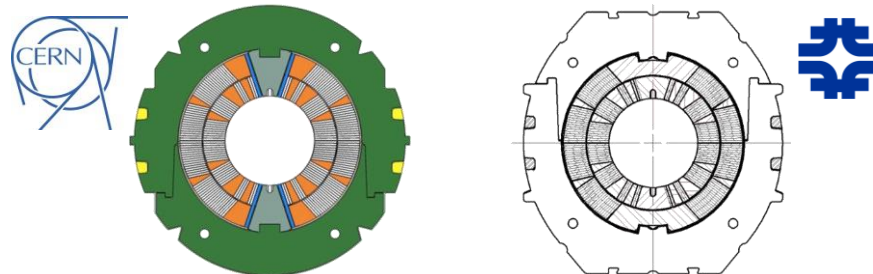


# Case study

## • 11 T Nb<sub>3</sub>Sn dipole for the LHC collimation upgrade



- Second long shutdown: increase of collimation efficiency
  - New collimation units
  - Some 8.3 T Nb-Ti dipoles replaced by **11 T Nb<sub>3</sub>Sn dipoles**
  - FNAL/CERN collaboration



## ● 11 T Nb<sub>3</sub>Sn dipole for the LHC collimation upgrade

### ● Introduction

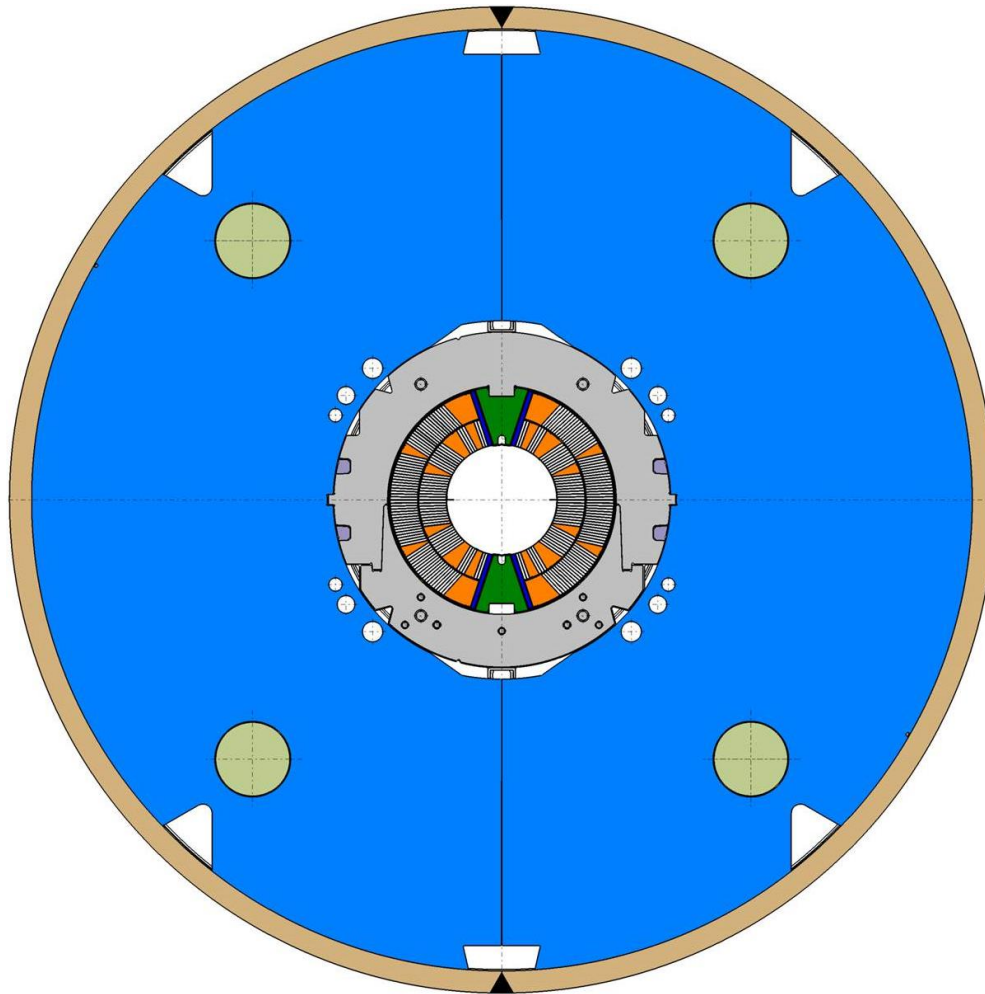
- The second phase of the LHC collimation upgrade will enable proton and ion beam operation at nominal and ultimate intensities.
- To improve the collimation efficiency by a factor 15–90, additional collimators are foreseen in the room temperature insertions and in the dispersion suppression (DS) regions around points 2, 3, and 7.
- To provide longitudinal space of about 3.5 m for additional collimators, a solution based on the substitution of a pair of 5.5-m-long 11 T dipoles for several 14.3-m-long 8.33 T LHC main dipoles (MB) is being considered.

### ● Goal

- Design a Nb<sub>3</sub>Sn superconducting **dipole** with an **60 mm aperture** and a operational field (80% of the current limit  $I_{ss}$ ) at **1.9 K** of **11 T**.

# Case study

- 11 T Nb<sub>3</sub>Sn dipole for the LHC collimation upgrade



## ● 11 T Nb<sub>3</sub>Sn dipole for the LHC collimation upgrade

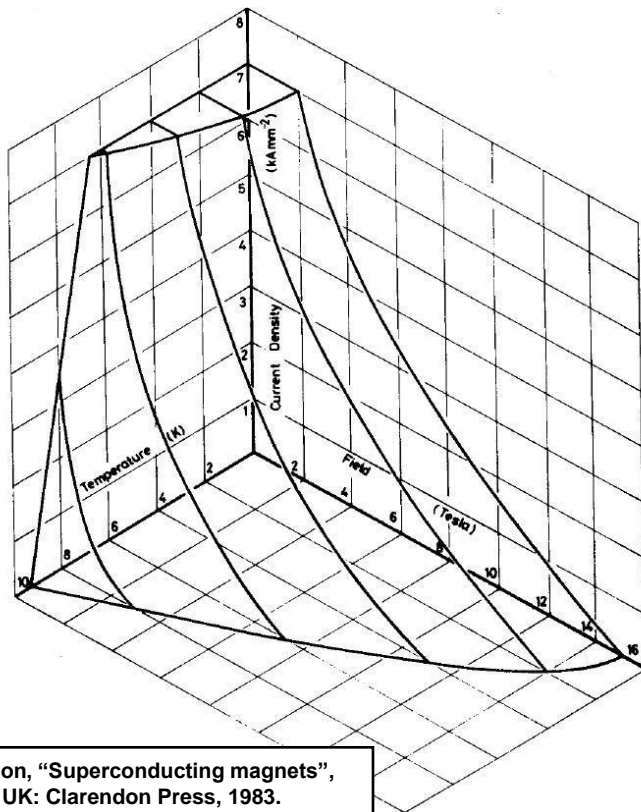
### ◆ Questions

1. Determine and plot critical curves ( $J_{sc}$  vs.  $B$ ) for Nb<sub>3</sub>Sn and Nb-Ti at 1.9 K
2. Determine coil filling factor  $\lambda$  ( $J_0 / J_{sc}$  ratio or  $A_{\text{non-Cu\_cable}} / A_{\text{insulated\_cable}}$ )
3. Compute load-line ( $J_{sc}$  vs.  $B$ ) for a
  1. Thick shell with  $\cos\theta$  current density distribution
  2. Sector coil (60°) with constant current density
4. Determine coil size, operational (80% of  $I_{ss}$ ), conditions, “short-sample” conditions, and margins for both approximations
  1.  $w$
  2.  $j_{sc\_ss}, j_{o\_ss}, B_{bore\_ss}, B_{peak\_ss}$
  3.  $j_{sc\_op}, j_{o\_op}, B_{bore\_op}, B_{peak\_op}$
  4.  $T, j_{sc}, B_{peak}$  margins
5. Compare “short sample”, “operational” conditions and margins if the same design uses Nb-Ti superconducting technology with the same coil size  $w$
6. Determine e.m forces  $F_x$  and  $F_y$  and the accumulated stress on the coil mid-plane in the operational conditions with both the thick shell and sector coil approximation
7. Evaluate dimension of collars, iron yoke, and shrinking cylinder, assuming that the support structure is designed to reach 90% of  $I_{ss}$

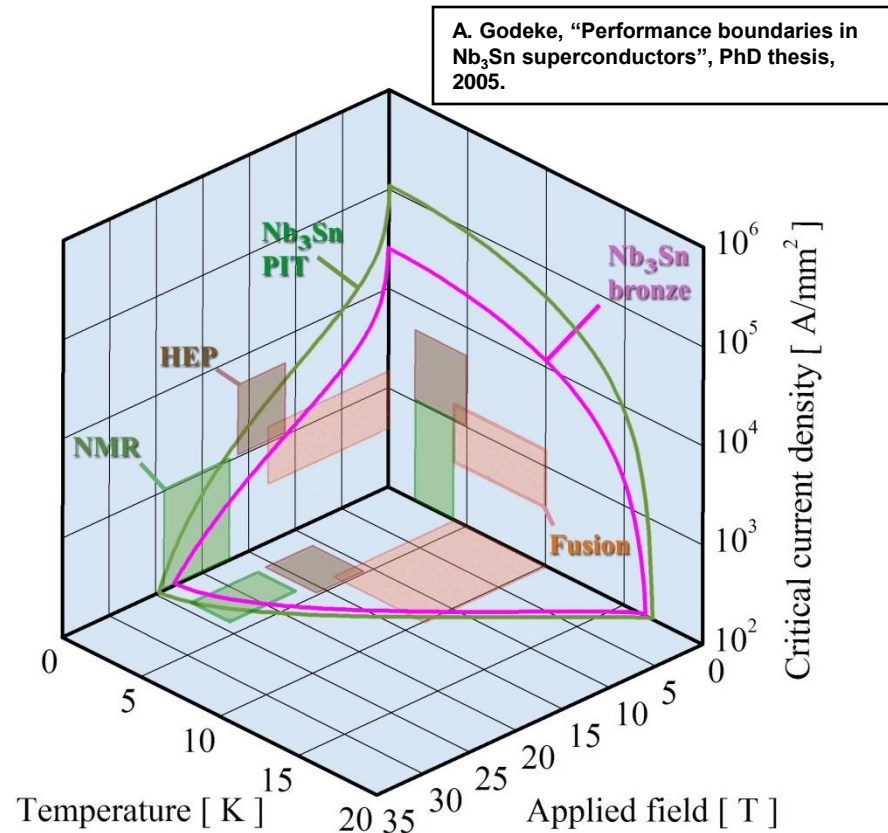
- 11 T Nb<sub>3</sub>Sn dipole for the LHC collimation upgrade
  - Question
    - Determine and plot critical curves ( $J_{sc}$  vs.  $B$ ) for Nb<sub>3</sub>Sn and Nb-Ti at 1.9 K

# Nb-Ti and Nb<sub>3</sub>Sn Critical surfaces

- The critical surface defines the boundaries between superconducting state and normal conducting state in the space defined by temperature, magnetic field, and current densities.
- The surface, determined experimentally, can be fitted with parameterization curves.



