



Review of light scalars

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No need for motivation at this workshop

1) The σ or $f_0(500)$

2) The $f_0(980)$

3) The κ or $K(800)$ and $a_0(980)$

4) Summary

5) Nature and classification.

Regge trajectory of the $f_0(500)$

I will focus on progress since
PDG2010

Following two points of view:

i) PDG

Consensual, conservative

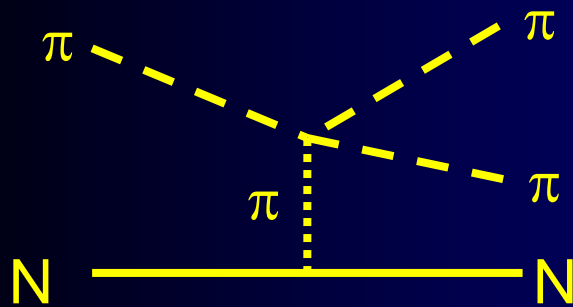
ii) My own

Probably closer to the dominant
view in the community
working on light scalars

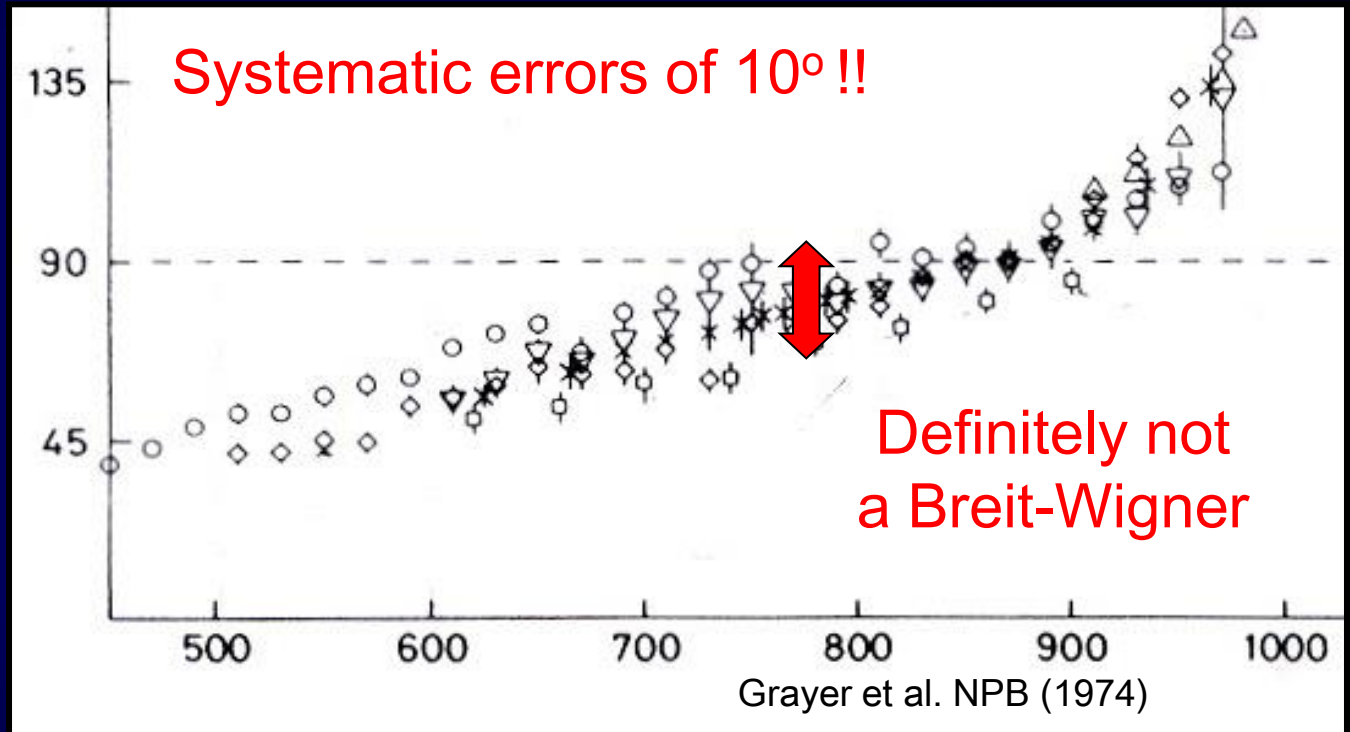
First proposed to explain NN attraction, but NN insensitive to details. Need other sources

1) From πN scattering

Initial state not well defined, model dependent off-shell extrapolations (OPE, absorption, A_2 exchange...) Phase shift ambiguities, etc...



Example: CERN-Munich
5 different $\pi\pi \rightarrow \pi\pi$
analysis of same
 $\pi p \rightarrow \pi\pi n$ data !!



2) From $K \rightarrow \pi\pi e\nu$ (" K_{l4} decays")

Geneva-Saclay (77), E865 (01)

Pions on-shell. Very precise, but $\delta_{00} - \delta_{11}$.

2010 NA48/2 data

3) Decays from heavier mesons

Fermilab E791, Focus, Belle, KLOE, BES,...

“Production” from J/Ψ , B- and D- mesons, and Φ radiative decays.

Very good statistics Clear initial states and different systematic uncertainties.

Strong experimental claims for wide and light σ around 500 MeV

“Strong” experimental claims for wide and light κ around 800 MeV

Very convincing for PDG, but personal caveats on parametrizations used, which may affect the precision and meaning of the pole parameters

PDG2002: “ σ well established”

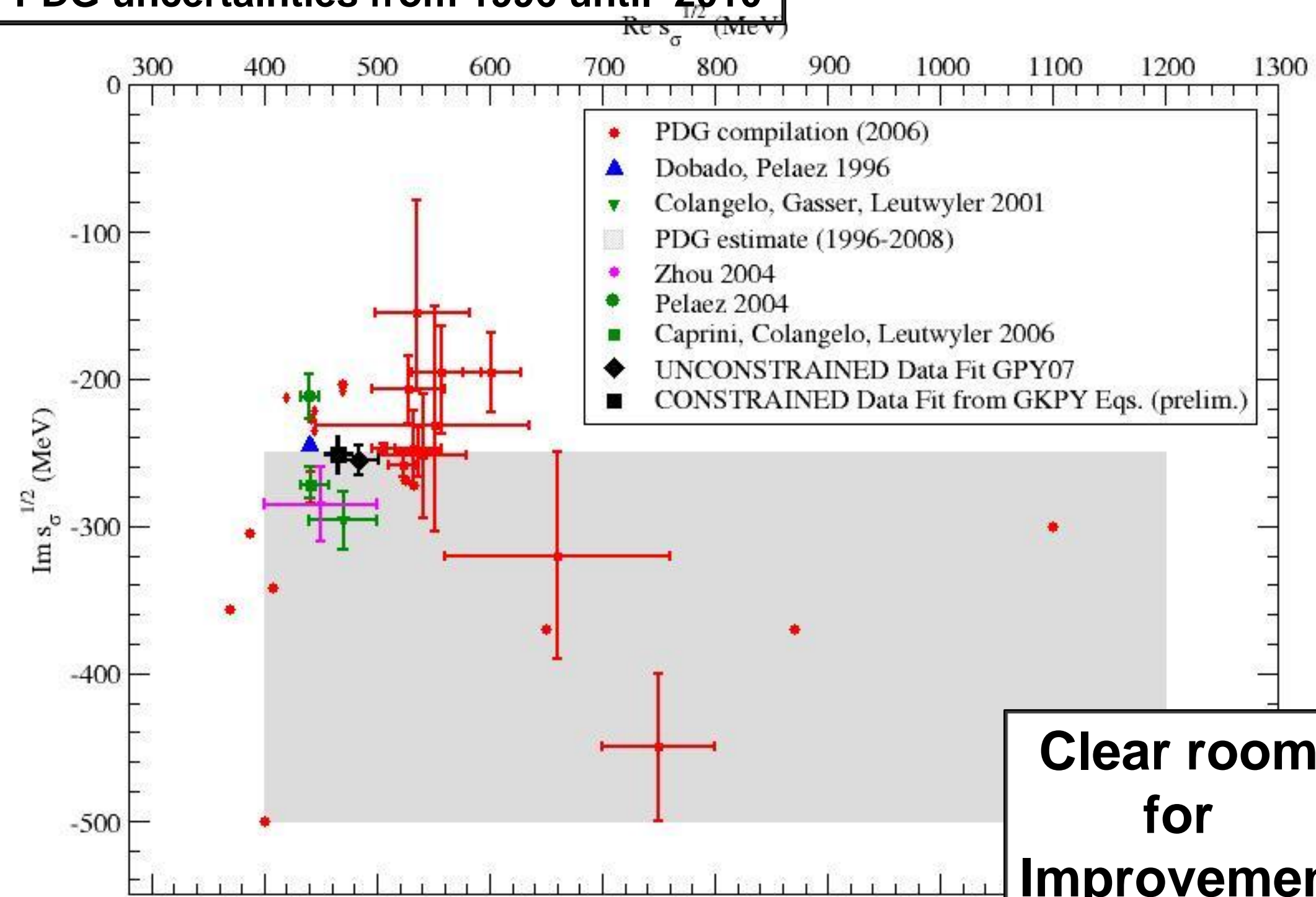
However, since 1996 until 2010 still quoted as

