



## Measurement of longitudinal parameter:

- Definition of longitudinal phase space
- Proton LINAC: Determination of mean energy
- Determination of longitudinal emittance
- Bunch length measurement for non-relativistic beams
- Bunch length measurement for relativistic beams
- Summary

## Longitudinal ↔ transverse correspondences:

- position relative to rf ↔ transverse center-of-mass
- bunch structure in time ↔ transverse profile in horizontal and vertical direction
- momentum or energy spread ↔ transverse divergence
- longitudinal emittance ↔ transverse emittance.

# Measurement of longitudinal Parameters



The longitudinal dynamics is described by the longitudinal emittance as given by:

➤ Spread of the bunches  $l$   
in time, length *or* rf-phase.

➤ Momentum spread  $\delta = \Delta p/p$ ,  
or energy spread  $\Delta W/W$

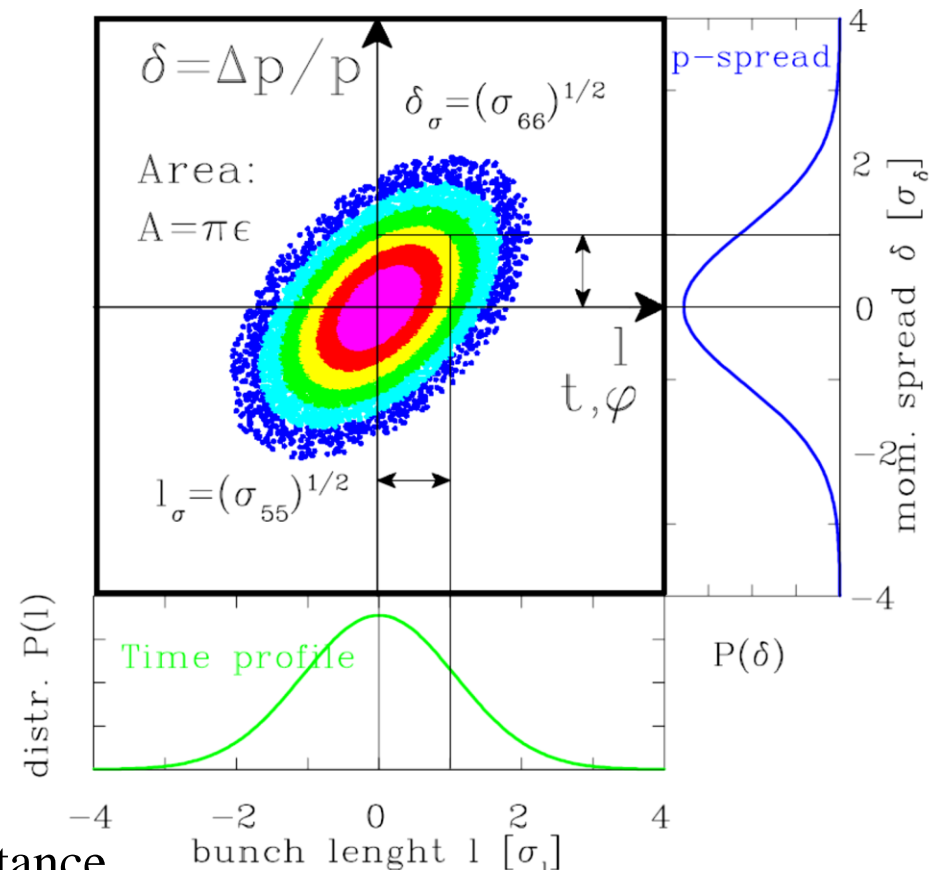
$$\Rightarrow \varepsilon_{long} = \frac{1}{\pi} \int_A dl \cdot d\delta$$

The normalized value is preserved:

$$\varepsilon_{long}^{norm} = \beta\gamma \cdot \varepsilon_{long}$$

## Discussed devices:

- Pick-ups for bunch length and emittance.
- Special detectors (low  $E_{kin}$  protons), streak cameras & ele.-optical modulation ( $e^-$ )



# The Bunch Position measured by a Pick-Up

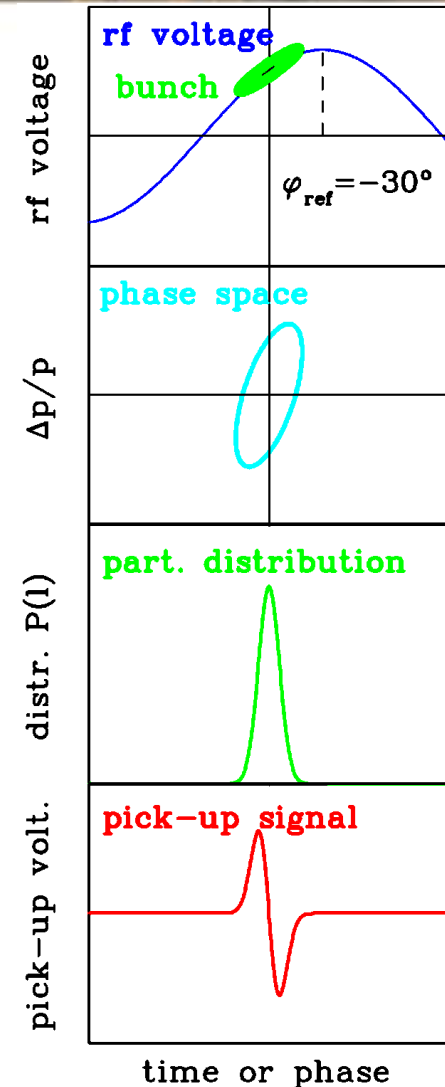
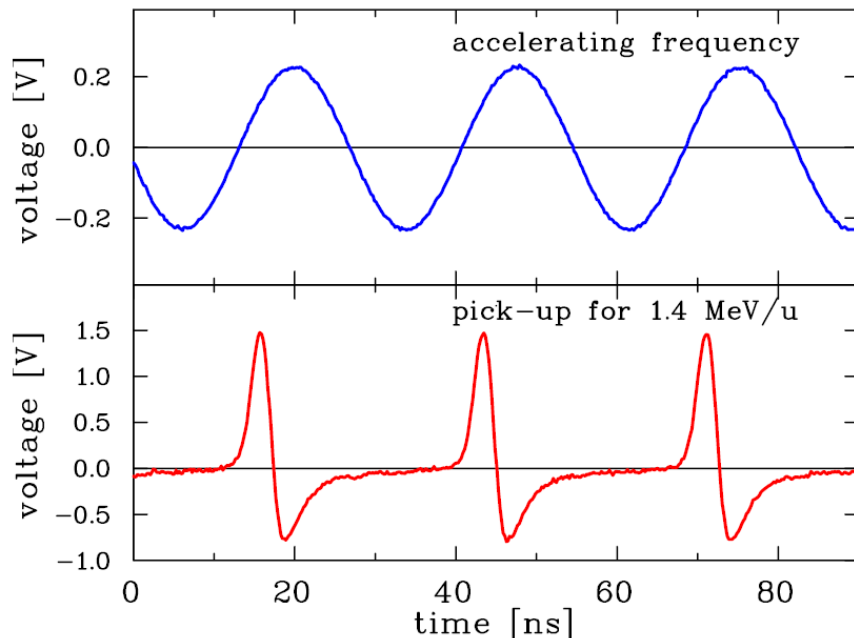
The *bunch position* is given relative to the accelerating rf.

e.g.  $\varphi_{ref} = -30^\circ$  inside a rf cavity

must be well aligned for optimal acceleration

Transverse correspondence: Beam position

**Example:** Pick-up signal and 36 MHz rf at GSI-LINAC:





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# Determination of non-relativistic mean Energy using Pick-Ups



The energy delivered by a LINAC is sensitive to the mechanics, rf-phase and amplitude.

For non-relativistic energies at proton LINACs time-of-flight (TOF) with two pick-ups is used:

$$\beta c = \frac{L}{NT + t_{\text{scope}}}$$

→ the velocity  $\beta$  is measured.

**Example:** Time-of-flight signal from two pick-ups at 1.4 MeV/u:

The reading is  $t_{\text{scope}} = 15.82(5)\text{ns}$  with

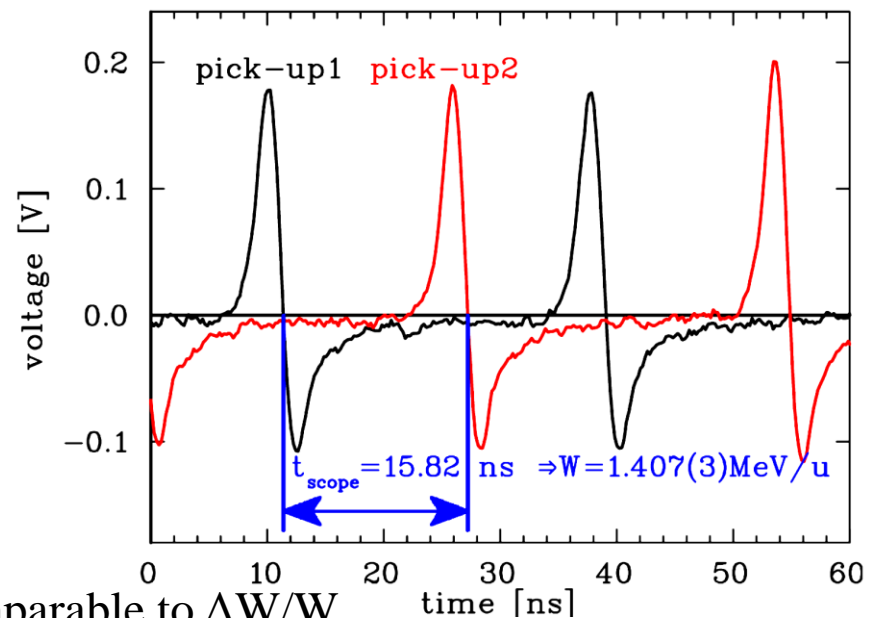
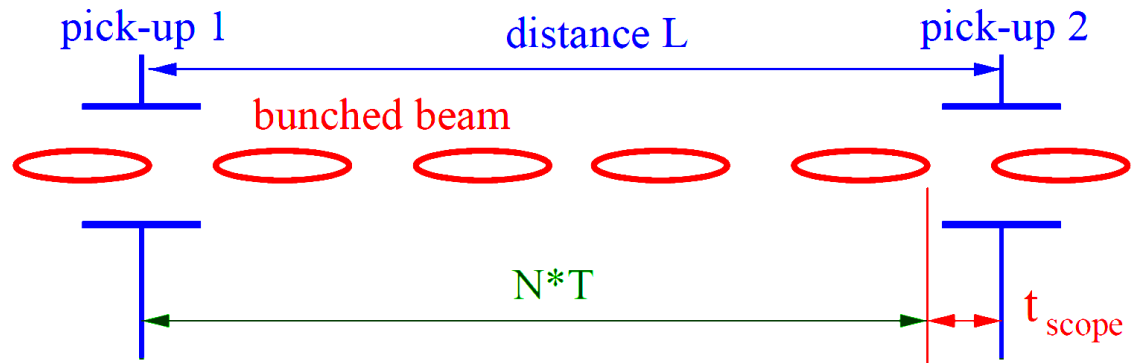
$f_{\text{rf}} = 36.136\text{MHz} \Leftrightarrow T = 27.673\text{ns}$

$L = 1.629\text{ m}$  and  $N = 3$

$\Rightarrow \beta = 0.05497(7)$

$\Rightarrow W = 1.407(3)\text{ MeV/u}$

The accuracy is typically 0.1 % i.e. comparable to  $\Delta W/W$



# Precision of TOF Measurement for non-relativistic Energy



The precision of TOF is given by the accuracy in time and distance reading:

$$\frac{\Delta\beta}{\beta} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t}{NT + t_{\text{scope}}}\right)^2}$$

Accuracy of scope reading  $\Delta t \approx 100$  ps, uncertainty in distance  $\Delta L \approx 1$  mm.

**Example:** GSI-LINAC:  $L = 3.25$  m and  $f_{rf} = 36$  MHz:

location (GSI-slang)		RFQ	IH1	IH2	AL4
energy $W$	[MeV/u]	0.12	0.75	1.4	11.4
velocity $\beta$	%	1.6	4.0	5.5	15.5
total TOF	[ns]	677	271	197	70
bunch spacing $\beta c / f_{rf}$	[cm]	13	33	45	129
Number of bunches $N$		25	9	7	2
resolution $\Delta W / W$	%	0.07	0.10	0.12	0.22

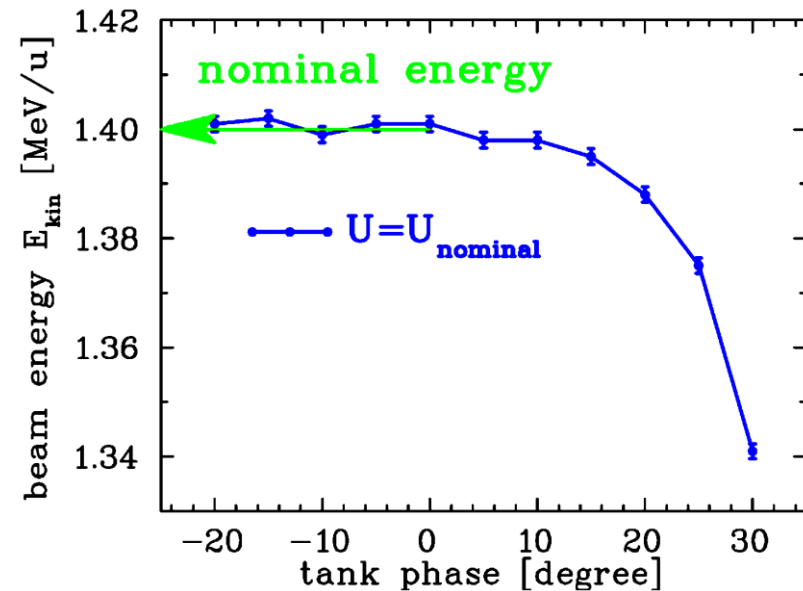
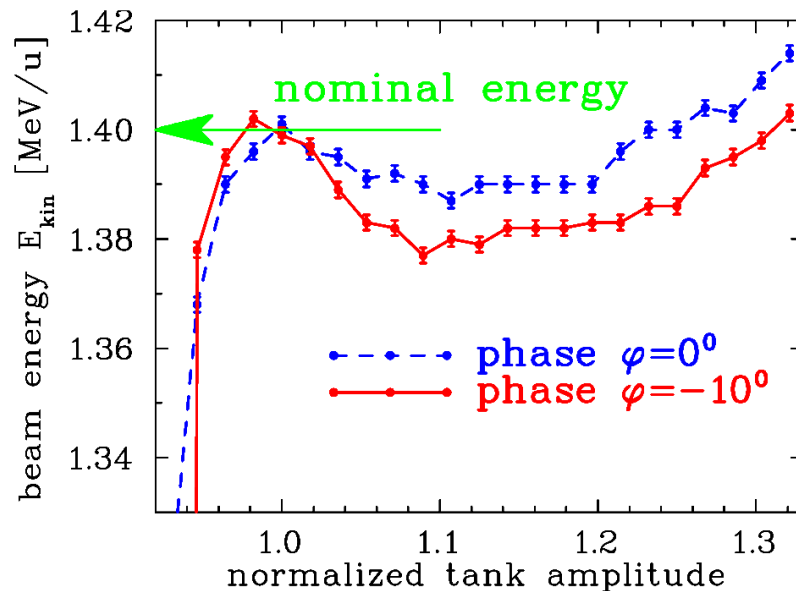
- The accuracy is typically 0.1 % (same order of magnitude as  $\Delta W / W$ )
- The length has to be matched to the velocity
- Due to the distance of  $\approx 3$  m, different solutions for the # of bunches  $N$  are possible  
→ A third pick-up has to be installed closed by, to get an unique solution.

# Cavity Alignment using a TOF Measurement



The mean energy is important for the matching between LINAC module. It depends on phase and amplitude of the rf wave inside the cavities.

**Example:** Energy at GSI LINAC (nominal energy 1.400 MeV/u):  
(distance between pick-ups:  $L = 1.97$  m  $\Rightarrow N = 4$  bunches)



➤ **Proton LINACs:** Amplitude and phase should be carefully aligned by precise TOF

➤ **Electron LINACs:** Due to relativistic velocity, TOF is not applicable.





