

Lecture 2: Magnets & training, plus fine filaments

Magnets

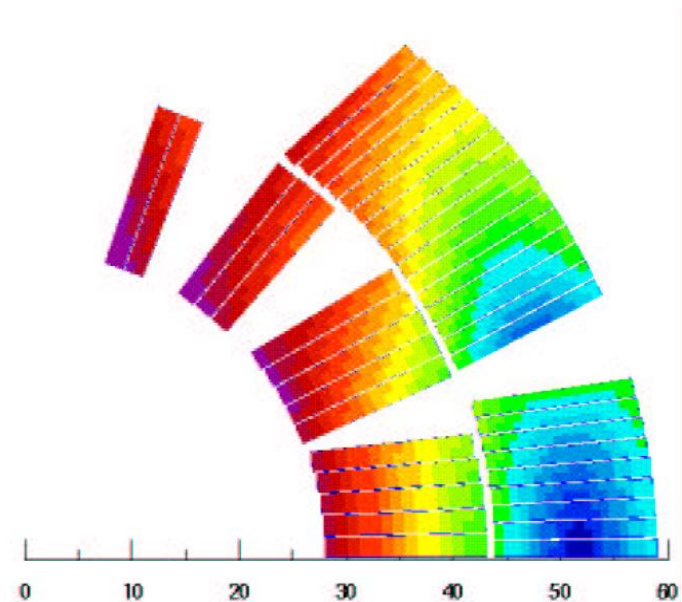
- magnetic fields above 2 Tesla
- coil shapes for solenoids, dipoles and quadrupoles
- engineering current density
- load lines

Degradation & Training

- causes of training - release of energy within the magnet
- reducing training - stability and minimum quench energy MQE
- need copper and fine filaments for low MQE

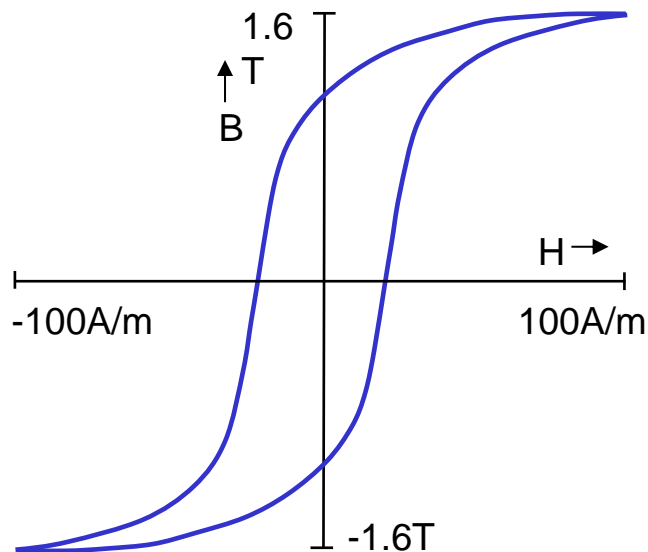
Flux Jumping

- need for fine filaments

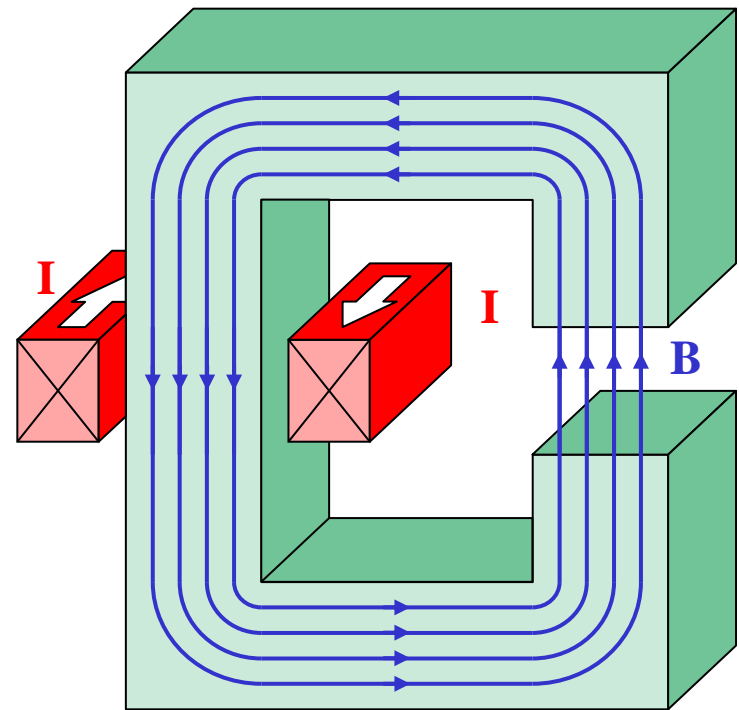


Fields and ways to create them: conventional

- conventional electromagnets have an iron yoke
 - reduces magnetic reluctance
 - reduces ampere turns required
 - reduces power consumption
- iron guides and shapes the field



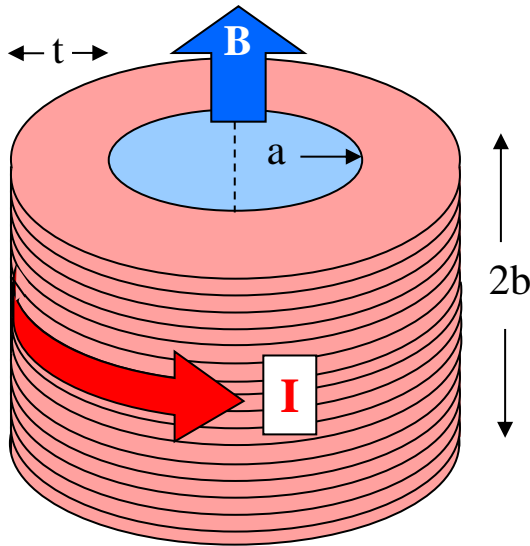
BUT iron saturates at ~ 2T



*Iron electromagnet
– for accelerators, motors,
transformers, generators etc*

for higher fields we cannot rely on iron
field must be created and shaped by the winding

Solenoids



- no iron - field shape depends only on the winding
- azimuthal current flow, eg wire wound on bobbin, axial field
- the field produced by an infinitely long solenoid is

$$B = \mu_o NI = \mu_o J_e t$$

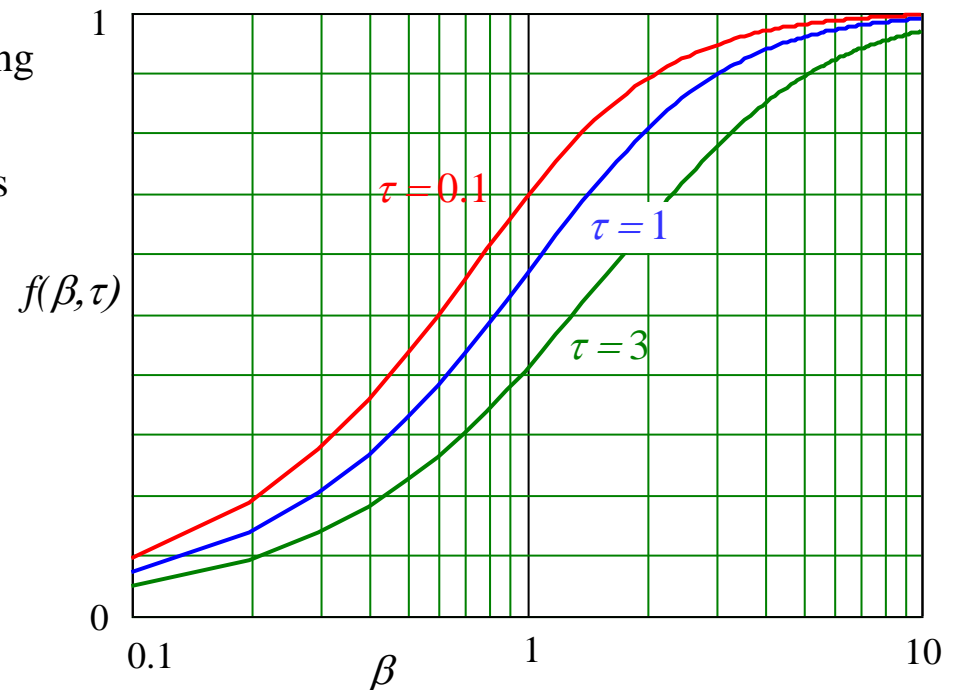
where N = number of turns/unit length, I = current ,
 J_e = engineering current density

- so high $J_e \Rightarrow$ thin compact economical winding
- in solenoids of finite length the central field is

$$B = \mu_o J_e t f(\beta, \tau)$$

where $\beta = b/a$ $\tau = t/a$

- field uniformity and the ratio of peak field to central field get worse in short fat solenoids



Superconducting solenoids



Delphi solenoid for HEP experiments at CERN

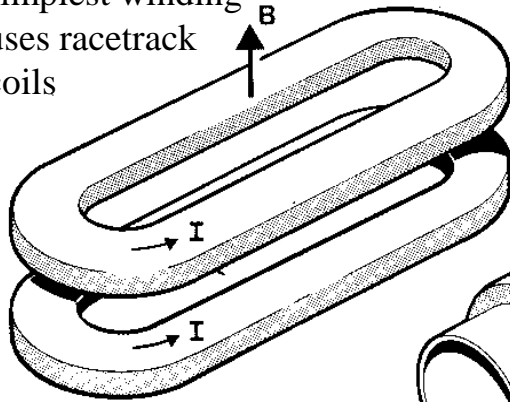
1.2T 5.5m dia 6.8m long 110MJ



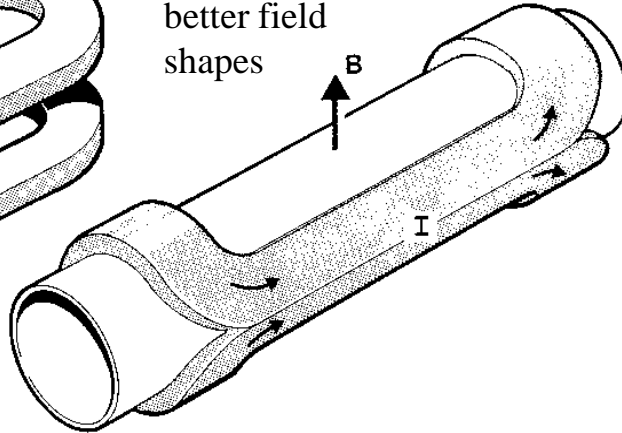
Superconducting
solenoid for
research

Accelerators need transverse fields

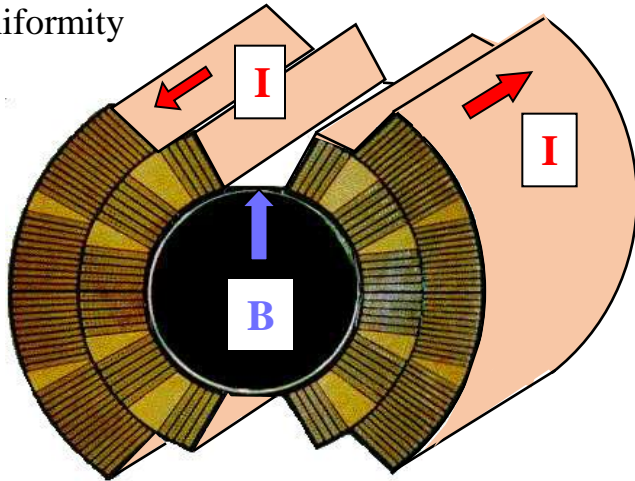
simplest winding
uses racetrack
coils



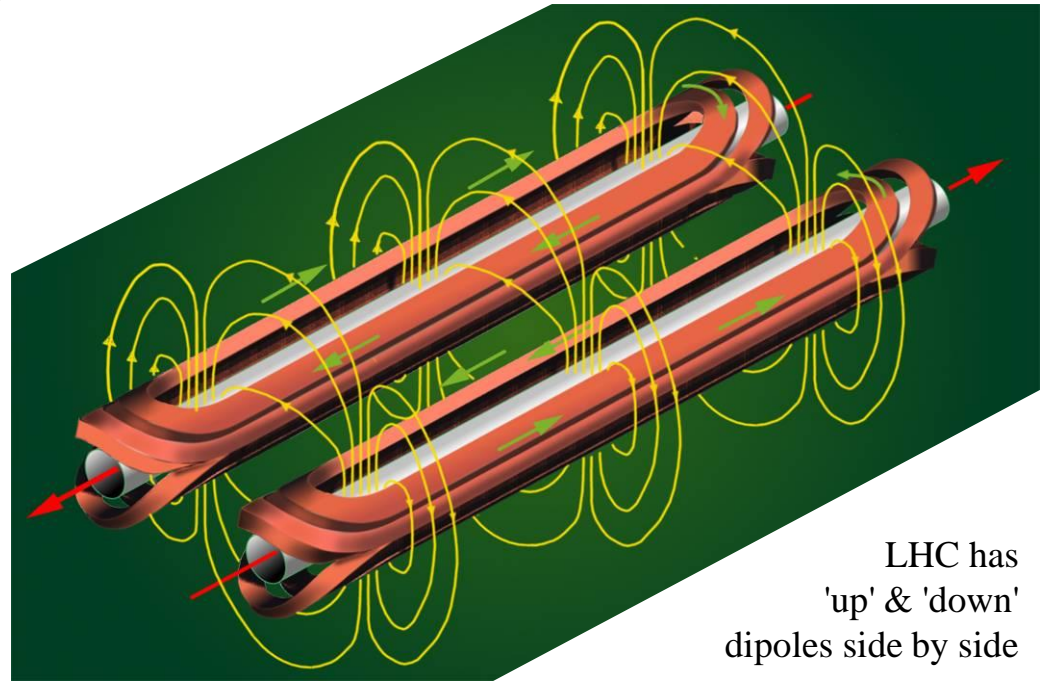
'saddle' coils make
better field
shapes



special winding cross
sections for good
uniformity



- some iron - but field shape is set mainly by the winding
- used when the long dimension is transverse to the field, eg accelerator magnets
- known as **dipole** magnets (because the iron version has 2 poles)