Mass= 400-1200 MeV

Width= 600-1000 MeV



PDG uncertainties from 1996 until 2010



**Clear room
for
Improvement**

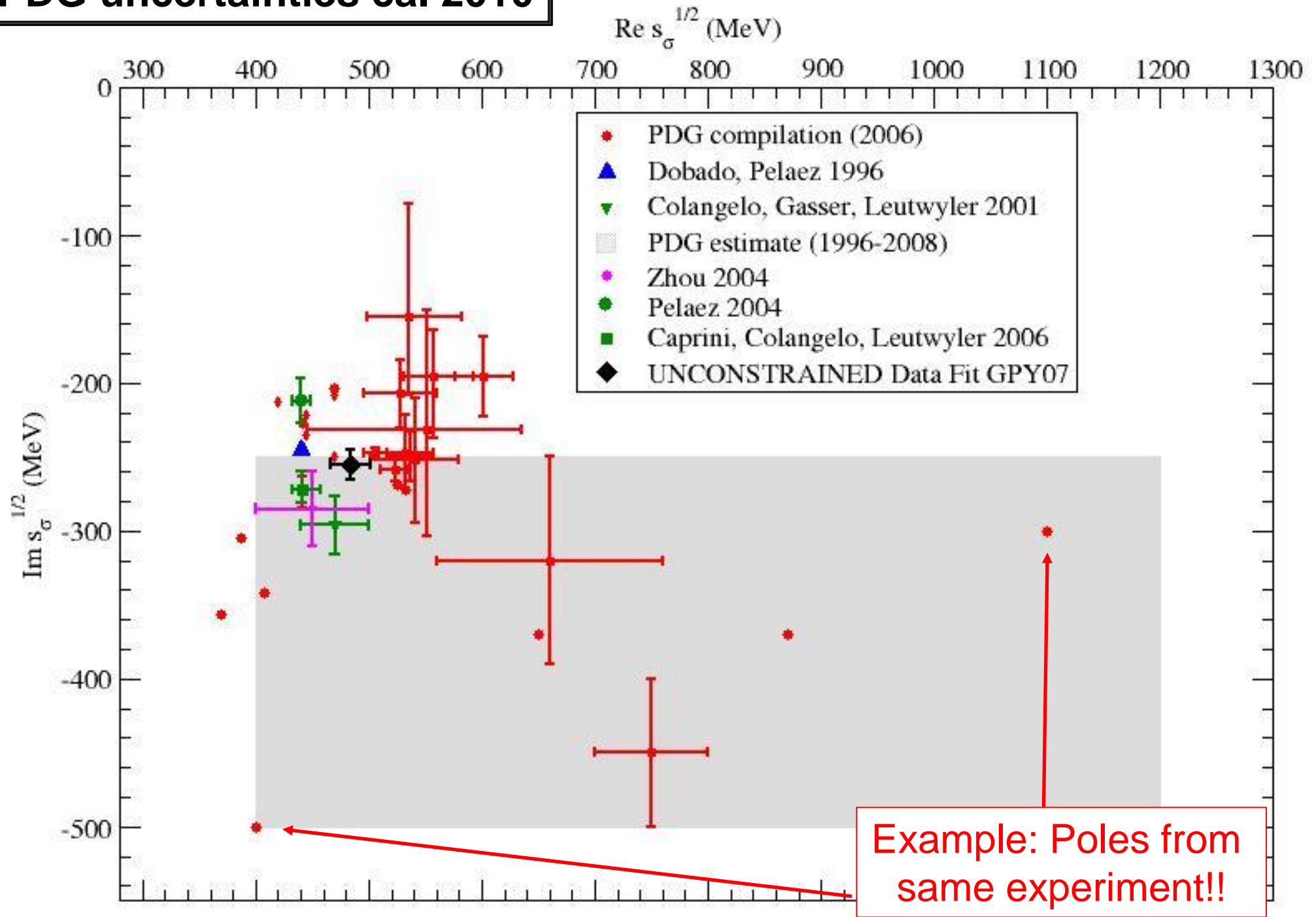
Part of the problem: The theory

Many old and new studies based on crude/simple models,

Strong model dependences

Suspicion: What you put in is what you get out??

PDG uncertainties ca. 2010



Part of the problem: The theory

Many old and new studies based on crude/simple models,

Strong model dependences

Suspicion: What you put in is what you get out??

Even experimental analysis using
WRONG theoretical tools contribute to confusion
(Breit-Wigners, isobars, K matrix,)

Lesson: For poles deep in the complex plane,
the correct analytic properties are essential

Analyticity constraints more powerful in scattering

Dispersive formalisms are the most precise and reliable

AND MODEL INDEPENDENT

The real improvement: Analyticity and Effective Lagrangians

- The 60's and early 70's: Strong constraints on amplitudes from ANALYTICITY in the form of dispersion relations

But poor input on some parts of the integrals and poor knowledge/understanding of subtraction constants = amplitudes at low energy values

- The 80's and early 90's: Development of Chiral Perturbation Theory (ChPT).
(Weinberg, Gasser, Leutwyler)

It is the effective low energy theory of QCD. Provides information/understanding on low energy amplitudes

- The 90's and early 2000's: Combination of Analyticity and ChPT

(Truong, Dobado, Herrero, Donoghe, JRP, Gasser, Leutwyler, Bijnens, Colangelo, Caprini, Zheng, Zhou, Pennington...)

- **Unitarized ChPT** (Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner, ...)

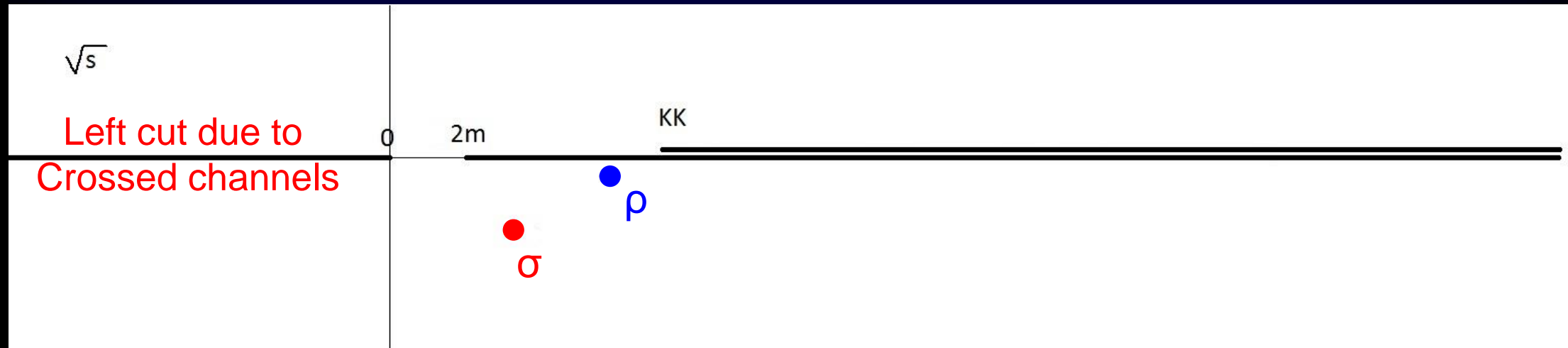
Use ChPT amplitudes inside left cut and subtraction constants of dispersion relation.

Relatively simple, although different levels of rigour. Generates all scalars

Crossing (left cut) approximated... so, not good for precision

Why so much worries about “the left cut”?

It is wrong to think in terms of analyticity in terms of \sqrt{s}



Since the partial wave is analytic in s



● Unitarized ChPT

90's Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner,...

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● Roy-like and GKPY equations.

70's Roy, Basdevant, Pennington, Petersen...

00's Ananthanarayan, Caprini, Colangelo, Gasser, Leutwyler, Moussallam, Decotes Genon, Lesniak, Kaminski, JRP...

Left cut implemented with precision . Use data on all waves + high energy .

Optional: ChPT predictions for subtraction constants

The most precise and model independent pole determinations

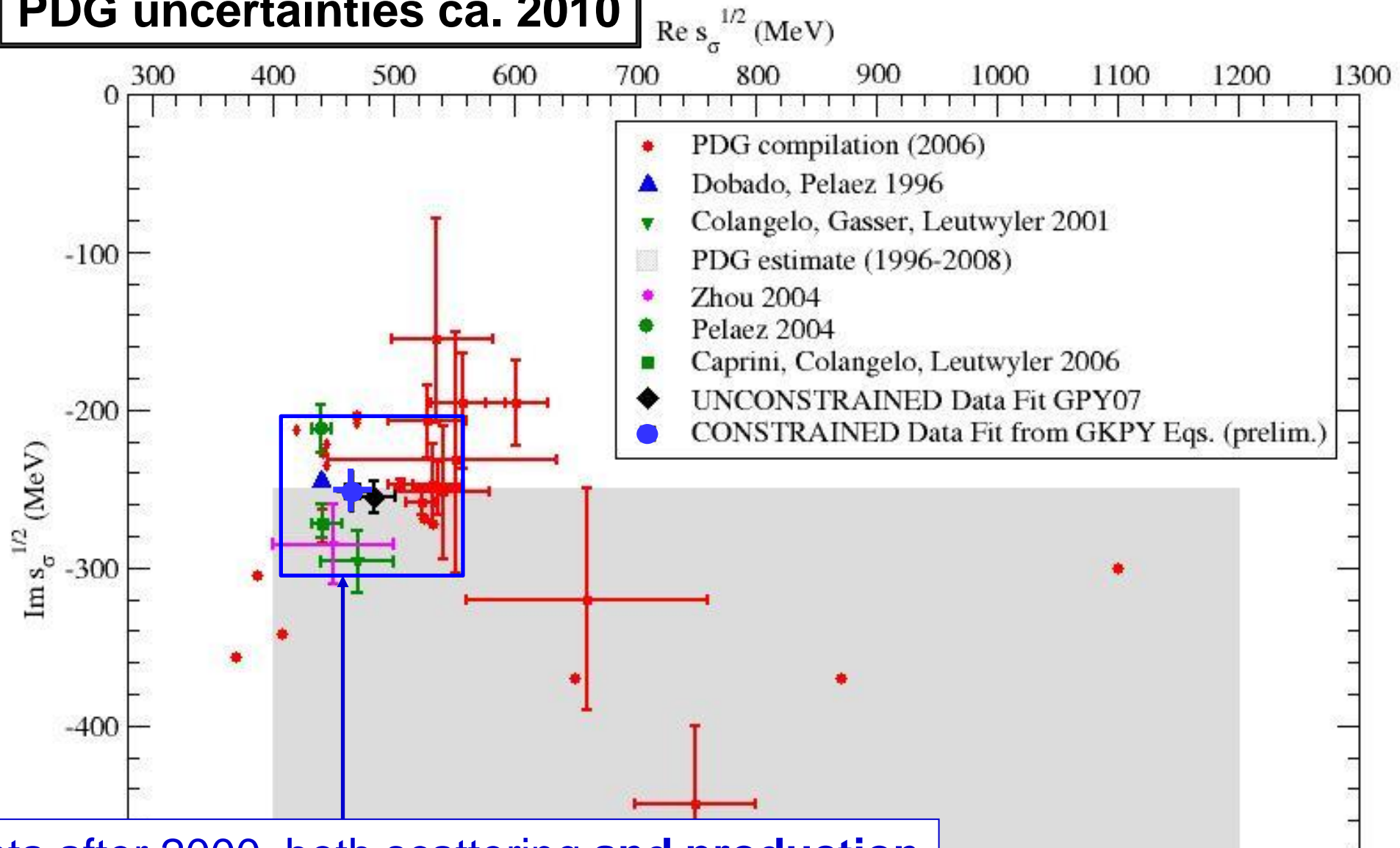
$f_0(600)$ and $\kappa(800)$ existence,
mass and width

firmly established with precision

For long, well known
for the “scalar community”

Yet to be acknowledged by PDG....

PDG uncertainties ca. 2010



Data after 2000, both scattering and production
Dispersive- model independent approaches
Chiral symmetry correct

Yet to be
acknowledged by
PDG....