M. Wilson, "Superconducting magnets",  
Oxford UK: Clarendon Press, 1983.



A. Godeke, "Performance boundaries in  
Nb<sub>3</sub>Sn superconductors", PhD thesis,  
2005.

# Measurements of the conductor critical current

- The critical current of a conductor is measured by winding a sample of the wire around a sample holder.
- To avoid premature quenching induced by Lorentz forces during ramping, the wire must be well supported
  - Stycast glue may be used to constrain the wire around the holder
- In case of  $\text{Nb}_3\text{Sn}$  wires, a sample holder made of titanium is used.
- Once the wire is cooled-down and placed in a given magnetic field, the current is increased until the transition occurs.



# Nb-Ti parameterization curve (LHC dipole)

- Nb-Ti parameterization

- Temperature and field dependence of  $B_{C2}$  and  $T_C$  are provided by Lubell's formulae:

$$B_{C2}(T) = B_{C20} \left[ 1 - \left( \frac{T}{T_{C0}} \right)^{1.7} \right] \quad T_C(B)^{1/1.7} = T_{C0} \left[ 1 - \left( \frac{B}{B_{C20}} \right)^{1/1.7} \right]$$

where  $B_{C20}$  is the upper critical flux density at zero temperature (14.5 T), and  $T_{C0}$  is critical temperature at zero field (9.2 K)

- Temperature and field dependence of  $J_c$  is given by Bottura's formula

$$\frac{J_C(B, T)}{J_{C, ref}} = \frac{C_{NbTi}}{B} \left[ \frac{B}{B_{C2}(T)} \right]^{\beta_{NbTi}} \left[ 1 - \frac{B}{B_{C2}(T)} \right]^{\beta_{NbTi}} \left[ 1 - \left( \frac{T}{T_{C0}} \right)^{1.7} \right]^{\gamma_{NbTi}}$$

where  $J_{C, Ref}$  is critical current density at 4.2 K and 5 T (3000 A/mm<sup>2</sup>) and  $C_{Nb-Ti}$  (27 T),  $\alpha_{Nb-Ti}$  (0.63),  $\beta_{Nb-Ti}$  (1.0), and  $\gamma_{Nb-Ti}$  (2.3) are fitting parameters.

# Nb<sub>3</sub>Sn parameterization curve (typical values for HEP magnets)

- Nb<sub>3</sub>Sn parameterization

- Temperature, field, and strain dependence of  $J_c$  is given by Summers' formula

$$J_c(B, T, \varepsilon) = \frac{C_{Nb_3Sn}(\varepsilon)}{\sqrt{B}} \left[ 1 - \frac{B}{B_{C2}(T, \varepsilon)} \right]^2 \left[ 1 - \left( \frac{T}{T_{C0}(\varepsilon)} \right)^2 \right]^2$$

$$\frac{B_{C2}(T, \varepsilon)}{B_{C20}} = \left[ 1 - \left( \frac{T}{T_{C0}(\varepsilon)} \right)^2 \right] \left\{ 1 - 0.31 \left( \frac{T}{T_{C0}(\varepsilon)} \right)^2 \left[ 1 - 1.77 \ln \left( \frac{T}{T_{C0}(\varepsilon)} \right) \right] \right\}$$

$$C_{Nb_3Sn}(\varepsilon) = C_{Nb_3Sn,0} \left( 1 - \alpha_{Nb_3Sn} |\varepsilon|^{1.7} \right)^{1/2}$$

$$B_{C20}(\varepsilon) = B_{C20m} \left( 1 - \alpha_{Nb_3Sn} |\varepsilon|^{1.7} \right)$$

$$T_{C0}(\varepsilon) = T_{C0m} \left( 1 - \alpha_{Nb_3Sn} |\varepsilon|^{1.7} \right)^{1/3}$$

where  $\alpha_{Nb_3Sn}$  is 900 for  $\varepsilon = -0.003$ ,  $T_{C0m}$  is 18 K,  $B_{C20m}$  is 27.6 T, and  $C_{Nb_3Sn,0}$  is a fitting parameter equal to 43100 A T<sup>1/2</sup> mm<sup>-2</sup> for a  $J_c = 2900$  A/mm<sup>2</sup> at 4.2 K and 12 T.

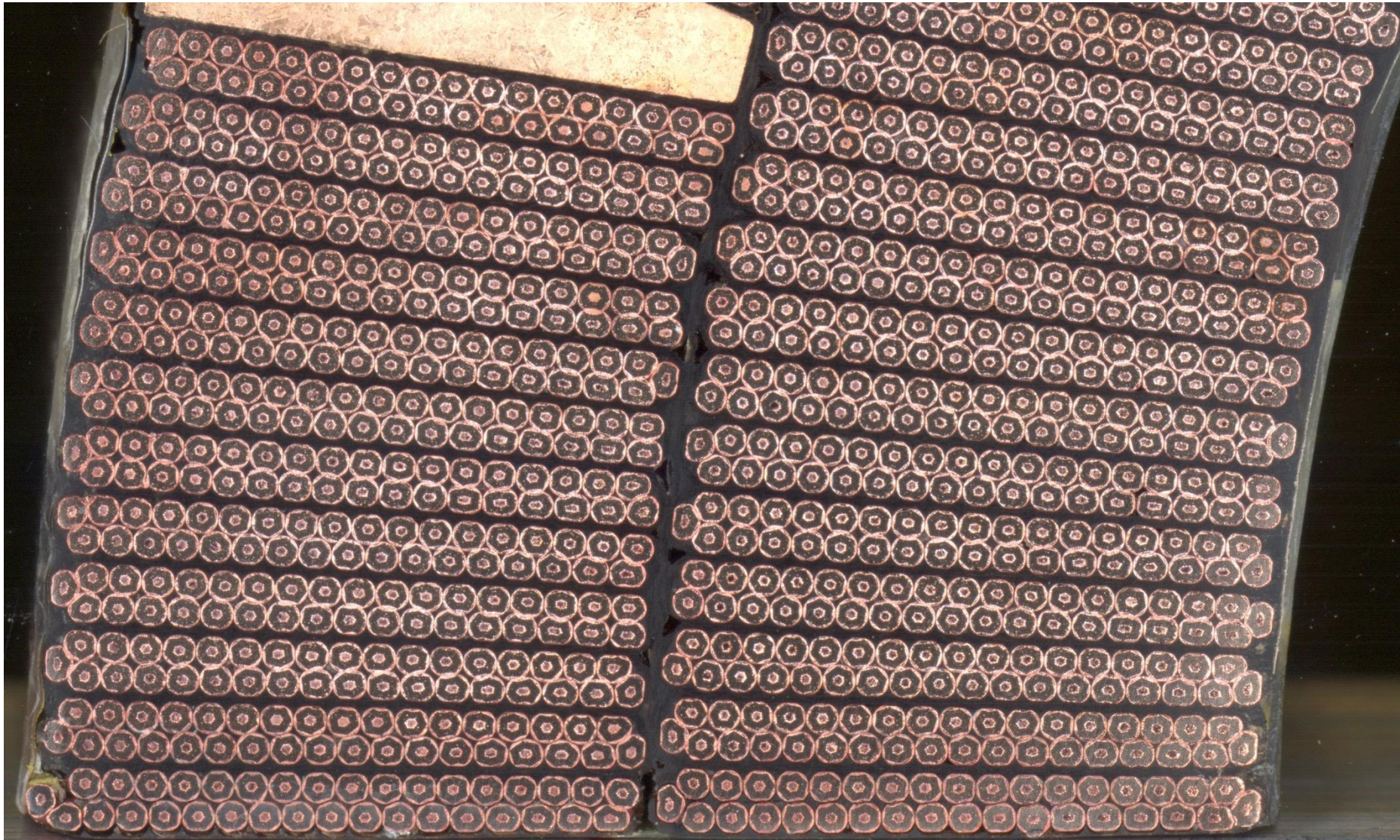
**Assume  $\varepsilon = 0.000$**

## ● References

- M.S. Lubell, "Empirical scaling formulas for critical current and critical fields for commercial NbTi," *IEEE Trans. Magn.*, Vol. MAG-19 No. 3, pp. 754–757, 1983.
- L. Bottura, "A practical fit for the critical surface of NbTi," *IEEE Trans. Appl. Supercond.*, Vol. 10, No. 1, pp. 1054–1057, 2000.
- L.T. Summers, M.W. Guinan, J.R. Miller and P.A. Hahn, "A model for the prediction of Nb<sub>3</sub>Sn critical current as a function of field, temperature, strain and radiation damage," *IEEE Trans. Magn.*, Vol. 27, No. 2, pp. 2041–2044, 1991.

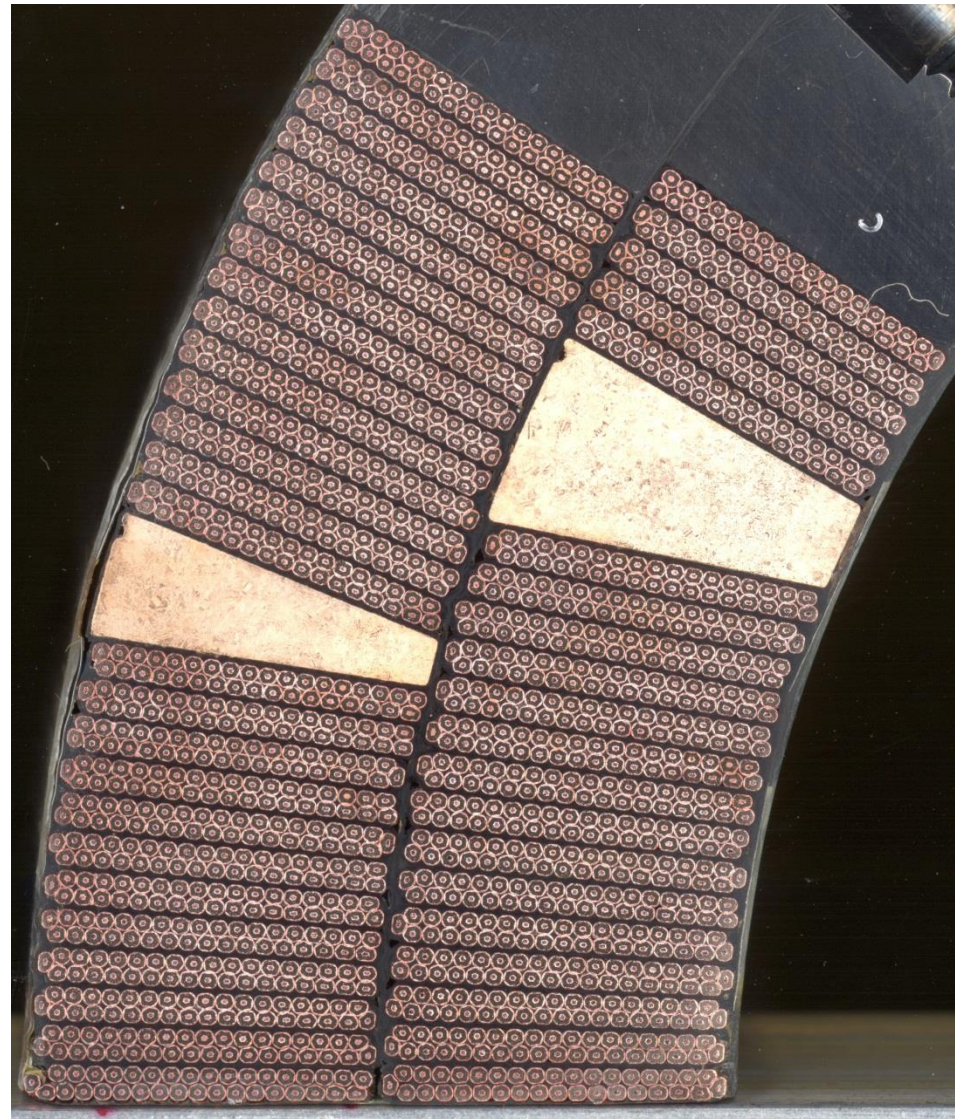
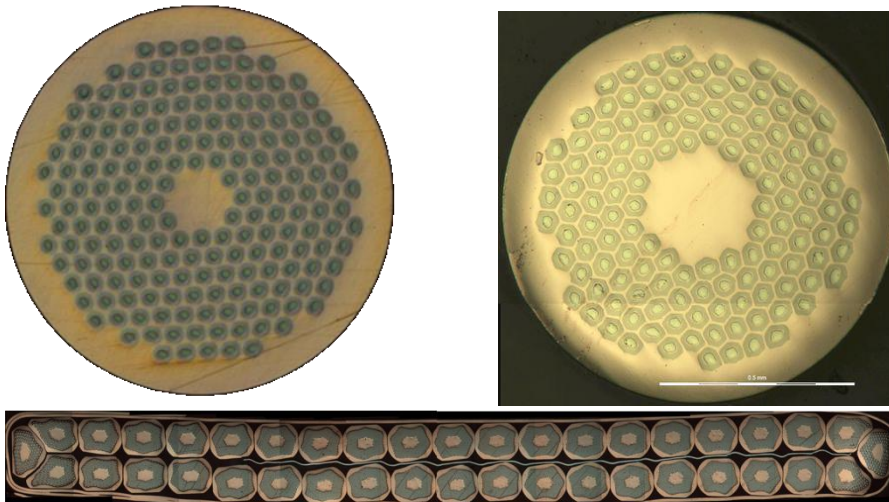
- 11 T Nb<sub>3</sub>Sn dipole for the LHC collimation upgrade
  - Question
    - Determine coil filling factor  $\lambda$  ( $J_0 / J_{sc}$  ratio or  $A_{\text{non-Cu\_cable}} / A_{\text{insulated\_cable}}$ )

