Transverse Beam Dynamics

JUAS tutorial 1

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1 Exercise: local radius, rigidity

We wish to design an electron ring with a radius of 200 m. Let us assume that only 50% of the circumference is occupied by bending magnets:

- What will be the local radius of bend ρ in these magnets if they all have the same strength?
- If the momentum of the electrons is 12 GeV/c, calculate the rigidity $B\rho$ and the field in the dipoles.

2 Exercise: particle momentum, geometry of a storage ring and thin lenses

The LHC storage ring at CERN will collide proton beams with a maximum momentum of $p = 7$ TeV/c per beam. The main parameters of this machine are:

Circumference	$C_0 = 26658.9$ m	
Particle momentum	$p = 7$ TeV/c	
Main dipoles	$B = 8.392$ T	$l_B = 14.2$ m
Main quadrupoles	$G = 235$ T/m	$l_q = 5.5$ m

- Calculate the magnetic rigidity of the design beam, the bending radius of the main dipole magnets in the arc and determine the number of dipoles that is needed in the machine.
- Calculate the k-strength of the quadrupole magnets and compare its focal length to the length of the magnet. Can this magnet be treated as a thin lens?
- What does the matrix for the quadrupoles look like?

3 Exercise: Hill equation

Solve the Hill´s equation:

$$y'' + k(s)y = 0$$

by substituting:

$$y = A\sqrt{\beta(s)}\cos[\phi(s) + \phi_0] \text{ with } \phi' = \frac{1}{\beta(s)}, \text{ and where } A \text{ and } \phi_0 \text{ are constants,}$$

demonstrating that a necessary condition is:

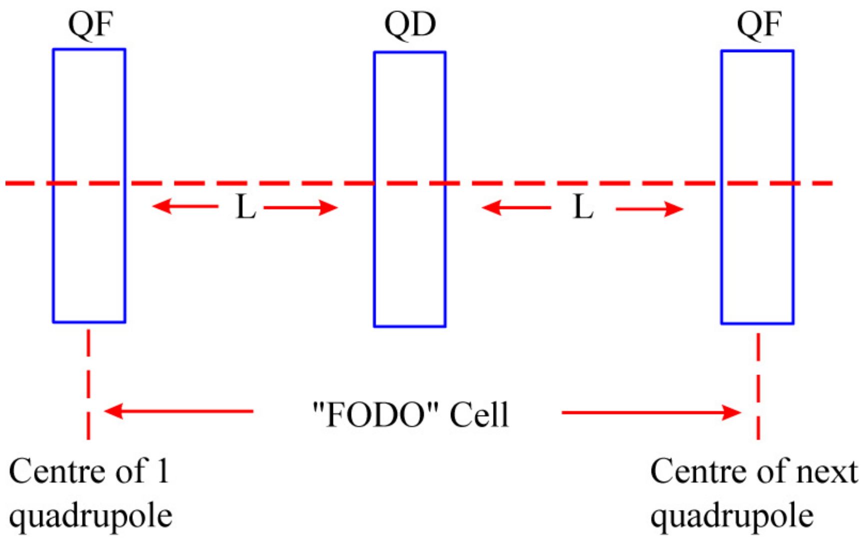
$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + k(s)\beta^2 = 1$$

4 Exercise: FODO lattice

A quadrupole doublet consists of two lenses of focal length f_1 and f_2 separated by a drift length L. Assume that the lenses are thin and show that the transport matrix of this system is

$$M = \begin{pmatrix} 1 - L/f_1 & L \\ -1/f^* & 1 - L/f_2 \end{pmatrix} \text{ where } \frac{1}{f^*} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$$

A FODO cell can be considered as the simplest block of the magnetic structure of modern accelerators and storage rings. It consists of a magnet structure of focusing (F) and defocusing (D) quadrupole lenses in alternating order (see schematic below). Its transfer matrix can be calculated using the matrix of the quadrupole doublet (above) with $f_1 = +2f$ and $f_2 = -2f$ followed (and multiplied) by another quadrupole doublet matrix with $f_1 = -2f$ and $f_2 = +2f$.



Show that the transfer matrix of a FODO system in thin lens approximation is as follows:

$$M_{FODO} = \begin{pmatrix} 1 - \frac{L^2}{2f^2} & 2L + \frac{L^2}{f} \\ -\frac{L}{2f^2}(1 - \frac{L}{2f}) & 1 - \frac{L^2}{2f^2} \end{pmatrix}$$