Some relevant DISPERSIVE POLE Determinations

(after 2010, also “according” to PDG)

● GKPY equations = Roy like with one subtraction

García Martín, Kaminski, JRP, Yndurain PRD83,074004 (2011)

R. Garcia-Martin , R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(2011).

Includes latest NA48/2 constrained data fit .One subtraction allows use of data only

NO ChPT input but good agreement with previous Roy Eqs.+ChPT results.

$$(457_{-15}^{+14}) - i(279_{-7}^{+11})\text{MeV}$$

● Roy equations

B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

An S0 Wave determination up to KK threshold with input from previous Roy Eq. works

$$(442_{-8}^{+5}) - i(274_{-5}^{+6})\text{MeV}$$

● Analytic K-Matrix model

G. Mennesier et al, PLB696, 40 (2010)

$$(452 \pm 13) - i(259 \pm 16)\text{MeV}$$

The consistency of dispersive approaches, and also with previous results implementing UNITARITY, ANALTICITY and chiral symmetry constraints by many other people ...

(Ananthanarayan, Caprini, Bugg, Anisovich, Zhou, Ishida Surotsev, Hannah, JRP, Kaminski, Oller, Oset, Dobado, Tornqvist, Schechter, Fariborz, Saninno, Zoou, Zheng, etc....)

Has led the PDG to neglect those works not fulfilling these constraints also restricting the sample to those consistent with NA48/2, Together with the latest results from heavy meson decays Finally quoting in the 2012 PDG edition...

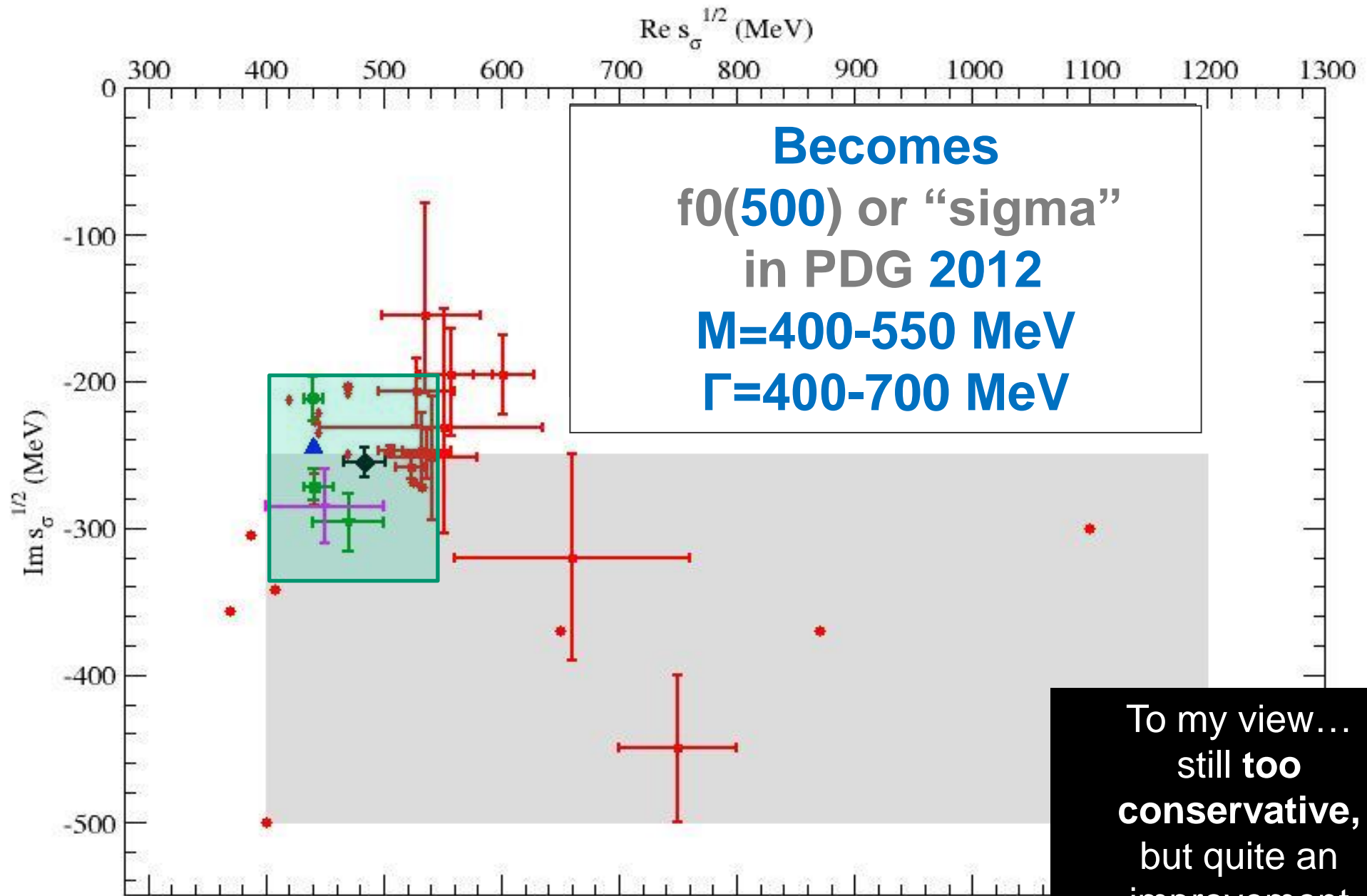
**$M=400-550$ MeV
 $\Gamma=400-700$ MeV**

More than 5 times reduction in the mass uncertainty and 40% reduction on the width uncertainty

Accordingly THE NAME of the resonance is changed to...

$f_0(500)$

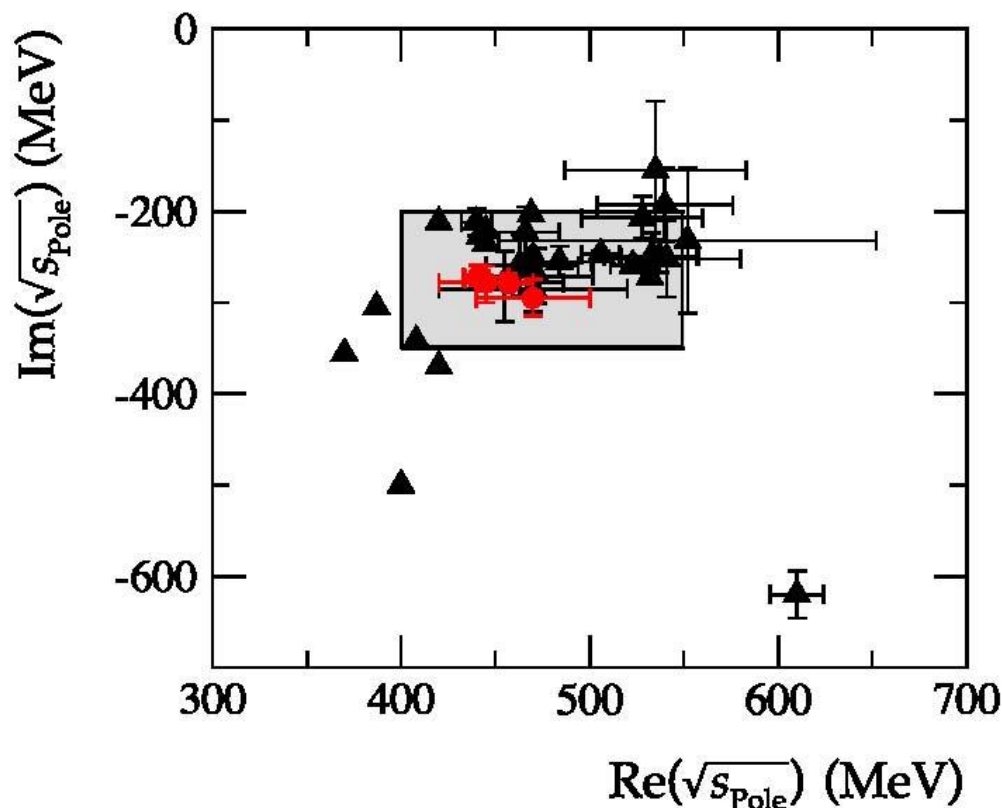
DRAMMATIC AND LONG AWAITED CHANGE ON “sigma” RESONANCE @ PDG!!



Actually, in
PDG 2012:
“Note on
scalars”

One might also take the more radical point of view and just average the most advanced dispersive analyses, Refs. [8–11], shown as solid dots in Fig. 1, for they provide a determination of the pole positions with minimal bias. This procedure leads to the much more restricted range of $f_0(500)$ parameters

$$\sqrt{s_{\text{Pole}}} = (446 \pm 6) - i(276 \pm 5) \text{ MeV} .$$



And, at the risk of being annoying....

Now I find somewhat bold to average those results, particularly the uncertainties

8. G. Colangelo, J. Gasser, and H. Leutwyler, NPB603, 125 (2001).
9. I. Caprini, G. Colangelo, and H. Leutwyler, PRL 96, 132001 (2006).
10. R. Garcia-Martin, R. Kaminski, JRP, J. Ruiz de Elvira, PRL107, 072001(2011).
11. B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

The dispersive approach is model independent.

Just analyticity and crossing properties

- Determine the amplitude at a given energy even if there were no data precisely at that energy.
- Relate different processes
- Increase the precision
- The actual parametrization of the data is irrelevant once inside integrals.

A precise $\pi\pi$ **scattering analysis** helps determining the σ and $f_0(980)$ parameters and is useful for any hadronic process containing several pions in the final state

Conformal expansion, 4 terms are enough. First, Adler zero at $m_\pi^2/2$

Average of $\pi N \rightarrow \pi \pi N$ data sets with enlarged errors, at 870- 970 MeV, where they are consistent within 10° to 15° error.