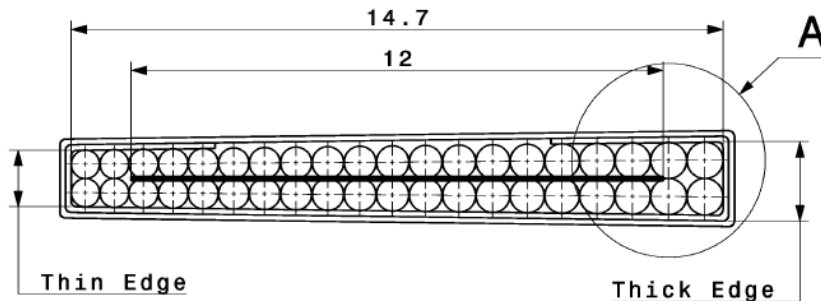
# Superconducting cables and coils



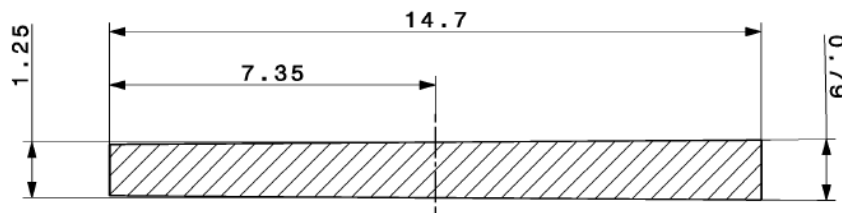
- The cable design parameters are:

- Number of wires  $N_{wire}$
- Wire diameter  $d_{wire}$
- Cable mid-thickness  $t_{cable}$
- Cable width  $w_{cable}$
- (Cu/non-Cu) ratio
- Insulation thickness
- Pitch angle
  - To be neglected in this comp.



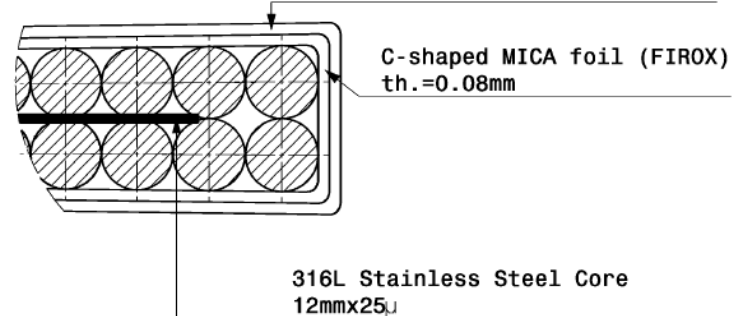


Enlarged and not to scale,  
for illustration purposes only



DIMENSION FOR  
CONDUCTOR WITHOUT INSULATION  
Scale:10:1

Detail A  
Scale: 20:1



UNREACTED CABLE DIMENSIONS	
Strand Type	Nb3Sn
Strand Diameter	0.7 mm
Number of strands	40 (2 x 20)
Width	14.7 mm
Mid-thickness	1.25 mm
Keystone Angle	0.79°
(Thin Edge Height)	(1.149 mm)
(Thick Edge Height)	(1.351 mm)
Inner Core	12 mm x 25 $\mu$
INSULATION THICKNESS	
Mica Layer	0.08 mm
Fibre braiding	0.075 mm
Total insulation thickness	0.155 mm (under compression 30MPa: 0.1 mm)

- *Cu to non-Cu ratio: 1.2*

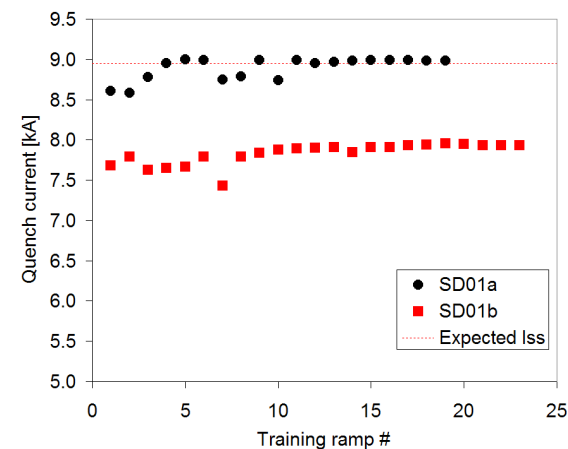
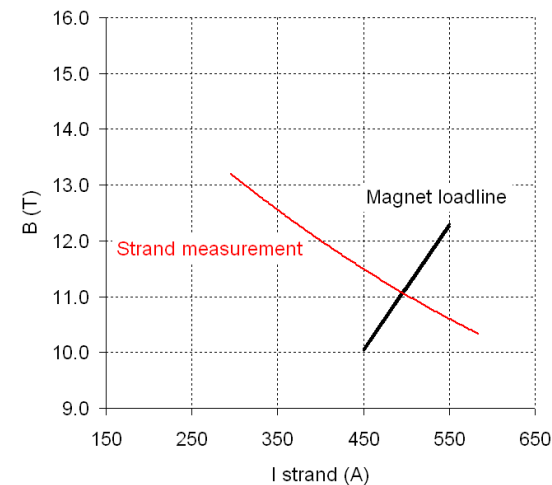
## ● 11 T Nb<sub>3</sub>Sn dipole for the LHC collimation upgrade

### ● Question

- Compute load-line ( $J_{sc}$  vs.  $B$ ) for a
  - Thick shell with  $\cos\theta$  current density distribution
  - Sector coil (60°) with constant current density
- Determine coil size, operational (80% of  $I_{ss}$ ), conditions, “short-sample” conditions, and margins for both approximations
  - $w$
  - $j_{sc_{ss}}, j_{o_{ss}}, B_{bore_{ss}}, B_{peak_{ss}}$
  - $j_{sc_{op}}, j_{o_{op}}, B_{bore_{op}}, B_{peak_{op}}$
  - $T, j_{sc}, B_{peak}$  margins
- Compare “short sample”, “operational” conditions and margins if the same design uses Nb-Ti superconducting technology with the same coil size  $w$

# Short sample and operational current

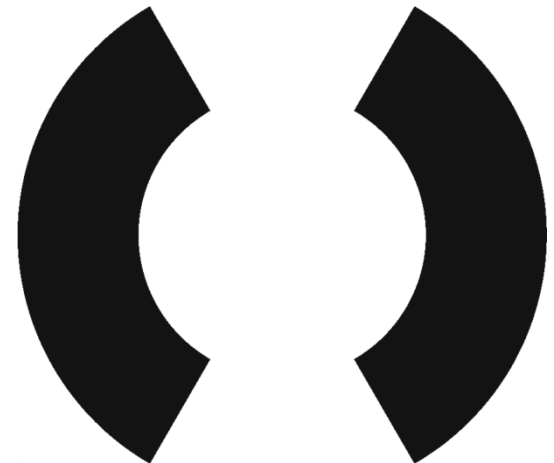
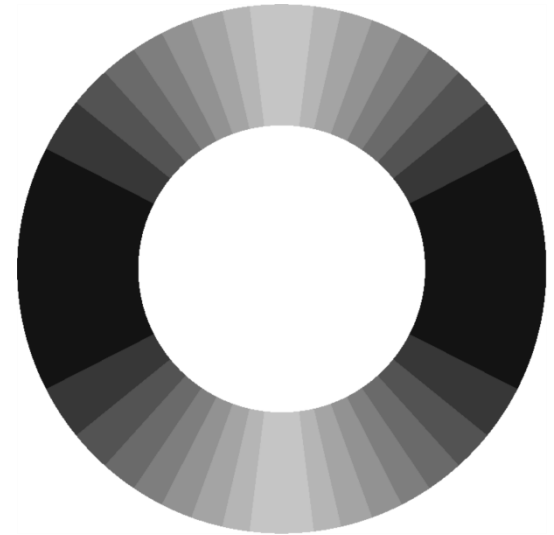
- Short sample current
  - The critical current is measured in few different conditions of temperature and field. By fitting the data with known parameterizations, the entire critical surface can be reconstructed.
- If the magnet reaches the maximum current computed through the intersection of the measured critical surface and the load line, i.e.  $I_{max} = I_{ss}$ , one can declare victory (at least from the quench performance point of view).
- If the magnet maximum current  $I_{max}$  is lower than  $I_{ss}$ , the quench performance is expressed in term of fraction of short sample ( $I_{max}/I_{ss}$ ).
- Usually magnets are designed to operate at  $I_{op} = 0.8 I_{ss}$  or below.