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LINAC: variation of bunch length  
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# Longitudinal Emittance by linear Transformation using a Buncher



## Longitudinal focusing:

Variation of the bunch shape by a rf-buncher

→ components 5 and 6 from 6-dim phase-space

Transversal corres.: Quadrupole variation

➤ Transfer matrix of buncher & drift:

$$R_{buncher} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, R_{drift} = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

with focal length:  $1/f = \frac{2\pi f_{rf}}{Apv^2} \cdot U$

➤ Variation of buncher amplitude  $U$

⇒ different bunch width at  $s_I$ :

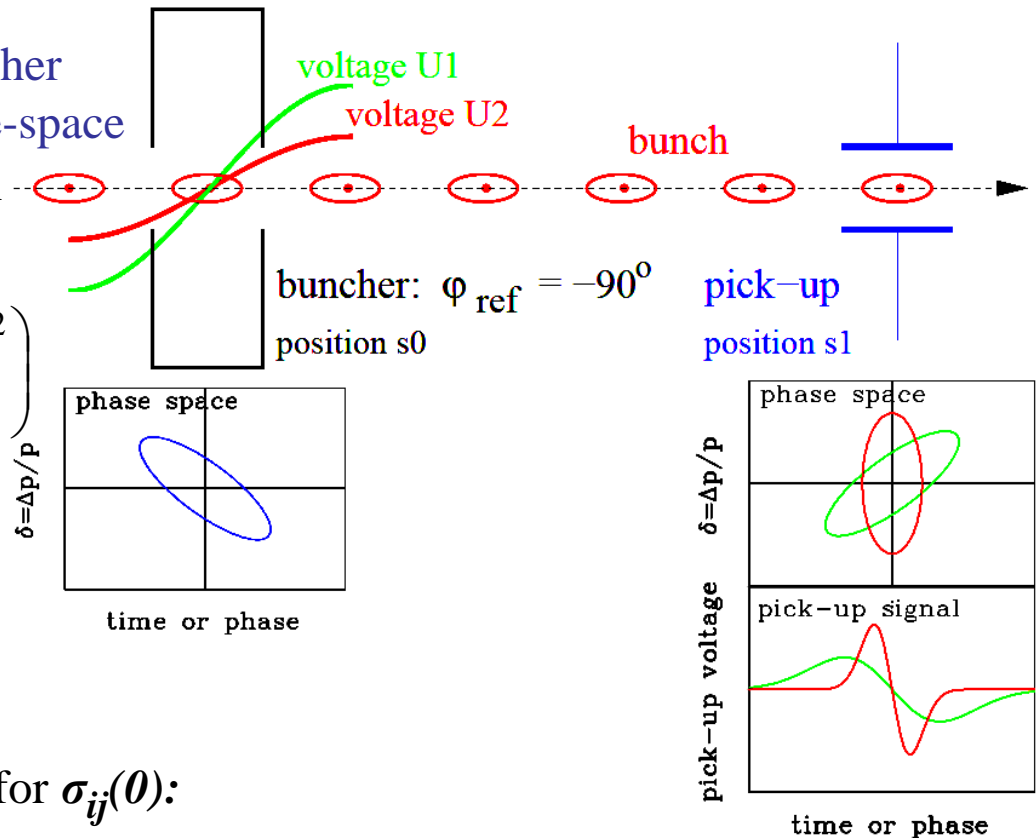
beam matrix  $\Delta t_{max}^2 = \sigma_{55}(1, f)$

➤ System of redundant linear equations for  $\sigma_{ij}(0)$ :

$$\sigma_{55}(1, f_1) = R_{55}^2(f_1) \cdot \sigma_{55}(0) + 2R_{55}(f_1)R_{56}(f_1) \cdot \sigma_{56}(0) + R_{56}^2(f_1) \cdot \sigma_{66}(0) \quad \text{focusing } f_1$$

:

$$\sigma_{55}(n, f_n) = R_{55}^2(f_n) \cdot \sigma_{55}(0) + 2R_{55}(f_n)R_{56}(f_n) \cdot \sigma_{56}(0) + R_{56}^2(f_n) \cdot \sigma_{66}(0) \quad \text{focusing } f_n$$

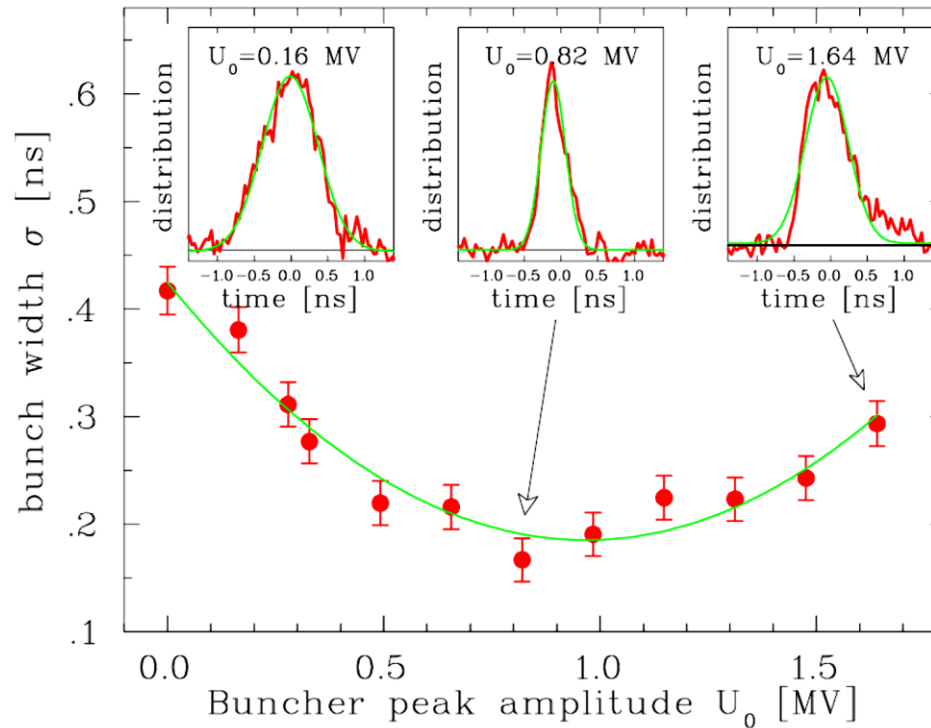


# Result of a longitudinal Emittance Measurement



## *Example* GSI LINAC:

The voltage at the single gap -resonator is varied for 11.4 MeV/u Ni<sup>14+</sup> beam, 31 m drift:

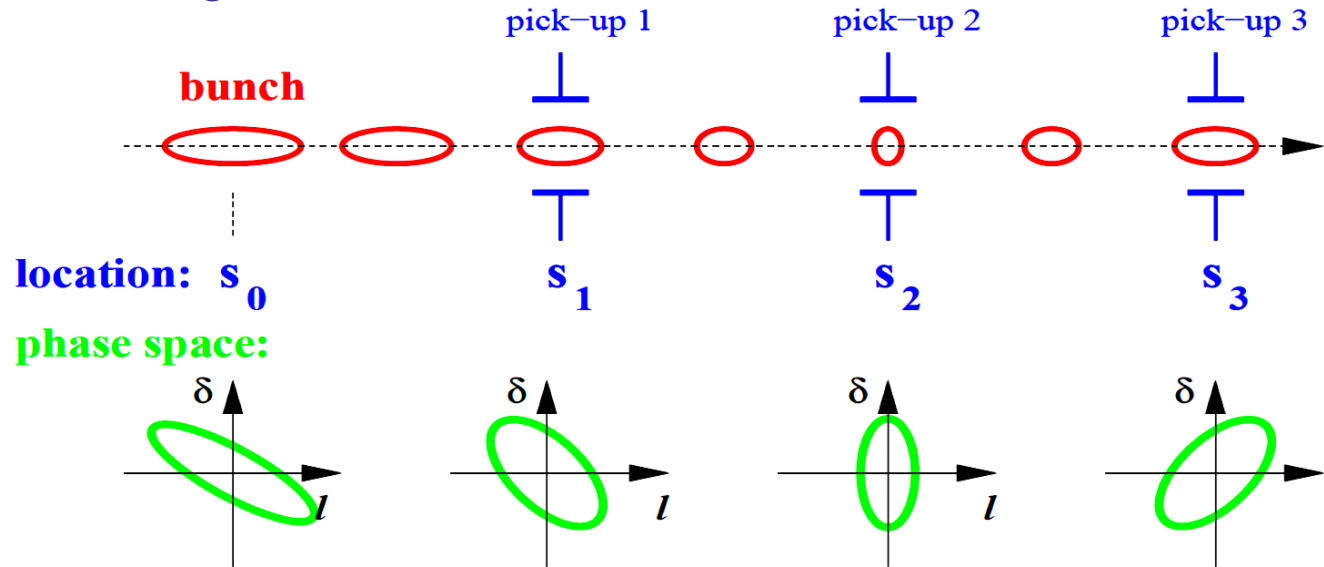


- The structure of short bunches can be determined with special monitor
- This example: The resolution is better than 50 ps or  $2^\circ$  for 108 MHz
- Typical bunch length at proton LINACs: 30 to 200 ps

# Longitudinal Emittance within a Transfer-Line



As for the 'three grid' method, the emittance can be determined in a transfer line.



The system of redundant linear equations with the transfer matrix  $\mathbf{R}(\mathbf{i})$  to location  $s_i$ :

$$\begin{aligned} \sigma_{55}(1) &= R_{55}^2(1) \cdot \sigma_{55}(0) + 2R_{55}(1)R_{56}(1) \cdot \sigma_{56}(0) + R_{56}^2(1) \cdot \sigma_{66}(0) & \mathbf{R}(1) : s_0 \rightarrow s_1 \\ &: \\ \sigma_{55}(n) &= R_{55}^2(n) \cdot \sigma_{55}(0) + 2R_{55}(n)R_{56}(n) \cdot \sigma_{56}(0) + R_{56}^2(n) \cdot \sigma_{66}(0) & \mathbf{R}(n) : s_0 \rightarrow s_n \end{aligned}$$

- Assumptions:**
- Bunches much longer than pick-up or relativistic E -field:  $E_{\perp} \gg E_{\parallel}$
  - Gaussian distribution without space-charge effects.

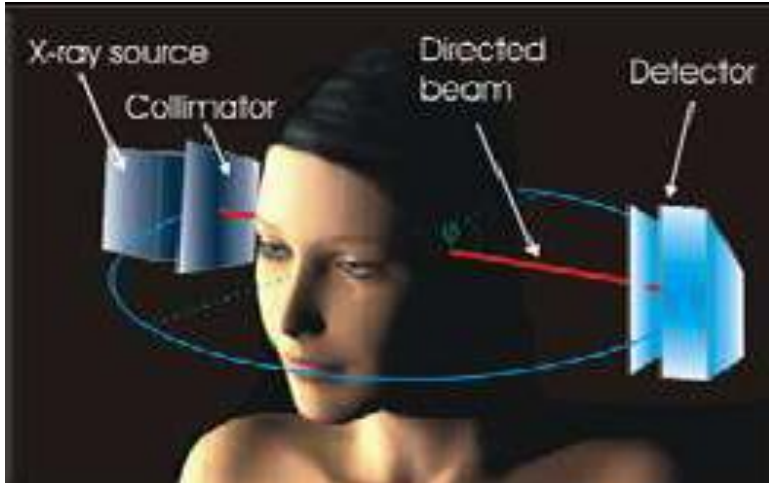
# Longitudinal Emittance using tomographic Reconstruction



Tomography is medical image method

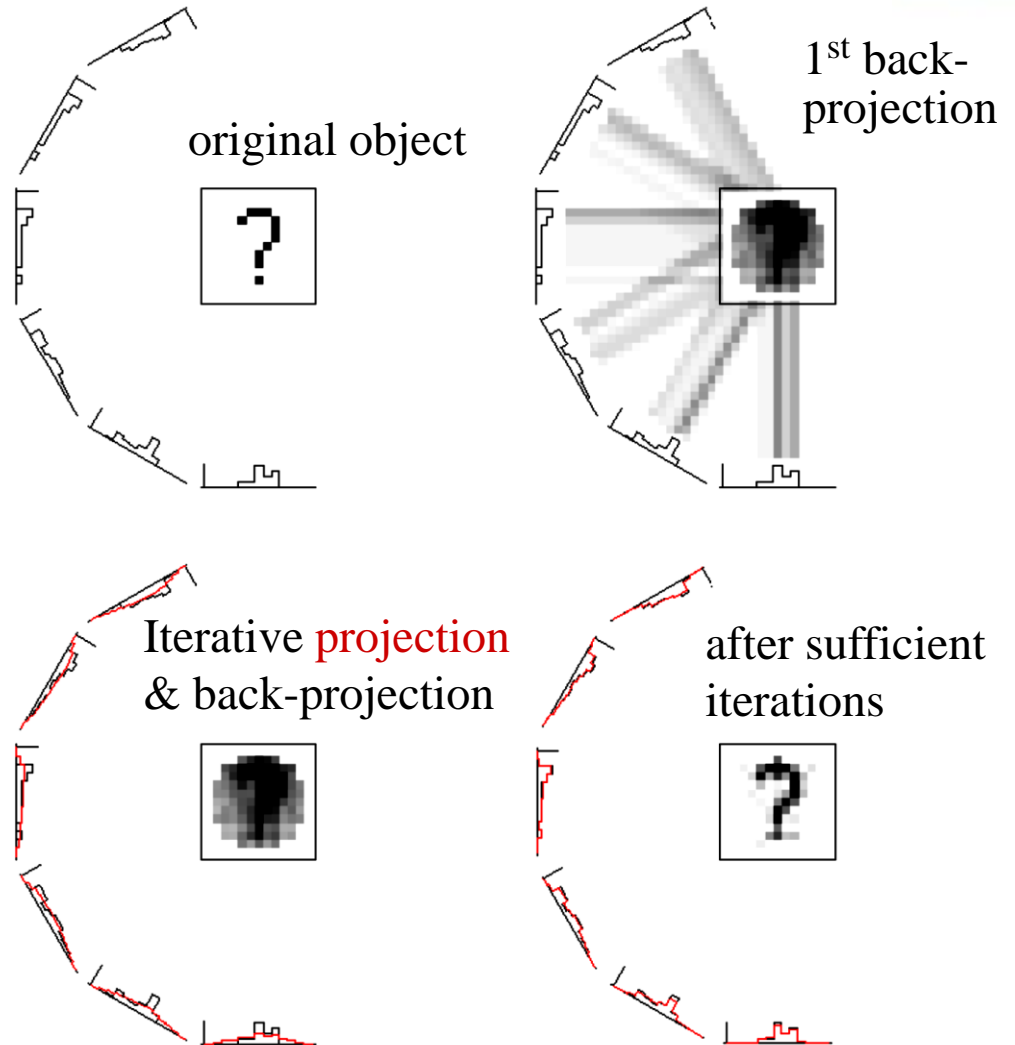
Tomography:

2-dim reconstruction of  
sufficient 1-dim projections



Algebraic back projection:

Iterative process by redistributing  
the 2-dim image and considering the  
differences to the previous iteration step.



# Longitudinal Emittance using tomographic Reconstruction



Tomography is medical image method

**Tomography:**

2-dim reconstruction of  
sufficient 1-dim projections

**Application at accelerators:**

Longitudinal emittance  
evolution in synchrotrons.