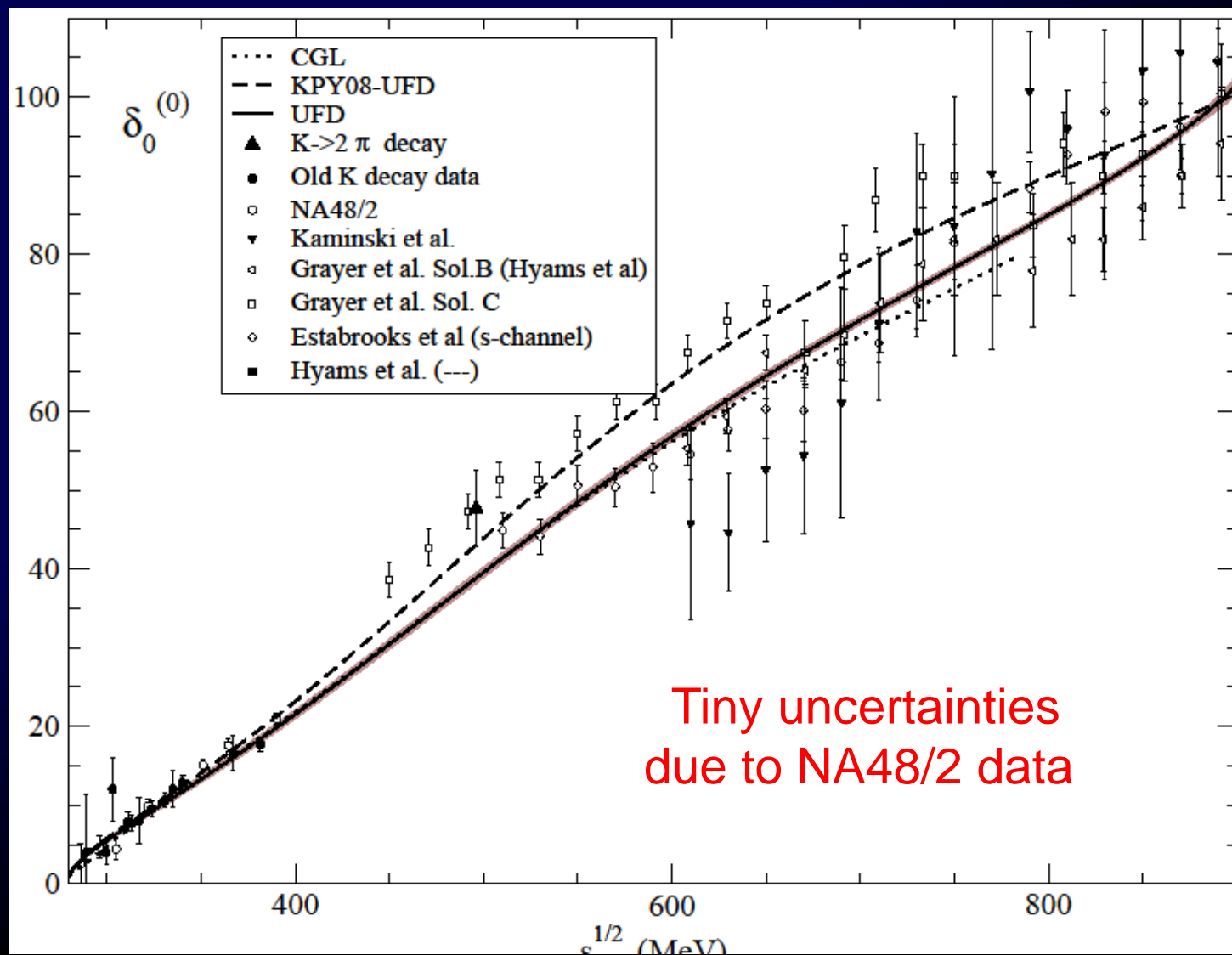
We use data on KI4 including the NEWEST:

NA48/2 results

Get rid of $K \rightarrow 2\pi$

Isospin corrections from Gasser to NA48/2

It does **NOT** HAVE
A BREIT-WIGNER
SHAPE



UNCERTAINTIES IN Standard ROY EQS. vs GKPY Eqs

Why are GKPY Eqs. relevant?

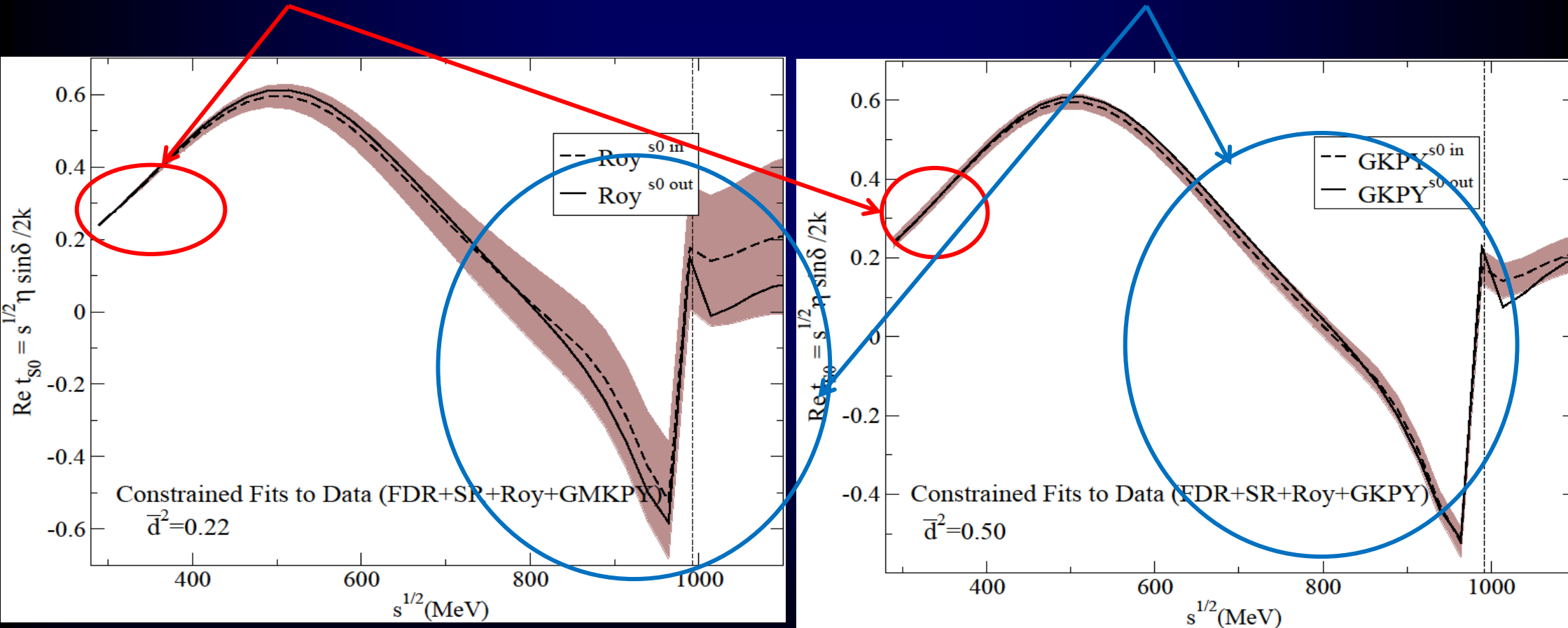
One subtraction yields better accuracy in $\sqrt{s} > 400$ MeV region

Roy Eqs.

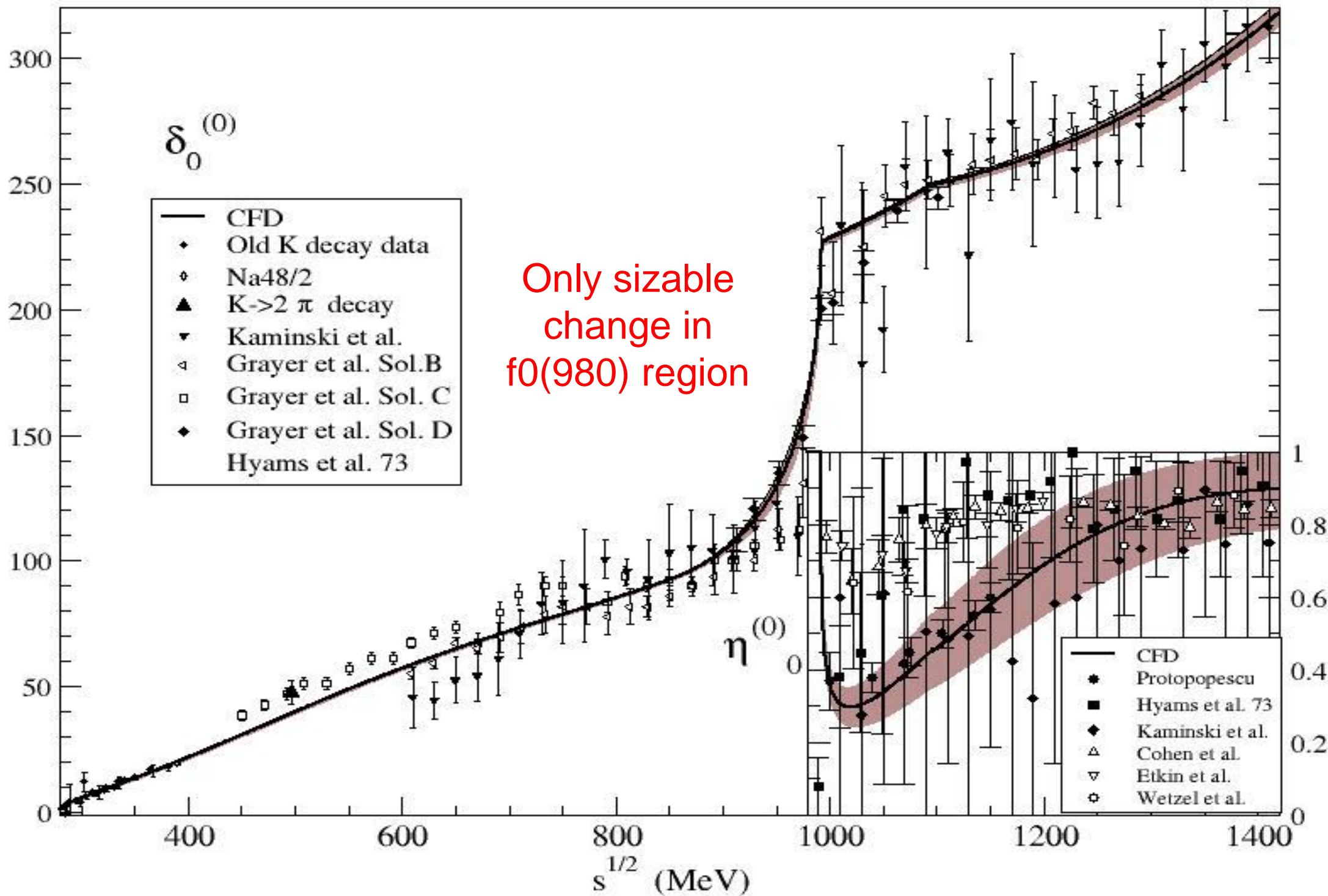
smaller uncertainty below ~ 400 MeV

GKPY Eqs,

smaller uncertainty above ~ 400 MeV



S0 wave: from UFD to CFD



Unfortunately, the PDG still quotes “Breit-Wigner parameters”, with consequences like this

PHYSICAL REVIEW D **87**, 052001 (2013)

Analysis of the resonant component in $\bar{B}^0 \rightarrow J/\psi \pi^+ \pi^-$

1. The signal function

The signal function for \bar{B}^0 is taken to be the coherent sum over resonant states that can decay into $\pi^+ \pi^-$, plus a possible nonresonant S -wave contribution³

$$S(s_{12}, s_{23}, \theta_{J/\psi}) = \sum_{\lambda=0, \pm 1} \left| \sum_i a_{\lambda}^{R_i} e^{i\phi_{\lambda}^{R_i}} \mathcal{A}_{\lambda}^{R_i}(s_{12}, s_{23}, \theta_{J/\psi}) \right|^2,$$

1)

The BW amplitude for a resonance decaying into spin-0 particles, labeled as 2 and 3, is

$$A_R(s_{23}) = \frac{1}{m_R^2 - s_{23} - im_R \Gamma(s_{23})},$$

TABLE III. Possible resonances in the $\bar{B}^0 \rightarrow J/\psi \pi^+ \pi^-$ decay mode.