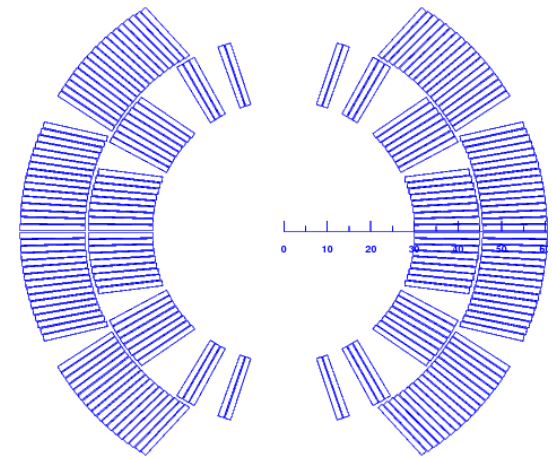
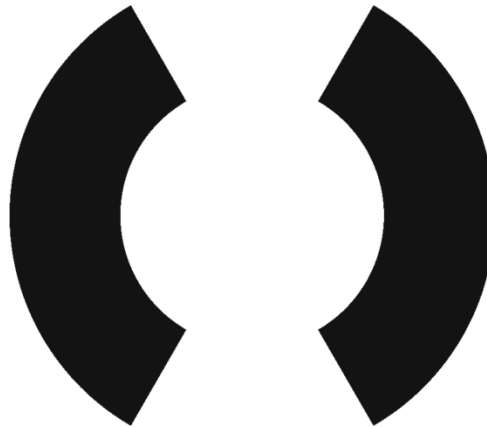
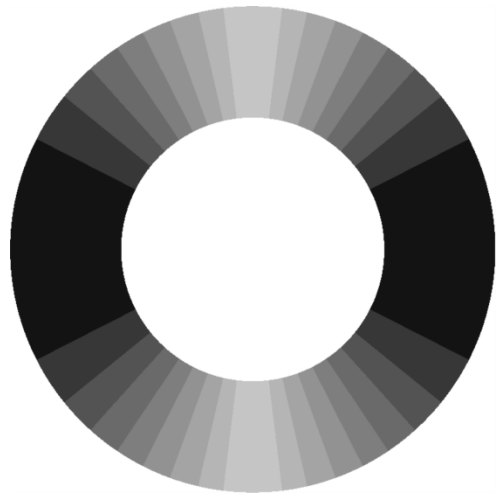
# Computation of the load line

## Approximations of practical winding cross-sections

- Thick shell
  - Current density  $J = J_0 \cos \theta$  (A per unit area) on a shell with a finite thickness
  
- Sector coil
  - Current density  $J = J_0$  (A per unit area) on a sector with a maximum angle  $\theta = 60^\circ$  for a dipole



# Approximations of practical winding cross-sections



# Computation of the load line

## Approximations of practical winding cross-sections

- Thick shell

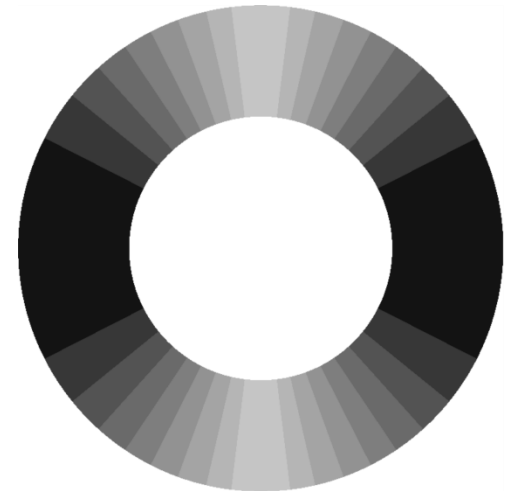
- Current density  $J = J_0 \cos \theta$  (A per unit area) on a shell with a finite thickness

$$B_{bore} = -\frac{j_0 \mu_0}{2} w$$

- Where,  $B_{bore}$  is the bore field,  $j_0$  is overall current density and  $w$  is the coil width
  - Ideal case
    - Conductor peak field  $B_{peak} = B_{bore}$**
    - Perfect field quality
      - No field errors
        - $b_3 = b_5 = b_7 = \dots = 0$

- Comparison:

- For solenoid
    - $B_1 = -j_0 \mu_0 w$
    - Twice more efficient than a dipole



# Computation of the load line

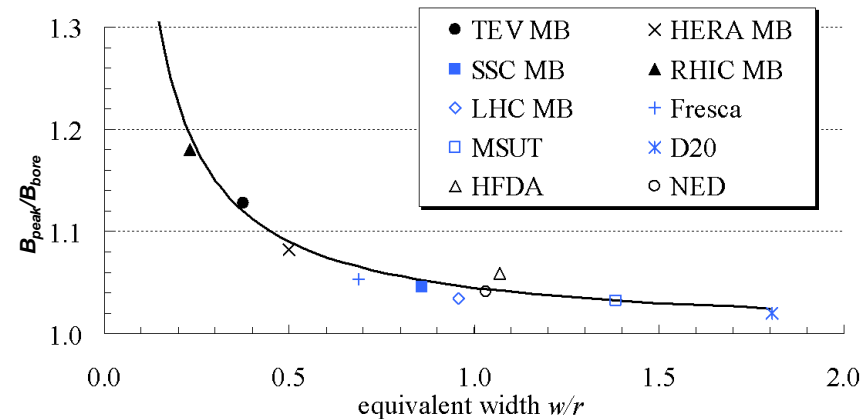
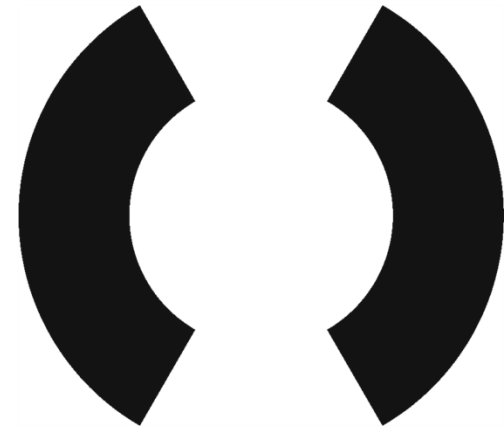
## Approximations of practical winding cross-sections

### • Sector coil

- Current density  $J = J_0$  (A per unit area) on a sector with a maximum angle  $\theta = 60^\circ$  for a dipole

$$B_{bore} = -\frac{2j_0\mu_0}{\pi} w \sin(60)$$

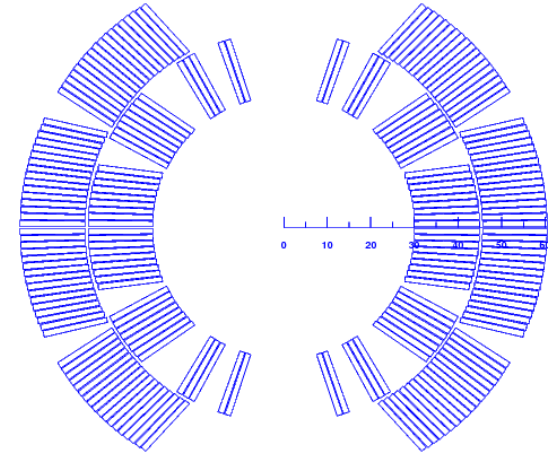
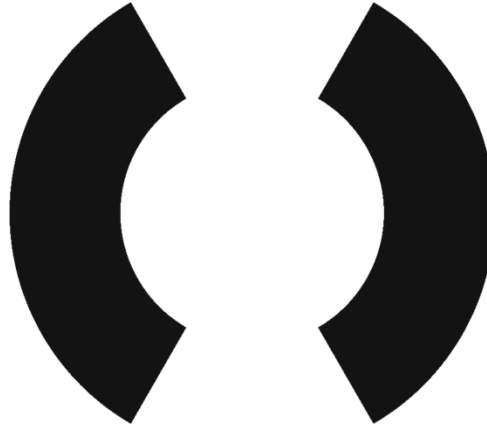
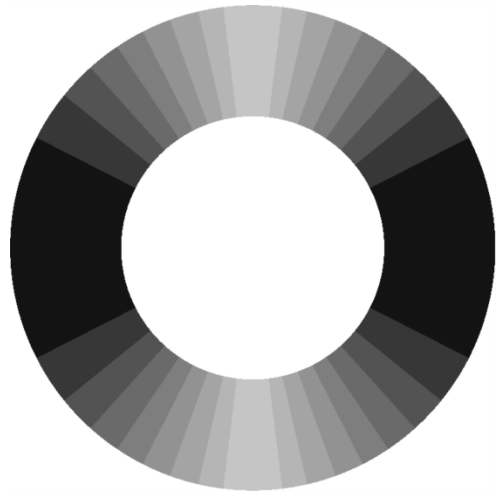
- Where,  $B_{bore}$  is the bore field,  $j_0$  is overall current density and  $w$  is the coil width
- “Less ideal” case
  - $B_{peak} = B_{bore} \cdot \sim 1.04$
  - “Not so perfect” field quality
    - $b_3 = 0$



L. Rossi, E. Todesco, “Electromagnetic design of superconducting quadrupoles”, Phys. Rev. ST Accel. Beams 9 (2006) 102401.

- With a  $w/r$  of  $30/30 = 1 \rightarrow 1.04$

# Comparison



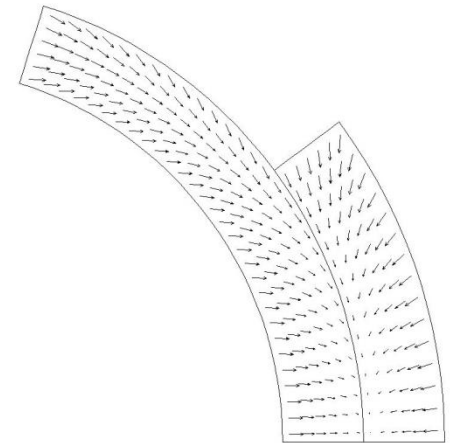
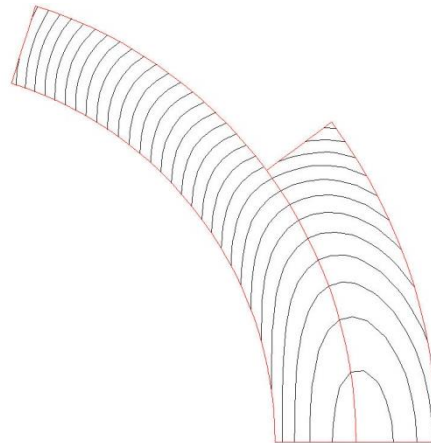
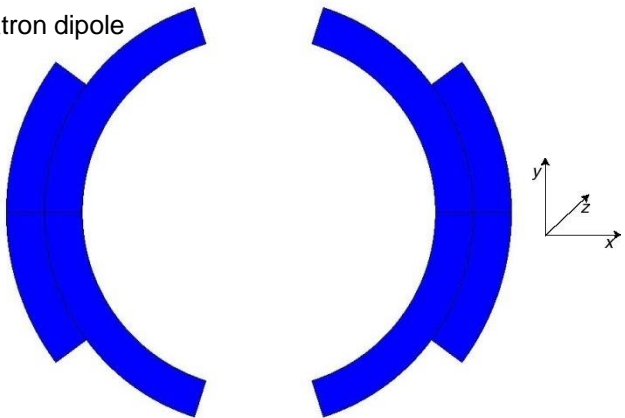
Roxie		
	mu	1.2566E-06
Degree	alpha	
A/m2	J0	796112011
lambda		0.324
A/m2	Jsc	2455676180
A/mm2	Jsc	2456
m	a1	0.03
m	a2	0.0598
m	w	0.0298
T	B1	13.726939
	Bpeak/B1	1.04019549
T	Bpeak	14.2787

- 11 T Nb<sub>3</sub>Sn dipole for the LHC collimation upgrade
  - Question
    - Determine e.m forces  $F_x$  and  $F_y$  and the accumulated stress on the coil mid-plane in the operational conditions with both the thick shell and sector coil approximation