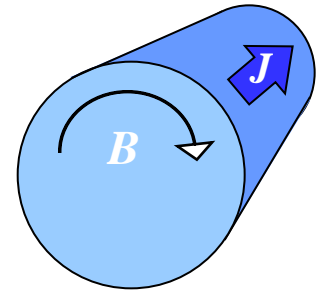


LHC has
'up' & 'down'
dipoles side by side

Dipole field from overlapping cylinders

Ampere's law for the field inside a cylinder carrying uniform current density

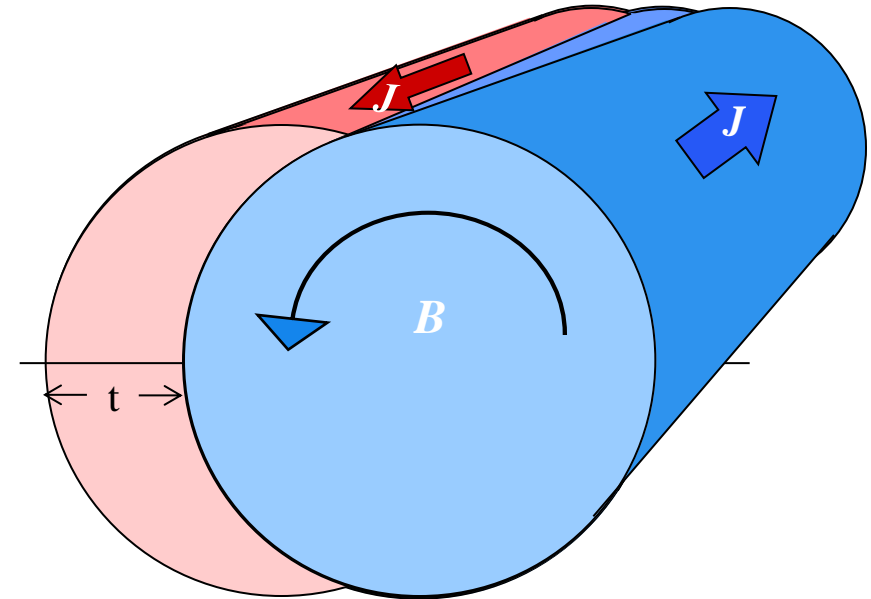
$$\oint B \cdot ds = 2\pi r B = \mu_o I = \mu_o \pi r^2 J \quad B = \frac{\mu_o J r}{2}$$



- two cylinders with opposite currents
- push them together
- currents cancel where they overlap \Rightarrow aperture
- fields in the aperture:

$$B_y = \frac{\mu_o J}{2} (-r_1 \cos \theta_1 + r_2 \cos \theta_2) = \frac{-\mu_o J t}{2}$$

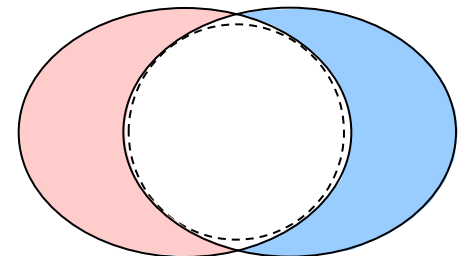
$$B_x = \frac{\mu_o J}{2} (-r_1 \sin \theta_1 + r_2 \sin \theta_2) = 0$$



- thus the overlapping cylinders give a perfect dipole field

$$B_y = \frac{-\mu_o J_e t}{2}$$

- same trick with ellipses
- circular aperture



Windings of distributed current density

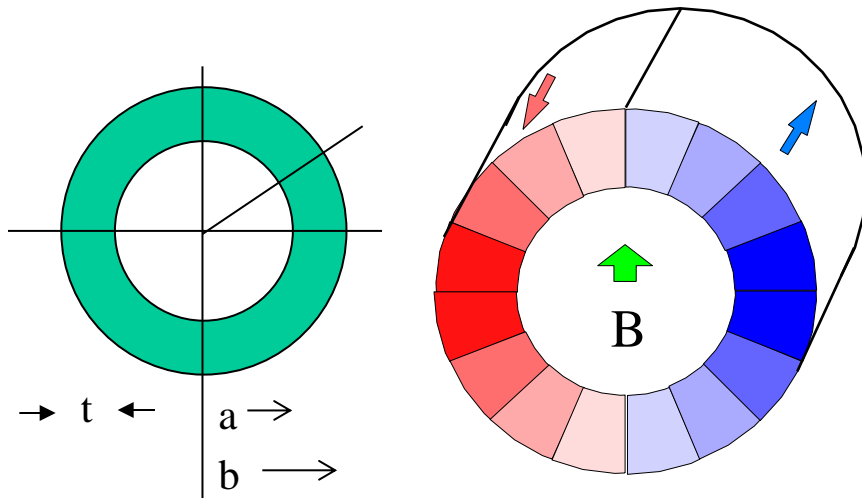
Analyse thin current sheets flowing on the surface of a cylinder using complex algebra. Let the **linear** current density (**Amps per m of circumference**) be $g_n = g_o \cos(n\theta)$ (**Am⁻¹**)

For $n = 1$ we find a pure dipole field inside the cylinder, $n = 2$ gives a quadrupole etc.

Now superpose many cylinders of increasing radius to get a thick walled cylinder carrying an (area) current density (**Am⁻²**) $J_n = J_o \cos(n\theta)$

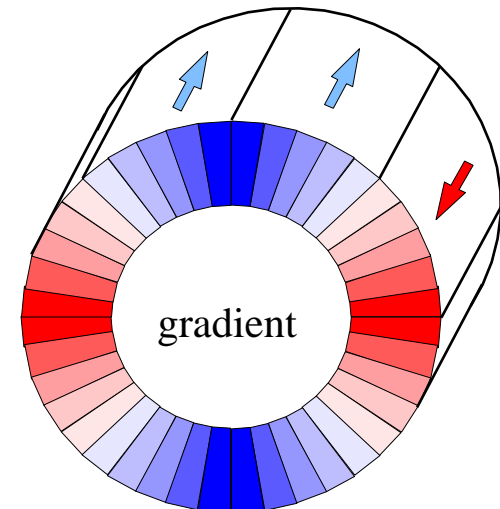
$$n=1 \quad J_1(\theta) = J_o \cos \theta$$

$$B_x = 0 \quad B_y = -\mu_o J_o (b-a)/2 = -\mu_o J_o t/2$$



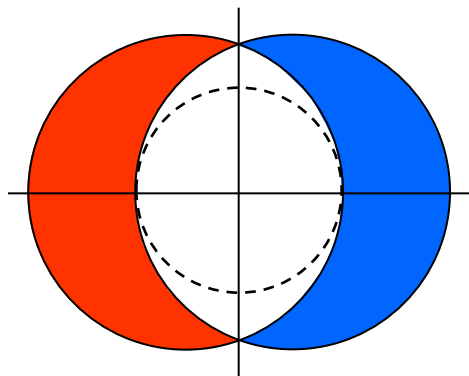
$$n=2 \quad J_2(\theta) = J_o \cos 2\theta$$

$$B_x = \frac{\mu_o J_o}{2} y \ln\left(\frac{b}{a}\right) \quad B_y = \frac{\mu_o J_o}{2} x \ln\left(\frac{b}{a}\right)$$



Summary of simplified dipole windings

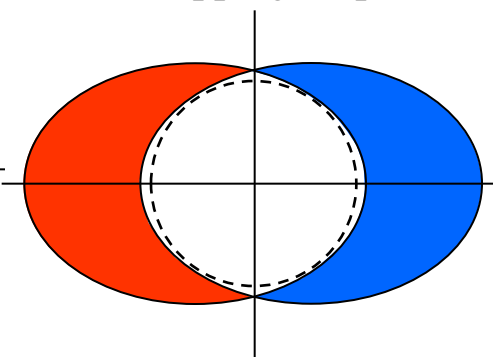
Overlapping circles



$$B = \frac{\mu_o}{2} J_o t$$

t = winding thickness J_o =
engineering current density

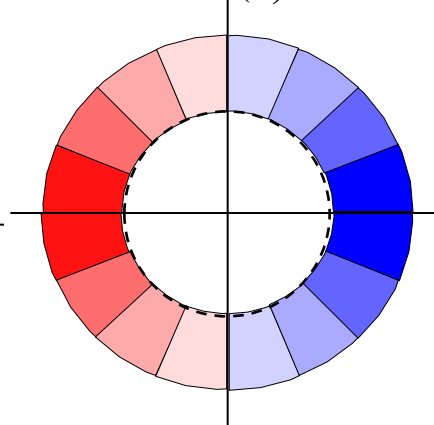
Overlapping ellipses



$$B = \mu_o J_o t \alpha$$

where $\alpha = c/(b+c)$
 c = height of ellipse
 b = width

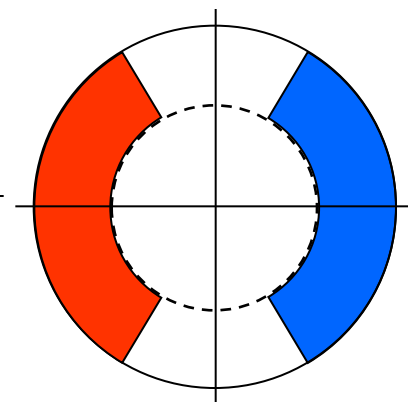
$\cos(\theta)$



$$B = \frac{\mu_o}{2} J_o t$$

best estimate
of forces

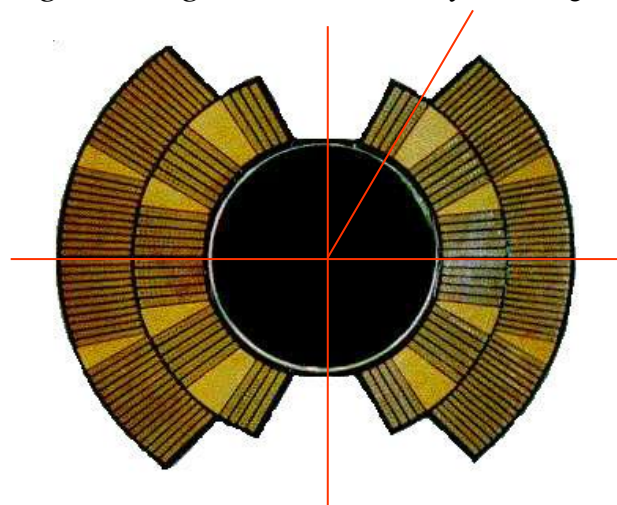
60° sector



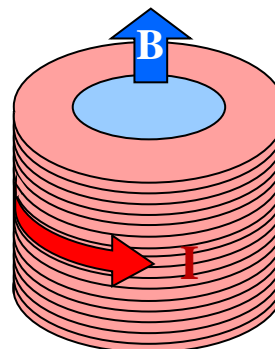
$$B = \frac{2\mu_o}{\pi} J_o t \sin(60^\circ)$$

