η-η' mixing: overview

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Purpose: to learn a few things on the η-η' system: the mixing angle(s), the gluonic content of the η' and the extraction of the mixing parameters from the η and η' transition form factors

Why? because of its relevance for present and future experimental analyses involving η and/or η' mesons: WASA, KLOE, MAMI, BES III, ...

Outline:

- *Notations for the mixing angle(s) and the gluonic content*
- *$V \to P\gamma$ analysis*
- *$J/\psi \to VP$ analysis*
- *Mixing parameters from the η and η' transition form factors*
- *Conclusions and Outlook*

• *Notation for the mixing angle: old scheme*

mixing of mass eigenstates

octet-singlet basis	quark-flavour basis

1 mixing angle

$$|\eta\rangle = \cos\theta_P|\eta_8\rangle - \sin\theta_P|\eta_0\rangle$$
$$|\eta'\rangle = \sin\theta_P|\eta_8\rangle + \cos\theta_P|\eta_0\rangle$$

with

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
$$|\eta_0\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$|\eta\rangle = \cos\phi_P|\eta_q\rangle - \sin\phi_P|\eta_s\rangle$$
$$|\eta'\rangle = \sin\phi_P|\eta_q\rangle + \cos\phi_P|\eta_s\rangle$$

and

$$|\eta_q\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
$$|\eta_s\rangle = s\bar{s}$$

$$\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ$$

Assumptions:

- no energy dependence
- $\Gamma_{\eta,\eta'} \ll m_{\eta,\eta'}$
- no mixing with other pseudoscalars (π^0, η_c, glueballs)

- *Notation for the mixing angles of the decay constants*

mixing of decay constants

octet-singlet basis

$$\langle 0|A^a_\mu|P(p)\rangle = i f^a_P p_\mu$$

with $A^a_\mu = \bar{q}\gamma_\mu\gamma_5 \frac{\lambda^a}{\sqrt{2}} q$

$f^a_P \; (a = 8, 0; P = \eta, \eta')$

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix}$$

2 mixing angles

2 decay constants

quark-flavour basis

$$\langle 0|A^i_\mu|P(p)\rangle = i f^i_P p_\mu$$

with

$$A^q_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)$$

and $A^s_\mu = \bar{s}\gamma_\mu\gamma_5 s$

$f^i_P \; (i = q, s; P = \eta, \eta')$

$$\begin{pmatrix} f^q_\eta & f^s_\eta \\ f^q_{\eta'} & f^s_{\eta'} \end{pmatrix} = \begin{pmatrix} f_q \cos\phi_q & -f_s \sin\phi_s \\ f_q \sin\phi_q & f_s \cos\phi_s \end{pmatrix}$$

- *Are all these mixing angles related?*

$$\mathcal{L} = \frac{1}{2}\partial_\mu \eta_B^T \mathcal{K} \partial^\mu \eta_B - \frac{1}{2}\eta_B^T \mathcal{M}^2 \eta_B \qquad \eta_B^T \equiv (\eta_8, \eta_1)$$

next-to-leading order corrections

$$\mathcal{K} = \begin{pmatrix} 1+\delta_8 & \delta_{81} \\ \delta_{81} & 1+\delta_1 \end{pmatrix} \qquad \mathcal{M}^2 = \begin{pmatrix} M_8^2 & M_{81}^2 \\ M_{81}^2 & M_1^2 \end{pmatrix}$$

$$\eta_B = {Z^{1/2}}^T \cdot \hat{\eta} \equiv {Z^{1/2}}^T \cdot \begin{pmatrix} \hat{\eta}_8 \\ \hat{\eta}_1 \end{pmatrix} \qquad Z^{1/2} = \begin{pmatrix} 1-\delta_8/2 & -\delta_{81}/2 \\ -\delta_{81}/2 & 1-\delta_1/2 \end{pmatrix}$$

$$Z^{1/2} \cdot \mathcal{K} \cdot {Z^{1/2}}^T = I_2 \qquad \widehat{\mathcal{M}}^2 = Z^{1/2} \cdot \mathcal{M}^2 \cdot {Z^{1/2}}^T$$

$$\hat{\eta} = R^T \cdot \eta_P \equiv R^T \cdot \begin{pmatrix} \eta \\ \eta' \end{pmatrix} \qquad R \equiv \begin{pmatrix} \cos\theta_P & -\sin\theta_P \\ \sin\theta_P & \cos\theta_P \end{pmatrix}$$