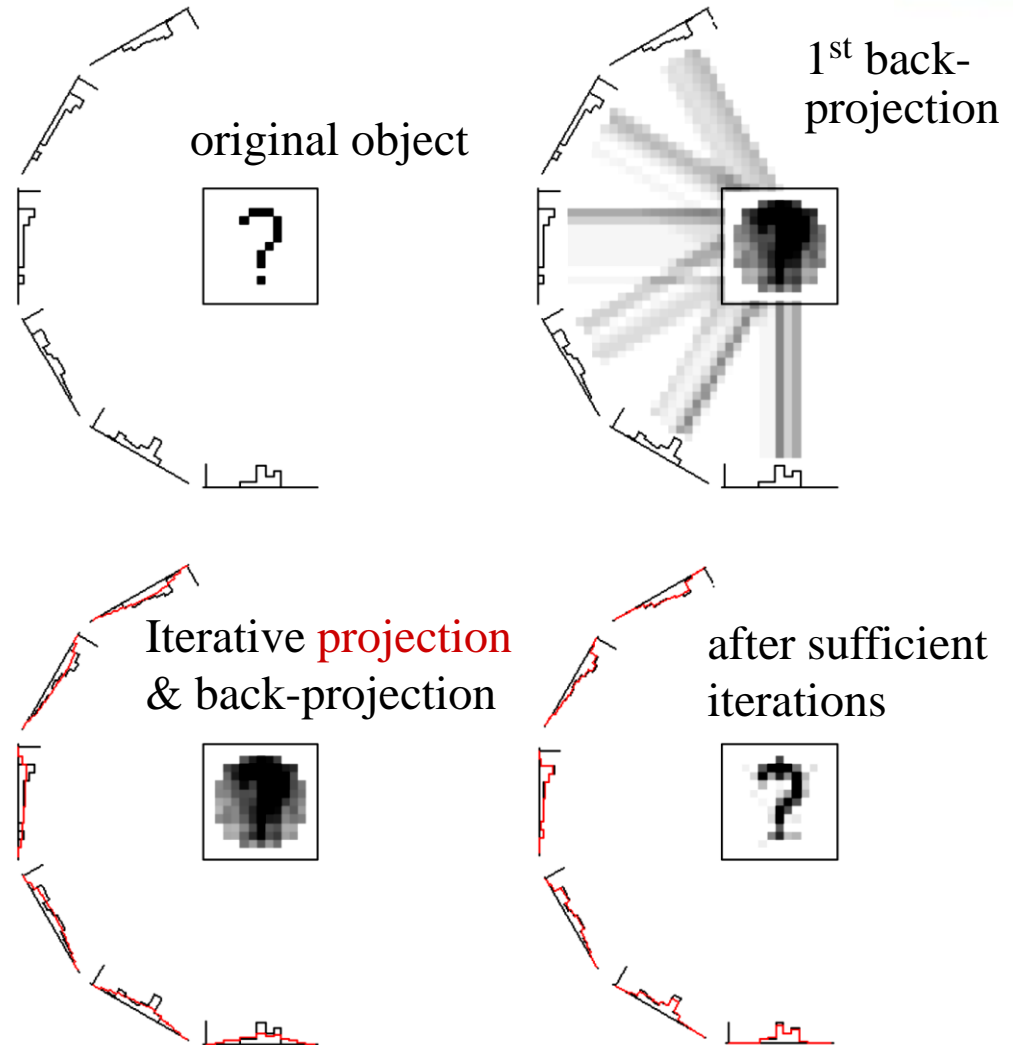
**Bunch observation:**

Each revolution,  
the bunch shape changes a bit  
due to synchrotron oscillations.

Fulfilled condition:  $f_{synch} \ll f_{ref}$ .

**Algebraic back projection:**

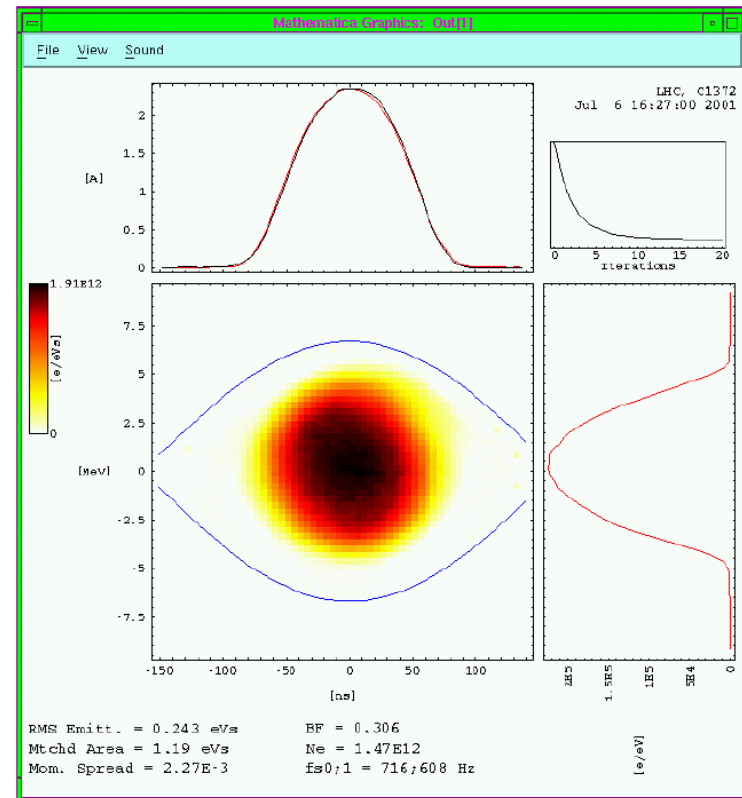
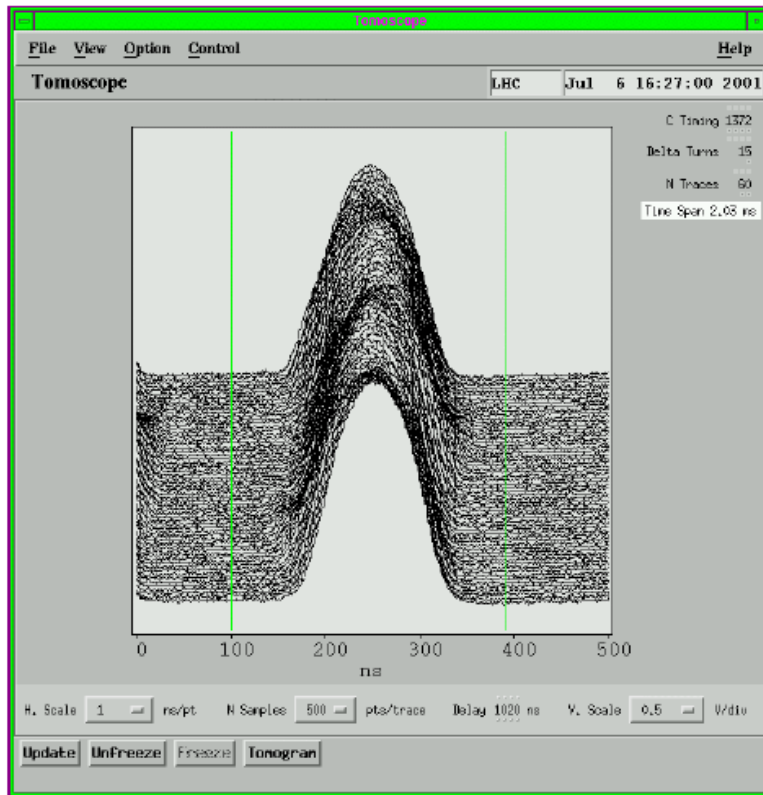
Iterative process by redistributing  
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# Results of tomographic Reconstruction at a Synchrotron I

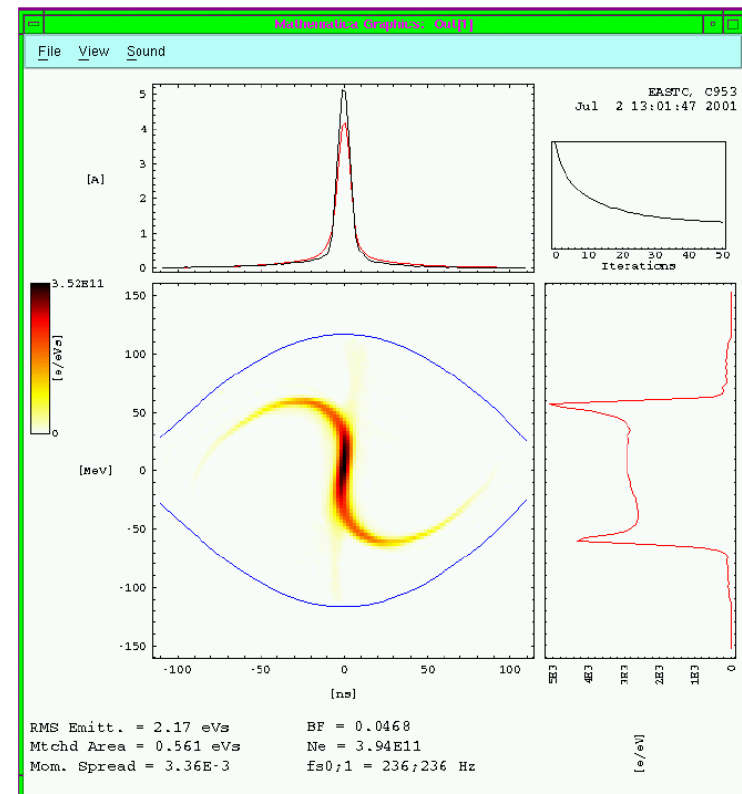
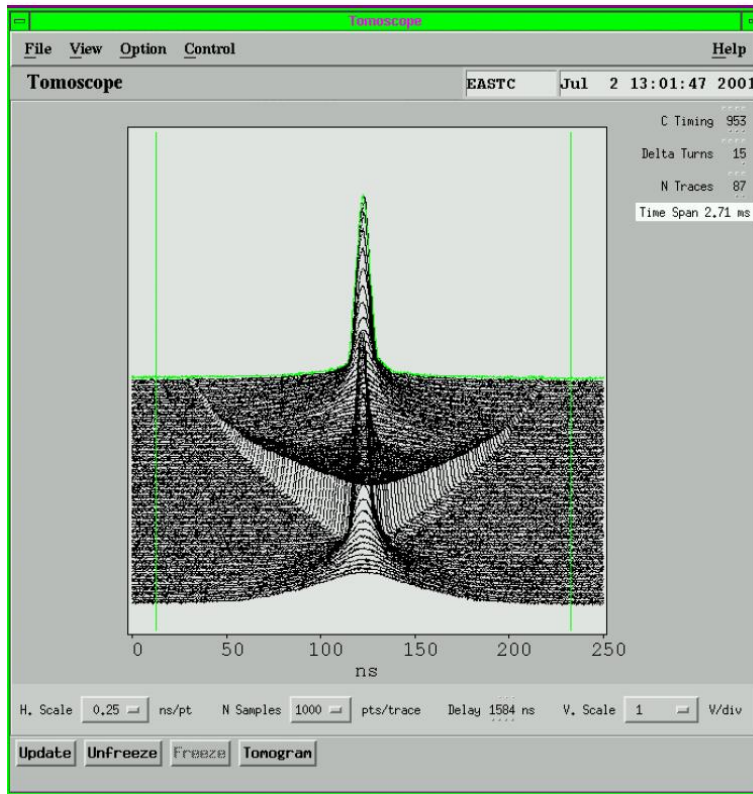
Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:



**Typical bucket filling. Important knowledge for bunch 'gymnastics'.**

# Results of tomographic Reconstruction at a Synchrotron II

Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:



**Mismatched bunch shown oscillations and filamentation due to ‘bunch-rotation’.**



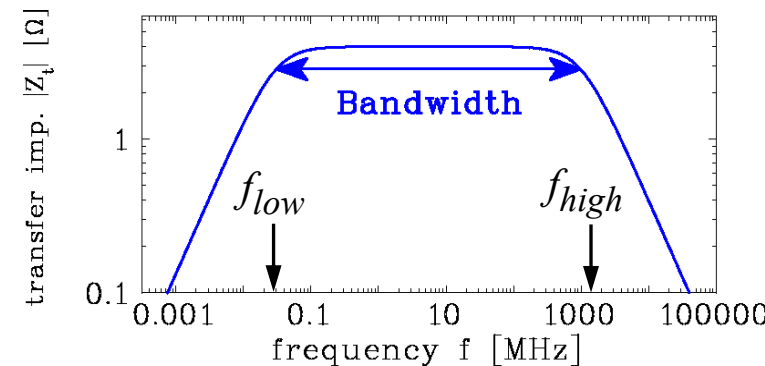
# Resistive Wall Current Monitor

Broadband observation of bunches can be performed with a resistive Wall Current Monitor

- Principle:**
- Ceramic gap bridged with  $n = 10 \dots 100$  resistors of  $R = 10 \dots 100 \, \Omega$
  - Measurement of voltage drop for  $R_{tot} = R/n = 1 \dots 10 \, \Omega$
  - Ferrite rings with high  $L \rightarrow$  forces low frequency components through resistors

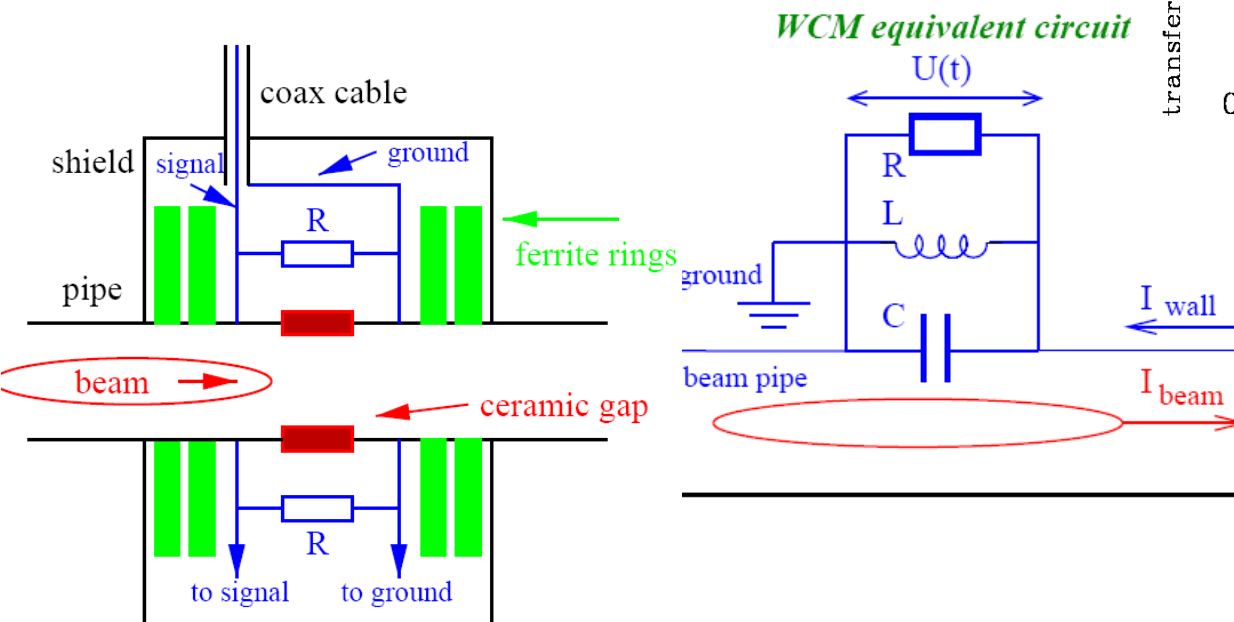
**Bandwidth:** typically  $f_{low} = R_{tot}/(2\pi L) \approx 10 \, \text{kHz}$   
 $f_{high} = 1/(2\pi R_{tot}C) \approx 1 \, \text{GHz}$