$$\frac{2}{\pi} \sin(60^\circ) = 0.55$$

best estimate
of peak field



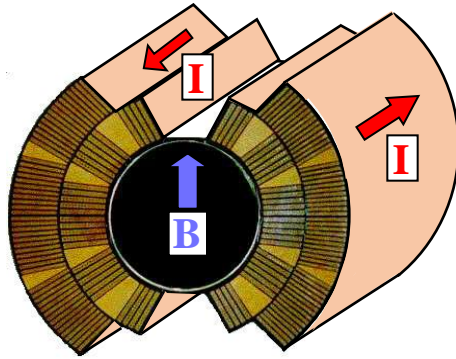
LHC
dipole
winding



recap solenoid

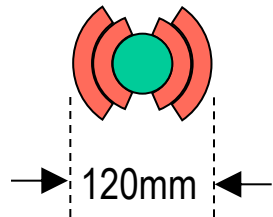
$$B = \mu_o J_e t f(\beta, \tau)$$

Importance of (engineering) current density in dipoles



LHC dipole

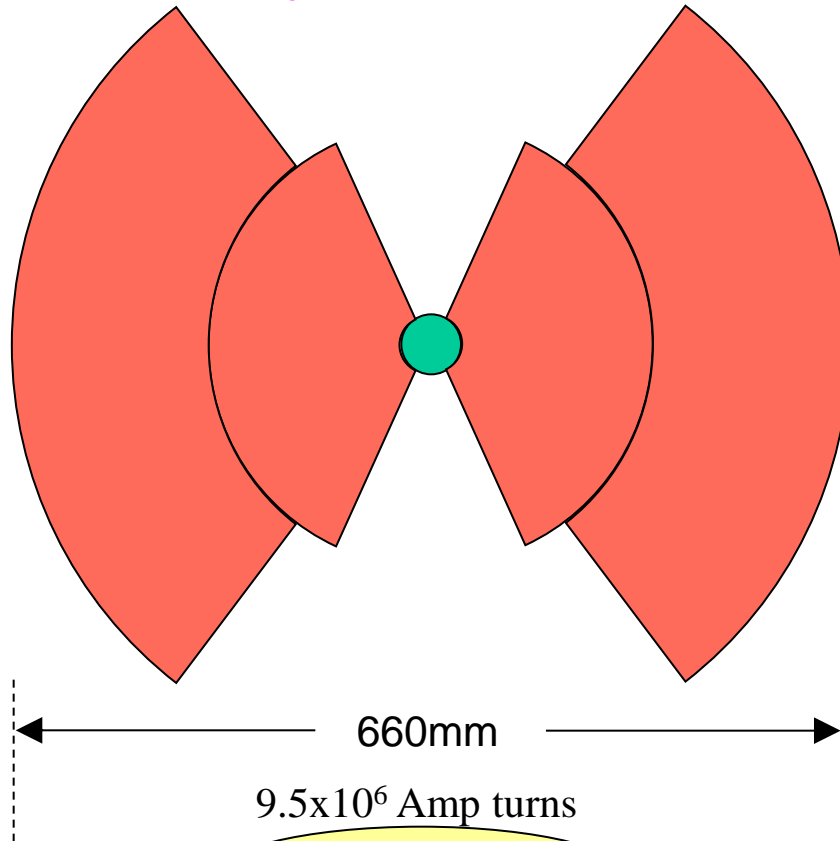
$$J_e = 375 \text{ Amm}^{-2}$$



$$9.5 \times 10^5 \text{ Amp turns}$$

$$= 1.9 \times 10^6 \text{ A.m per m}$$

$$J_e = 37.5 \text{ Amm}^{-2}$$



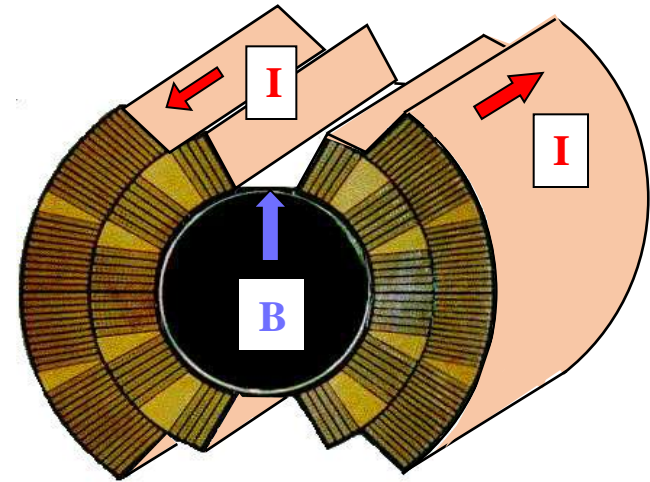
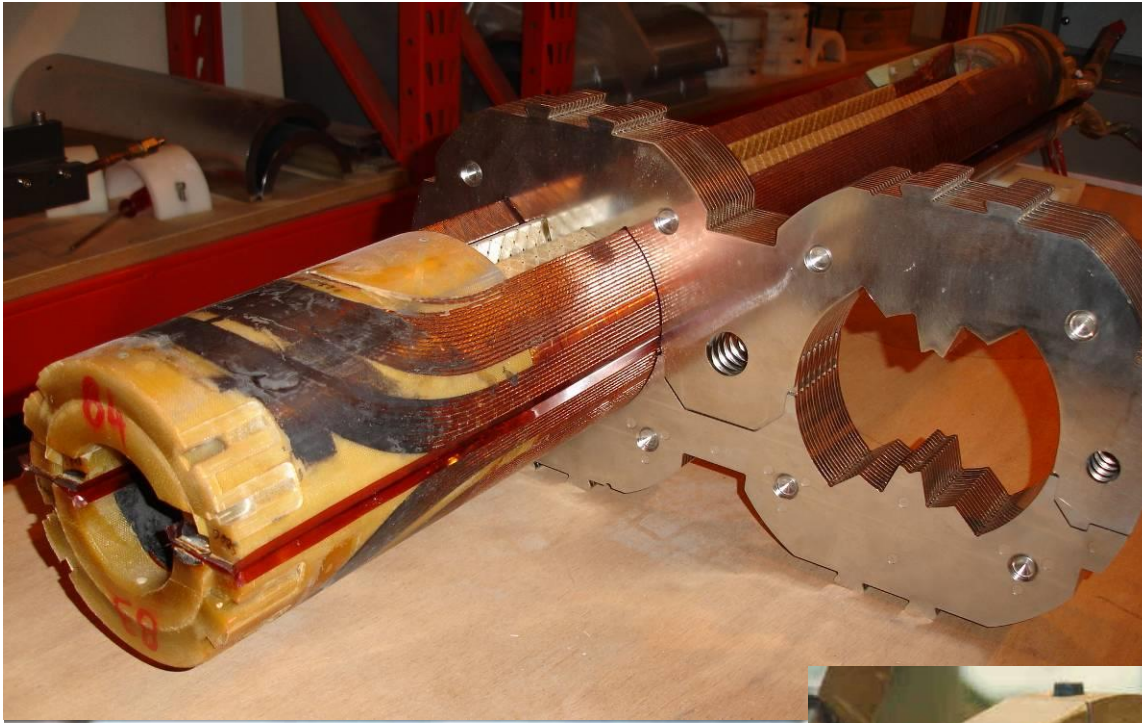
$$9.5 \times 10^6 \text{ Amp turns}$$

$$= 1.9 \times 10^7 \text{ A.m per m}$$

field produced
by a perfect
dipole is

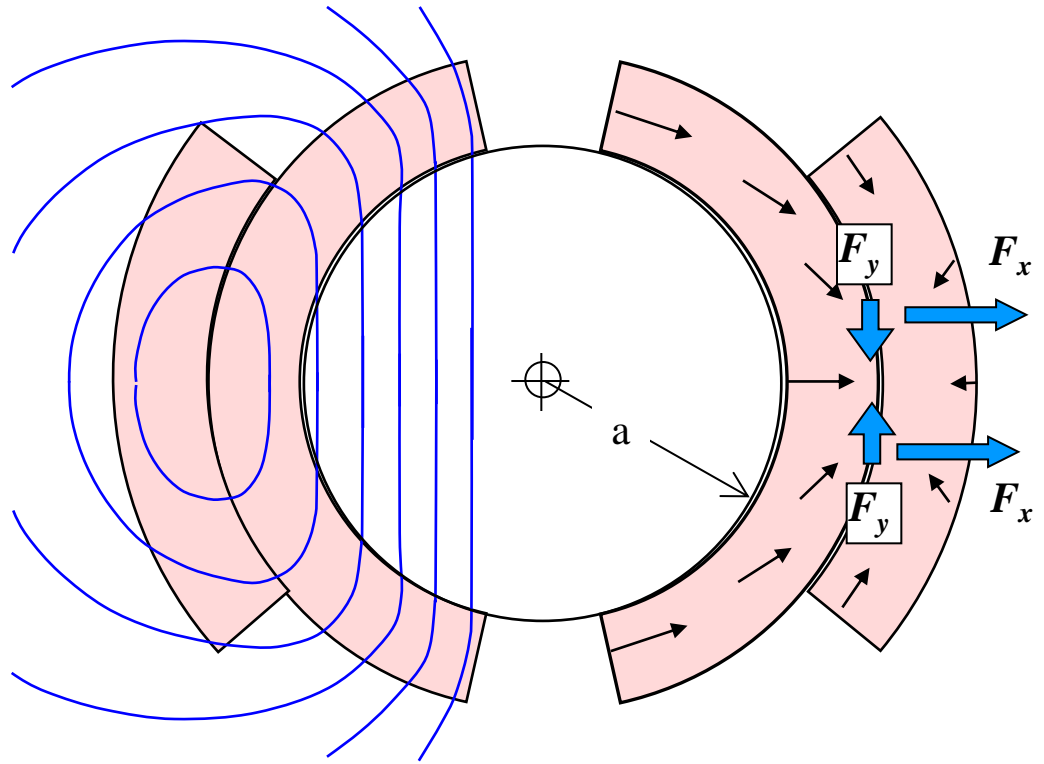
$$B = \mu_o J_e \frac{t}{2}$$

Dipole Magnets



Electromagnetic forces in dipoles

- forces in a dipole are horizontally outwards and vertically towards the median plane
- unlike a solenoid, the bursting forces cannot be supported by tension in the winding
- the outward force must be supported by an external structure
- both forces cause compressive stress and shear in the conductor and insulation
- apart from the ends, there is no tension in the conductor
- simple analysis for thin windings

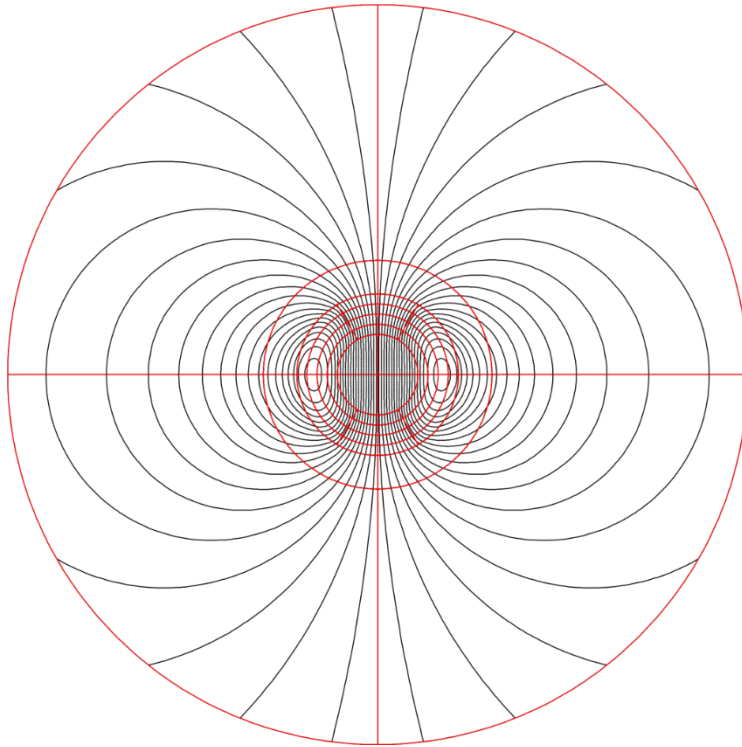


$$F_x = \frac{B_i^2}{2\mu_o} \frac{4a}{3}$$

$$F_y = -\frac{B_i^2}{2\mu_o} \frac{4a}{3}$$

Estimating the iron shield thickness

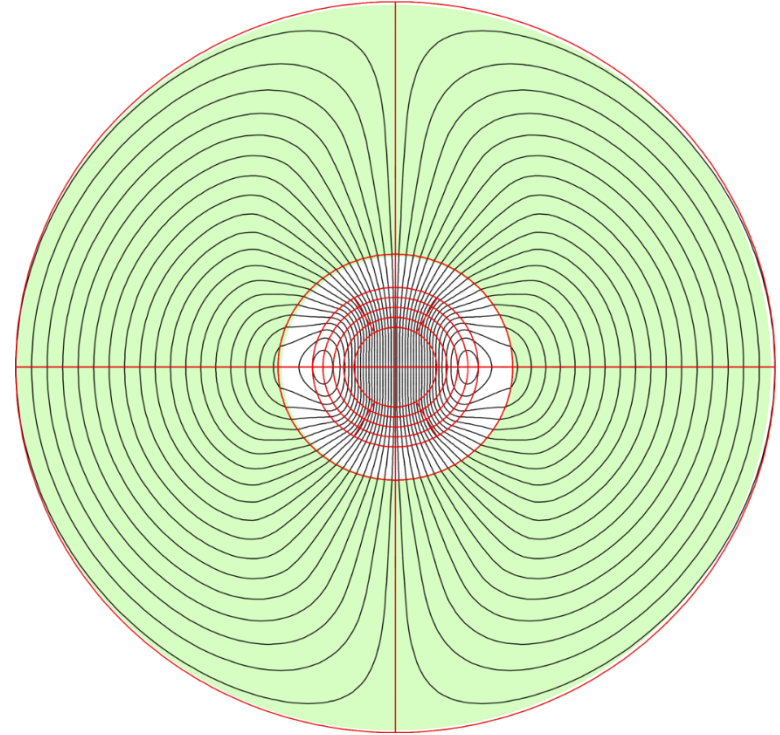
no iron



some flux returns close to the coil
flux through $\frac{1}{2}$ coil aperture

$$\phi_c = B_o a \quad a = \text{coil radius}$$

with iron



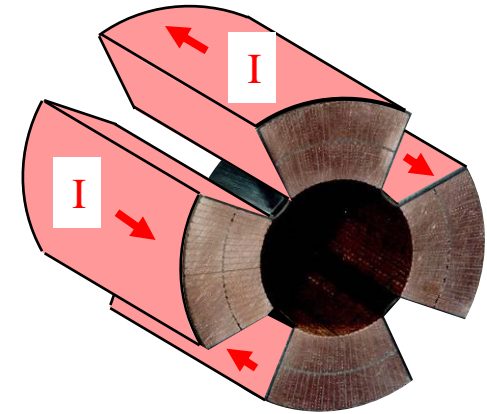
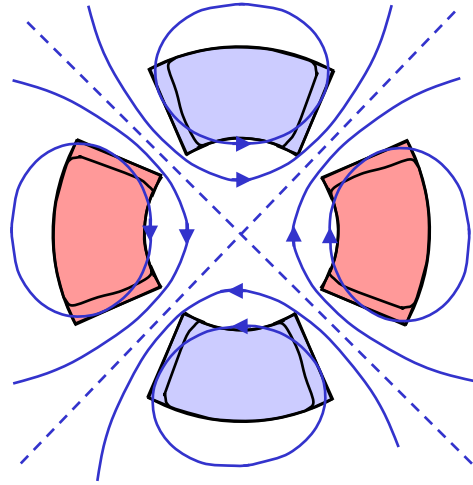
almost all flux returns through the iron
return flux through iron (one side)