$$\widehat{\mathcal{M}}^2 = R^T \cdot \mathcal{M}_D^2 \cdot R$$

- *Are all these mixing angles related?*

$$\eta_B = (R\cdot Z^{1/2})^T \cdot \eta_P$$

$$R\cdot Z^{1/2} = \begin{pmatrix} \cos\theta_P(1-\delta_8/2)+\sin\theta_P\delta_{81}/2 & -\sin\theta_P(1-\delta_1/2)-\cos\theta_P\delta_{81}/2 \\ \sin\theta_P(1-\delta_8/2)-\cos\theta_P\delta_{81}/2 & \cos\theta_P(1-\delta_1/2)-\sin\theta_P\delta_{81}/2 \end{pmatrix}$$

lesson 1: @leading order in Large Nc ChPT only 1 mixing angle must be used

lesson 2: @next-to-leading order the mixing structure is more complicated...

$$f_P^a \;=\; f[(F^\dagger)^{-1}\mathcal{K}]_P^a$$

$$\mathcal{K} = F^\dagger I_2 F \qquad\qquad \mathcal{M}^2 = F^\dagger \mathcal{M}_D^2 F$$

$$F = R\cdot({Z^{1/2}}^T)^{-1} \qquad f_P^a = f[R\cdot({Z^{1/2}}^T)^{-1}]_P^a \qquad \eta_P = R\cdot({Z^{1/2}}^T)^{-1}\cdot\eta_B$$

lesson 3: the mixing structure of the decay constants and of the fields is the same!

• *Are all these mixing angles related?*

To first order in δ:

$$f_\eta^8/f = \cos\theta_P(1+\delta_8/2) - \sin\theta_P\delta_{81}/2 \ ,$$
$$f_\eta^0/f = -\sin\theta_P(1+\delta_1/2) + \cos\theta_P\delta_{81}/2 \ ,$$
$$f_{\eta'}^8/f = \sin\theta_P(1+\delta_8/2) + \cos\theta_P\delta_{81}/2 \ ,$$
$$f_{\eta'}^0/f = \cos\theta_P(1+\delta_1/2) + \sin\theta_P\delta_{81}/2 \ ,$$

to compare with

$$\{f_P^a\} = \begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8\cos\theta_8 & -f_0\sin\theta_0 \\ f_8\sin\theta_8 & f_0\cos\theta_0 \end{pmatrix}$$

decay constants:

$$f_8 = f(1+\delta_8/2)$$
$$f_0 = f(1+\delta_1/2)$$

mixing angles:

$$\theta_8 = \theta_P + \arctan(\delta_{81}/2)$$
$$\theta_0 = \theta_P - \arctan(\delta_{81}/2)$$

lesson 4: @next-to-leading order in Large Nc ChPT 2 mixing angles must be used!

lesson 5: f_η and $f_{\eta'}$ do not exist!

• *Notation for the mixing angle(s): new scheme*

In Large Nc ChPT:

$$\delta_8 = \frac{8L_5}{F^2}\overset{\circ}{M}{}_8^2, \quad \delta_{81} = \frac{8L_5}{F^2}\overset{\circ}{M}{}_{81}^2, \quad \delta_1 = \frac{8L_5}{F^2}\overset{\circ}{M}{}_1^2 + \Lambda_1$$

octet-singlet basis:

$$f_8^2 = \frac{4f_K^2 - f_\pi^2}{3}, \qquad f_0^2 = \frac{2f_K^2 + f_\pi^2}{3} + f_\pi^2\Lambda_1,$$

$$f_8 f_0 \sin(\theta_8 - \theta_0) = -\frac{2\sqrt{2}}{3}(f_K^2 - f_\pi^2),$$

OZI rule violating parameter

quark-flavour basis:

$$f_q^2 = f_\pi^2 + \frac{2}{3}f_\pi^2\Lambda_1, \qquad f_s^2 = 2f_K^2 - f_\pi^2 + \frac{1}{3}f_\pi^2\Lambda_1,$$

$$f_q f_s \sin(\phi_q - \phi_s) = \frac{\sqrt{2}}{3}f_\pi^2\Lambda_1,$$

Approximate relations valid for $\phi_q = \phi_s \equiv \phi$

$$f_8 = \sqrt{\frac{1}{3}f_q^2 + \frac{2}{3}f_s^2}, \qquad \theta_8 = \phi - \arctan\left(\frac{\sqrt{2}f_s}{f_q}\right)$$

$$f_0 = \sqrt{\frac{2}{3}f_q^2 + \frac{1}{3}f_s^2}, \qquad \theta_0 = \phi - \arctan\left(\frac{\sqrt{2}f_q}{f_s}\right)$$