**Application:** Broadband bunch observation.



$$\frac{1}{Z_t} = \frac{1}{R_{tot}} + \frac{1}{i\omega L} + i\omega C$$

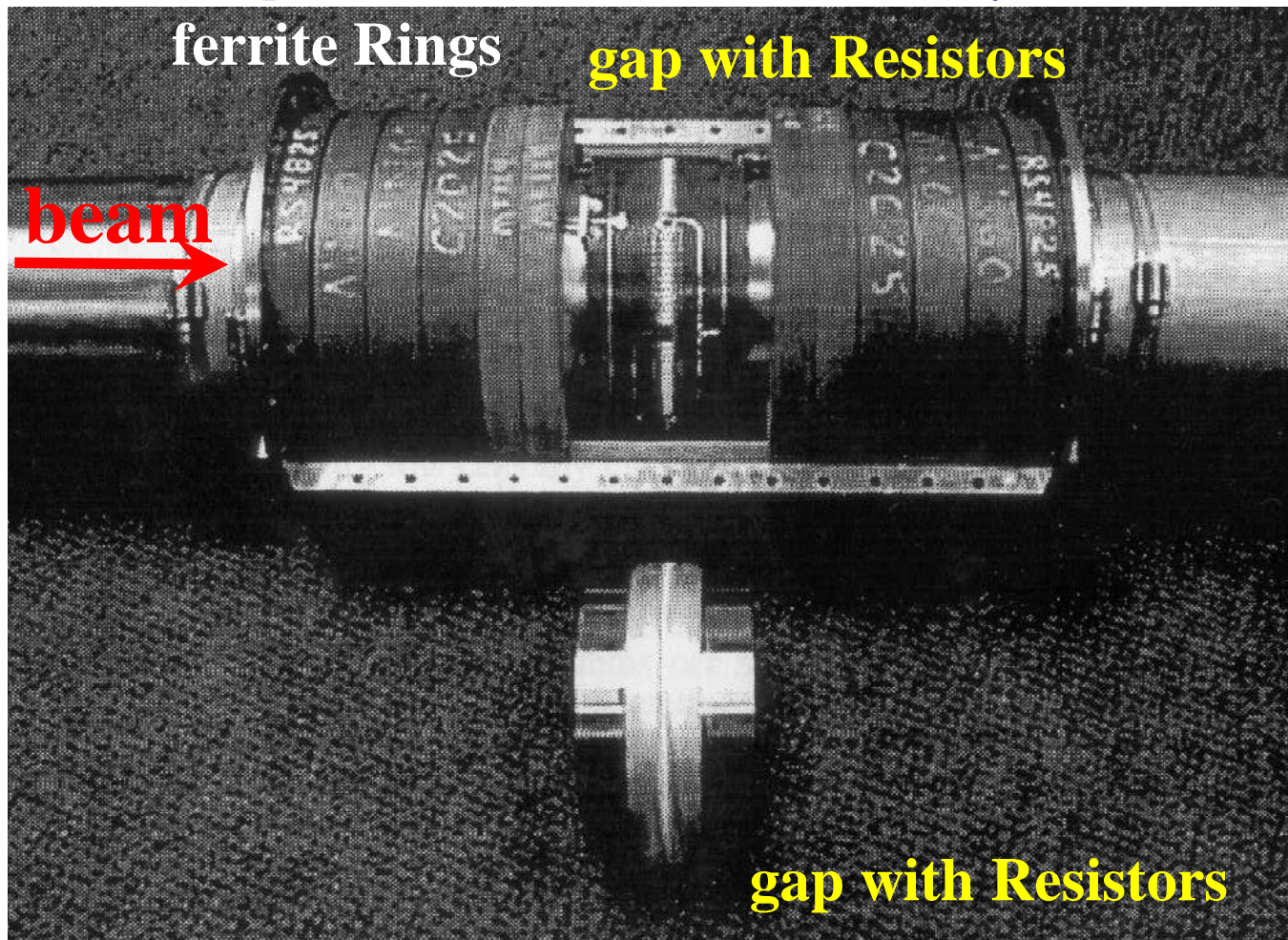
Within bandwidth:  $Z_t \cong R_{tot}$



# Resistive Wall Current Monitor



*Example:* Realization at Fermi-Laboratory





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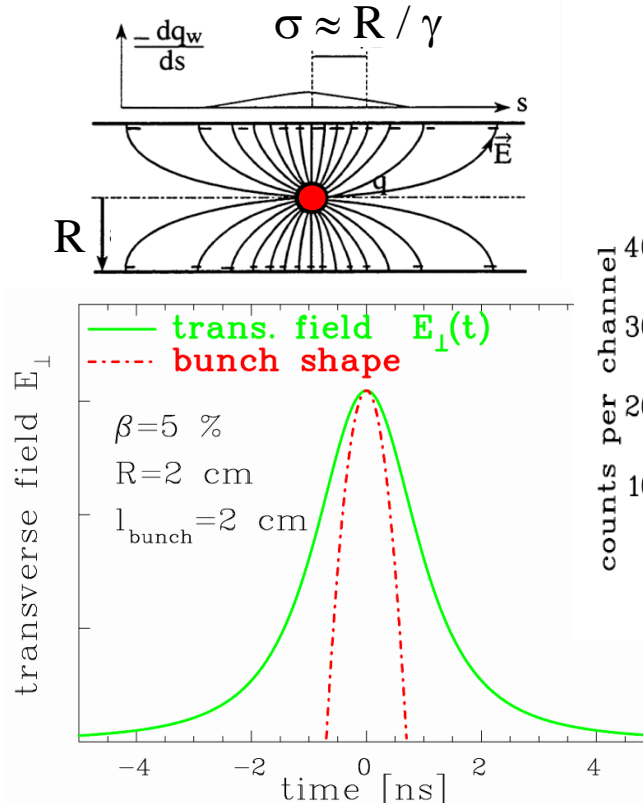
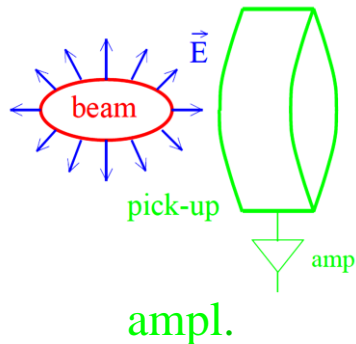
# Bunch Structure at low $E_{kin}$ : Not possible with Pick-Ups

## Pick-ups are used for:

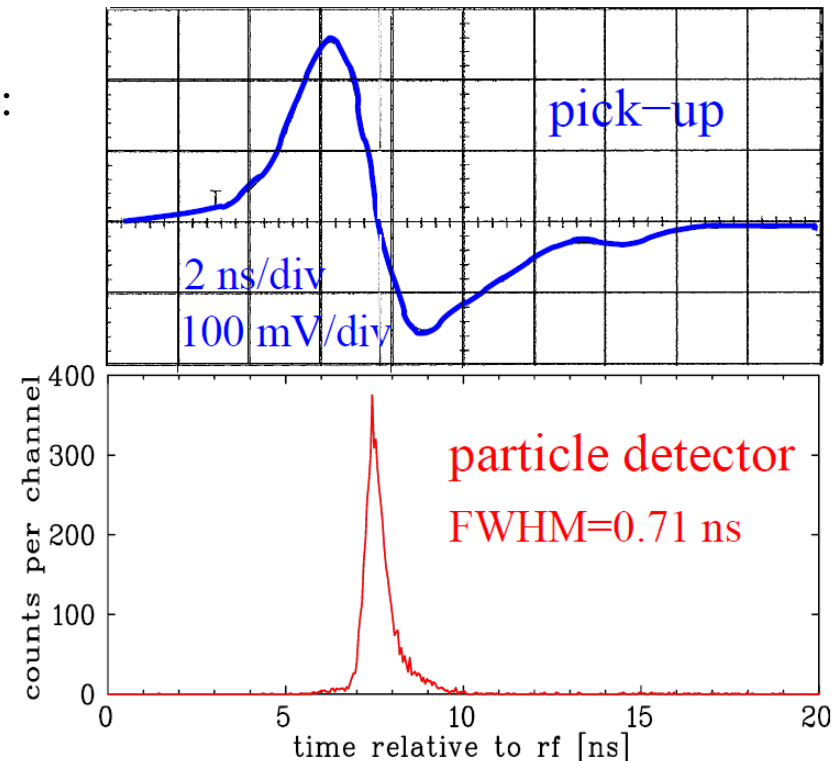
- precise for bunch-center relative to rf
- course image of bunch shape

## But:

For  $\beta \ll 1 \rightarrow$  long.  $E$ -field significantly modified:



*Example:* Comparison pick-up – particle counter:  
 $\text{Ar}^{1+}$  with 1.4 MeV/u ( $\beta = 5.5\%$ )



$\Rightarrow$  the pick-up signal is insensitive to bunch 'fine-structure'

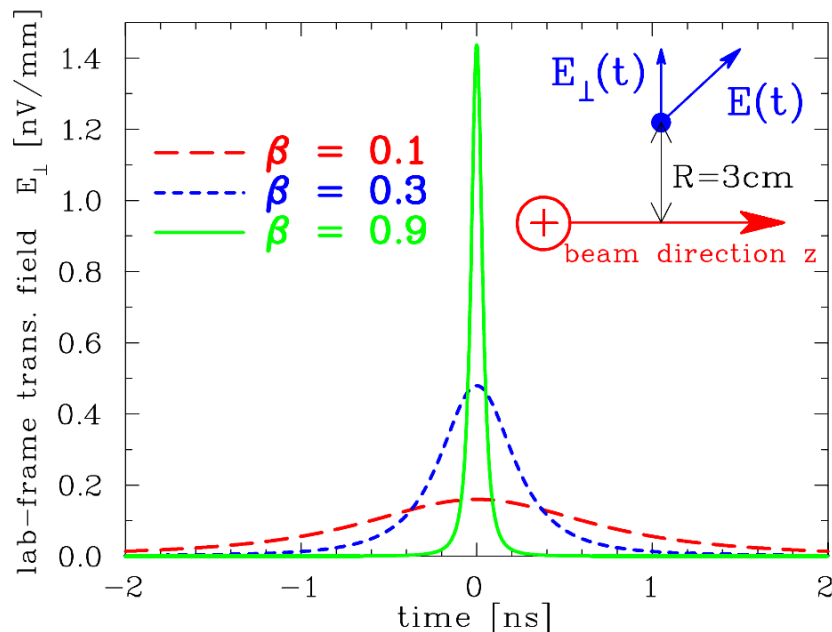
# Low Velocity Effect: General Consideration

Lorentz transformation of single point-like charge:

Lorentz boost *and* transformation of time:  $E_{\perp}(t) = \gamma \cdot E'_{\perp}(t')$  and  $t \rightarrow t'$

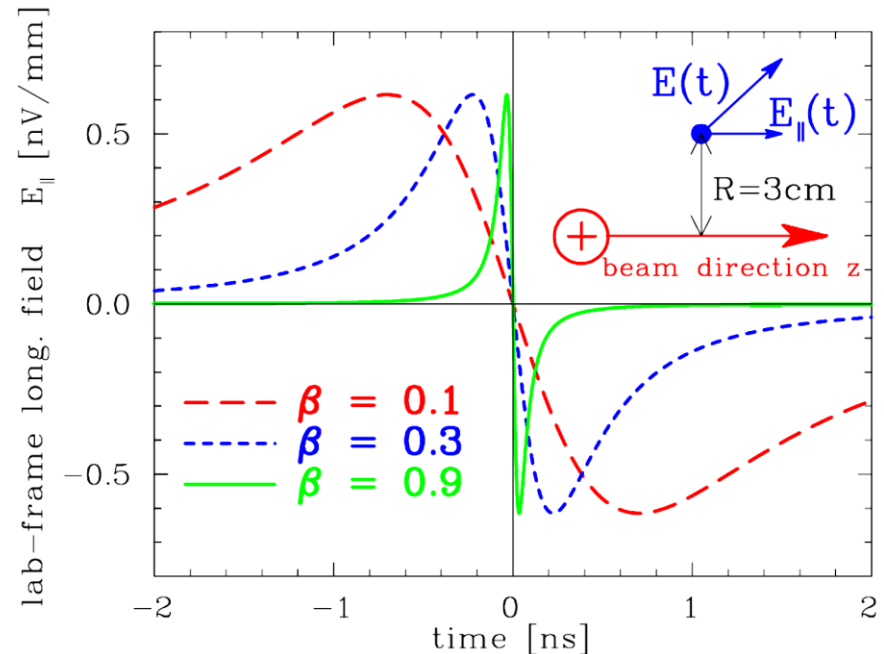
Trans.  $E_{\perp}$  lab.-frame of a point charge:

$$E_{\perp}(t) = \frac{e}{4\pi\epsilon_0} \cdot \frac{\gamma R}{\left[R^2 + (\gamma\beta ct)^2\right]^{3/2}}$$



Long.  $E_{\parallel}$  lab.-frame of a point charge:

$$E_{\parallel}(t) = -\frac{e}{4\pi\epsilon_0} \cdot \frac{\gamma\beta ct}{\left[R^2 + (\gamma\beta ct)^2\right]^{3/2}}$$



# Broadband coaxial Faraday Cups for Bunch Structure



The bunch structure can be observed with cups, having a bandwidth up to several GHz.

Bandwidth and rise time:  $\text{BW [GHz]} = 0.3/t_{\text{rise}}[\text{ns}]$

Impedance of a  
coaxial transmission line:

$$Z_0 = \frac{Z_c}{2\pi} \cdot \ln \frac{r_{\text{outer}}}{r_{\text{inner}}}$$

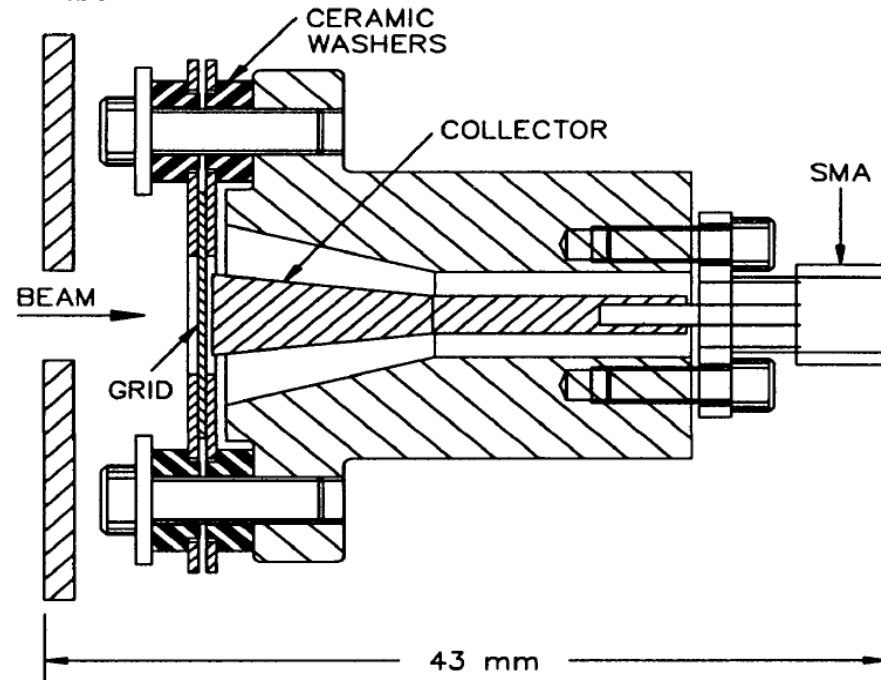
$$\text{with } Z_c = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

→ impedance matching to prevent  
for reflections

$$\text{Voltage reflection: } \rho_V = \frac{Z - Z_0}{Z + Z_0}$$

$$\text{Voltage Standing Wave Ratio: } \text{VSWR} = \frac{Z}{Z_0} = \frac{1 + \rho_V}{1 - \rho_V}$$

$Z = Z_0$ : no reflection.  $Z = 0 \Rightarrow \rho_V = -1$ : short circuit.  $Z = \infty \Rightarrow \rho_V = 1$ : open circuit.



# Realization of a Broadband coaxial Faraday Cup





# Bunch Structure using secondary Electrons for low $E_{\text{kin}}$ Protons

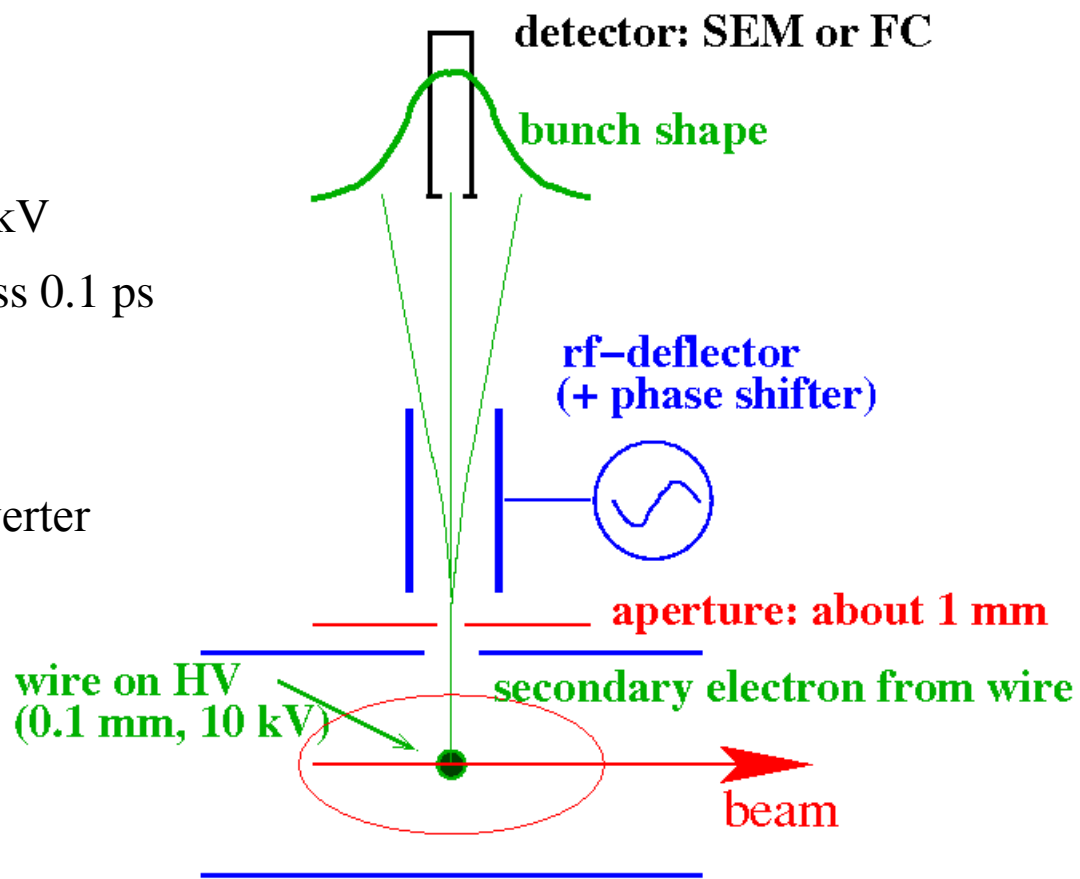


Secondary  $e^-$  liberated from a wire carrying the time information.