$$\phi_i = B_{sat} t \quad t = \text{iron thickness}$$

$$\phi_i \sim \phi_c \quad \text{so}$$

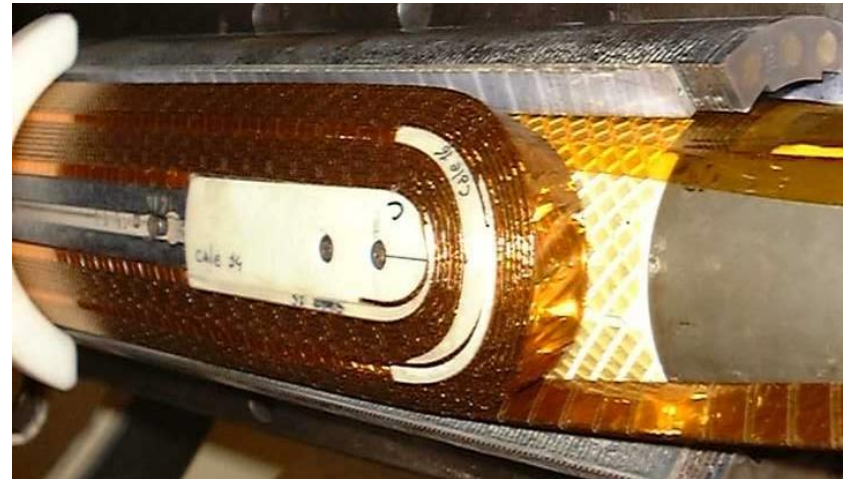
$$t \sim a B_o / B_{sat}$$

Quadrupole windings

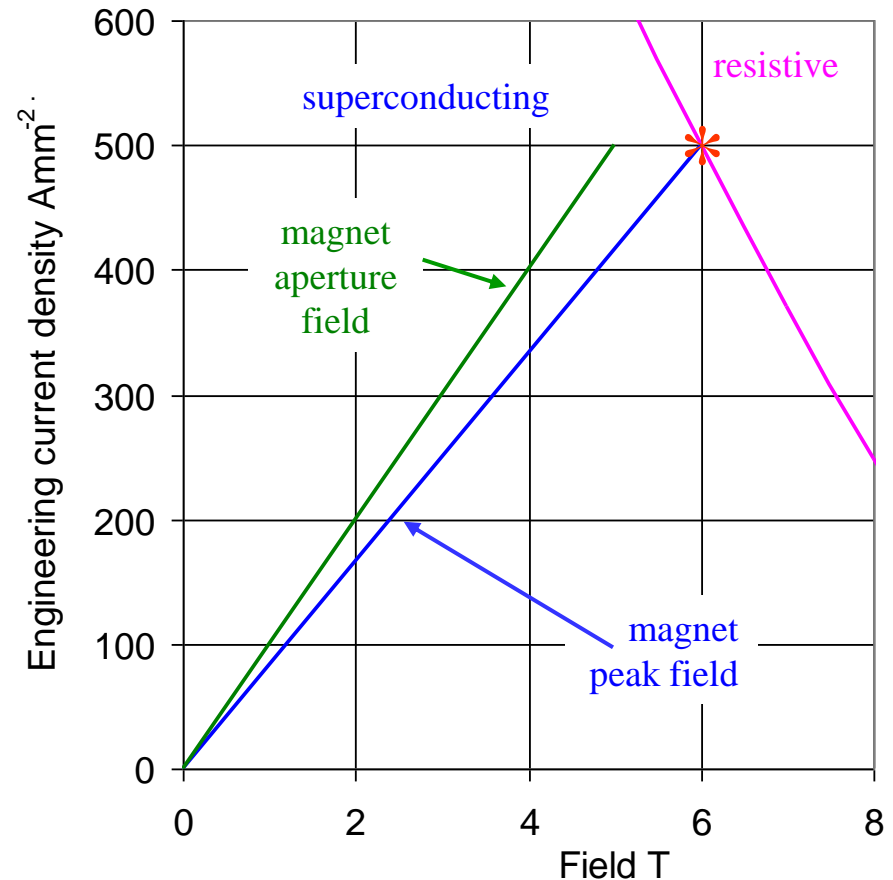
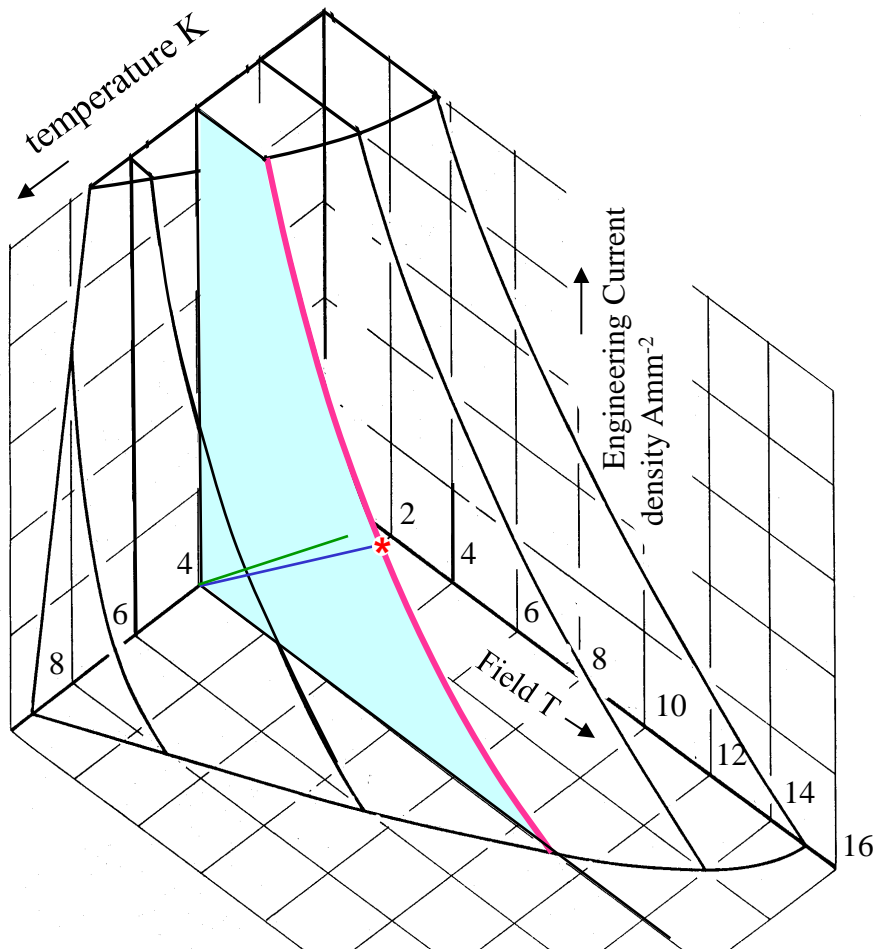


$$B_x = ky$$

$$B_y = kx$$



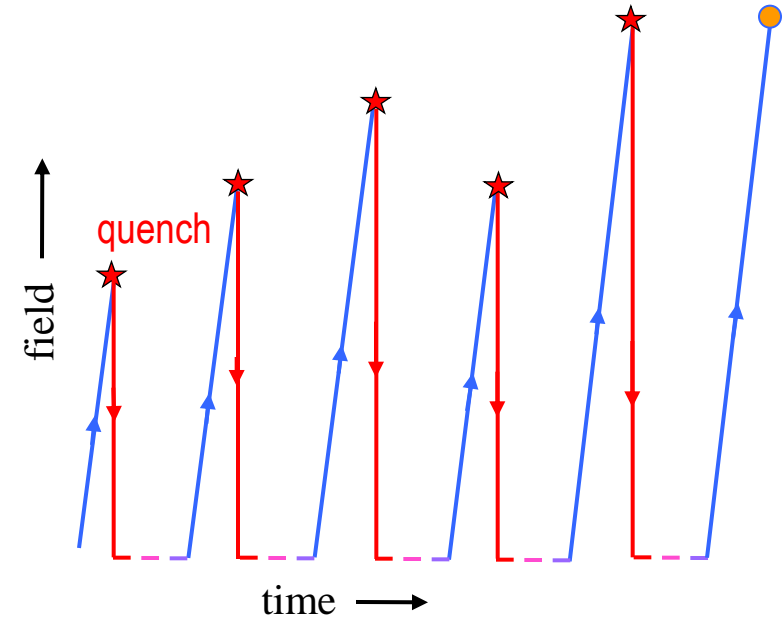
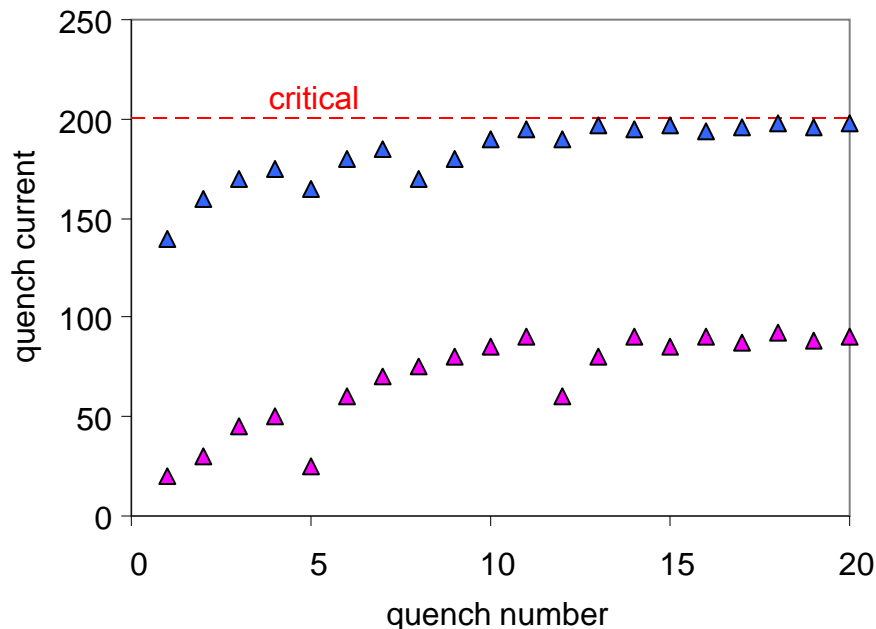
Critical surface and magnet load lines



- load line relates magnet field to current
- peak field > aperture (useful) field
- we expect the magnet to go resistive '*quench*' where the peak field load line crosses the critical current line *

Degraded performance and 'training' of magnets

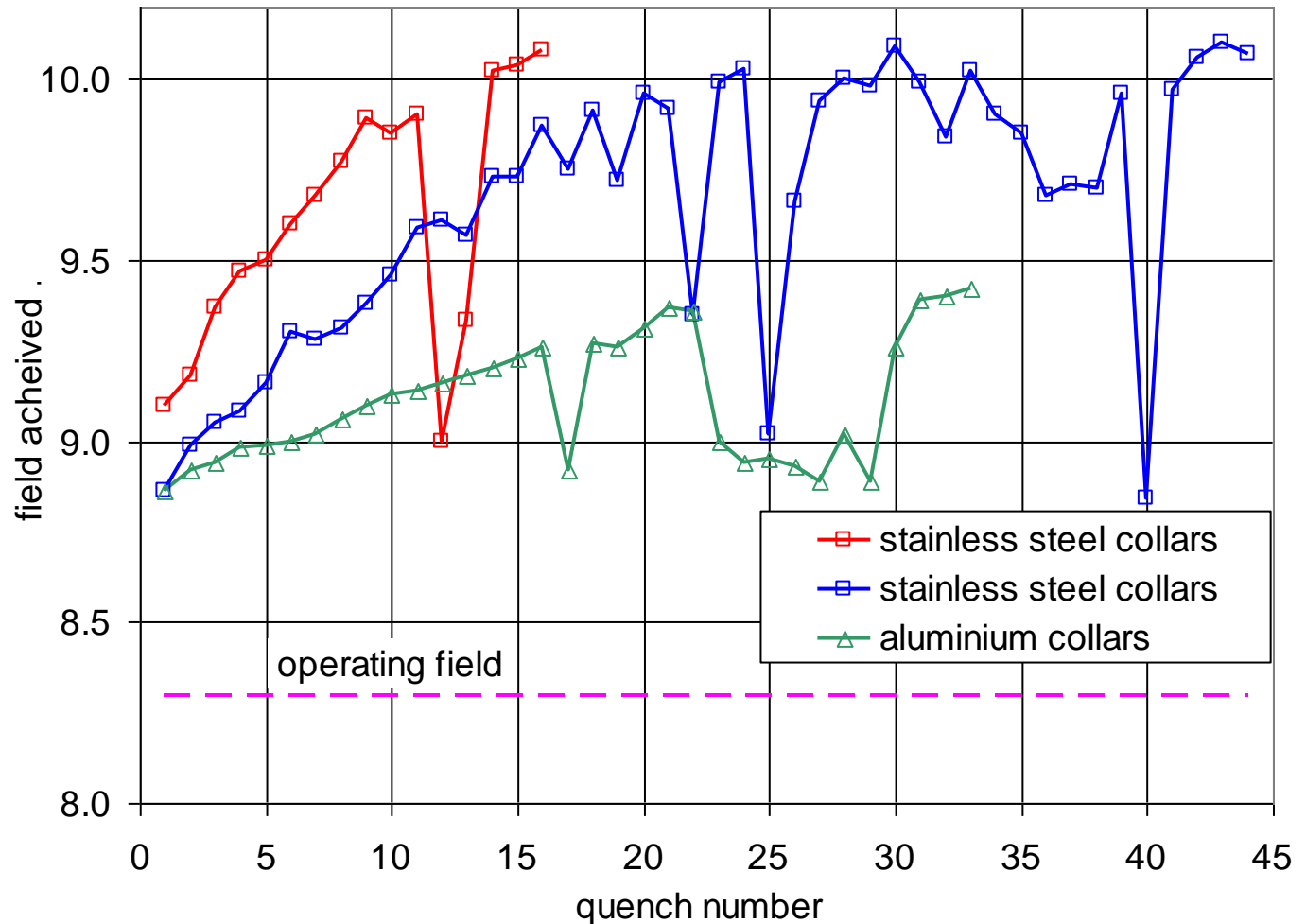
- early disappointment for magnet makers when they ramped up the magnet current for the first time
- instead of going up to the critical line, it 'quenched' (went resistive) at less than the expected current
- at the next try it did better
- known as *training*