SU(3) breaking effect

lesson 6: in experimental analyses always use the quark-flavour basis

- *Study of the η-η' system in the two mixing angle scheme*

η,η'→γγ decays

R. E., J.-M. Frère, JHEP 06, 029 (2005)

The interpolating fields η and η' are related with the axial-vector currents

$$\eta(x) = \frac{1}{m_\eta^2}\frac{f_{\eta'}^0 \partial^\mu A_\mu^8(x) - f_{\eta'}^8 \partial^\mu A_\mu^0(x)}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0}\,, \qquad \eta'(x) = \frac{1}{m_{\eta'}^2}\frac{f_\eta^0 \partial^\mu A_\mu^8(x) - f_\eta^8 \partial^\mu A_\mu^0(x)}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0}\,.$$

This leads to

$$\Gamma(\eta\to\gamma\gamma) = \frac{\alpha^2 m_\eta^3}{96\pi^3}\left(\frac{f_{\eta'}^0 - 2\sqrt{2} f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0}\right)^2 = \frac{\alpha^2 m_\eta^3}{96\pi^3}\left(\frac{c\theta_0/f_8 - 2\sqrt{2} s\theta_8/f_0}{c\theta_0 c\theta_8 + s\theta_8 s\theta_0}\right)^2\,,$$

$$\Gamma(\eta'\to\gamma\gamma) = \frac{\alpha^2 m_{\eta'}^3}{96\pi^3}\left(\frac{f_\eta^0 - 2\sqrt{2} f_\eta^8}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0}\right)^2 = \frac{\alpha^2 m_{\eta'}^3}{96\pi^3}\left(\frac{s\theta_0/f_8 + 2\sqrt{2} c\theta_8/f_0}{c\theta_0 c\theta_8 + s\theta_8 s\theta_0}\right)^2\,.$$

VPγ decays

We extend our analysis to the couplings of the radiative decays $V\to(\eta,\eta')\gamma$ and $\eta'\to V\gamma$ with $V=\rho,\omega,\phi$.

The form factors $F_{VP\gamma}(0,0)$ are fixed by the AVV triangle anomaly

Using their analytic properties

$$F_{VP\gamma}(0,0) = \frac{f_V}{m_V} g_{VP\gamma} + \cdots \quad \text{(Vector Meson Dominance)}$$

where the vertex couplings $g_{VP\gamma}$ are the on-shell V-P electromagnetic form factors

$$\langle P(p_P)|J_\mu^{\rm EM}|V(p_V,\lambda)\rangle|_{(p_V-p_P)^2=0} = -g_{VP\gamma}\epsilon_{\mu\nu\alpha\beta}p_P^\nu p_V^\alpha \varepsilon_V^\beta(\lambda)\,.$$

• *Study of the η-η' system in the two mixing angle scheme*

Our best results for the mixing parameters are

$$f_8 = (1.51 \pm 0.05) f_\pi \ , \qquad \theta_8 = (-23.8 \pm 1.4)^\circ \ ,$$
$$f_0 = (1.29 \pm 0.04) f_\pi \ , \qquad \theta_0 = (-2.4 \pm 1.9)^\circ \ ,$$

in the octet-singlet basis, and

$$f_q = (1.09 \pm 0.03) f_\pi \ , \qquad \phi_q = (39.9 \pm 1.3)^\circ \ ,$$
$$f_s = (1.66 \pm 0.06) f_\pi \ , \qquad \phi_s = (41.4 \pm 1.4)^\circ \ ,$$

in the quark-flavour basis.

- in the octet-singlet basis a two mixing angle scheme is needed to describe experimental data in a better way;
- in the quark-flavour basis a one mixing angle description of data is enough at the current experimental accuracy.

At the present accuracy, our results satisfy the approximate relations

$$f_8 = \sqrt{1/3 f_q^2 + 2/3 f_s^2} \ , \qquad \theta_8 = \phi - \arctan(\sqrt{2} f_s / f_q) \ ,$$
$$f_0 = \sqrt{2/3 f_q^2 + 1/3 f_s^2} \ , \qquad \theta_0 = \phi - \arctan(\sqrt{2} f_q / f_s) \ .$$

• *Notation for the gluonic content: phenomenological parametrization*

We work in a basis consisting of the states

$$|\eta_q\rangle \equiv \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle \qquad |\eta_s\rangle = |s\bar{s}\rangle \qquad |G\rangle \equiv |\mathrm{gluonium}\rangle$$

The physical states η and η' are assumed to be the linear combinations

$$\begin{aligned} |\eta\rangle &= X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle + Z_\eta|G\rangle \ , \\ |\eta'\rangle &= X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|G\rangle \ , \end{aligned}$$

with $X^2_{\eta(\eta')} + Y^2_{\eta(\eta')} + Z^2_{\eta(\eta')} = 1$ and thus $X^2_{\eta(\eta')} + Y^2_{\eta(\eta')} \leq 1$