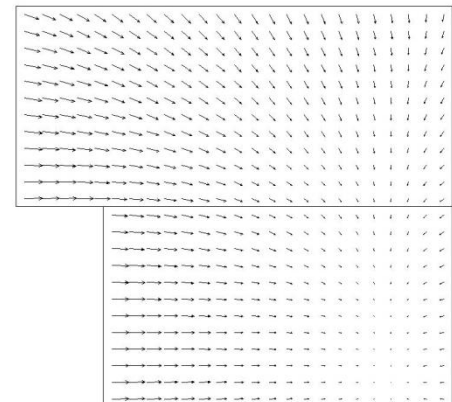
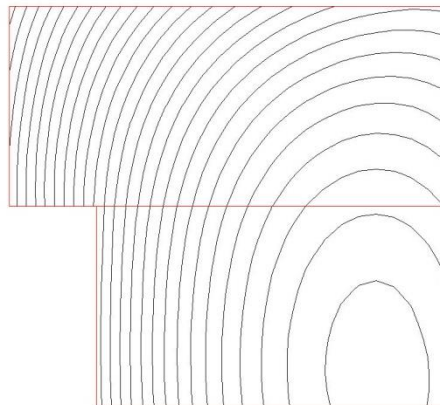
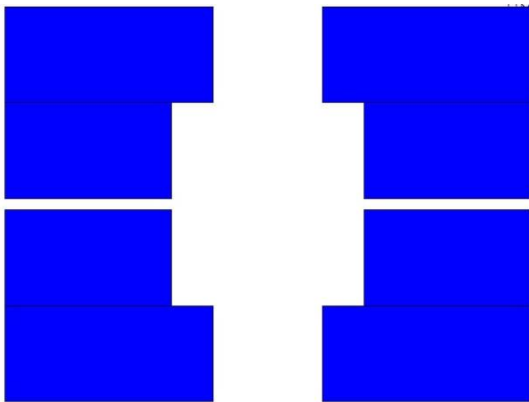
# E.m. forces and stresses

- The e.m. forces in a dipole magnet tend to push the coil
  - Towards the mid plane in the vertical-azimuthal direction ( $F_y, F_\theta < 0$ )
  - Outwards in the radial-horizontal direction ( $F_x, F_r > 0$ )

Tevatron dipole



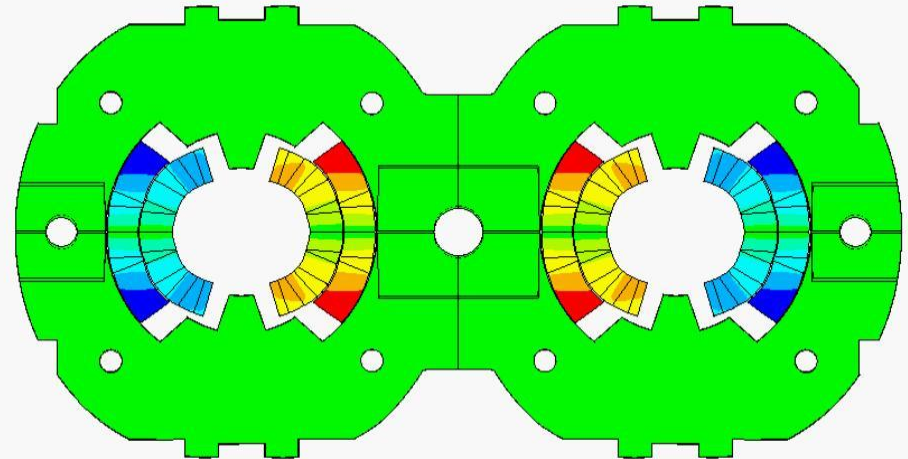
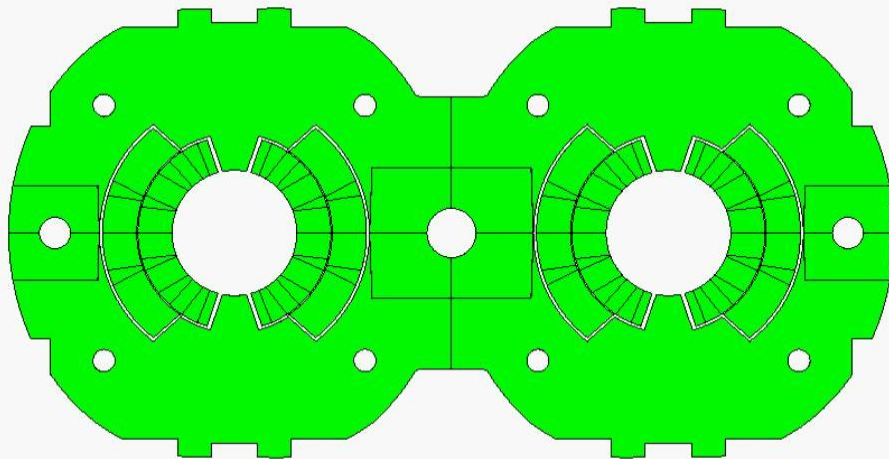
HD2



# E.m. forces and stresses

LHC dipole at 0 T

LHC dipole at 9 T



Displacement scaling = 50

- Usually, in a dipole or quadrupole magnet, the highest stresses are reached at the mid-plane, where all the azimuthal e.m. forces accumulate (over a small area).

# E.m. forces and stresses

## Thick shell approximation

- For a thick shell, with an inner radius  $a_1$ , an outer radius  $a_2$  and an overall current density  $j = j_0 \cos \theta$ , each block (quadrant) see

- Horizontal force outwards

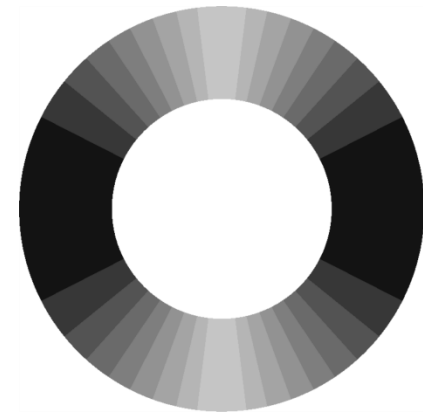
$$F_x = \frac{\mu_0 J_0^2}{2} \left[ \frac{7}{54} a_2^3 + \frac{1}{9} \left( \ln \frac{a_2}{a_1} + \frac{10}{3} \right) a_1^3 - \frac{1}{2} a_2 a_1^2 \right]$$

- Vertical force towards the mid-plane

$$F_y = -\frac{\mu_0 J_0^2}{2} \left[ \frac{2}{27} a_2^3 + \frac{2}{9} \left( \ln \frac{a_1}{a_2} - \frac{1}{3} \right) a_1^3 \right]$$

- In case of frictionless and “free-motion” conditions, no shear, and infinitely rigid radial support, the forces accumulated on the mid-plane produce a stress of

$$\sigma_{\theta\_mid-plane} = \int_0^{\pi/2} f_{\theta} r d\theta = -\frac{\mu_0 J_0^2}{2} \frac{r}{2} \left[ (a_2 - r) + \frac{r^3 - a_1^3}{3r^2} \right]$$



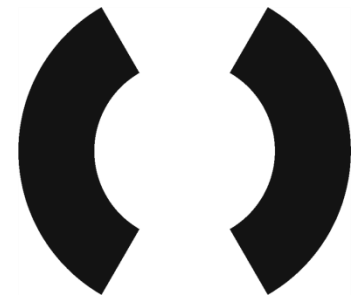
# E.m. forces and stresses

## Sector coil approximation

- For a dipole sector coil, with an inner radius  $a_1$ , an outer radius  $a_2$  and an overall current density  $j_o$ , each block (quadrant) see

- Horizontal force outwards

$$F_x = + \frac{2\mu_0 J_0^2}{\pi} \frac{\sqrt{3}}{2} \left[ \frac{2\pi - \sqrt{3}}{36} a_2^3 + \frac{\sqrt{3}}{12} \ln \frac{a_2}{a_1} a_1^3 + \frac{4\pi + \sqrt{3}}{36} a_1^3 - \frac{\pi}{6} a_2 a_1^2 \right]$$



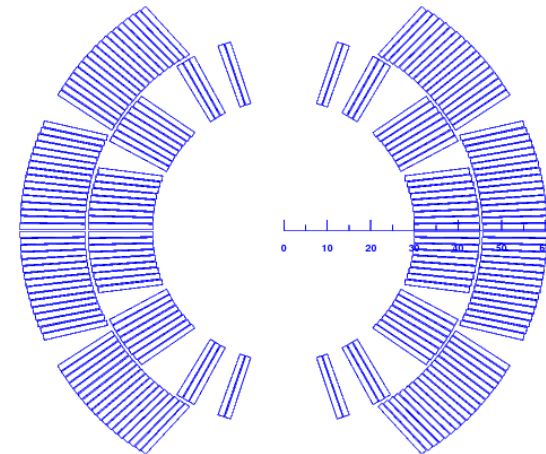
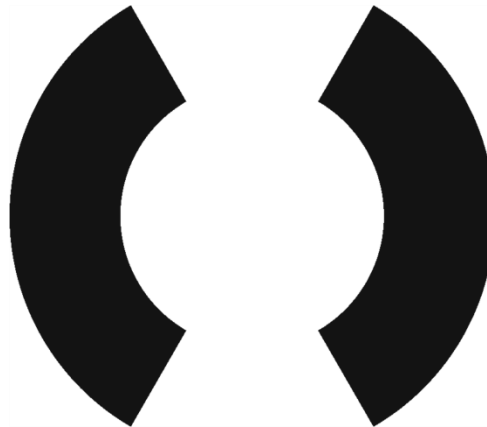
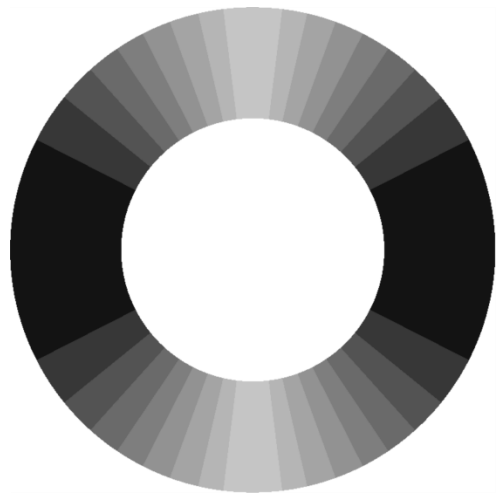
- Vertical force towards the mid-plane

$$F_y = - \frac{2\mu_0 J_0^2}{\pi} \frac{\sqrt{3}}{2} \left[ \frac{1}{12} a_2^3 + \frac{1}{4} \ln \frac{a_1}{a_2} a_1^3 - \frac{1}{12} a_1^3 \right]$$

- In case of frictionless and “free-motion” conditions, no shear, and infinitely rigid radial support, the forces accumulated on the mid-plane produce a stress of

$$\sigma_{\theta\_mid-plane} = \int_0^{\pi/2} f_{\theta} r d\theta = - \frac{\mu_0 J_0^2}{2} \frac{r}{2} \left[ (a_2 - r) + \frac{r^3 - a_1^3}{3r^2} \right]$$

# Comparison



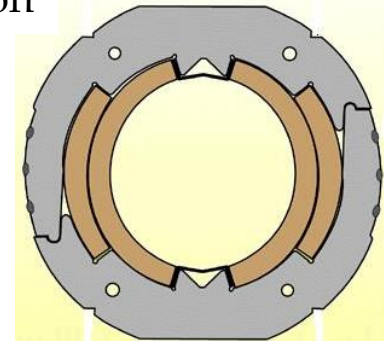
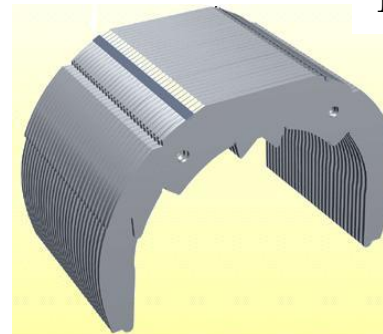
Roxie		
	mu	1.2566E-06
Degree	alpha	
A/m2	J0	796112011
lambda		0.324
A/m2	Jsc	2455676180
A/mm2	Jsc	2456
m	a1	0.03
m	a2	0.0598
m	w	0.0298
T	B1	13.726939
	Bpeak/B1	1.04019549
T	Bpeak	14.2787
N/m	Fx (quad)	4127000
N/m	Fy (quad)	-3294600
N/m	Fx tot	8254000

- 11 T Nb<sub>3</sub>Sn dipole for the LHC collimation upgrade
  - Question
    - Evaluate dimension of collars, iron yoke, and shrinking cylinder, assuming that the support structure is designed to reach 90% of  $I_{ss}$

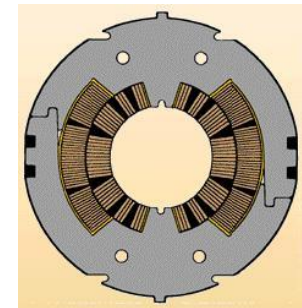
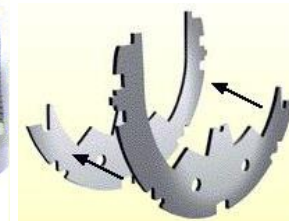
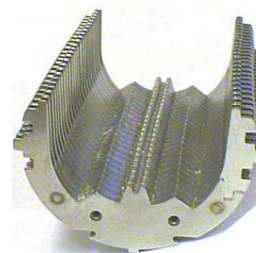
# Support structure Collars

- Collars were implemented for the first time in the Tevatron dipoles.
- Since then, they have been used in all but one (RHIC) the accelerator magnets and in most of the R&D magnets.
- They are composed by stainless-steel or aluminum laminations few mm thick.
- By clamping the coils, the collars provide
  - coil pre-stressing;
  - rigid support against e.m. forces (it can be self-supporting or not);
  - precise cavity (tolerance  $\pm 20 \mu\text{m}$ ).