- after a *quench*, the stored energy of the magnet is dissipated in the magnet, raising its temperature way above critical
- you must wait for it to cool down and then try again
- well made magnets ▲ are better than poorly made ▲

'Training' of magnets

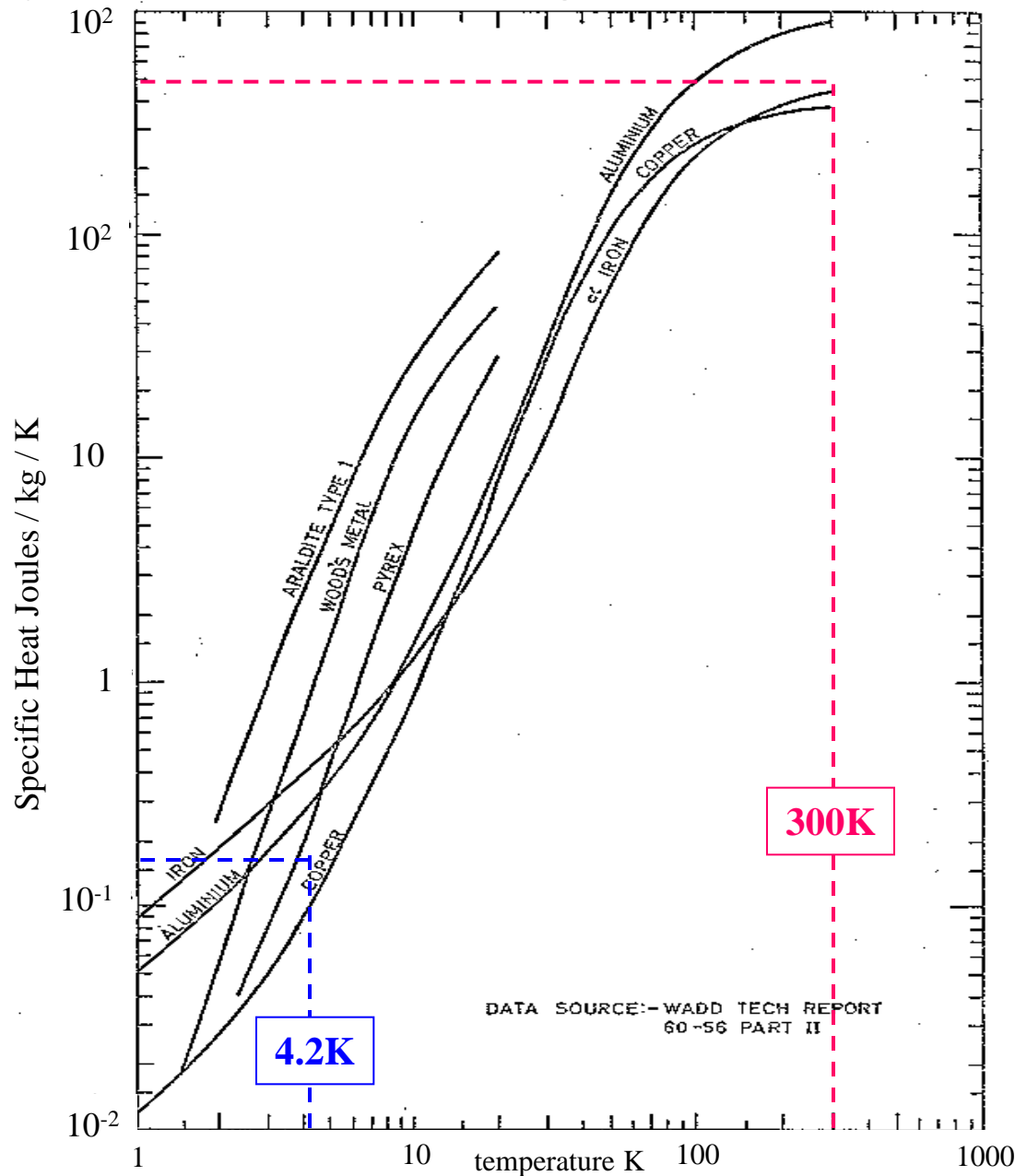
- it's better than the old days, but training is still with us
- it seems to be affected by the construction technique of the magnet
- it can be wiped out if the magnet is warmed to room temperature
- 'de-training' is the most worrisome feature



Training of LHC short prototype dipoles (from A. Siemko)

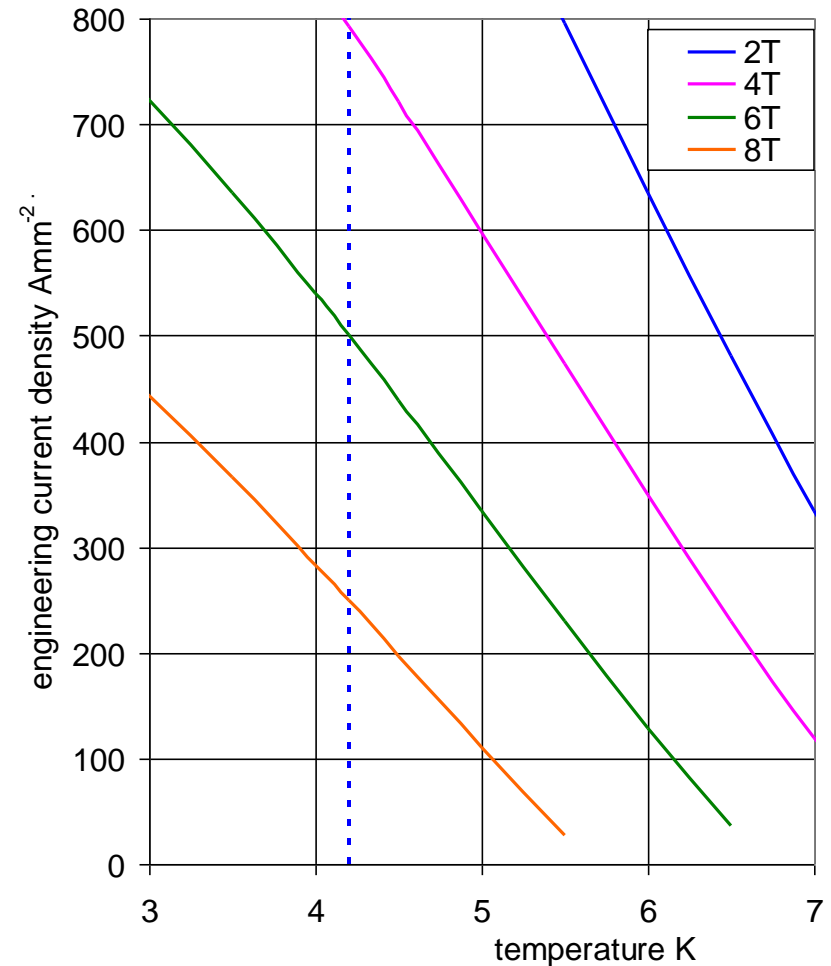
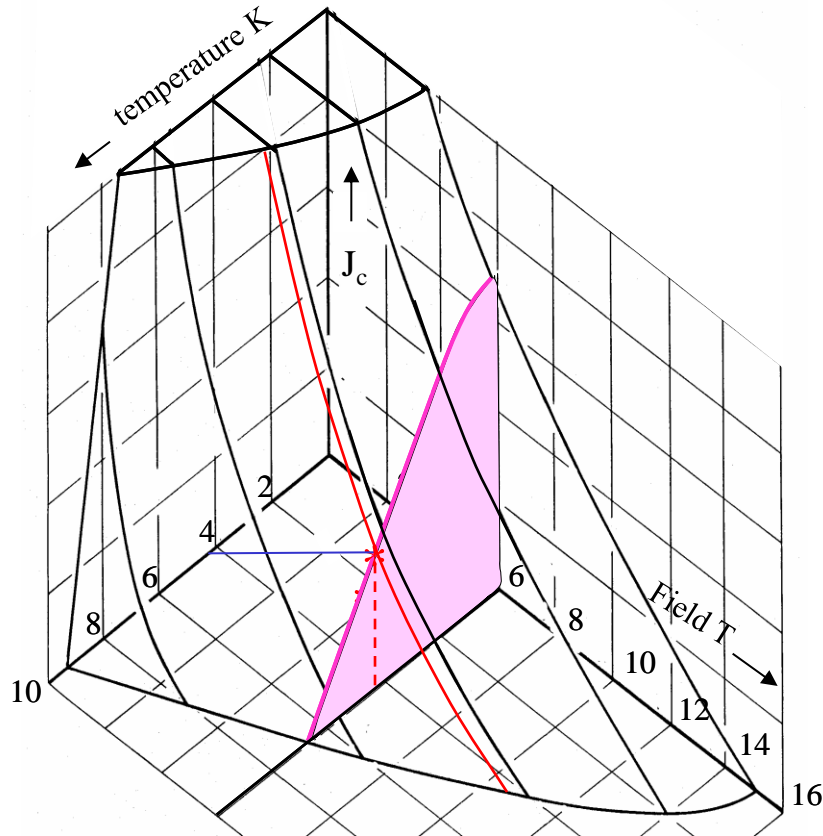
Causes of training:

(1) low specific heat



- the specific heat of all substances falls with temperature
- at 4.2K, it is ~2,000 times less than at room temperature
- a given release of energy within the winding thus produce a temperature rise 2,000 times greater than at room temperature
- the smallest energy release can therefore produce catastrophic effects

Causes of training: (2) J_c decreases with temperature

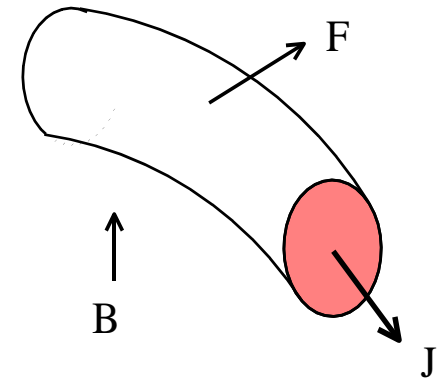


at any field, J_c of NbTi falls ~ linearly with temperature

- so any temperature rise drives the conductor towards the resistive state

Causes of training: (3) conductor motion

Conductors in a magnet are pushed by the electromagnetic forces. Sometimes they move suddenly under this force - the magnet 'creaks' as the stress comes on. A large fraction of the work done by the magnetic field in pushing the conductor is released as frictional heating



work done per unit length of conductor if it is pushed a distance δz

$$W = F.\delta z = B.I.\delta z$$

frictional heating per unit volume

$$Q = B.J.\delta z$$

typical numbers for NbTi:

$$B = 5\text{T} \quad J_{\text{eng}} = 5 \times 10^8 \text{ A.m}^{-2}$$

$$\text{so if } \delta = 10 \mu\text{m}$$

$$\text{then } Q = 2.5 \times 10^4 \text{ J.m}^{-3}$$

$$\text{Starting from } 4.2\text{K} \quad \theta_{\text{final}} = 7.5\text{K}$$

can you
engineer a
winding to
better than
10 μm ?