→ Bunch Shape Monitor (BSM)

## Working principle:

- insertion of a 0.1 mm wire at  $\approx 10$  kV
- emission of secondary  $e^-$  within less 0.1 ps
- secondary  $e^-$  are accelerated
- toward an rf-deflector
- rf-deflector as 'time-to-space' converter
- detector with a thin slit
- slow shift of the phase
- resolution  $\approx 1^\circ < 10$  ps
- Measurements are comparable to that obtained with particle detectors.

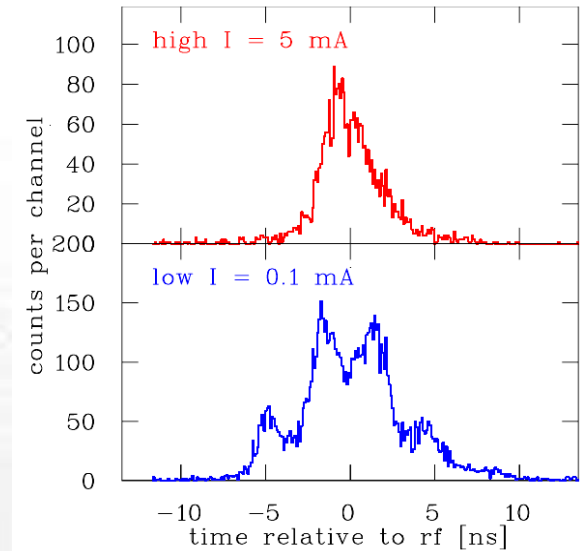
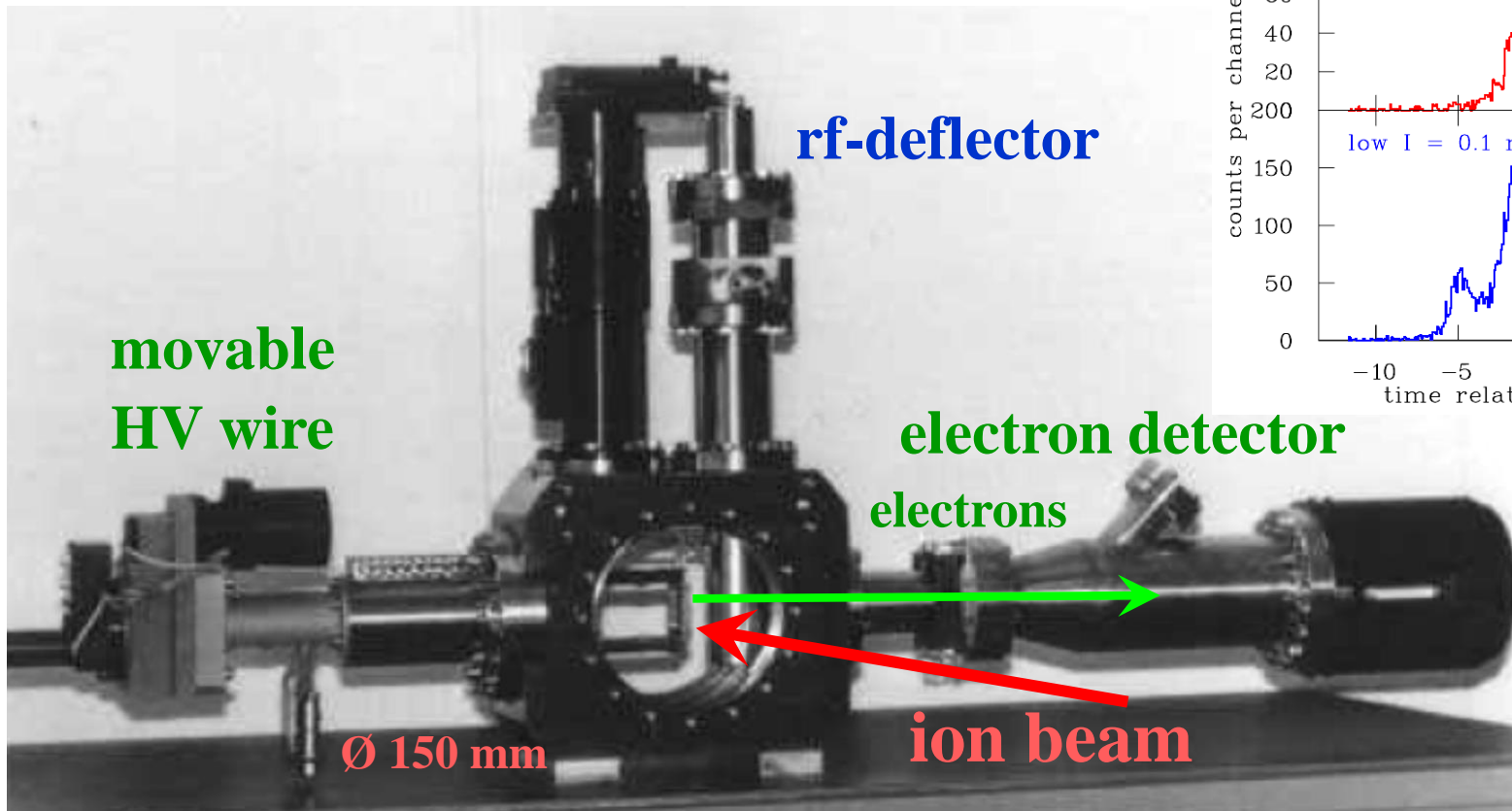


SEM: secondary electron multiplier

# Realization of Bunch Shape Monitor at CERN LINAC2



*Example:* The bunch shape at 120 keV/u for 120 keV/u:





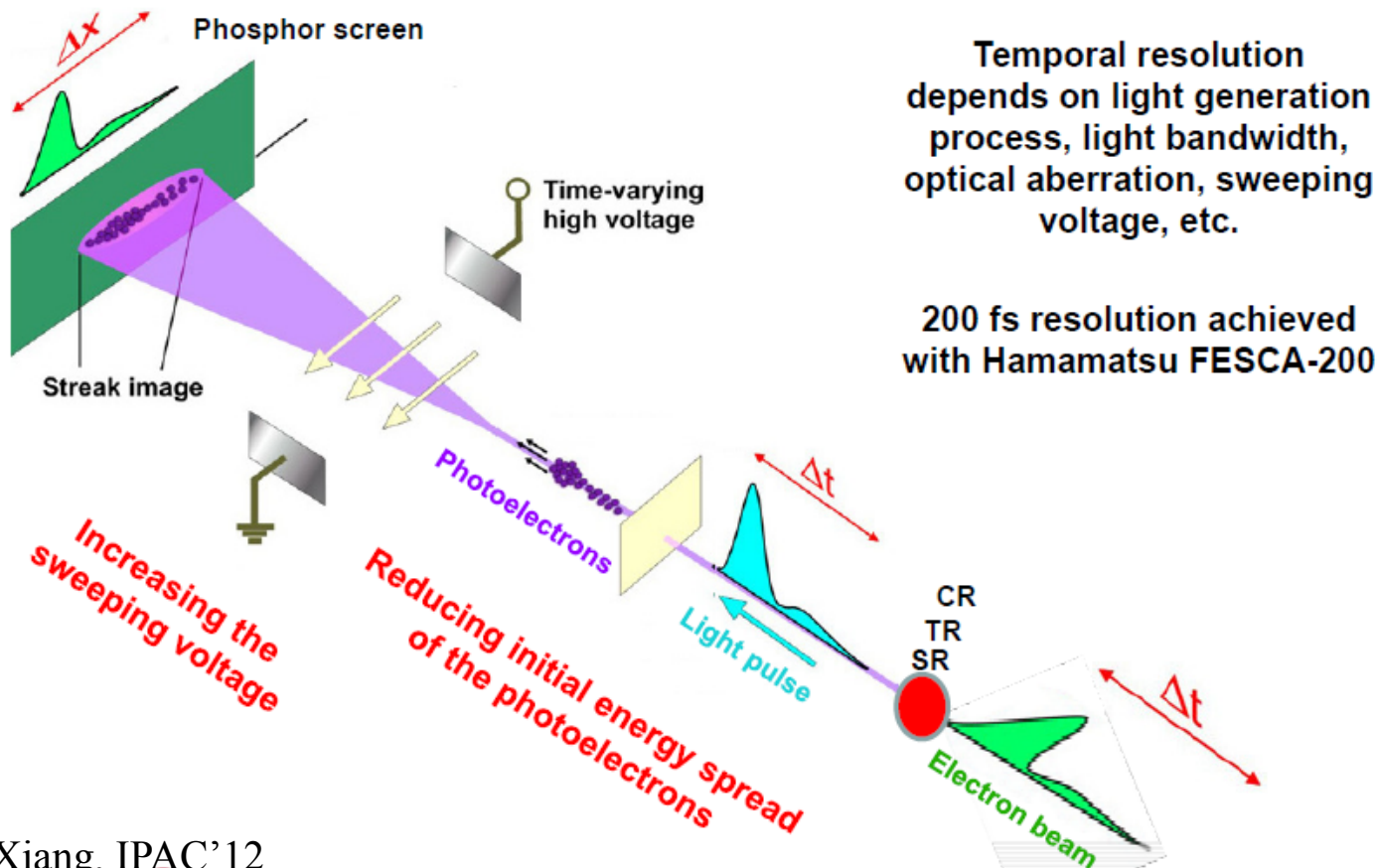
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Determination of particle arrival
- **Bunch length measurement for relativistic beams**  
**Synchrotron light monitor and electro-optical modulation of a laser beam**
- **Summary**

# Bunch Length Measurement for relativistic $e^-$

Electron bunches are too short ( $\sigma_t < 300$  ps) to be covered by the bandwidth of pick-ups ( $f < 1$  GHz  $\Leftrightarrow t_{rise} > 300$  ps) for structure determination.

→ Time resolved observation of synchr. light with a streak camera: Resolution  $\approx 1$  ps.