Tevatron

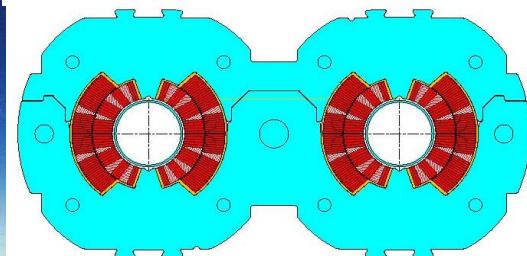
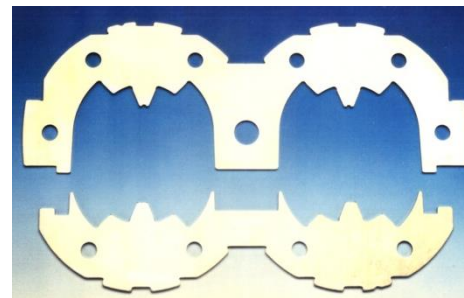


SSC



LHC

MJB Plus, Inc., [2]

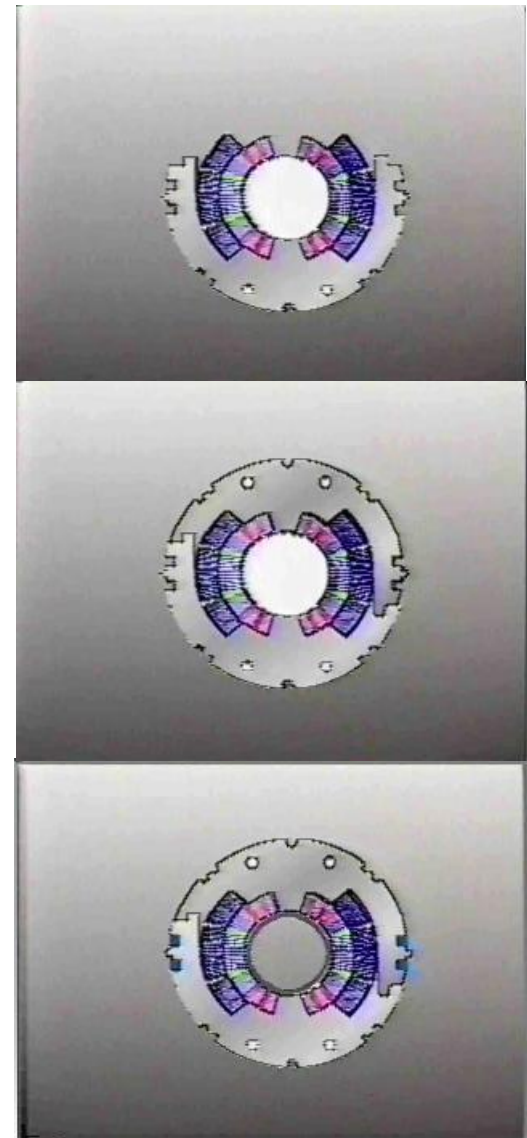


L. Rossi, [1]

# Support structure

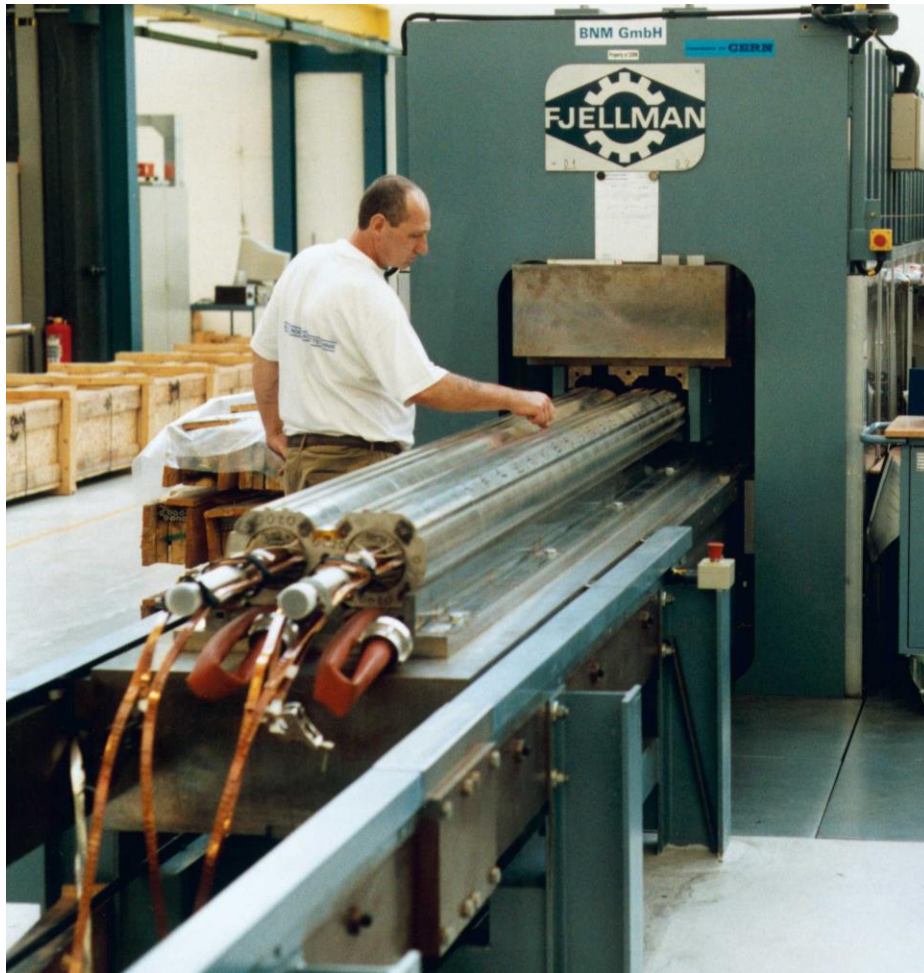
## Collars

- Collaring procedure
  - Collars are pre-assembled in packs (several cm long) and placed around the coil.
    - The collar laminations are divided in “short” and “long”.
  - Since the uncompressed coil is oversized with respect to the collar cavity dimension, at the beginning of the collaring procedure the collars are not locked (open).
  - The coil/collar pack is then introduced into a collaring press.
  - The pressure of the press is increased until a nominal value.
  - Collars are locked with keys, rods or welded, and the press released.
  - Once the collaring press is released, the collar experience a “spring back” due to the clearance of the locking feature and deformation.