Causes of training: (4) resin cracking

Try to stop wire movement by impregnating the winding with epoxy resin. But resin contracts more than metal, so it goes into tension. Almost all organic materials become brittle at low temperature.

brittleness + tension \Rightarrow cracking \Rightarrow energy release

Calculate strain energy in resin caused by differential thermal contraction

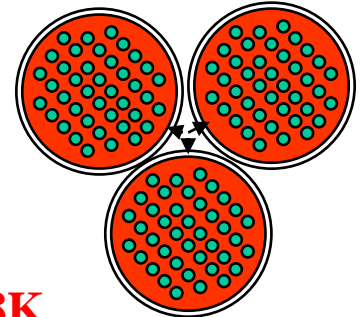
σ = tensile stress Y = Young's modulus ν = Poisson's ratio

ϵ = differential strain due to cooling = contraction (resin - metal)

typically: $\epsilon = (11.5 - 3) \times 10^{-3}$ $Y = 7 \times 10^9 \text{ Pa}$ $\nu = 1/3$

uniaxial strain $Q_1 = \frac{\sigma^2}{2Y} = \frac{Y\epsilon^2}{2}$ $Q_1 = 2.5 \times 10^5 \text{ J.m}^{-3}$ **$\theta_{\text{final}} = 16\text{K}$**

triaxial strain $Q_3 = \frac{3\sigma^2(1-2\nu)}{2Y} = \frac{3Y\epsilon^2}{2(1-2\nu)}$ $Q_3 = 2.3 \times 10^6 \text{ J.m}^{-3}$ **$\theta_{\text{final}} = 28\text{K}$**



cracking releases most of this stored energy as heat

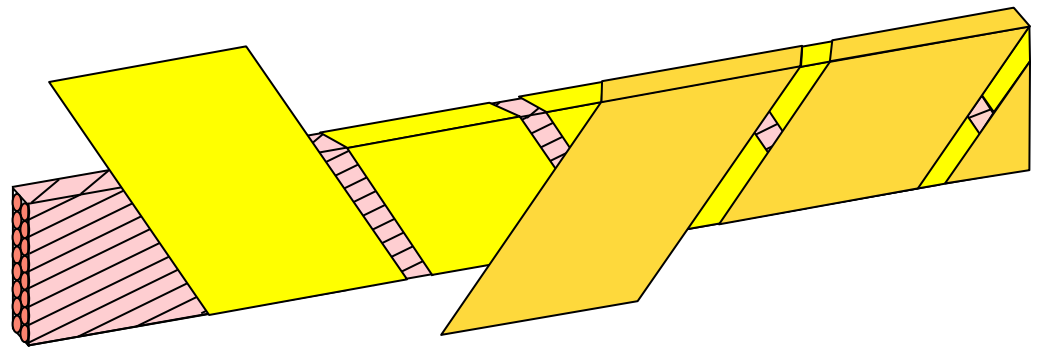
Interesting fact: magnets impregnated with paraffin wax show almost no training although the wax is full of cracks after cooldown.

Presumably the wax breaks at low σ before it has had chance to store up any strain energy

How to reduce training?

1) Reduce the disturbances occurring in the magnet winding

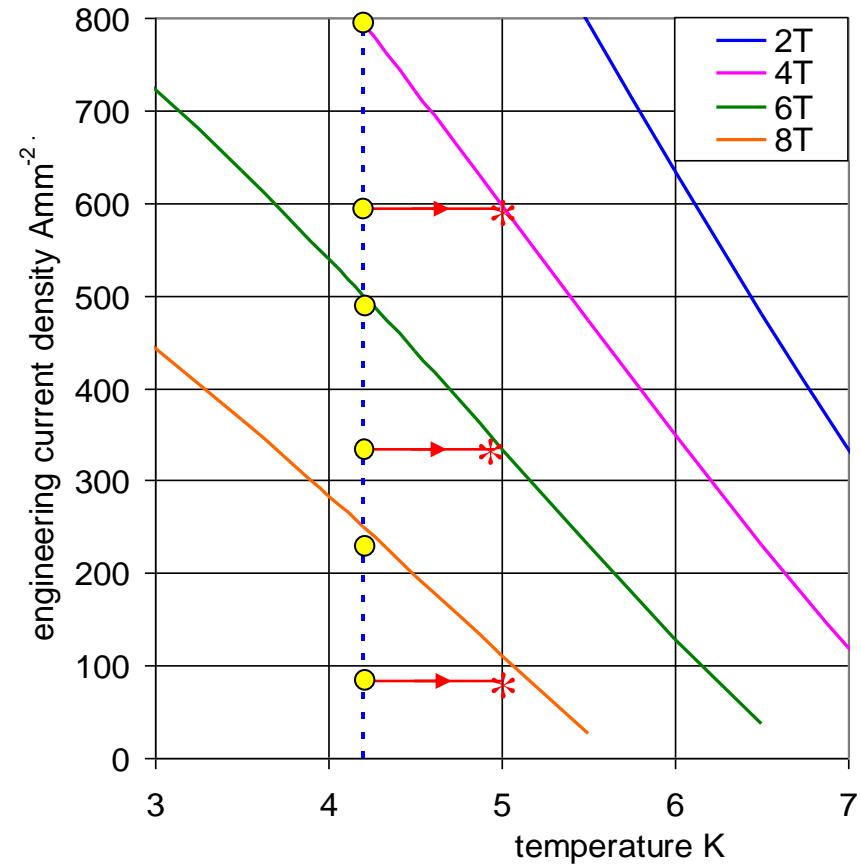
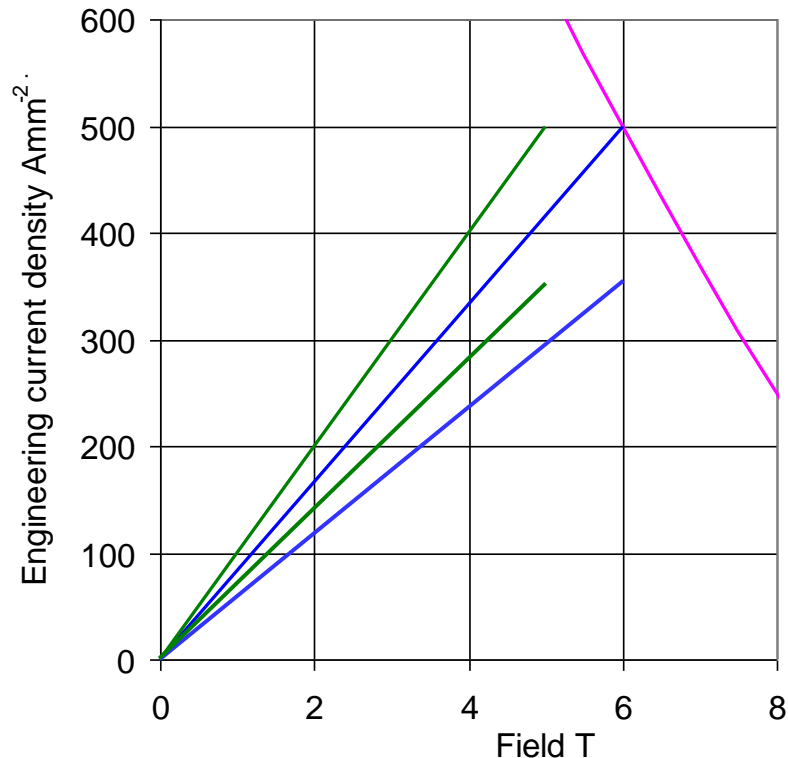
- make the winding fit together exactly to reduce movement of conductors under field forces
- pre-compress the winding to reduce movement under field forces
- if using resin, minimize the volume and choose a crack resistant type
- match thermal contractions, eg fill epoxy with mineral or glass fibre
- impregnate with wax - but poor mechanical properties
- most accelerator magnets are insulated using a Kapton film with a very thin adhesive coating on the outer face
 - away from the superconductor
- allows liquid helium to penetrate the cable



How to reduce training?

2) Make the conductor able to withstand disturbances without quenching

- increase the **temperature margin** $\sim 0.8\text{K}$
- operate at lower current
- but need more winding to make same field



- harder at high fields than at low fields
- higher critical temperature - HTS?

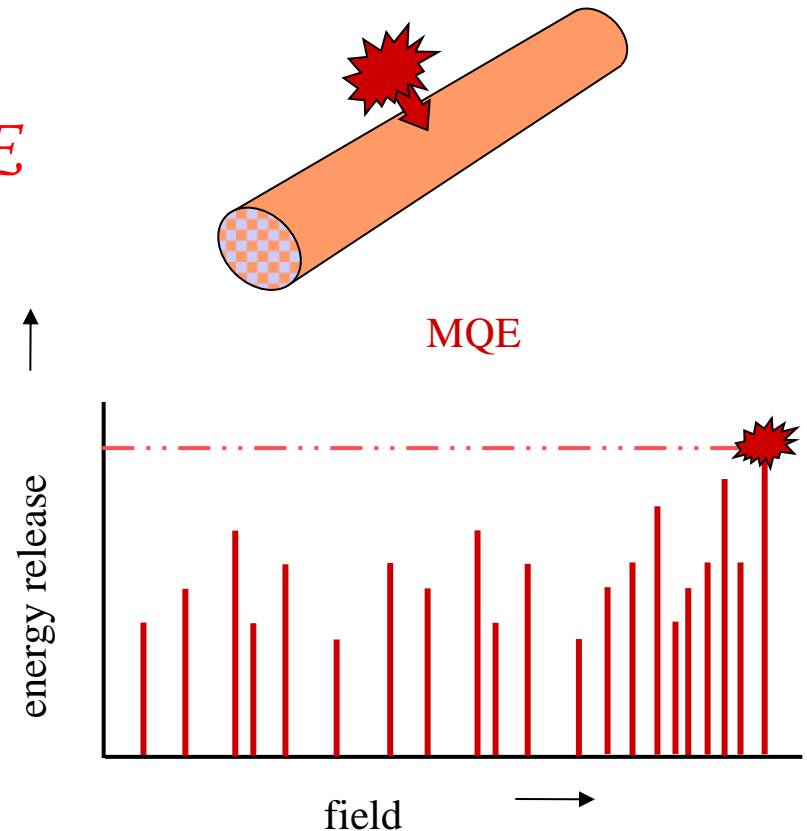
How to reduce training?

2) Make the conductor able to withstand disturbances without quenching

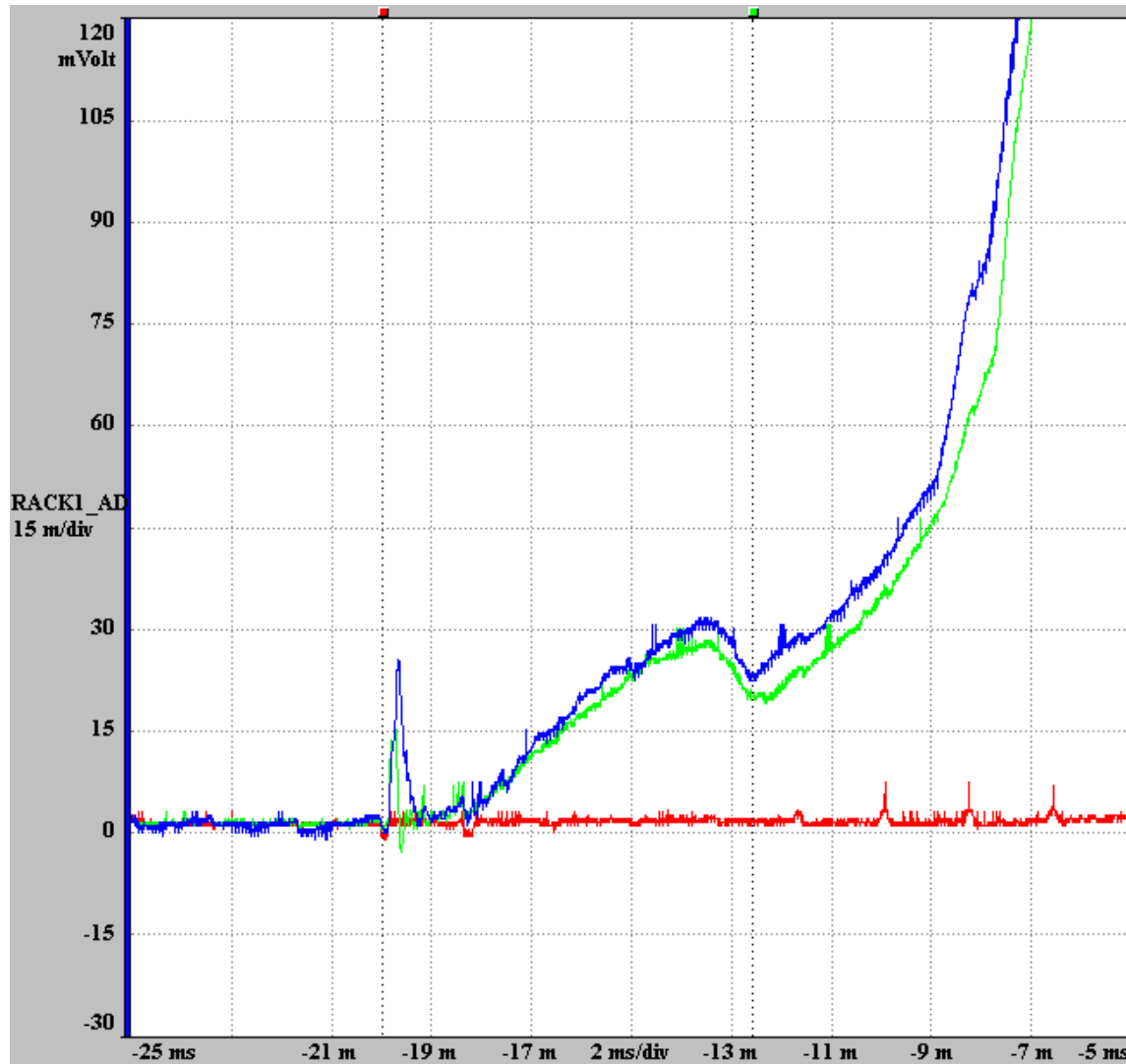
- increase the temperature margin
- increase the cooling - more cooled surface - better heat transfer - superfluid helium
- increase the specific heat - experiments with $\text{Gd}_2\text{O}_2\text{S}$ HoCu_2 etc
- most of this may be characterized by a single number

Minimum Quench Energy MQE

- defined as the energy input at a point in very short time which is just enough to trigger a quench.
- energy input $>$ MQE \Rightarrow quench
- energy input $<$ MQE \Rightarrow recovery
- energy disturbances occur at random as a magnet is ramped up to field
- for good magnet performance we want a high MQE