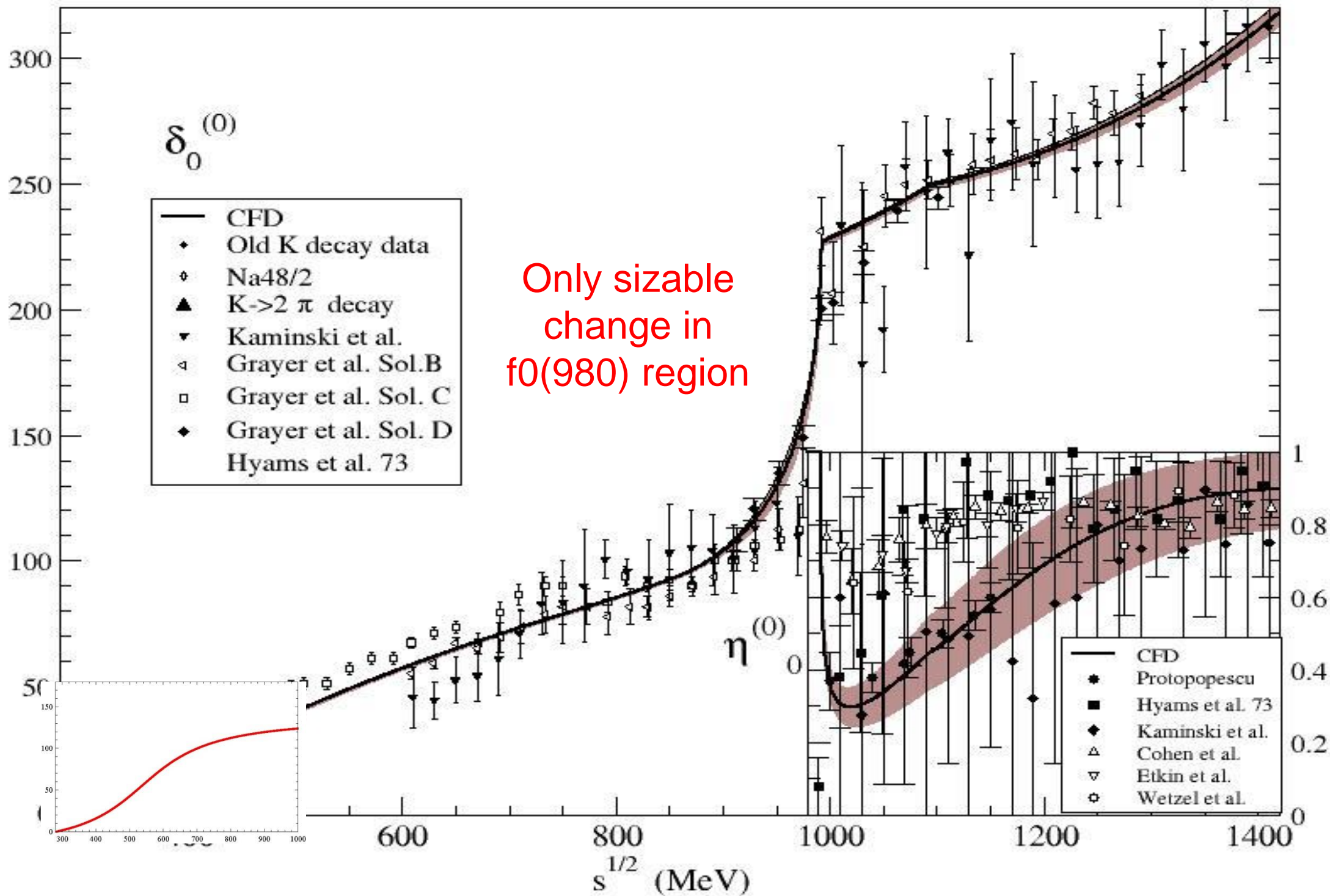
Resonance	Spin	Helicity	Resonance formalism
$f_0(500)$	0	0	BW
$\rho(770)$	1	0, ± 1	BW
$\omega(782)$	1	0, ± 1	BW

TABLE IV. Breit-Wigner resonance parameters.

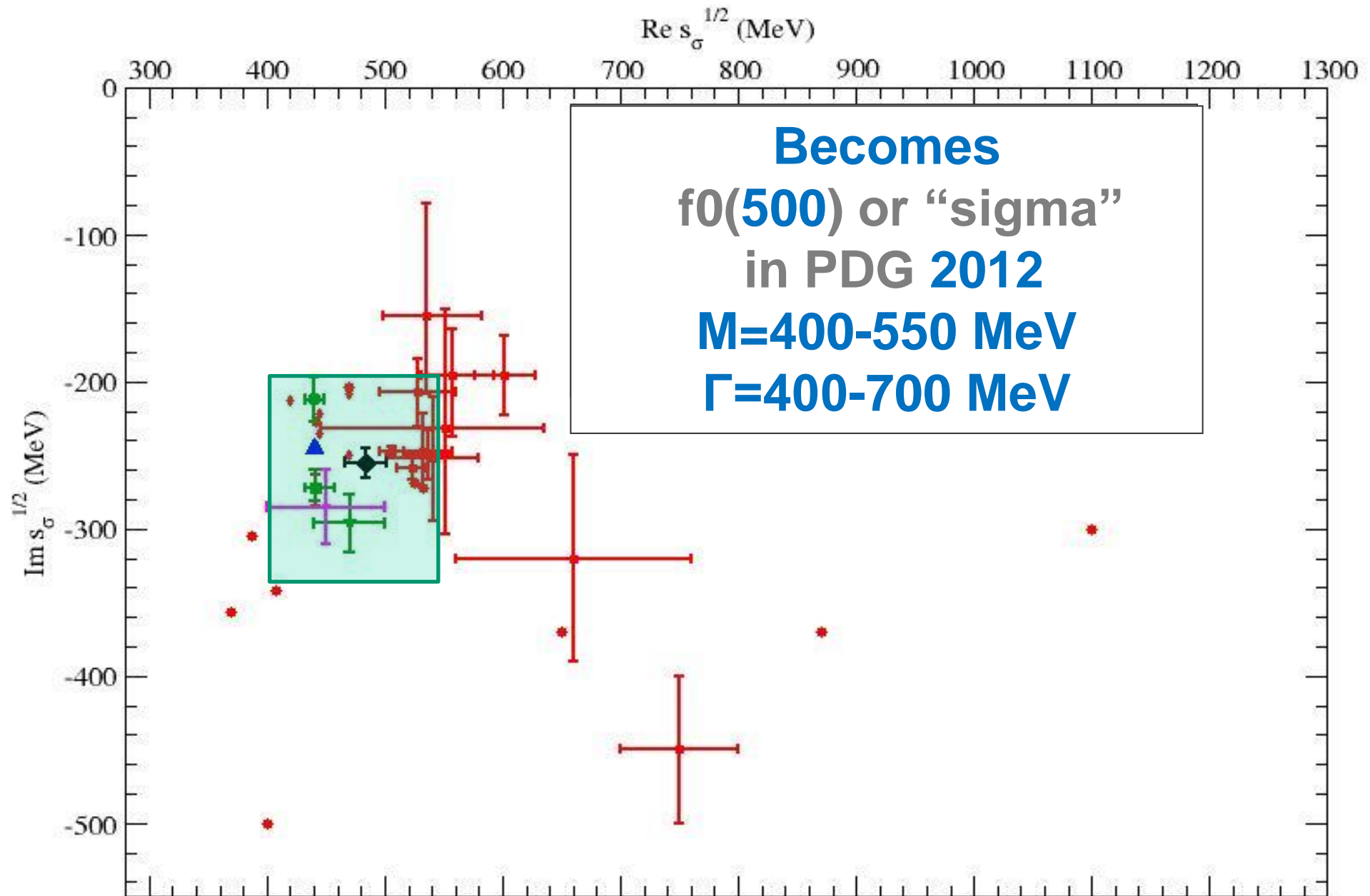
Resonance	Mass (MeV)	Width (MeV)	Source
$f_0(500)$	513 ± 32	335 ± 67	CLEO [27]
$\rho(770)$	775.49 ± 0.34	149.1 ± 0.8	PDG [15]
$\omega(782)$	782.65 ± 0.12	8.49 ± 0.08	PDG [15]