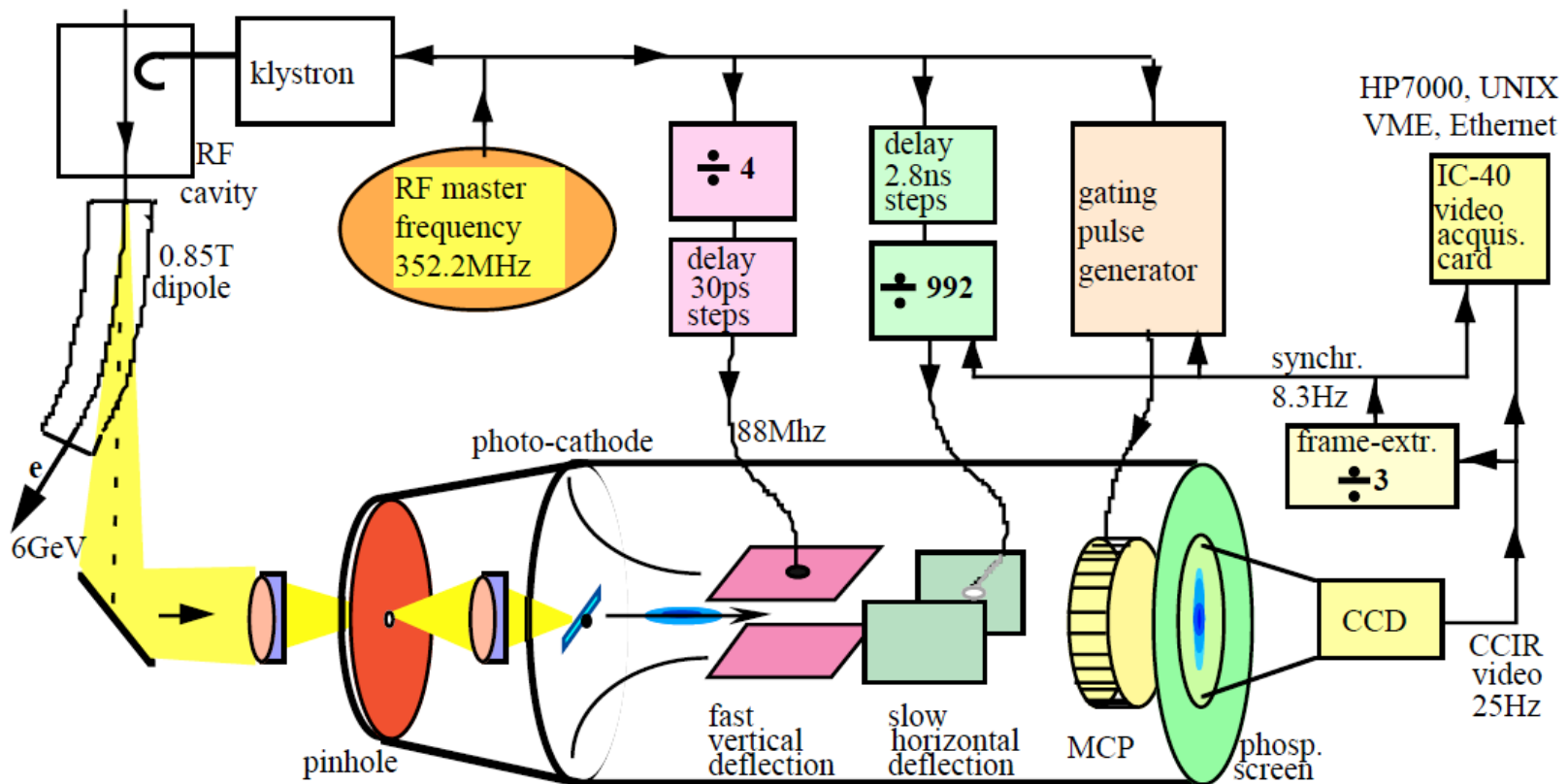


From D. Xiang, IPAC'12

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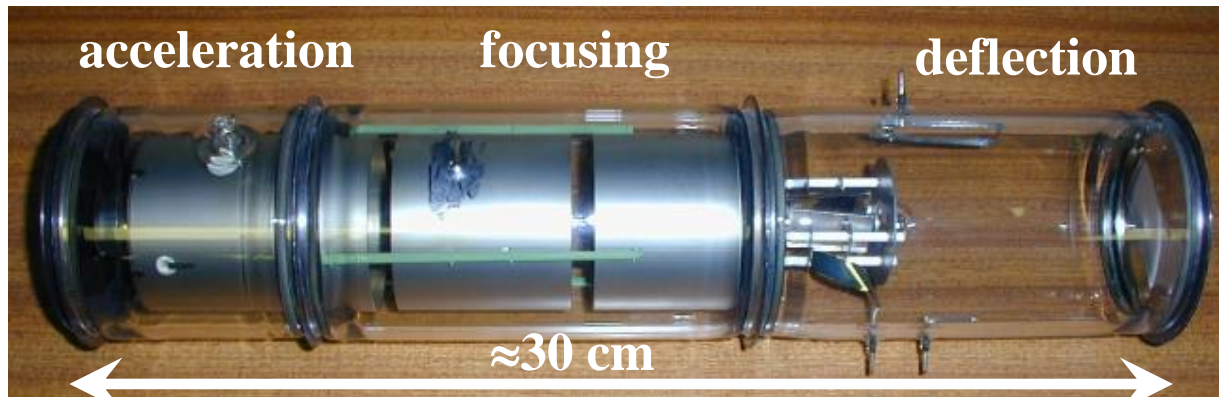
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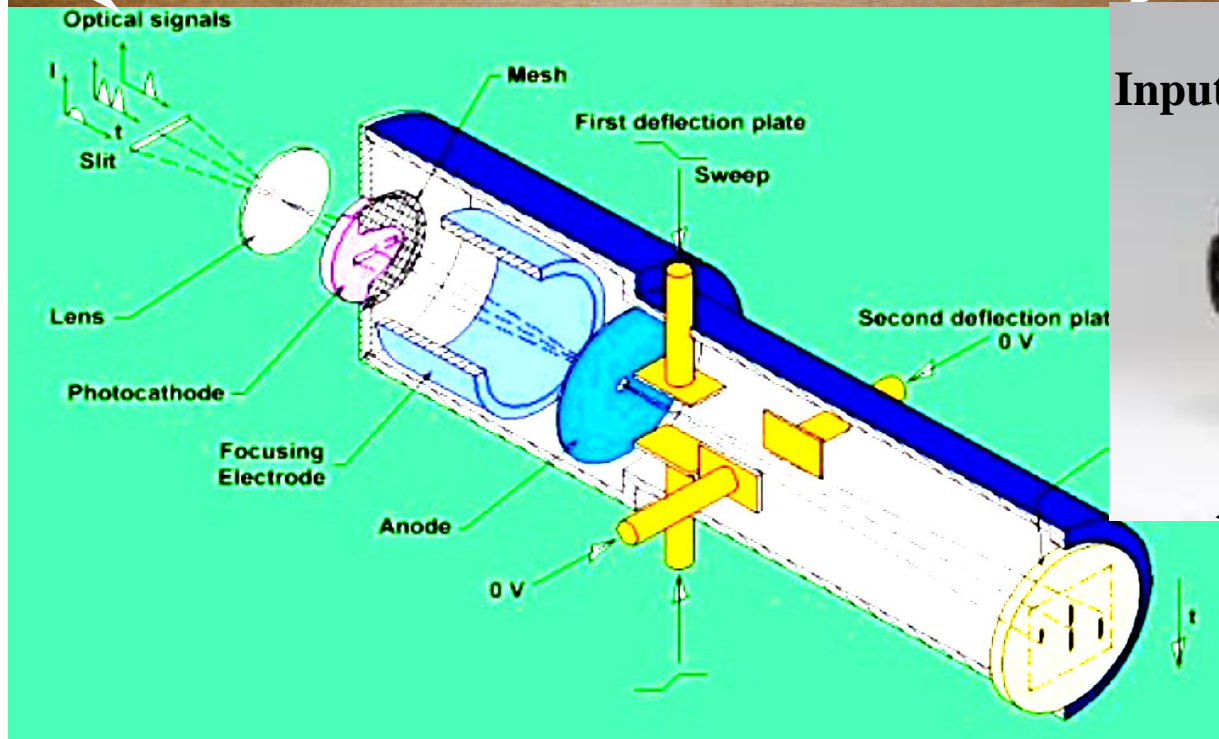


# Technical Realization of Streak Camera



Hardware of a streak camera

Time resolution down to 0.5 ps:



Input optics    Streak tube    CCD

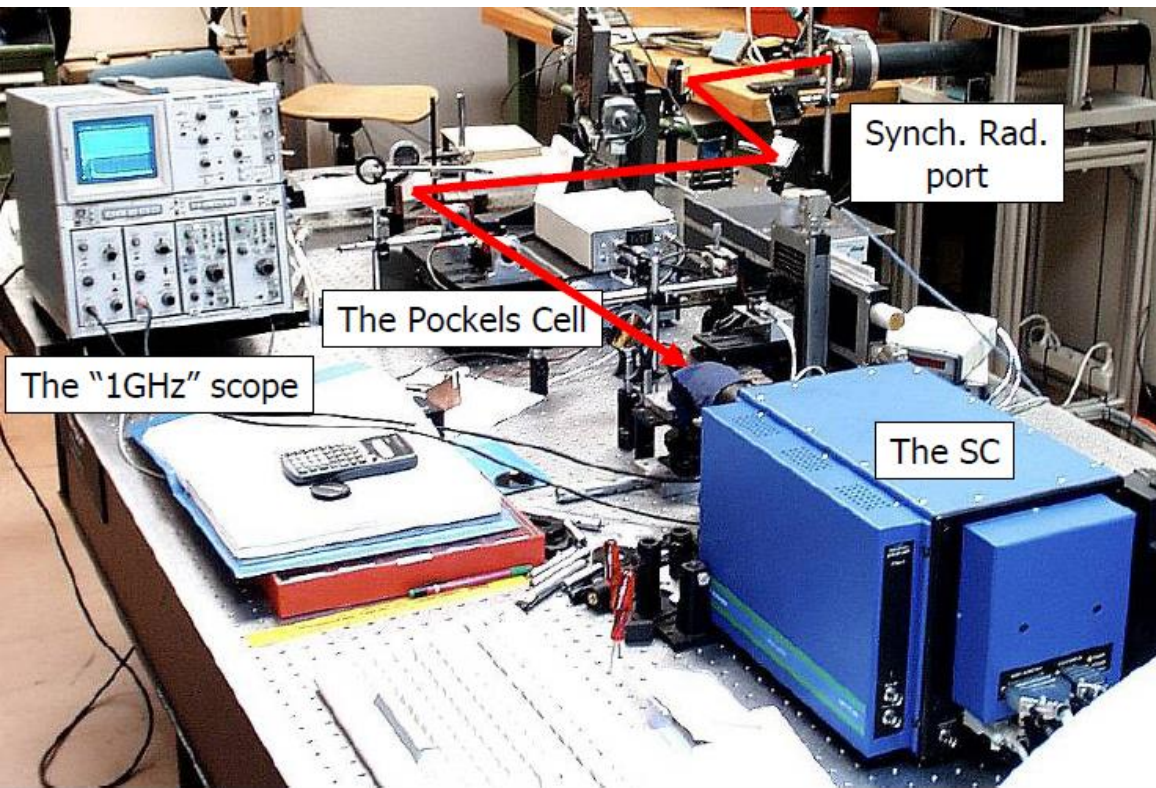


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Hardware of a streak camera

Time resolution down to 0.5 ps:



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The Streak Camera setup at ELETTRA, Trieste, Italy

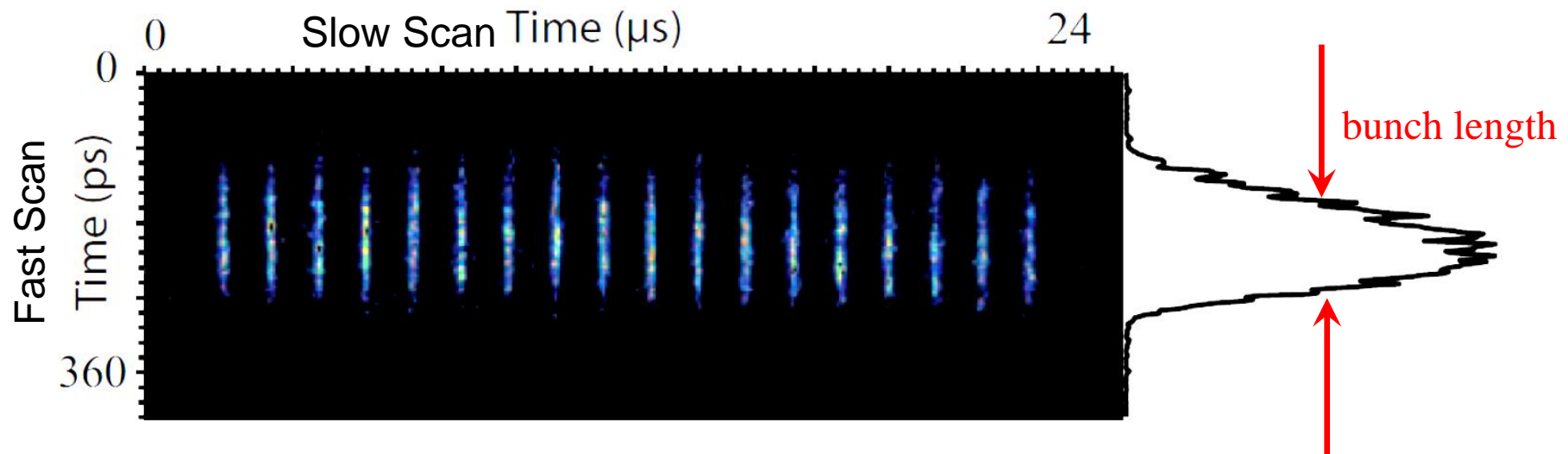


# Results of Bunch Length Measurement by a Streak Camera



The streak camera delivers a fast scan in vertical direction (here 360 ps full scale) and a slower scan in horizontal direction (24  $\mu\text{s}$ ).

**Example:** Bunch length at the synch. Light source SOLEIL for  $U_{rf} = 2$  MV  
for slow direction 24  $\mu\text{s}$  and scaling for fast scan 360 ps: measure  $\sigma_t = 35$  ps.



# The Artist View of a Streak Camera



## FARADAY CUP 1998

*Purpose.* To recognize and encourage innovative achievements in the field of accelerator beam instrumentation.

*Award.* The Faraday Cup Award consists of a US\$ 5000 prize and a certificate to be presented at the next Beam Instrumentation Workshop. Winners participating in the BIW will be given a \$1000 travel allowance.

*Eligibility.* Nominations are open to contributors of all nations regardless of the geographical location at which the work was done.

The Award goes normally to one person, but may be shared by recipients having contributed to the same accomplishment. It will normally be awarded to scientists in the early stage of their career. Nominations of candidates shall remain active for 2 competitions.

*Establishment and support.* The Award was established in 1991 with the support of the Beam Instrumentation Workshop Organizing Committee.

*Rules.* The Faraday Cup shall be awarded for an outstanding contribution to the development of an innovative beam diagnostics instrument of proven workability. The Faraday Cup is only awarded for published contribution and delivered performance—as opposed to theoretical performance. Rules are available on request.

*Award Committee.* The Beam Instrumentation Workshop Organizing Committee.

*Nominations.* The nomination package shall include the name of the candidate, relevant publications, a statement outlining his/her personal contribution and that of others, two letters from co-workers familiar with the candidate and his contribution. Two master copies suitable for photocopying of this package must be submitted not later than the 15th of November 1997 to Steven Smith c/o BIW'98 Secretariat, SLAC, Stanford University, Stanford CA 94305-4085, U.S.A..



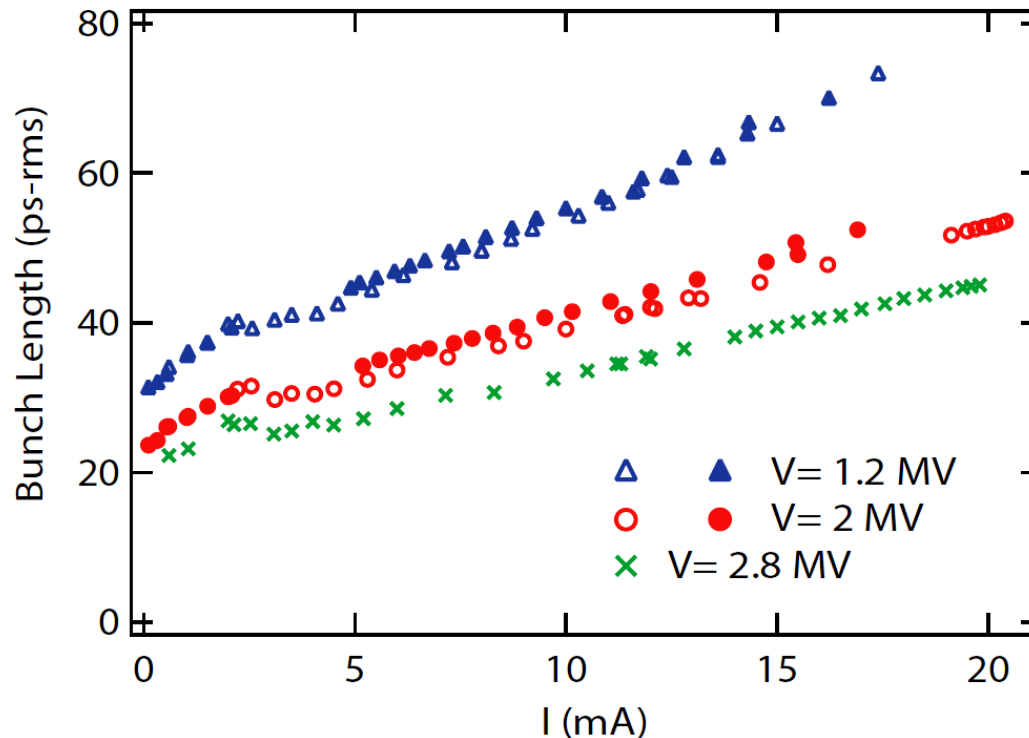
# The Importance of Bunch Length by Streak Camera



Short bunches are desired by the synchrotron light users for time resolved spectroscopy.  
The bunch focusing is changed by the rf-amplitude.

**Example:** Bunch length  $\sigma_t$  as a function of stored current

(space-charge de-focusing, impedance broadening) for different rf-amplitudes at SOLEIL:



# Bunch length measurement by electro-optical methods



For Free Electron Lasers → bunch length below 1 ps is achieved

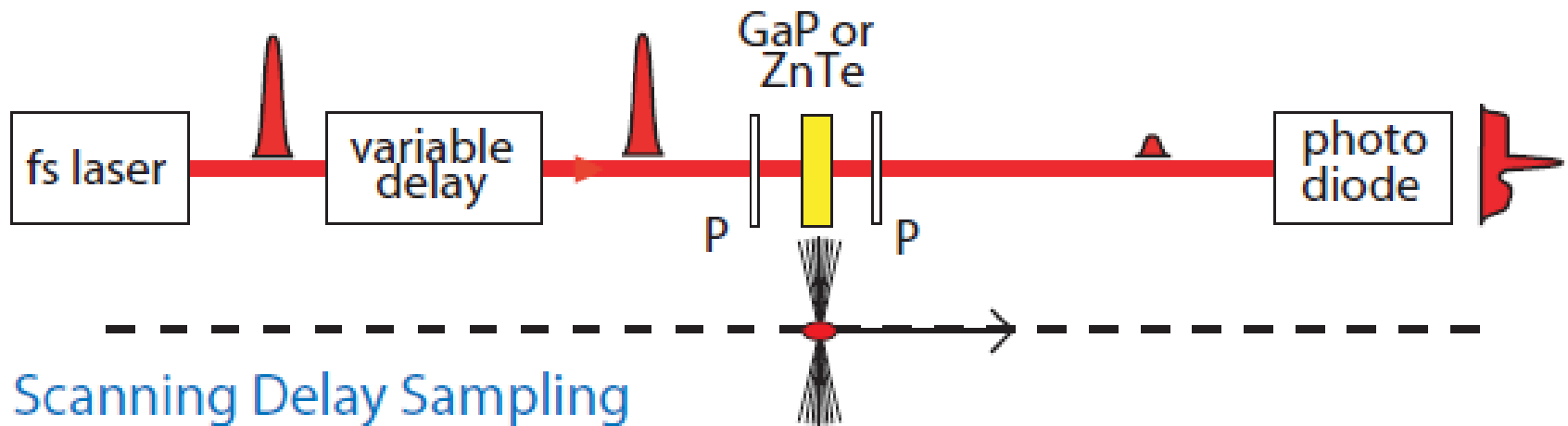
→ below resolution of streak camera

→ short laser pulses with  $t \approx 10$  fs and electro-optical modulator

Electro optical modulator: birefringent, rotation angle depends on external electric field

Relativistic electron bunches: transverse field  $E_{\perp, lab} = \gamma E_{\perp, rest}$  carries the time information

Scanning of delay between bunch and laser → time profile after several pulses.