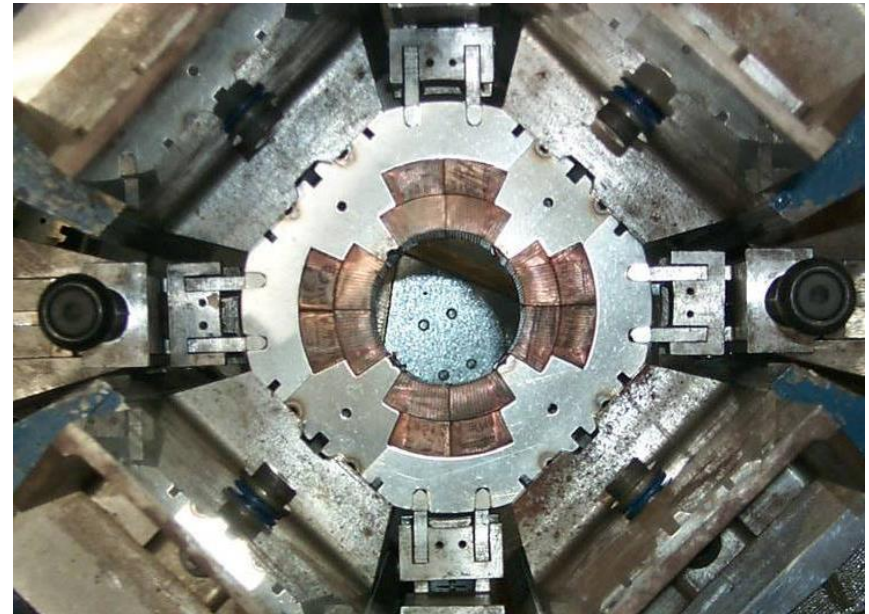


# Support structure Collars

Collaring of a dipole magnet



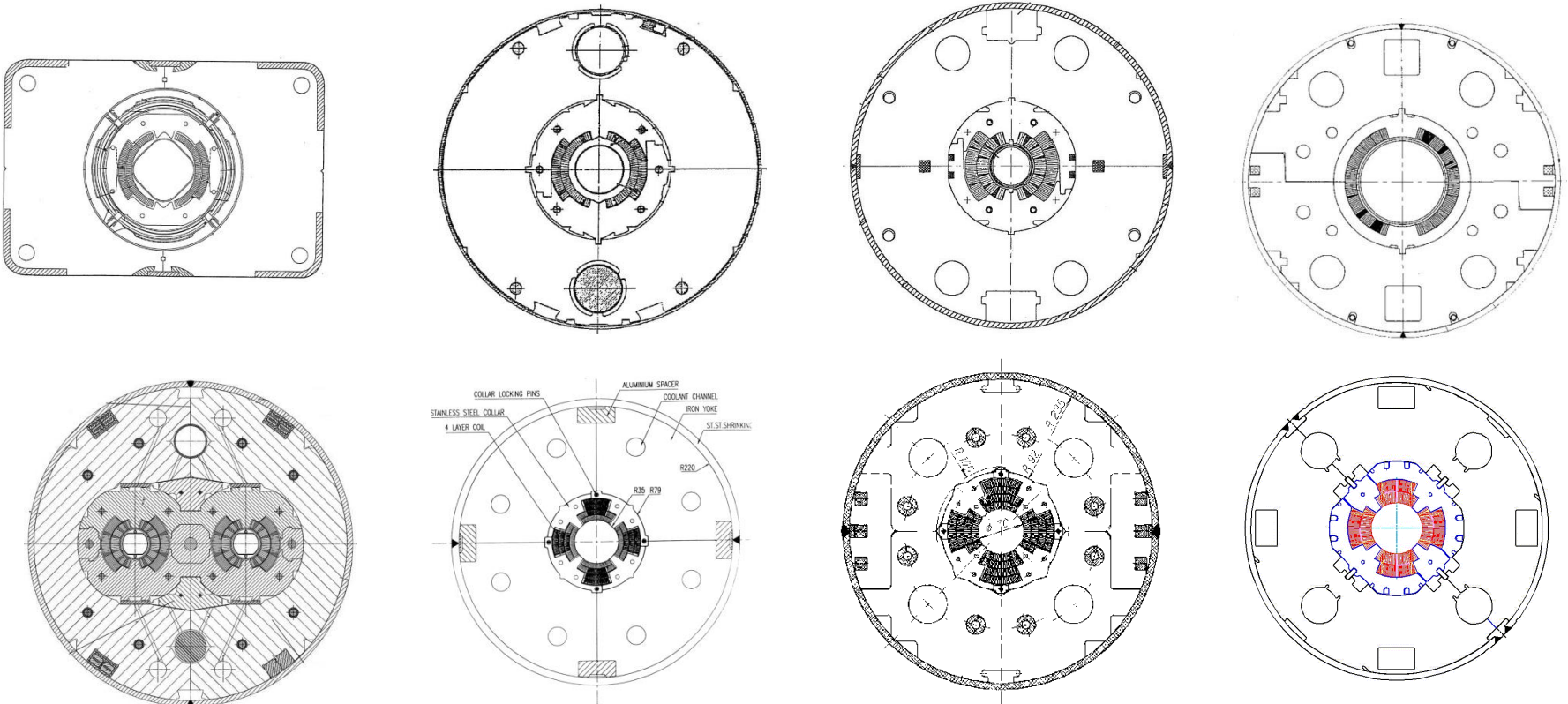
Collaring of a quadrupole magnet



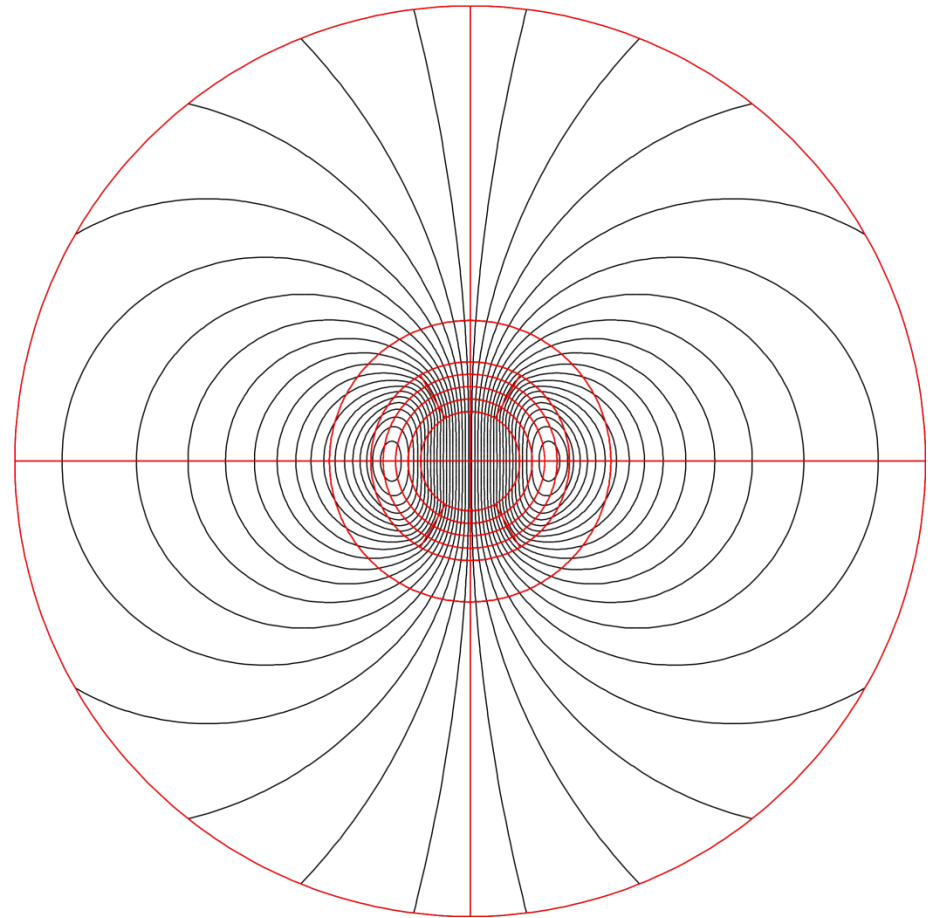
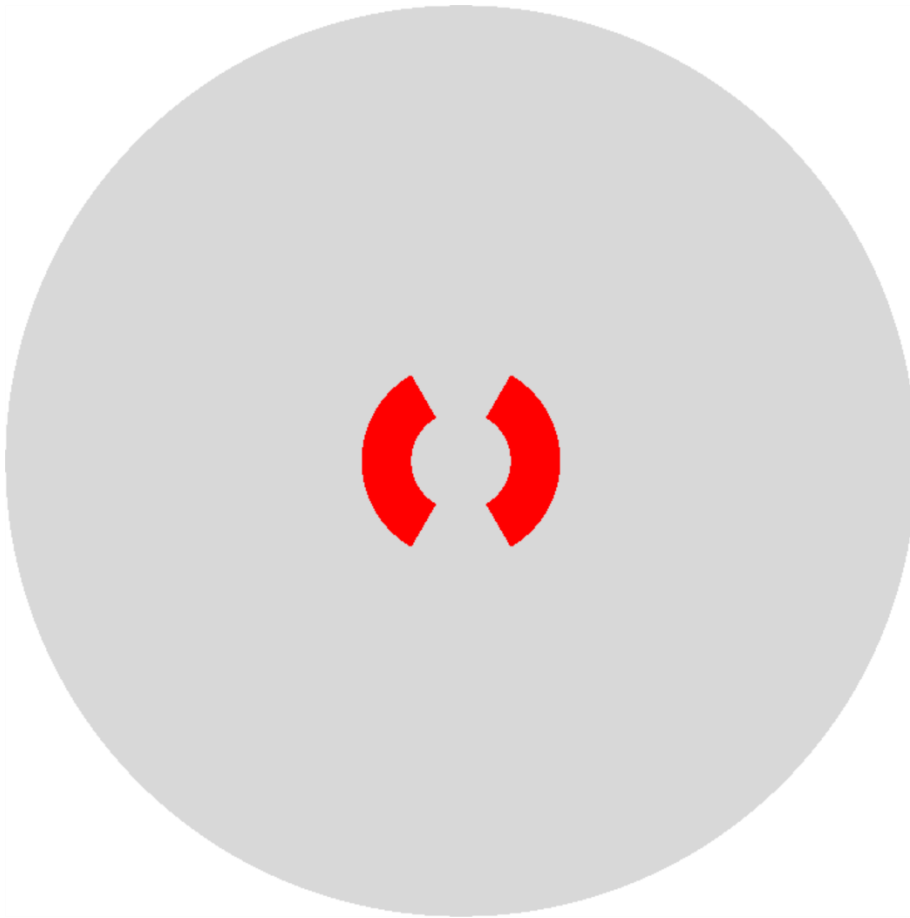
# Dimension of the support structure

## Collars

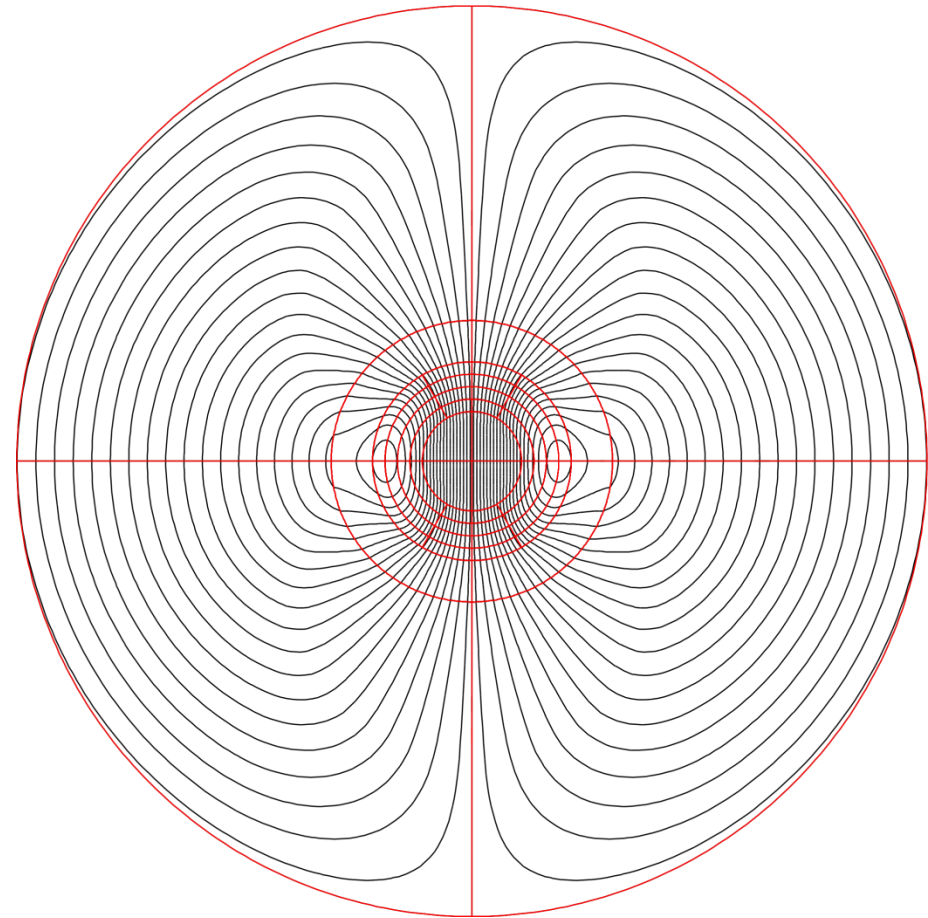
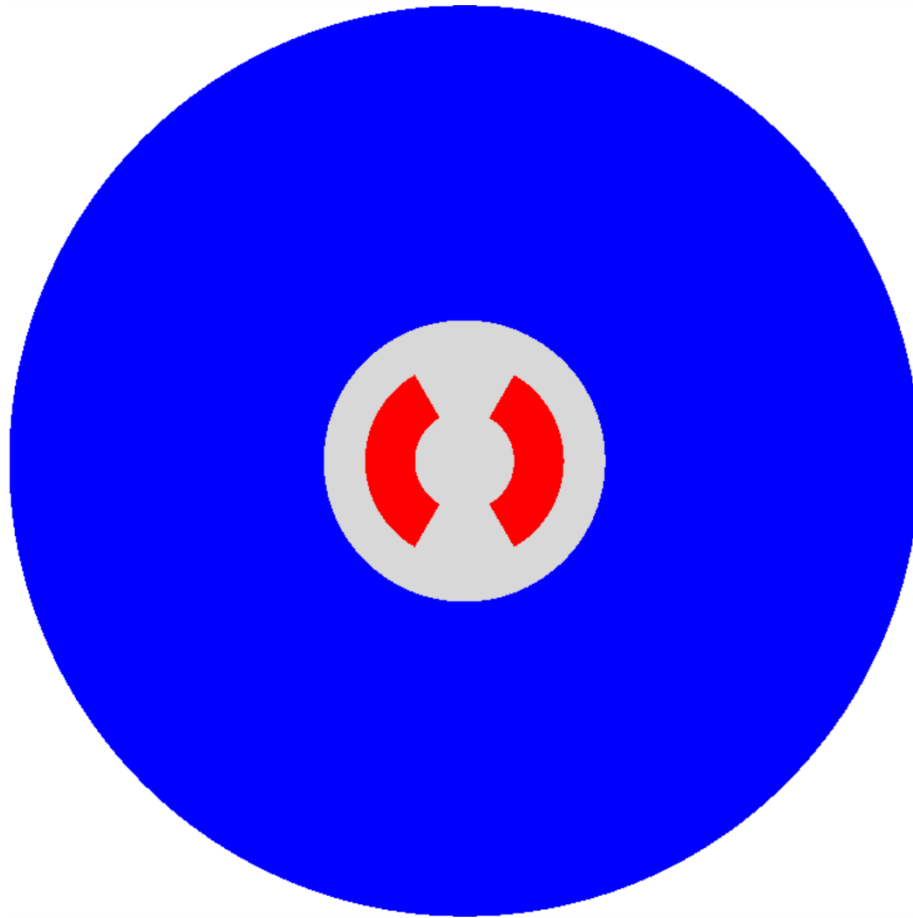
- We assume a 25 mm thick collar
  - Images not in scale