I know there are very smart people at the PDG trying to fight this BW nonsense

S0 wave: from UFD to CFD



“sigma” Summary



1) The σ or $f_0(500)$

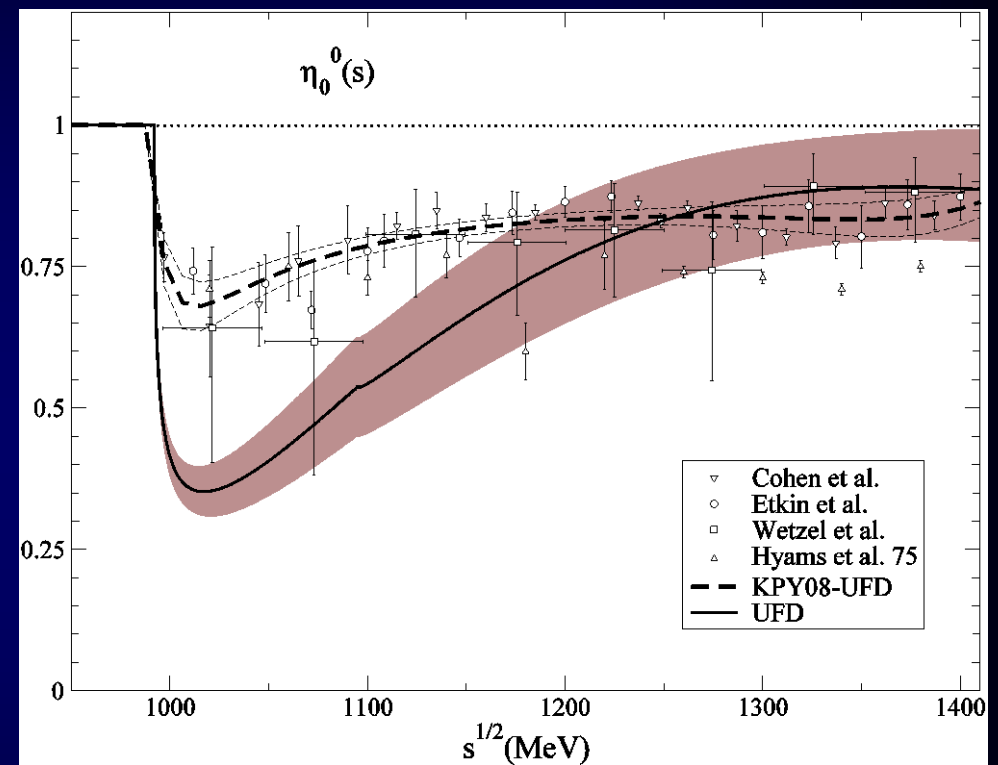
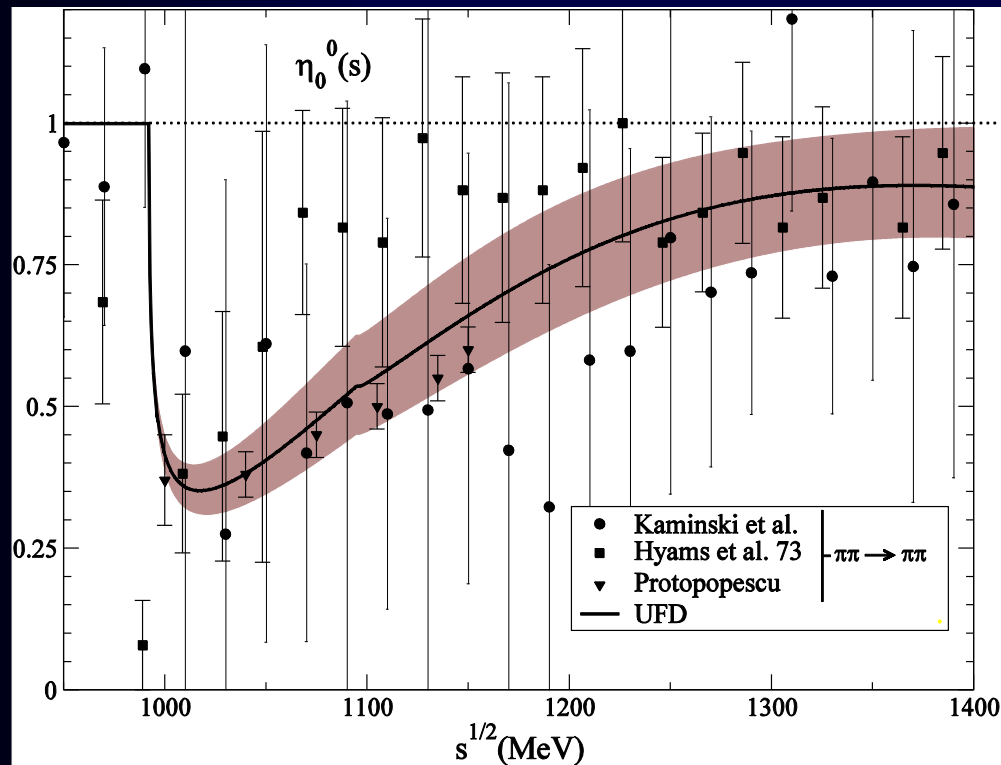
2) The $f_0(980)$

DIP vs NO DIP inelasticity scenarios

Longstanding controversy between inelasticity data sets : (Pennington, Bugg, Zou, Achasov....)

Some of them prefer a “dip” structure...

... whereas others do not



GKPY Eqs. disfavors the non-dip solution

García Martín, Kaminski, JRP, Yndurain PRD83,074004 (2011)

Garcia-Martin , Kaminski, JRP, Ruiz de Elvira, PRL107, 072001(2011)

Confirmation from Roy Eqs.

B. Moussallam, Eur. Phys. J. C71, 1814 (2011)

Some relevant recent DISPERSIVE POLE Determinations of the $f_0(980)$ (after QCHS-2010, also “according” to PDG)

- GKPY equations = Roy like with one subtraction

García Martín, Kaminski, JRP, Yndurain PRD83,074004 (2011)

Garcia-Martin , Kaminski, JRP, Ruiz de Elvira, PRL107, 072001(2011)

$$(996 \pm 7) - i(25_{-6}^{+10}) \text{ MeV}$$

- Roy equations

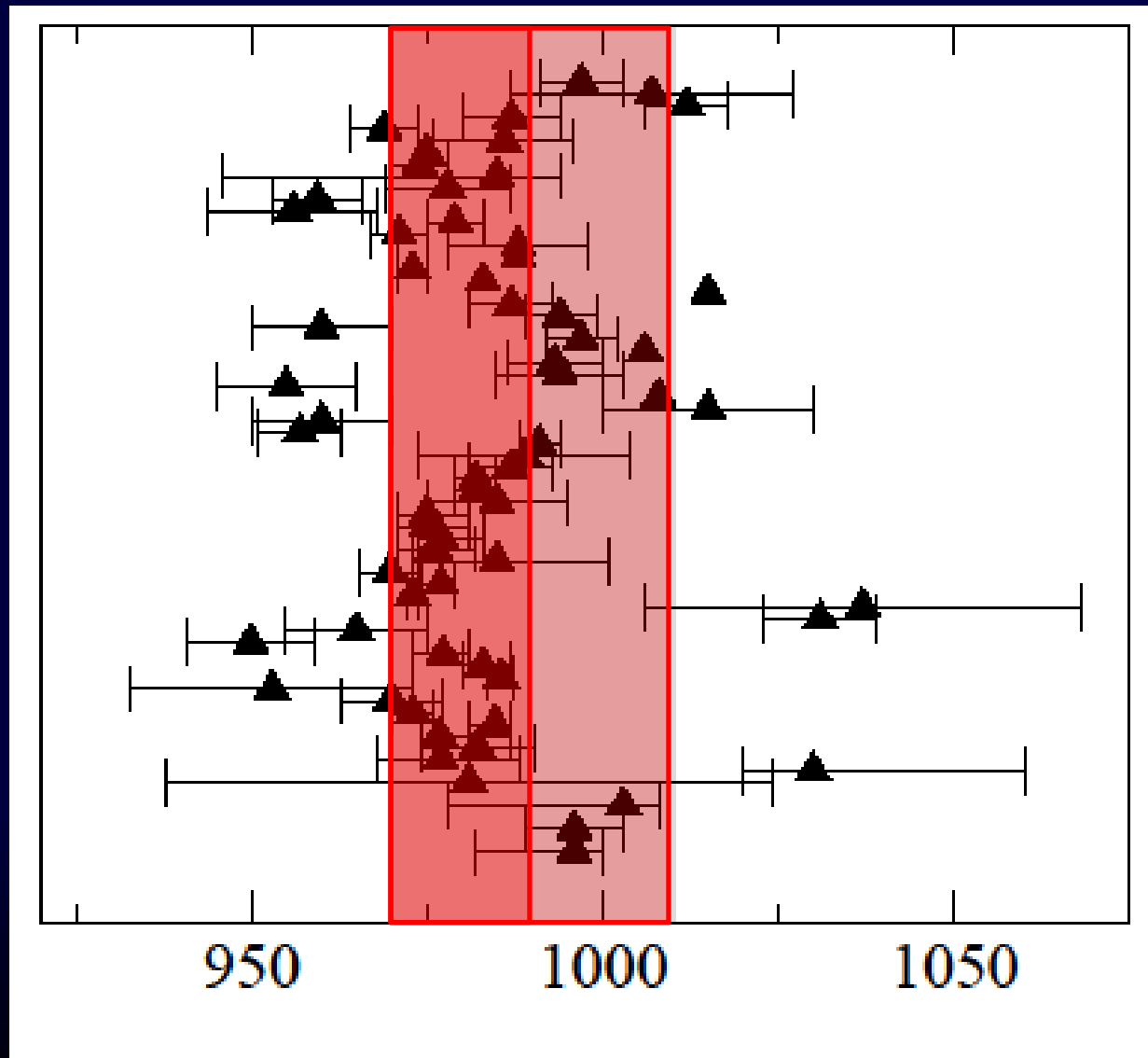
$$(996_{-14}^{+4}) - i(24_{-3}^{+11}) \text{ MeV}$$

B. Moussallam, Eur. Phys. J. C71, 1814 (2011).

The dip solution favors somewhat higher masses slightly above KK threshold
and reconciles widths from production and scattering

Thus, PDG12 made a small correction for the $f_0(980)$ mass
& more conservative uncertainties

$$M = 980 \pm 10 \text{ MeV} \rightarrow M = 990 \pm 20 \text{ MeV}$$



1) The σ or $f_0(500)$

2) The $f_0(980)$

3) The κ or $K(800)$ and $a_0(980)$

No changes on the a_0 mass and width at the PDG for the $a_0(980)$

- Still “omitted from the summary table” since, “needs confirmation”

But, all sensible implementations of unitarity, chiral symmetry, describing the data find a pole between 650 and 770 MeV with a 550 MeV width or larger.

As for the sigma, and the most sounded determination comes from a Roy-Steiner dispersive formalism, consistent with UChPT

Decotes Genon et al 2006

Since 2009 two EXPERIMENTAL results are quoted from D decays @ BES2

Surprisingly BES2 gives a pole position of $(764 \pm 63_{-54}^{+71}) - i(306 \pm 149_{-85}^{+143})\text{MeV}$

But AGAIN!! PDG goes on giving their Breit-Wigner parameters!! More confusion!!

Fortunately, the PDG mass and width averages are dominated by the Roy-Steiner result

$$(682 \pm 29) - i(273 \pm 22)\text{MeV}$$

Summary

For quite some time now the use of analyticity, unitarity, chiral symmetry, etc... to describe scattering and production data has allowed to establish the existence of light the σ and κ

These studies, together with more reliable and precise data, have allowed for PRECISE determinations of light scalar pole parameters

The PDG 2012 edition has FINALLY acknowledged the consistency of theory and experiment and the rigour and precision of the latest results, fixing, to a large extent, the very unsatisfactory compilation of σ results

Unfortunately, some traditional but inadequate parametrizations, long ago discarded by the specialists, are still being used in the PDG for the σ and the κ

But with the addition of new members to the PDG I expect a more “cleaning up” in the PDG for other scalar resonances soon

- 1) Scalar Mesons: motivation & perspective
- 2) The σ or $f_0(500)$
- 3) The $f_0(980)$
- 4) The κ or $K(800)$ and $a_0(980)$
- 5) Nature and classification.