From S.P.Jamison et al., EPAC 2006



# Pockels Effect

Crystal with different index of refraction on orthogonal planes

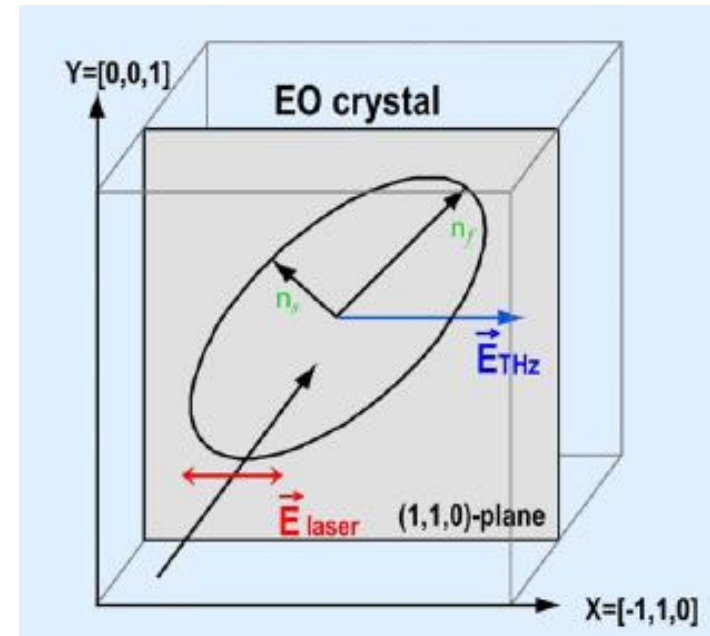
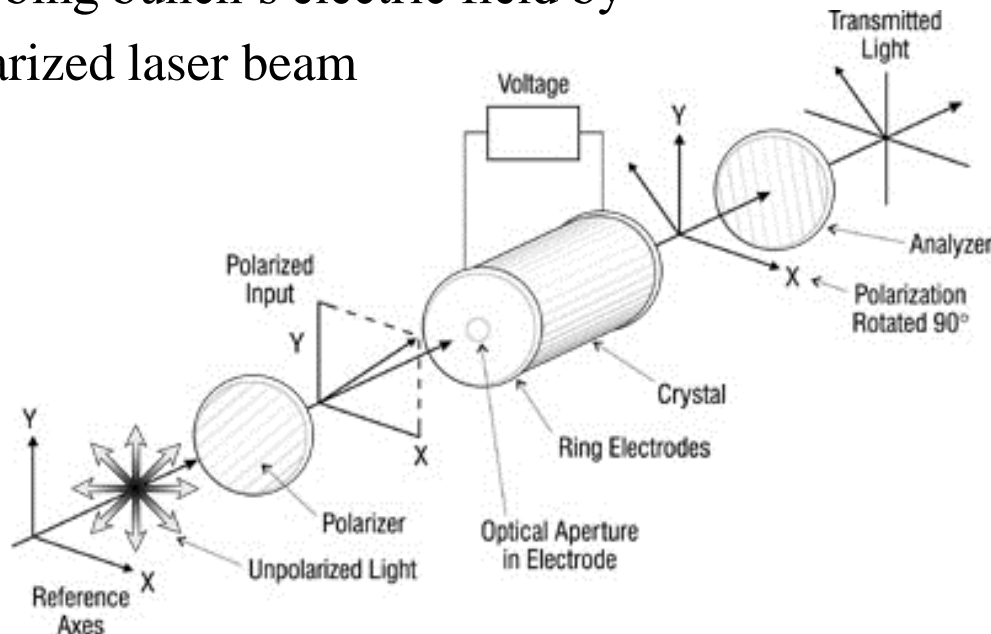
⇒ Bire-fringent

⇒ external electric field changes the phase between orthogonal wave:

‘Pockels effect’ with polarization  $\mathbf{P} = \epsilon_0 \chi \mathbf{E}$

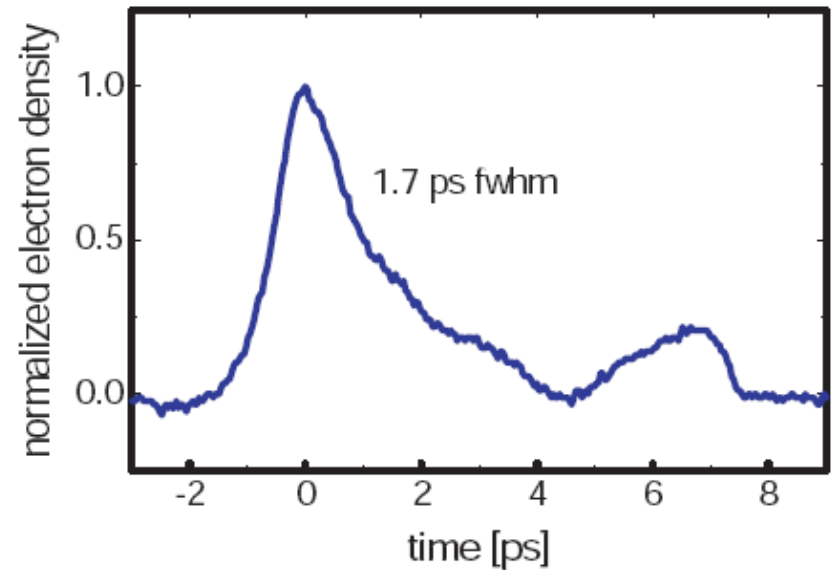
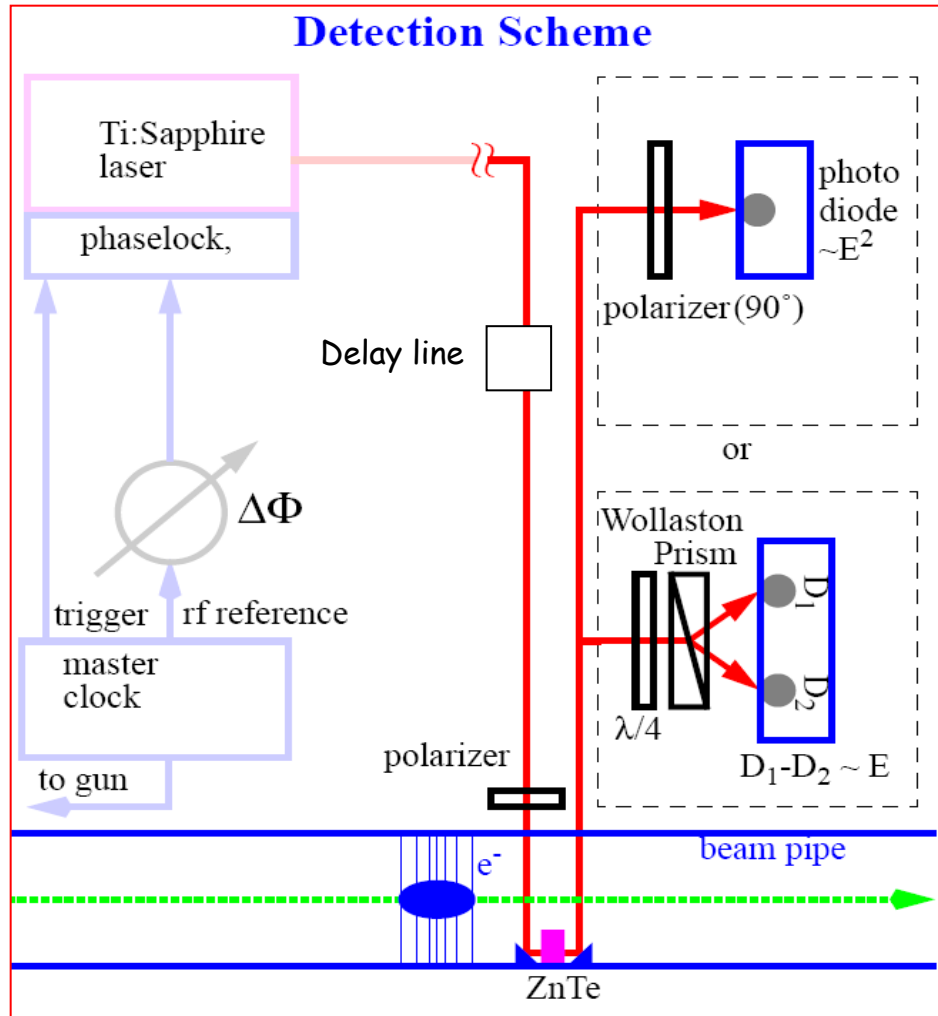
## Diagnostics method:

Probing bunch’s electric field by  
polarized laser beam



# Realization of EOS Scanning

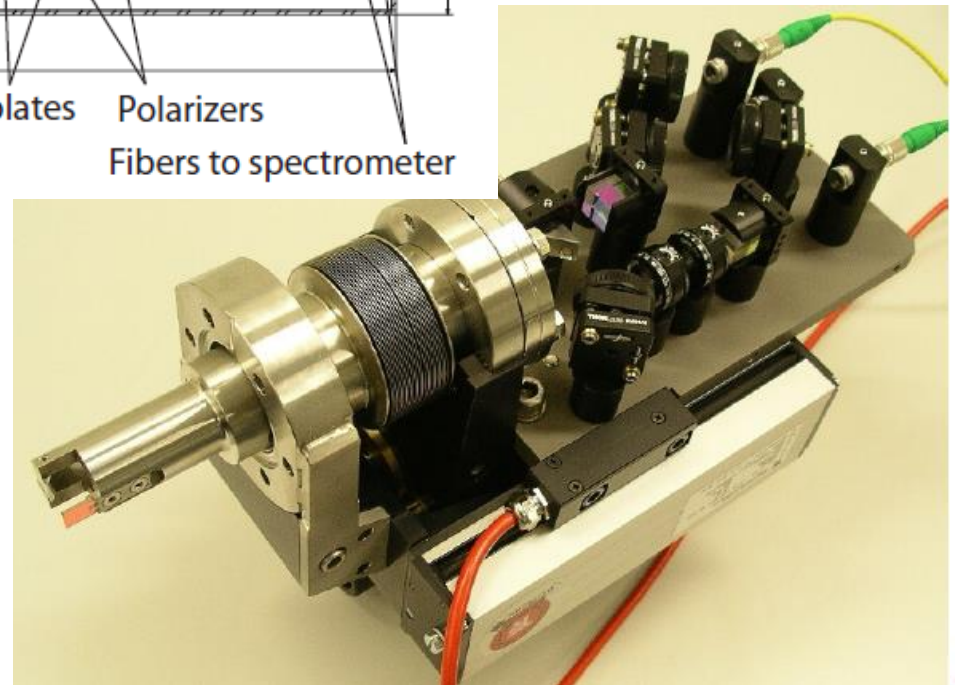
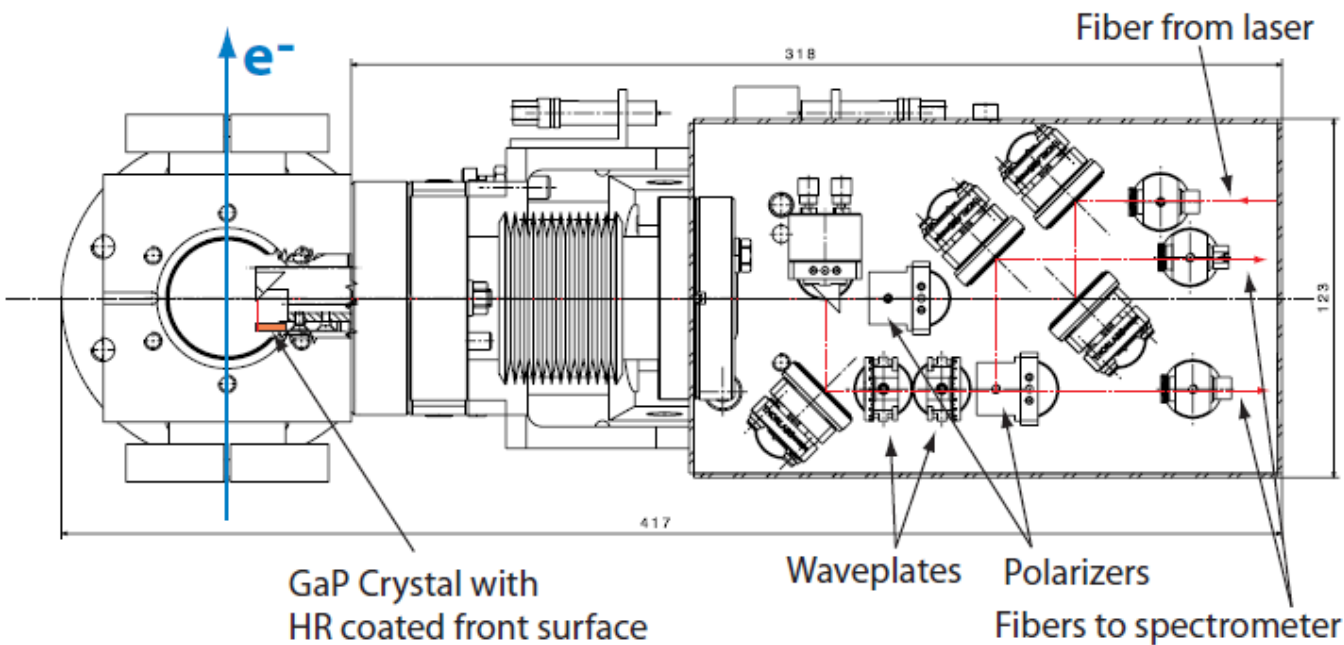
## Setup of a scanning EOS method



Using 12fs Ti:Al<sub>2</sub>O<sub>3</sub> laser at 800nm and ZnTe crystal 0.5mm thick and a beam of 46MeV, 200pC, 2ps.