# Without iron yoke



# With iron yoke

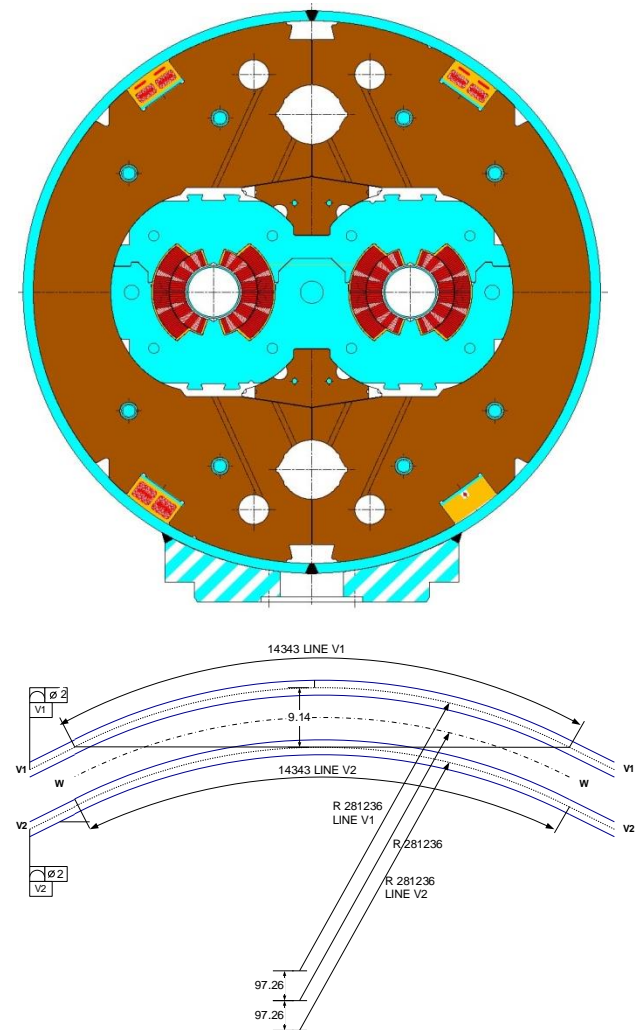


- The iron yoke thickness can be estimated with  $rB \sim t_{\text{iron}} B_{\text{sat}}$

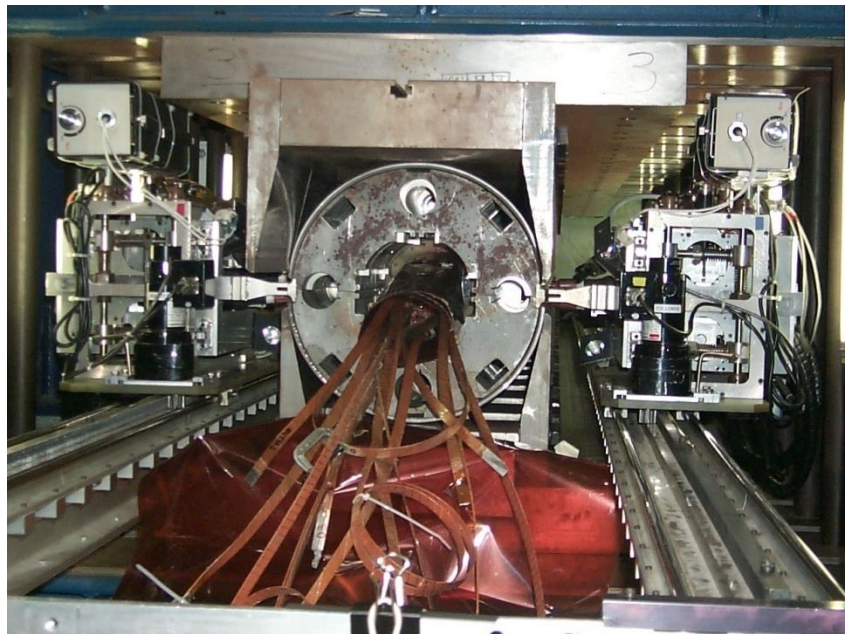
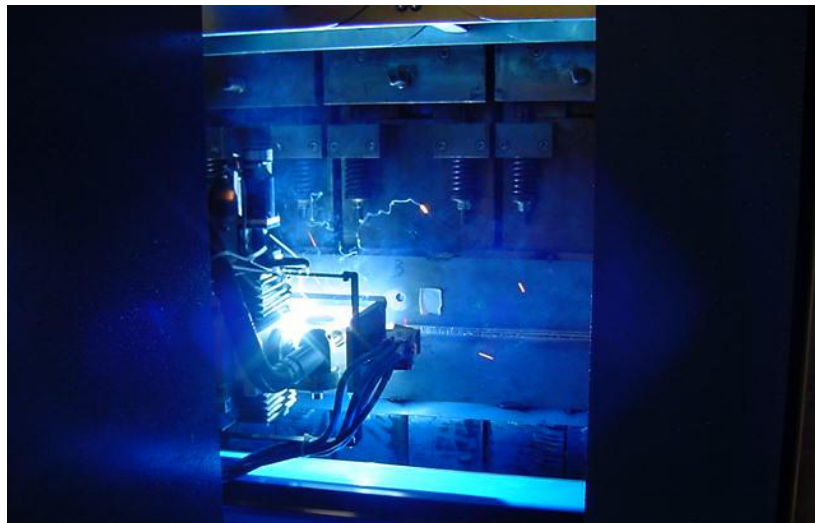
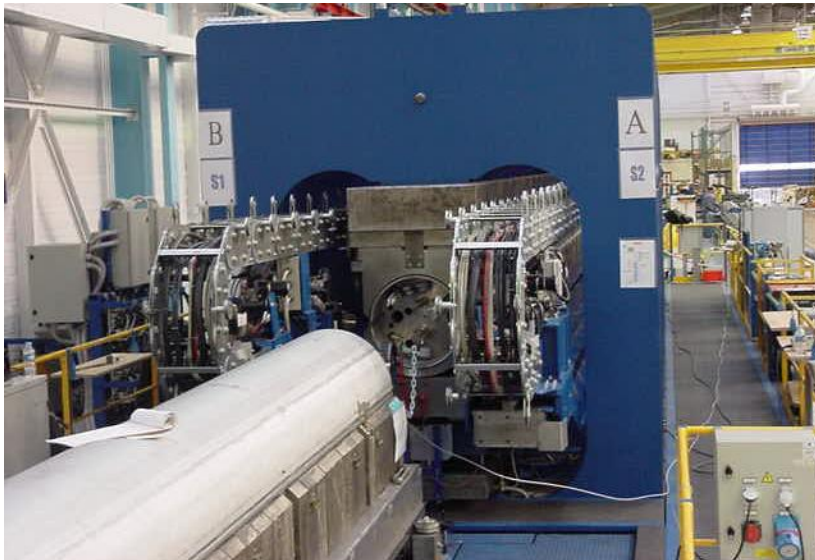
# Support structure

## Shell (or shrinking cylinder)

- The cold mass is contained within a shell (or shrinking cylinder).
- The shell constitutes a containment structure for the liquid Helium.
- It is composed by two half shells of stainless steel welded around the yoke with high tension (about 150 MPa for the LHC dipole).
  - With the iron yoke, it contributes to create a rigid boundary to the collared coil.
- If necessary, during the welding process, the welding press can impose the desired curvature on the cold mass.
  - In the LHC dipole the nominal sagitta is of 9.14 mm.



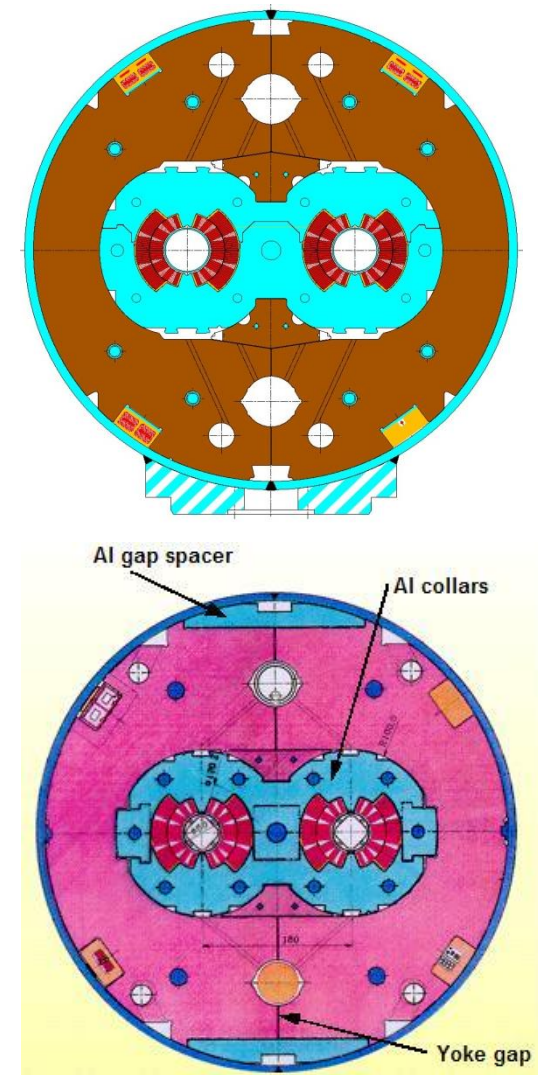
# Support structure Shell (or shrinking cylinder)



# Support structure

## Shell (or shrinking cylinder)

- The shell tension provided by the welding may contribute to the overall support of the collared coil.
- An often (SSC, LHC) implemented approach is the line-to-line fit.
  - When the yoke is put around the collared coil, a gap (vertical or horizontal) remains between the two halves; this gap is due to the collar deformation induced by coil pre-stress.
  - After welding, the shell tension closes the gap, and good contact is provided between yoke and collar.
  - After cool-down, despite the higher thermal contraction of the collared coil with respect to iron, the gap remain closed (high rigidity), and the collared coil in good contact with the yoke.
- Aluminum spacer may be used to control the yoke gap.



# Dimension of the support structure

- We assume that the shell will close the yoke halves with the same force as the total horizontal e.m. force at 90% of  $I_{ss}$
- We assume an azimuthal shell stress after cool-down of
  - $\sigma_{\text{shell}} = 200 \text{ MPa}$