Regge trajectory of the $f_0(500)$

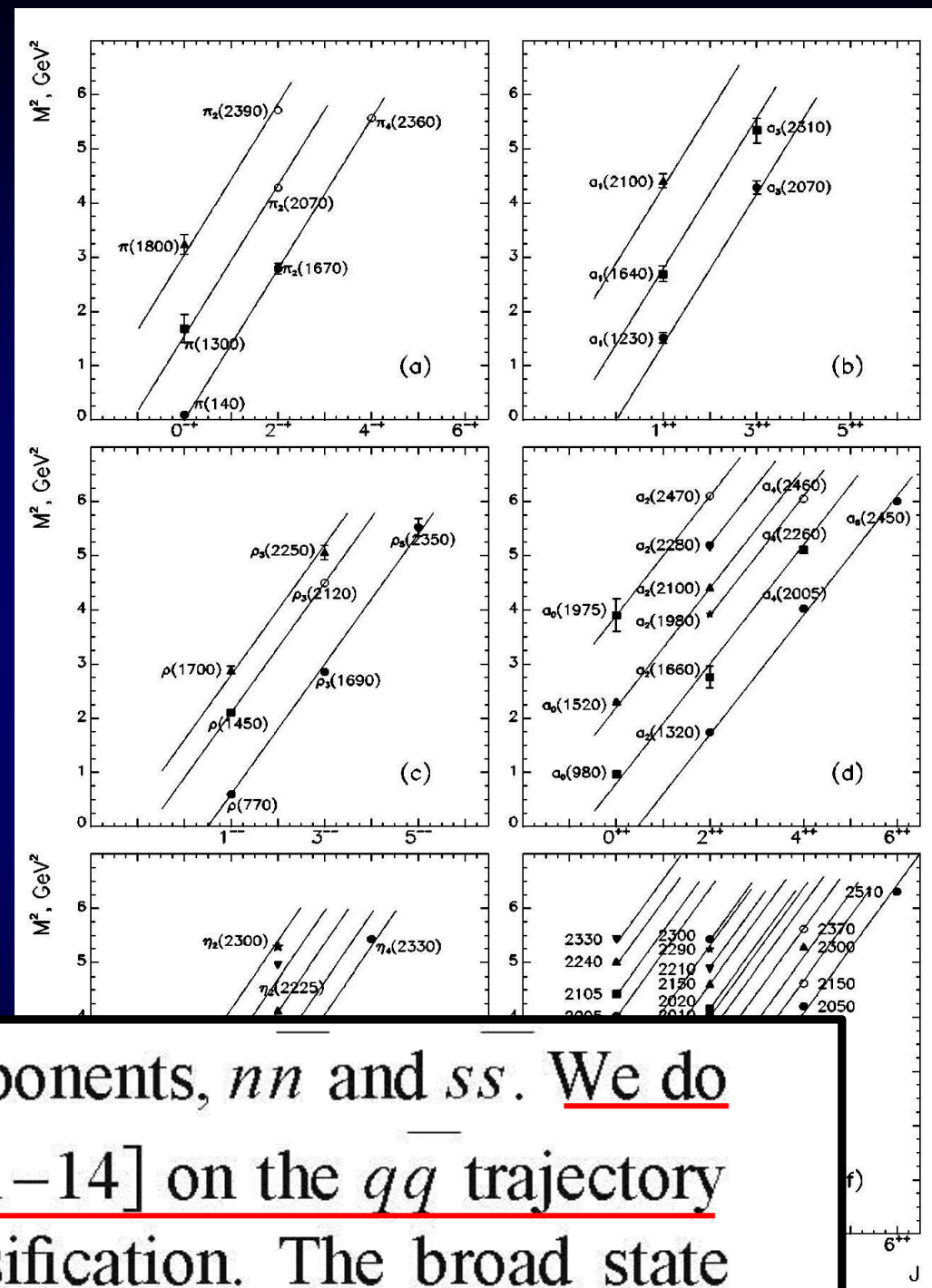
In collaboration with J.Nebreda, A. Szczepaniak and T. Londergan
Phys. Lett. B 729 (2014) 9–14

Another feature of QCD as a confining theory is that hadrons are classified in almost linear (J, M^2) trajectories

Roughly, this can be explained by a quark-antiquark pair confined at the ends of a string-like/flux-tube configuration.

The trajectories can also be understood from the analytic extension to the complex angular momentum plane (Regge Theory)

However, light scalars, and particularly the $f_0(500)$ do not fit in.



are doubled due to two flavor components, nn and ss . We do not put the enigmatic σ meson [11–14] on the qq trajectory supposing σ is alien to this classification. The broad state

An elastic partial wave amplitude near a Regge pole reads

Where α is the “trajectory” and β the “residue”

$$t_l(s) = \frac{\beta(s)}{l - \alpha(s)} + f(l, s)$$

If the amplitude is dominated by the pole, unitarity implies:

$$\text{Im } \alpha(s) = \rho(s)\beta(s).$$

Imposing the threshold behavior q^{2l} and other constraints from the analytic extension to the complex plane,

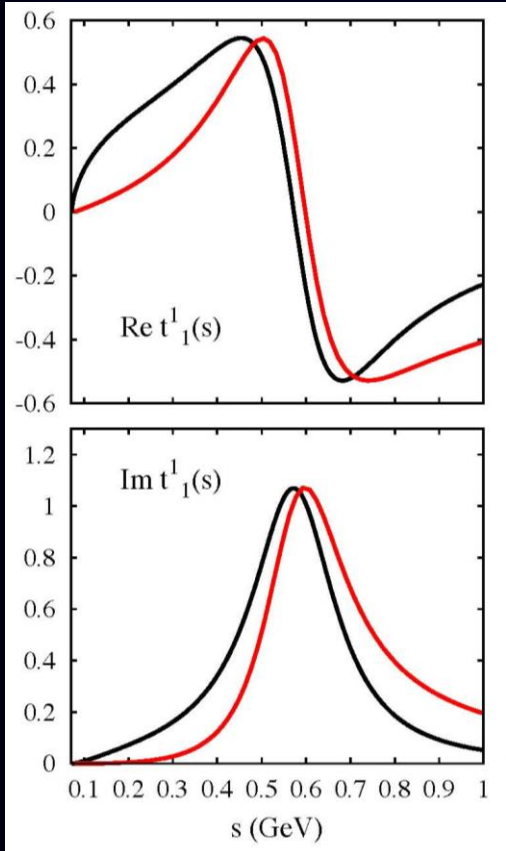
$$\beta(s) = \frac{\hat{s}^{\alpha(s)}}{\Gamma(\alpha(s) + \frac{3}{2})} \gamma(s)$$

This leads to a set of dispersion relations constraining the trajectory and residue

$$\text{Re}\alpha(s) = \alpha_0 + \alpha' s + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}\alpha(s')}{s'(s' - s)}$$

$$\text{Im}\alpha(s) = \rho(s)b_0 \frac{\hat{s}^{\alpha_0 + \alpha' s}}{|\Gamma(\alpha(s) + \frac{3}{2})|} \exp\left(-\alpha' s [1 - \log(\alpha' s_0)] + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}\alpha(s') \log \frac{\hat{s}}{\hat{s}'} + \arg \Gamma(\alpha(s') + \frac{3}{2})}{s'(s' - s)}\right)$$

The scalar case requires a small modification to include the Adler zero



When we iteratively solve the previous equations fitting only the pole and residue of the $\rho(770)$ obtained from the model independent GKPY approach...

We recover a fair representation of the amplitude

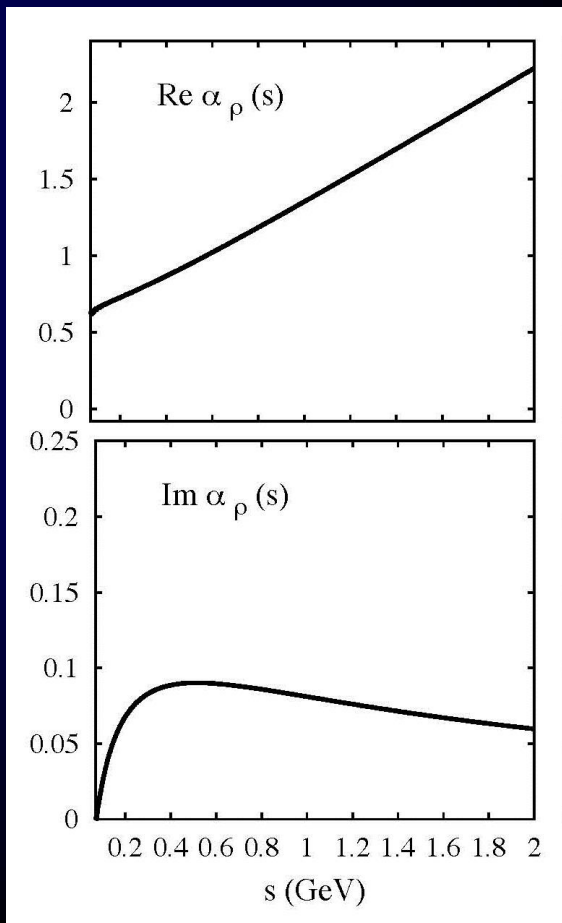
But we also obtain a “prediction” for the Regge rho trajectory, which is:

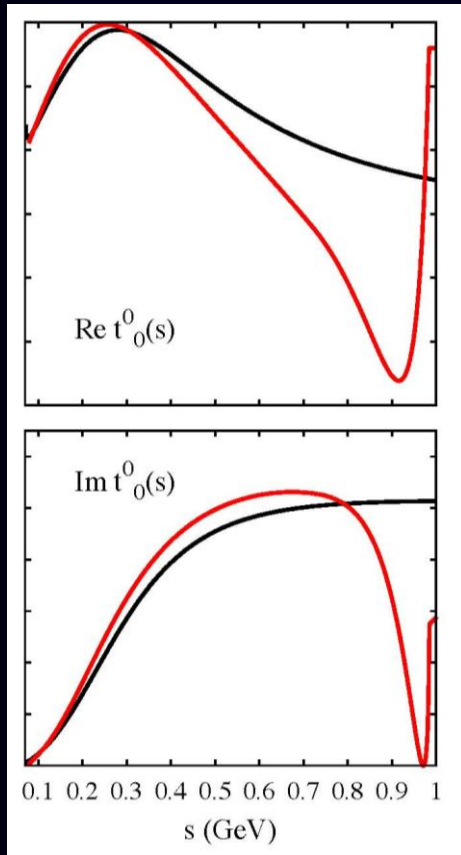
- 1) Almost real
- 2) Almost linear: $\alpha(s) \sim \alpha_0 + \alpha' s$

THIS IS A RESULT, NOT INPUT

- 3) The intercept $\alpha_0 = 0.52$
- 4) The slope $\alpha' = 0.913 \text{ GeV}^{-2}$

Remarkably consistent with the literature, taking into account our approximations





Since the approach works remarkably well for the rho, we repeat it for the f0(500). We fit the pole obtained from GKPY to a single pole-Regge like amplitude