X. Yan *et al*, PRL 85, 3404 (2000)

# Hardware of a compact EOS Scanning Setup



From B. Steffen et al. PSI, Swiss, DIPAC'09

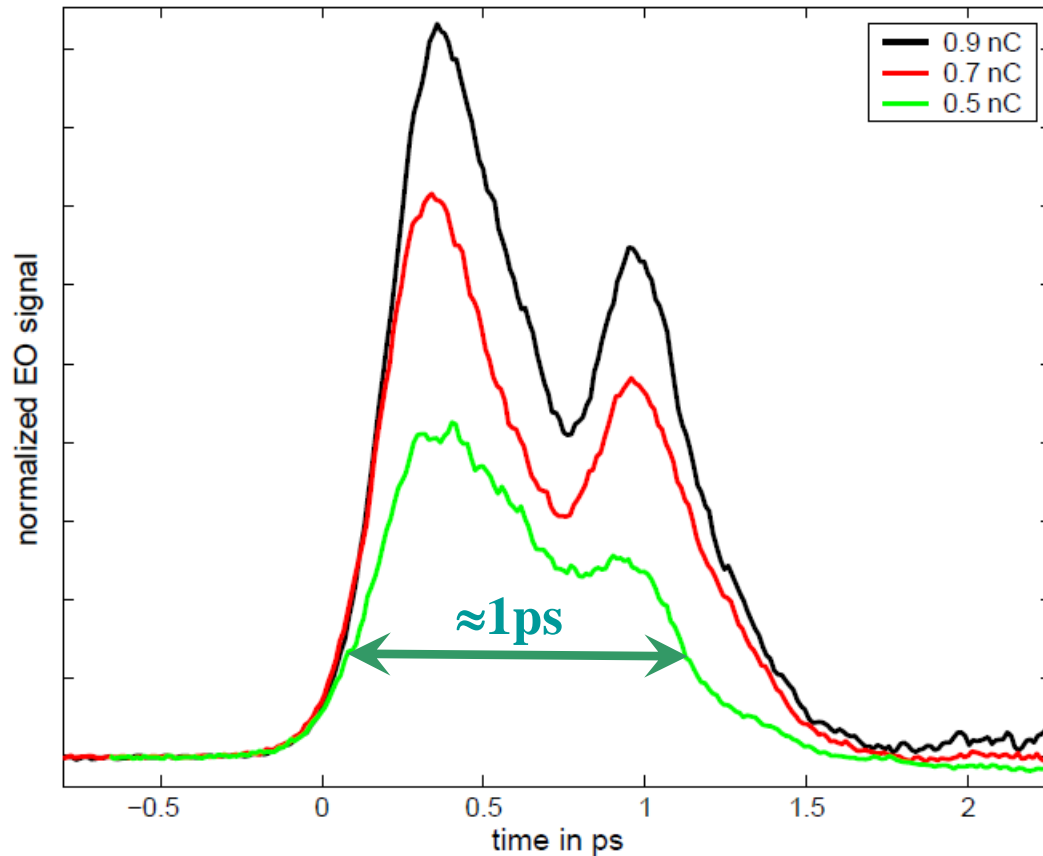


# Measurement of Bunch Shape at FEL-Facility

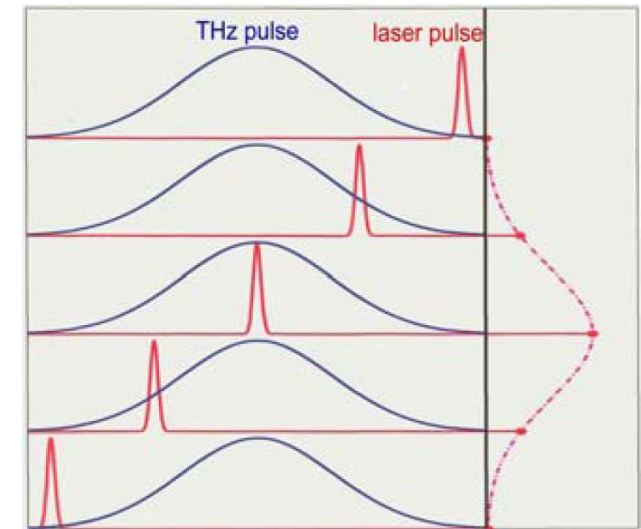


**Example:** Bunch length at FEL test facility FLASH

Bunch shape dependence on bunch charge



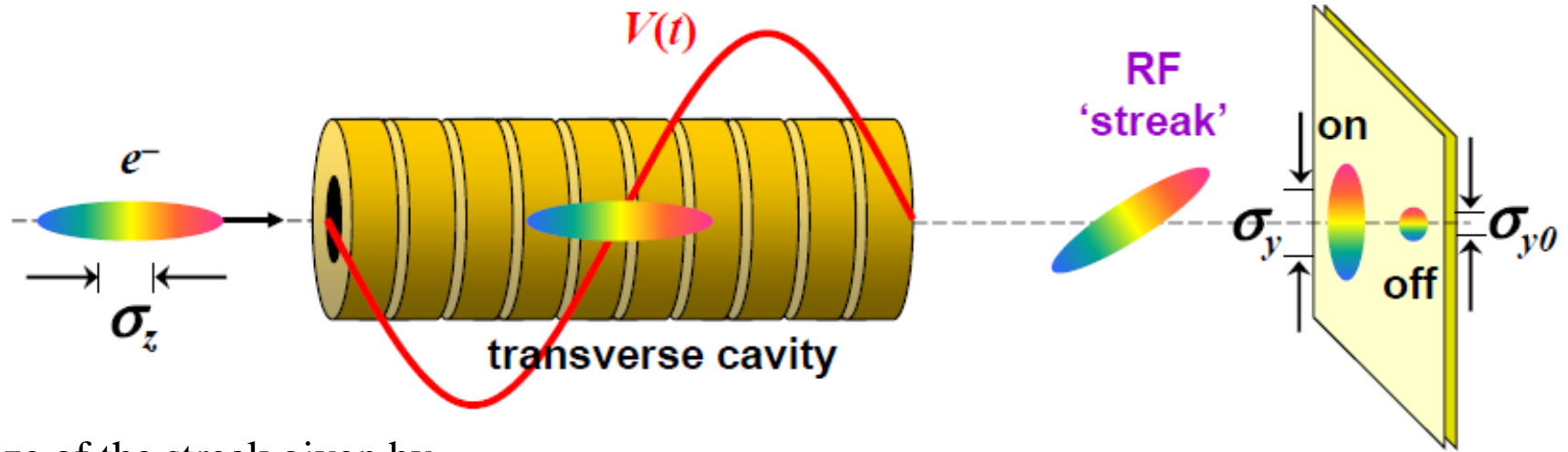
Scanning of the short laser pulse relative to bunch:



Results at FLASH, Hamburg, see B. Steffen et al., FEL Conf. Stanford, p. 549, 2005.

# Bunch Length by rf-Deflection: Principle

Transversal deflection of the bunch i.e. time-to-space conversion

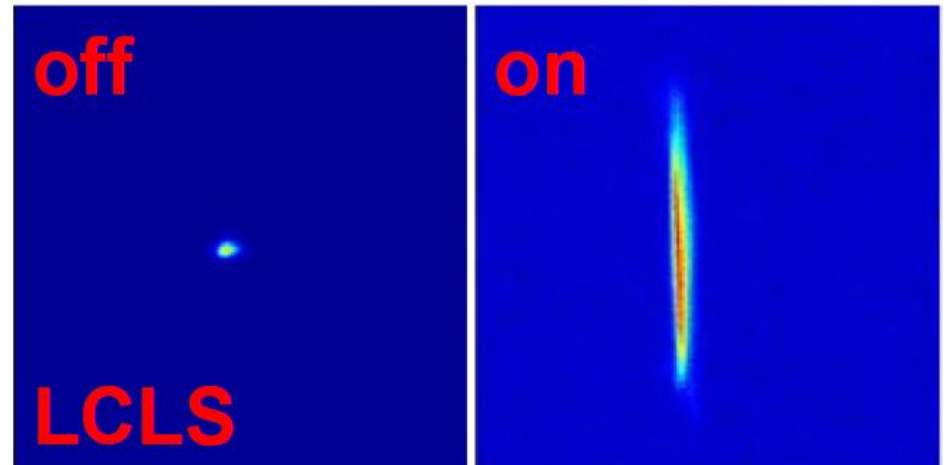


Size of the streak given by

$$\sigma_y = \sqrt{\sigma_{y0}^2 + R_{35} \cdot k \cdot \sigma_z^2}$$

$k$  is determined by the rf-power

$$k = \frac{2\pi e \cdot U_{rf}}{\lambda_{rf} E}$$



From D. Xiang, IPAC'12

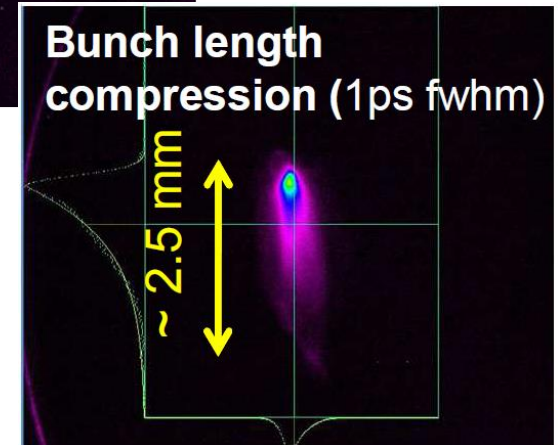
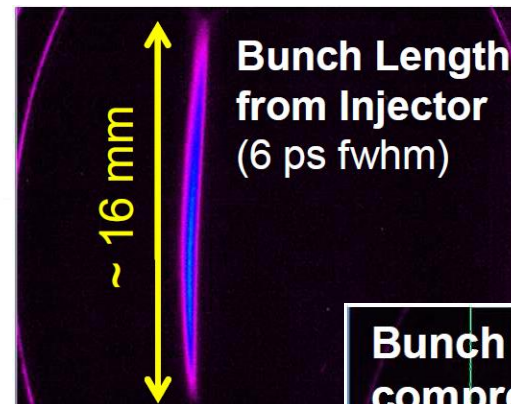
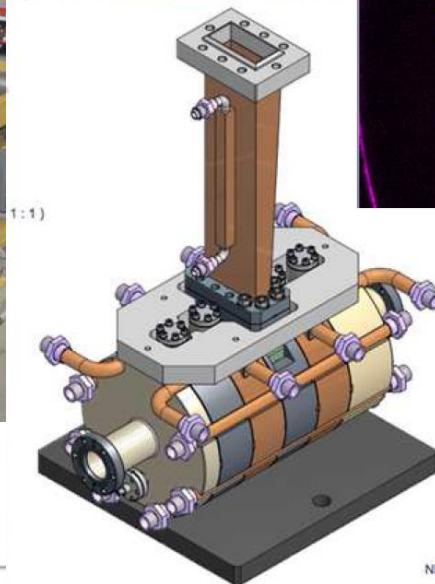
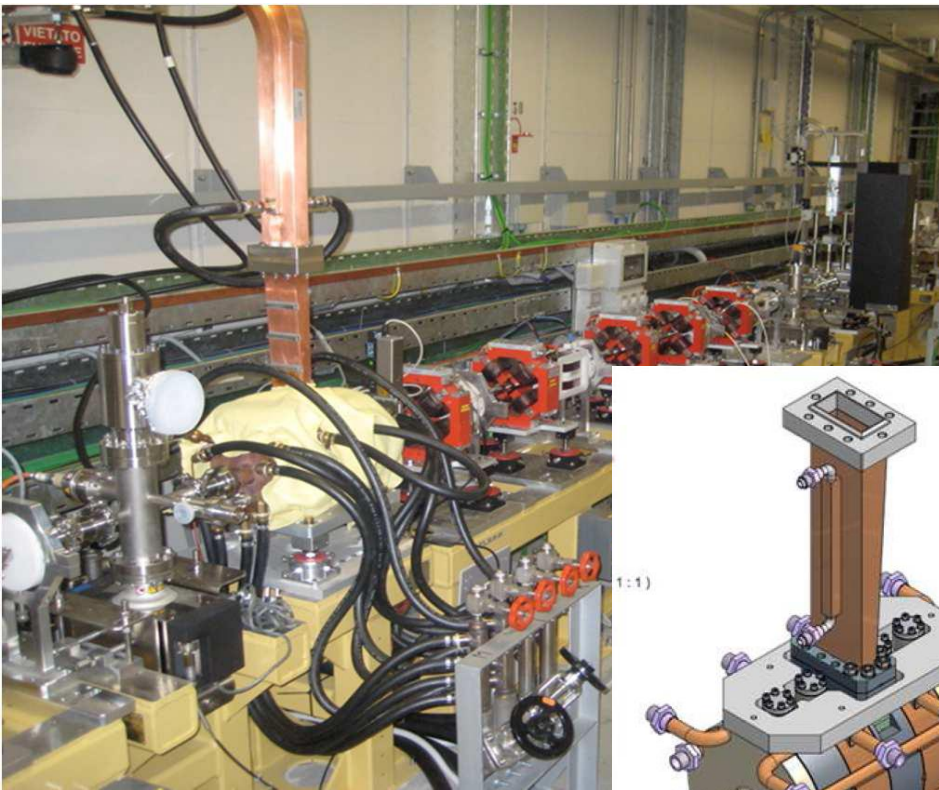
# Bunch Length by rf-Deflection: Hardware



Transversal deflection of the bunch  
i.e. time-to-space conversion

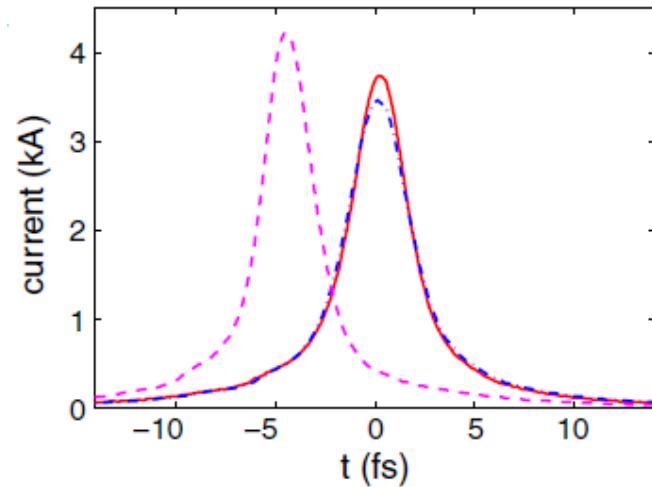
*Example:* Cavity at FERMI, Trieste, Italy

Beam energy	320 MeV
Typical beam size	0.2 mm
Length	0.5 m
Frequency	2.998 GHz
Max. rf power	5 MW
Total trans. volt.	4.9 MV
Time resolution	70 fs



From M. Veronese, BIW'12

# Bunch Length by rf-Deflection: Results



*Example:* Short bunch generation at Free Electron Laser Facility LCLS, Stanford, USA

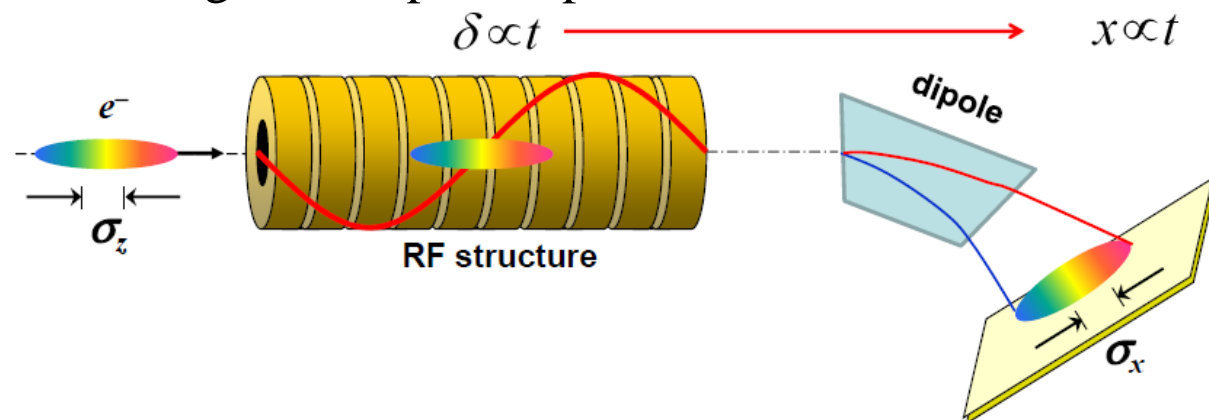
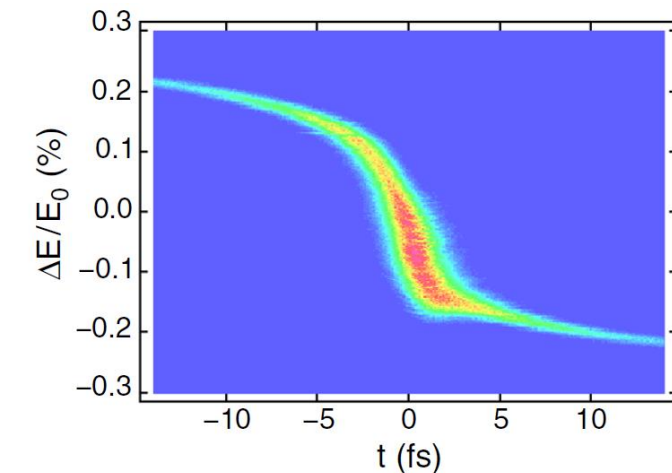
**Time resolution down to (true) fs range!**

**Vertical deflection** with cavity  $\rightarrow$  time spectrum

**Horizontal bending**

$\rightarrow$  energy (momentum) distribution via dispersion

$\Rightarrow$  Longitudinal phase space determined!



From D. Xiang, PRST-AB, 13, 094001 (2010)



# Summary of longitudinal Measurements



## Longitudinal $\leftrightarrow$ transverse correspondences:

- position relative to rf  $\leftrightarrow$  transverse center-of-mass
- bunch structure in time  $\leftrightarrow$  transverse profile in space
- momentum or energy spread  $\leftrightarrow$  transverse divergence.

## Determination uses:

- Broadband pick-ups:**
- position relative to rf, mean energy
  - emittance at transfer lines or synchrotron via tomography  
assumption: bunches longer than pick-up.

- Particle detectors:**
- TOF or secondary  $e^-$  from wire  
→ for non-relativistic proton beams  
reason:  $E$ -field does not reflect bunch shape.

- Streak cameras:**
- time resolved monitoring of synchrotron radiation  
→ for relativistic  $e^-$ -beams,  $t_{\text{bunch}} < 1\text{ns}$   
reason: too short bunches for rf electronics.

- Laser scanning:**
- Electro-optical modulation of short laser pulse  
→ highest time resolution.