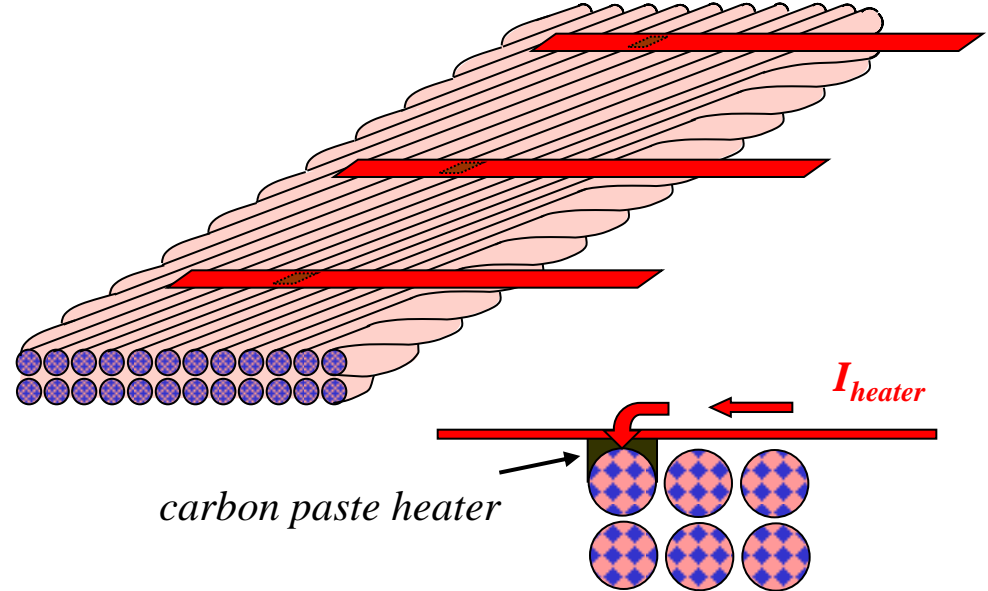
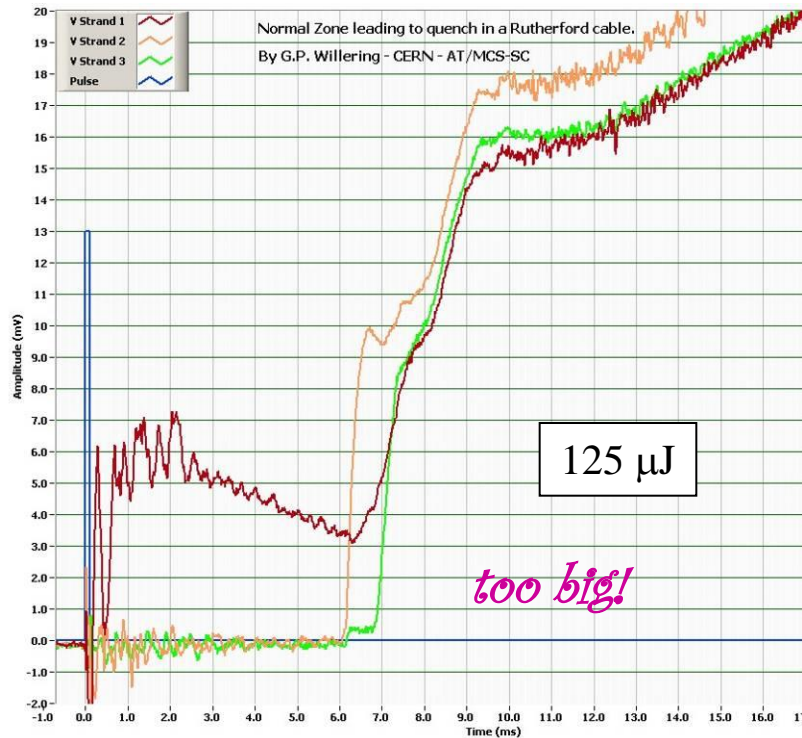


Quench initiation by a disturbance

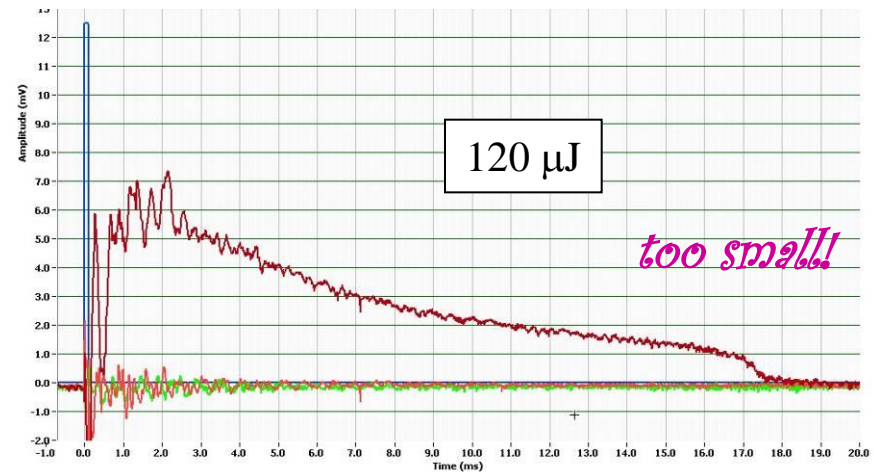


- CERN picture of the internal voltage in an LHC dipole just before a quench
- note the initiating spike - conductor motion?
- after the spike, conductor goes resistive, then it almost recovers
- but then goes on to a full quench
- this disturbance was more than the MQE

Measuring the MQE for a cable

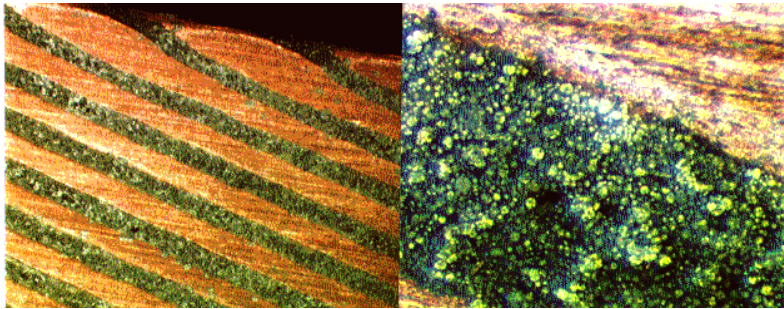


- pass a small pulse of current from the copper foil to the superconducting wire
- generates heat in the carbon paste contact
- how much to quench the cable?
- find the **Minimum Quench Energy MQE**

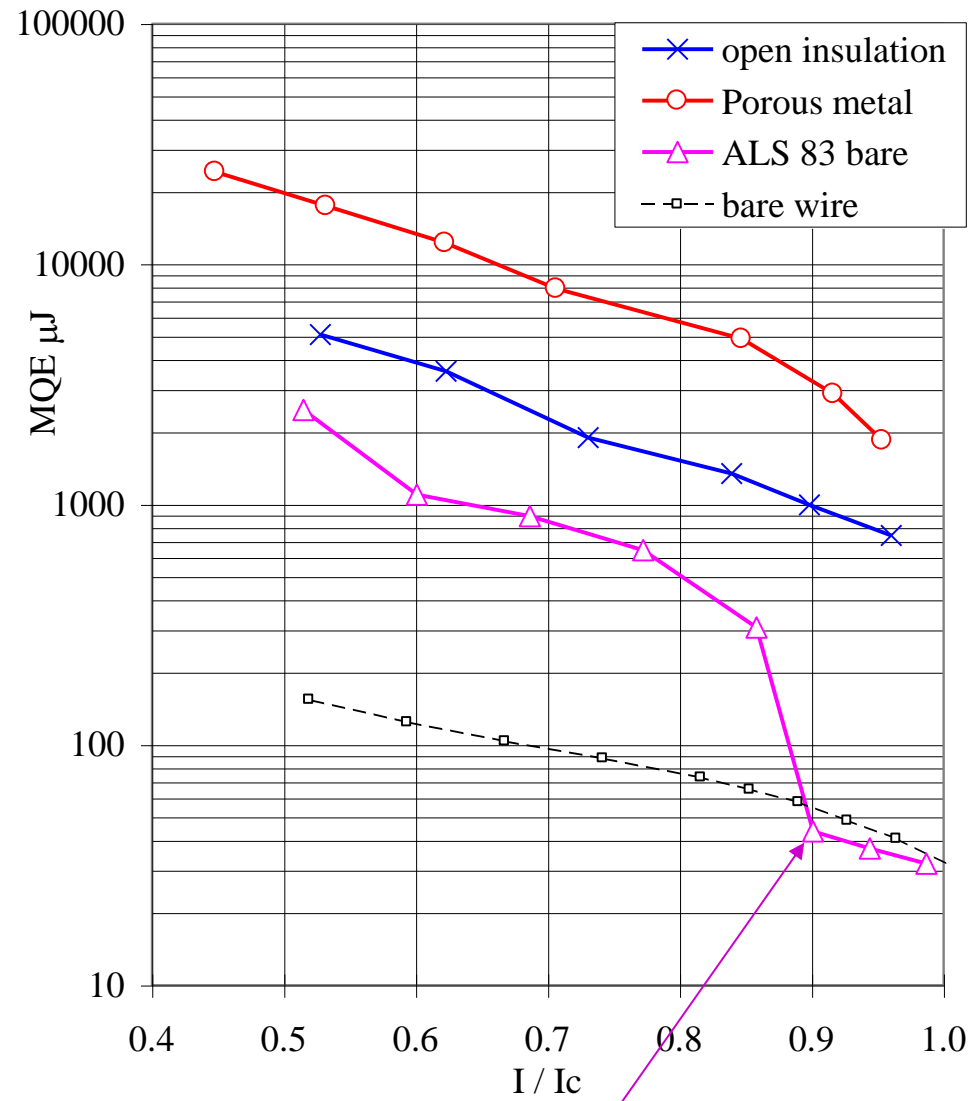
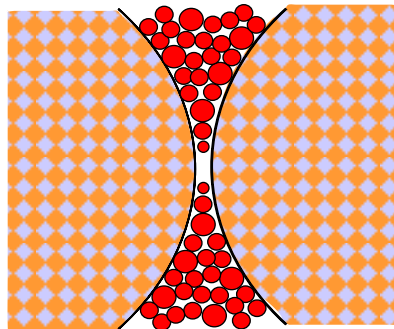


Different cables have different MQEs

- similar cables with different cooling
- better cooling gives higher MQE
- high MQE is best because it is harder to quench the magnet

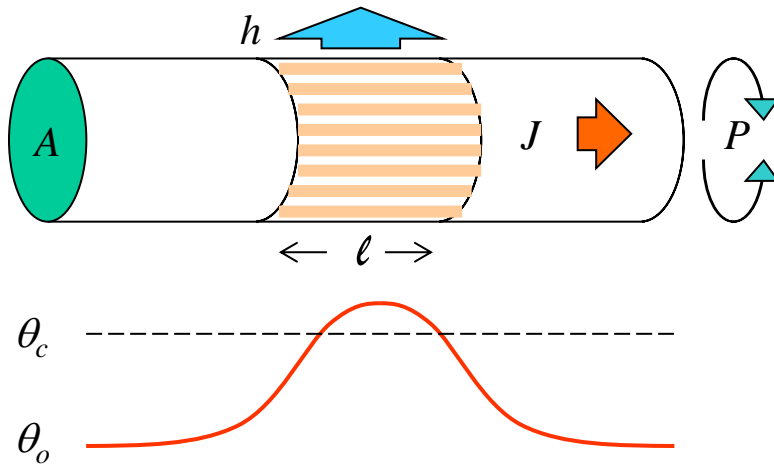


- experimental cable with porous metal heat exchanger
- excellent heat transfer to the liquid helium coolant



40 μJ is a pin dropping 40mm

Factors affecting the Minimum Quench Energy



- heat a short zone of conductor \Rightarrow resistive
- heat conducted out $>$ generation \Rightarrow zone shrinks
- heat conducted out $<$ generation \Rightarrow zone grows
- boundary between the two conditions is the **minimum propagating zone MPZ**
- large MPZ \Rightarrow stability against disturbances

Very approximate heat balance

$$\frac{2kA(\theta_c - \theta_o)}{l} + hPl(\theta_c - \theta_o) = J_c^2 \rho Al$$

so length
of MPZ

$$l = \left\{ \frac{2k(\theta_c - \theta_o)}{J_c^2 \rho - \frac{hP}{A}(\theta_c - \theta_o)} \right\}^{\frac{1}{2}}$$

where: k = thermal conductivity ρ = resistivity A = cross sectional area of conductor
 h = heat transfer coefficient to coolant – if there is any in contact
 P = cooled perimeter of conductor