A significant gluonic admixture in a state is possible only if

$$Z^2_{\eta(\eta')} = 1 - X^2_{\eta(\eta')} - Y^2_{\eta(\eta')} > 0$$

Assumptions:

- no mixing with π^0 (isospin symmetry)
- no mixing with η_c states
- no mixing with radial excitations

• *Notation for the gluonic content*

In absence of gluonium (standard picture)

$$Z_{\eta(\eta')} \equiv 0$$

$$\begin{aligned} |\eta\rangle &= \cos\phi_P|\eta_q\rangle - \sin\phi_P|\eta_s\rangle \\ |\eta'\rangle &= \sin\phi_P|\eta_q\rangle + \cos\phi_P|\eta_s\rangle \end{aligned}$$

with $X_\eta = Y_{\eta'} \equiv \cos\phi_P$ and $X^2_{\eta(\eta')} + Y^2_{\eta(\eta')} = 1$

$X_{\eta'} = -Y_\eta \equiv \sin\phi_P$

where ϕ_P is the η-η' mixing angle in the quark-flavour basis related to its octet-singlet analog through

$$\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ$$

Similarly, for the vector states ω and ϕ the mixing is given by

$$\begin{aligned} |\omega\rangle &= \cos\phi_V|\omega_q\rangle - \sin\phi_V|\phi_s\rangle \\ |\phi\rangle &= \sin\phi_V|\omega_q\rangle + \cos\phi_V|\phi_s\rangle \end{aligned}$$

where ω_q and ϕ_s are the analog non-strange and strange states of η_q and η_s, respectively.

- *Euler angles*

In presence of gluonium,

$$|\eta\rangle = X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle + Z_\eta|G\rangle$$
$$|\eta'\rangle = X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|G\rangle$$
$$|\iota\rangle = X_\iota|\eta_q\rangle + Y_\iota|\eta_s\rangle + Z_\iota|G\rangle$$

glueball-like state η(1440)? → $|\iota\rangle$

Normalization:

$$X_\eta^2 + Y_\eta^2 + Z_\eta^2 = 1$$
$$X_{\eta'}^2 + Y_{\eta'}^2 + Z_{\eta'}^2 = 1$$
$$X_\iota^2 + Y_\iota^2 + Z_\iota^2 = 1$$

Orthogonality:

$$X_\eta X_{\eta'} + Y_\eta Y_{\eta'} + Z_\eta Z_{\eta'} = 0$$
$$X_\eta X_\iota + Y_\eta Y_\iota + Z_\eta Z_\iota = 0$$
$$X_{\eta'} X_\iota + Y_{\eta'} Y_\iota + Z_{\eta'} Z_\iota = 0$$

→ 3 independent parameters: ϕ_P, $\phi_{\eta G}$ and $\phi_{\eta' G}$

$$\begin{pmatrix} \eta \\ \eta' \\ \iota \end{pmatrix} = \begin{pmatrix} c\phi_{\eta\eta'}c\phi_{\eta G} & -s\phi_{\eta\eta'}c\phi_{\eta G} & -s\phi_{\eta G} \\ s\phi_{\eta\eta'}c\phi_{\eta' G} - c\phi_{\eta\eta'}s\phi_{\eta' G}s\phi_{\eta G} & c\phi_{\eta\eta'}c\phi_{\eta' G} + s\phi_{\eta\eta'}s\phi_{\eta' G}s\phi_{\eta G} & -s\phi_{\eta' G}c\phi_{\eta G} \\ s\phi_{\eta\eta'}s\phi_{\eta' G} + c\phi_{\eta\eta'}c\phi_{\eta' G}s\phi_{\eta G} & c\phi_{\eta\eta'}s\phi_{\eta' G} - s\phi_{\eta\eta'}c\phi_{\eta' G}s\phi_{\eta G} & c\phi_{\eta' G}c\phi_{\eta G} \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \\ G \end{pmatrix}$$