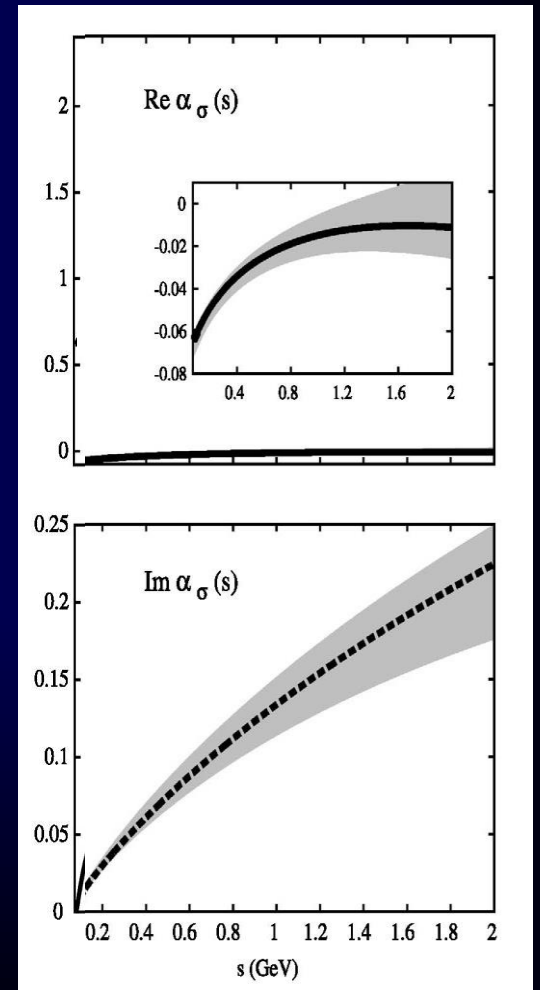
Again we recover a fair representation of the amplitude, even better than for the rho

And we obtain a “prediction” for the Regge sigma trajectory, which is:

- 1) NOT real
- 2) NOT evidently linear

3) Intercept $\alpha_\sigma(0) = -0.090^{+0.004}_{-0.012}$,

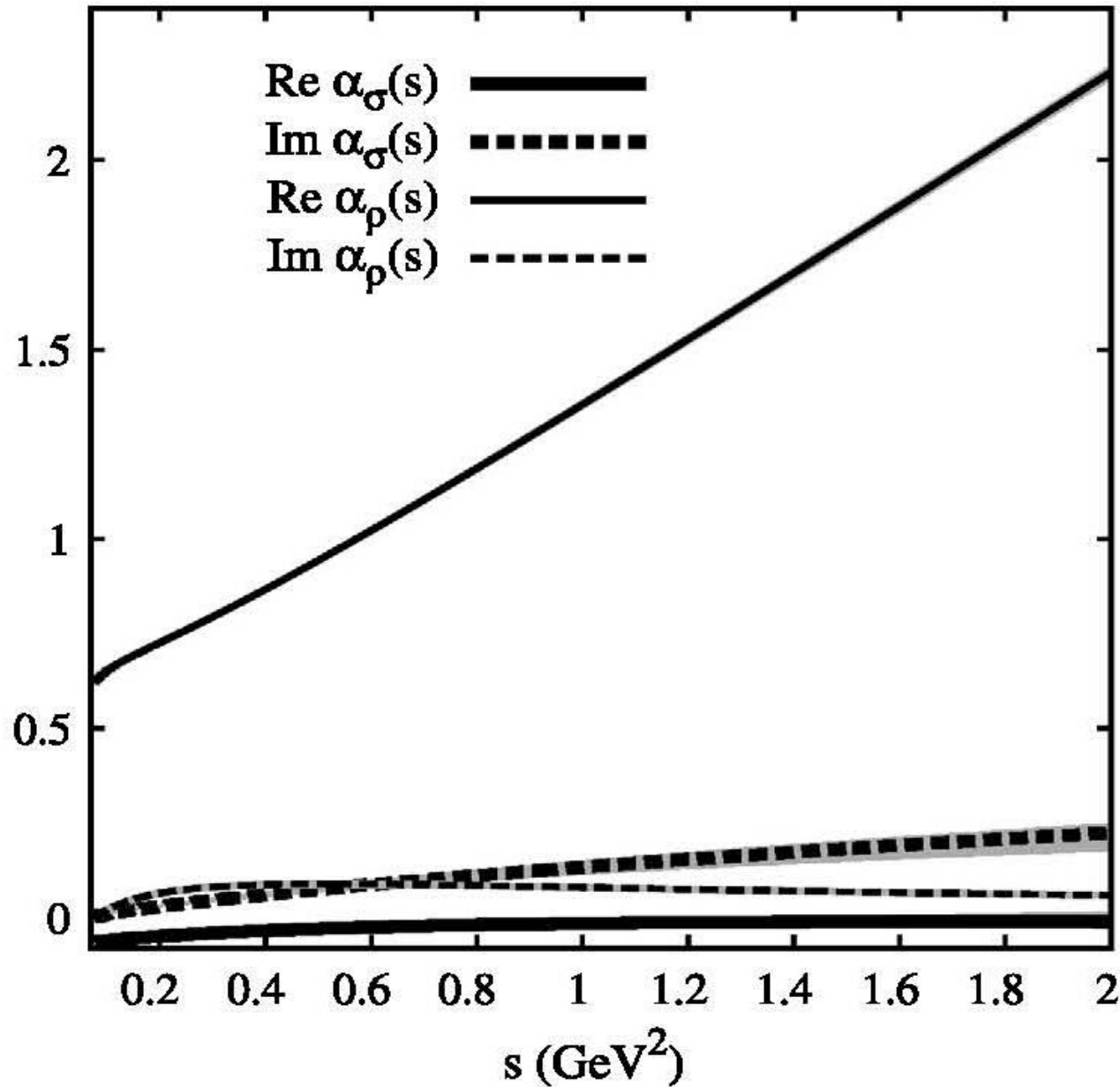
4) Slope $\alpha'_\sigma \simeq 0.002^{+0.050}_{-0.001} \text{ GeV}^{-2}$.



Two orders of magnitude flatter than other hadrons
The sigma does NOT fit the usual classification

Results: σ vs. ρ trajectories

Using the same scale....



No evident
Regge partners
for the $f_0(500)$

If not-ordinary...

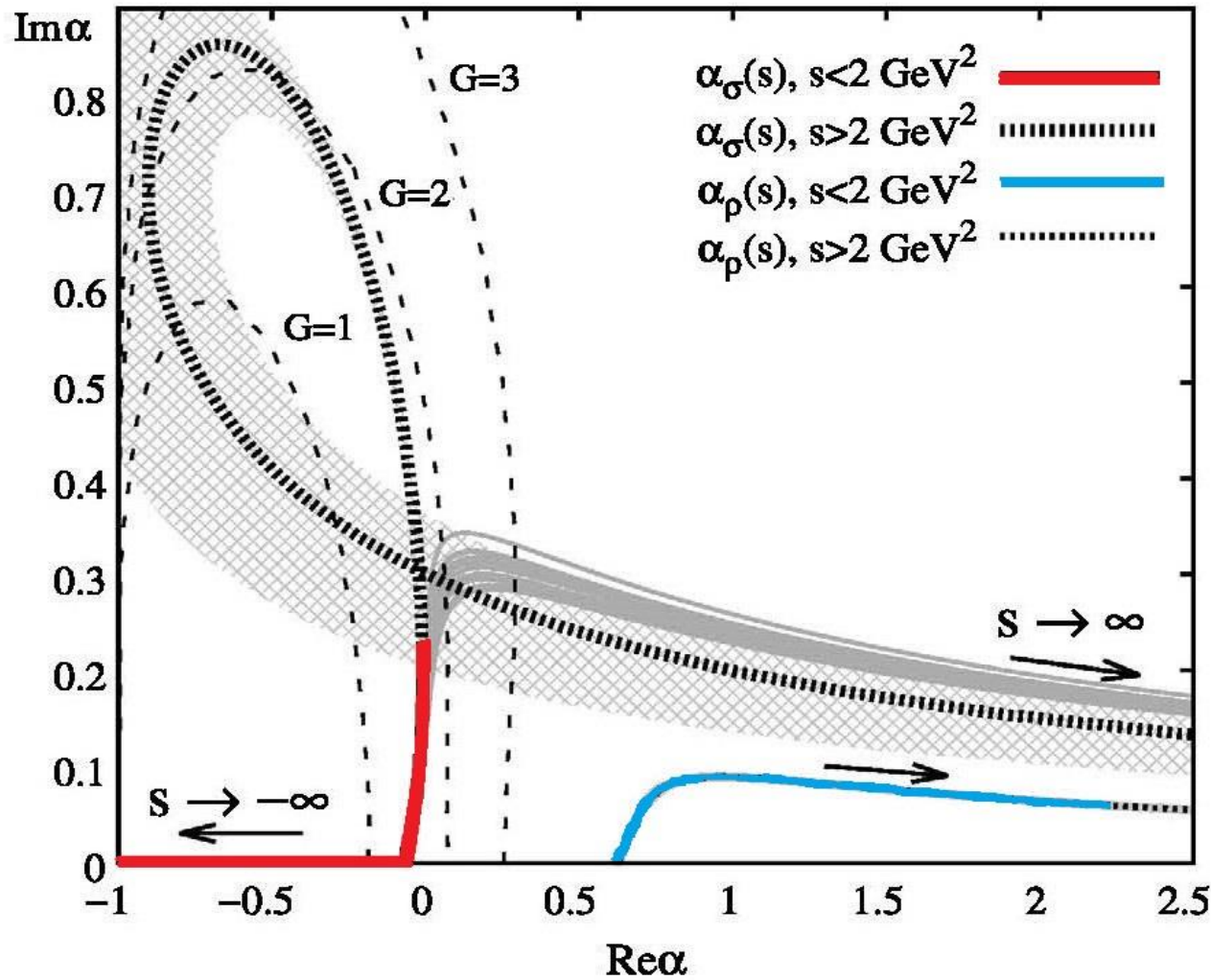
What then?

Can we identify the dynamics of the trajectory?

Not quite yet... but...

Plotting the trajectories in the complex J plane...

Striking similarity with Yukawa potentials at low energy $V(r) \propto \exp(-r/a)/r$



Our result is mimicked with $a=0.5 \text{ GeV}^{-1}$ to compare with S-wave $\pi\pi$ scattering length 1.6 GeV^{-1}

σ rather small !!!
(recent claims by Oller)

Non-ordinary σ trajectory

Ordinary ρ trajectory

The extrapolation of our trajectory also follows a Yukawa but deviates at very high energy

Summary

- Analytic constraints on Regge trajectories as integral equations
- **Fitting JUST the pole position and residue of an isolated resonance,**
yields its Regge trajectory parameters
- ρ trajectory: COMES OUT LINEAR, with universal parameters
- σ trajectory: NON-LINEAR.
 - Trajectory slope **two orders of magnitude smaller**
 - No partners.
- If we force the σ trajectory to have universal slope, data description ruined
- At low energies, striking similarities with trajectories of Yukawa potential