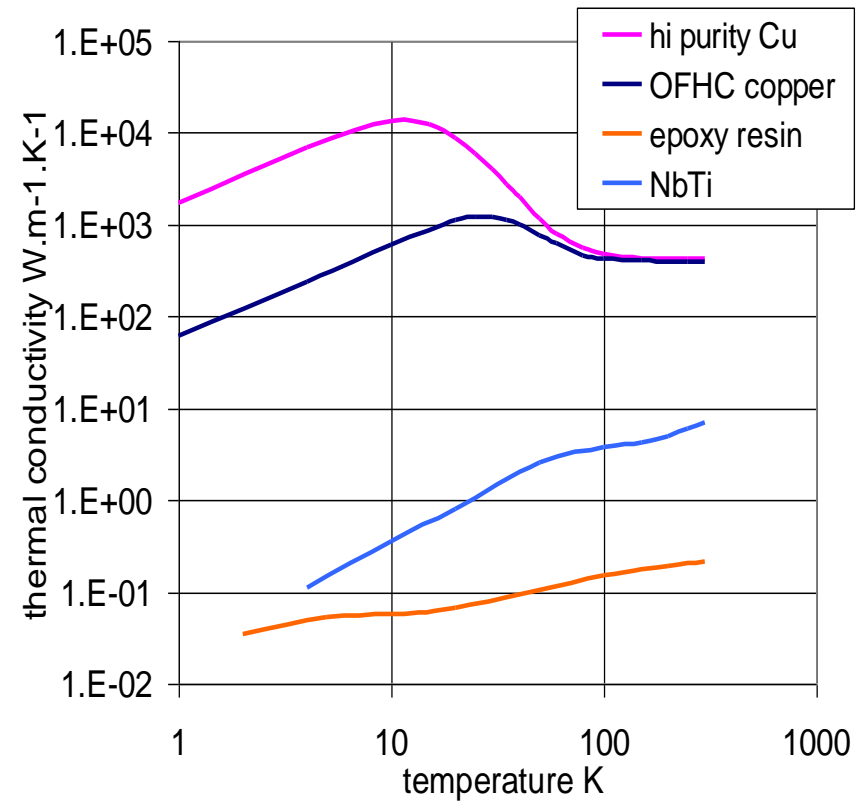
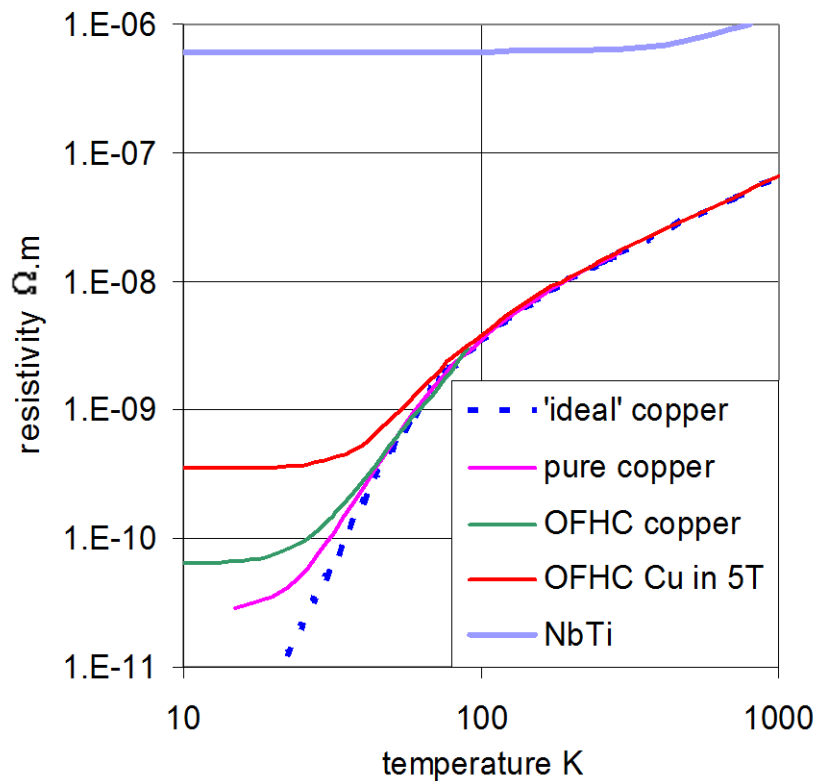
Energy to set up MPZ is the Minimum Quench Energy

long MPZ \Rightarrow large MQE

How to make a long MPZ \Rightarrow large MQE

$$l = \left\{ \frac{2k(\theta_c - \theta_o)}{J_c^2 \rho - \frac{hP}{A}(\theta_c - \theta_o)} \right\}^{\frac{1}{2}}$$

- make thermal conductivity k large
- make resistivity ρ small
- make heat transfer hP/A large (but \Rightarrow low J_{eng})

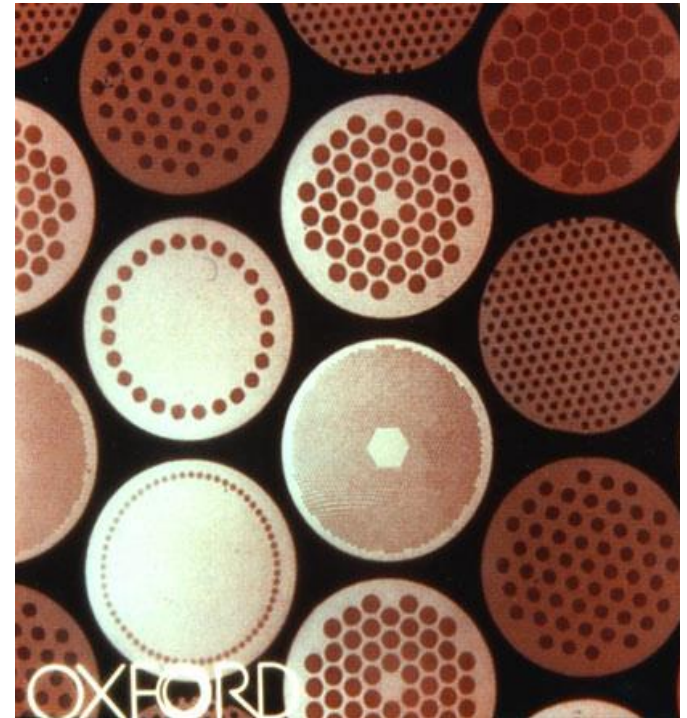


Large MPZ \Rightarrow large MQE \Rightarrow less training

$$l = \left\{ \frac{2k(\theta_c - \theta_o)}{J_c^2 \rho - \frac{hP}{A}(\theta_c - \theta_o)} \right\}^{\frac{1}{2}}$$

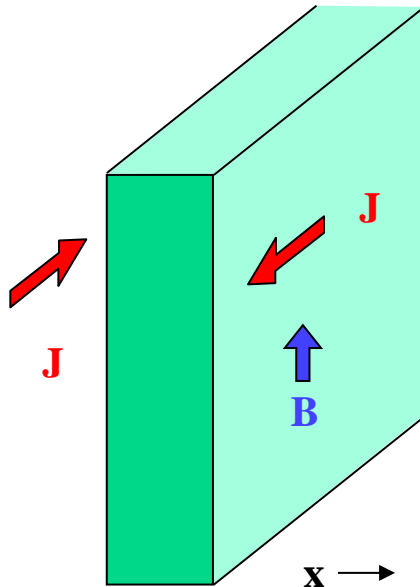
- make thermal conductivity k large
- make resistivity ρ small
- make heat transfer term hP/A large

- NbTi has high ρ and low k
- copper has low ρ and high k
- mix copper and NbTi in a filamentary composite wire
- make NbTi in *fine filaments* for intimate mixing
- maximum diameter of filaments $\sim 50\mu\text{m}$
- make the windings porous to liquid helium
 - superfluid is best
- fine filaments also eliminate flux jumping (see later slides)



Another cause of training: flux jumping

- recap: changing magnetic fields induce screening currents in superconductors
- **screening currents** are in addition to **transport currents**, which come from the power supply
- like eddy currents but don't decay because no resistance,



- usual model is a superconducting slab in a changing magnetic field B_y
- assume it's infinitely long in the z and y directions - simplifies to a 1 dim problem
- dB/dt induces an electric field E which causes screening currents to flow at critical current density J_c
- known as the **critical state model** or **Bean model**
- in the 1 dim infinite slab geometry, Maxwell's equation says

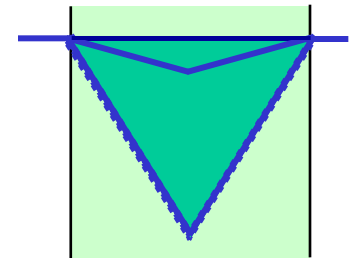
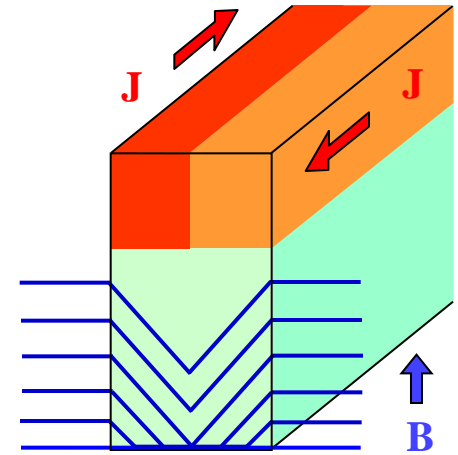
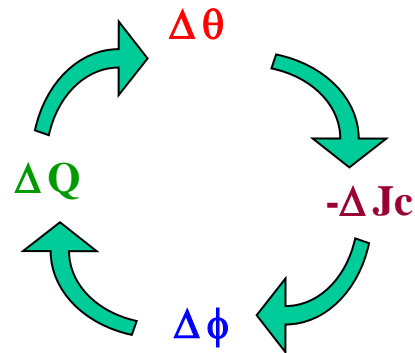
$$\frac{\partial B_y}{\partial x} = -\mu_o J_z = \mu_o J_c$$

- so uniform J_c means a constant field gradient inside the superconductor

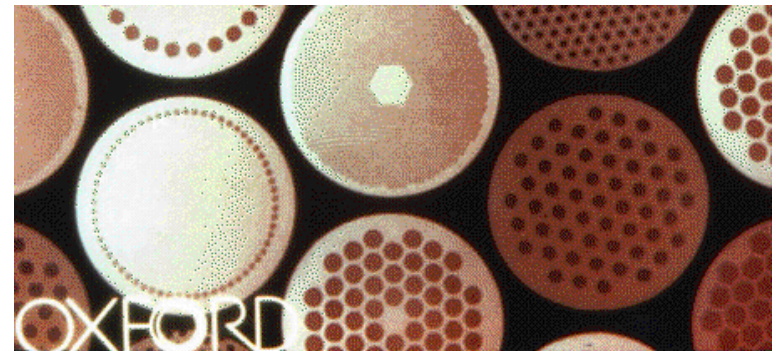
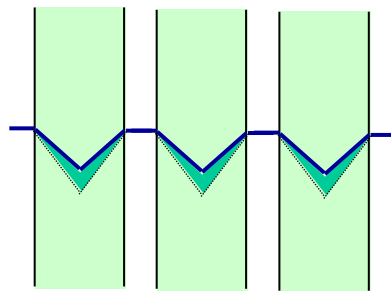
Flux Jumping

a magnetic thermal feedback instability

- screening currents
- temperature rise
- reduced critical current density
- flux motion
- energy dissipation
- temperature rise



- cure flux jumping by weakening a link in the feedback loop
- fine filaments reduce $\Delta \phi$ for a given $-\Delta J_c$
- for NbTi the stable diameter is $\sim 50\mu\text{m}$



Flux jumping: the numbers for NbTi

criterion for
stability against
flux jumping
 a = half width of
filament

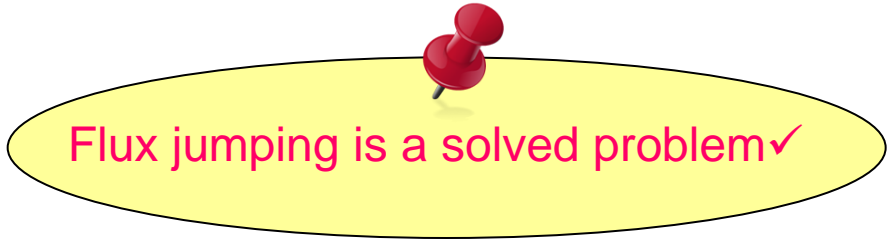
$$a = \frac{1}{J_c} \left\{ \frac{3\gamma C(\theta_c - \theta_o)}{\mu_o} \right\}^{\frac{1}{2}}$$

typical figures for NbTi at 4.2K and 1T
 J_c critical current density = $7.5 \times 10^9 \text{ Am}^{-2}$
 γ density = $6.2 \times 10^3 \text{ kg.m}^{-3}$
 C specific heat = $0.89 \text{ J.kg}^{-1}\text{K}^{-1}$
 θ_c critical temperature = 9.0K

so $a = 33\mu\text{m}$, ie $66\mu\text{m}$ diameter filaments

Notes:

- least stable at low field because J_c is highest
- instability gets worse with decreasing temperature because J_c increases and C decreases
- criterion gives the size at which filament is just stable against infinitely small disturbances
 - still sensitive to moderate disturbances, eg mechanical movement
- better to go somewhat smaller than the limiting size
- in practice $50\mu\text{m}$ diameter seems to work OK



Flux jumping is a solved problem✓

Concluding remarks

- superconducting magnets can make higher fields than conventional because they don't need iron which saturates at 2T - although iron is often used for shielding
- to get different field shapes you have to shape the winding (not the iron)
- practical winding shapes are derived from the ideal overlapping ellipses or $J = J_o \cos \theta$
- engineering current density is important for a compact economic magnet design
- expected magnet performance is given by the intersection of the load line and critical surface
- degraded performance and training are still a problem for magnets - and de-training is worse
- improve training by good winding construction
 - ⇒ no movement, low thermal contraction, no cracking
- improve training by making the conductor have a high MQE
 - temperature margin, high conductivity, good cooling
 - NbTi in good contact with copper ⇒ fine filaments
- changing fields induce screening currents in all superconductors ⇒ flux jumping
- flux jumping did cause degraded magnet performance but fine filaments have now cured it