- *Euler angles*

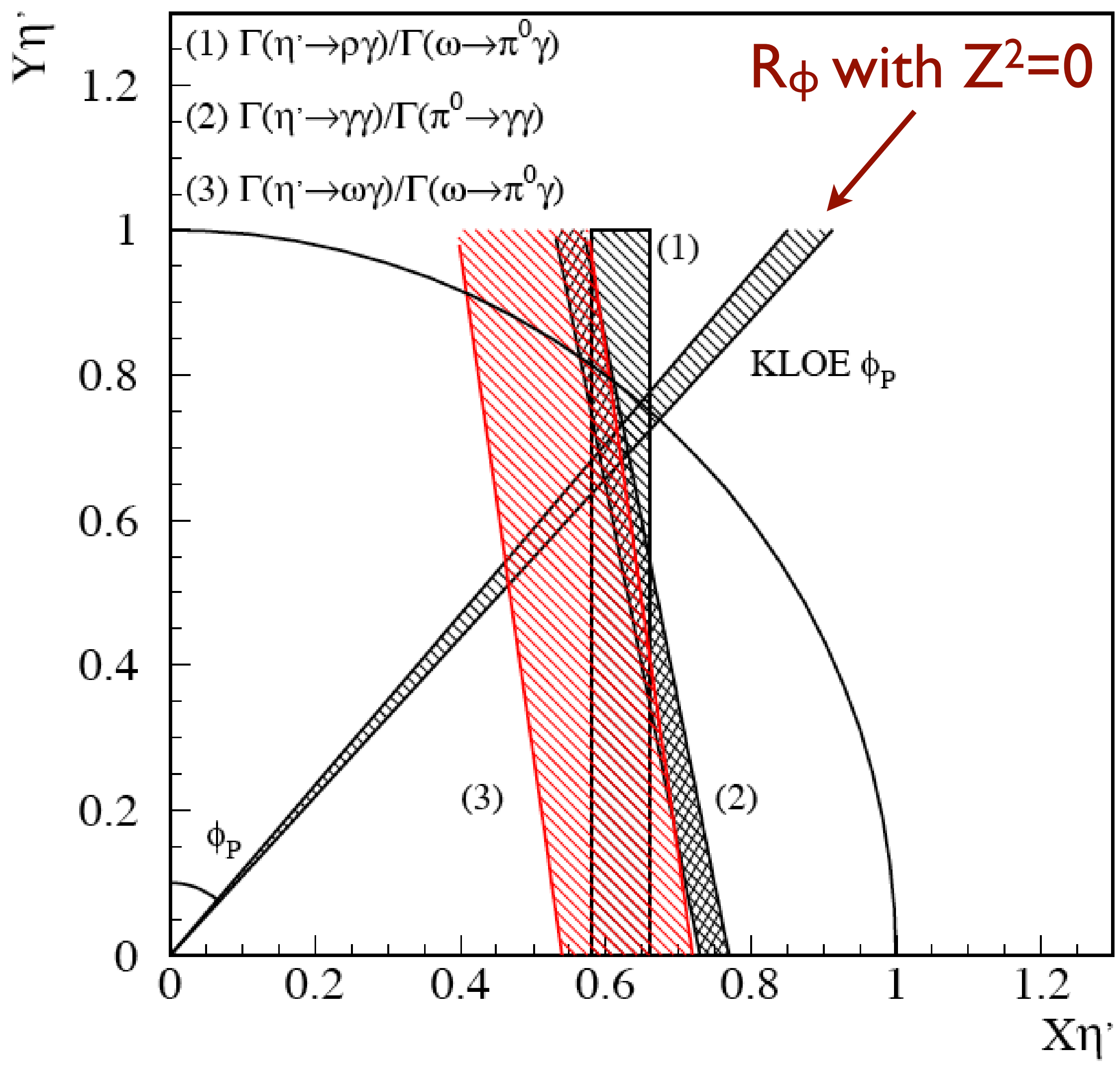
$$X_\eta = \cos\phi_P \cos\phi_{\eta G} \,, \quad X_{\eta'} = \sin\phi_P \cos\phi_{\eta' G} - \cos\phi_P \sin\phi_{\eta G} \sin\phi_{\eta' G} \,,$$
$$Y_\eta = -\sin\phi_P \cos\phi_{\eta G} \,, \quad Y_{\eta'} = \cos\phi_P \cos\phi_{\eta' G} + \sin\phi_P \sin\phi_{\eta G} \sin\phi_{\eta' G} \,,$$
$$Z_\eta = -\sin\phi_{\eta G} \,, \quad Z_{\eta'} = -\sin\phi_{\eta' G} \cos\phi_{\eta G} \;.$$

In the limit $\phi_{\eta G}=0$:

$$X_\eta = \cos\phi_P \;, \qquad Y_\eta = -\sin\phi_P \;, \qquad Z_\eta = 0 \;,$$
$$X_{\eta'} = \sin\phi_P \cos\phi_{\eta' G} \;, \quad Y_{\eta'} = \cos\phi_P \cos\phi_{\eta' G} \;, \quad Z_{\eta'} = -\sin\phi_{\eta' G} \;.$$

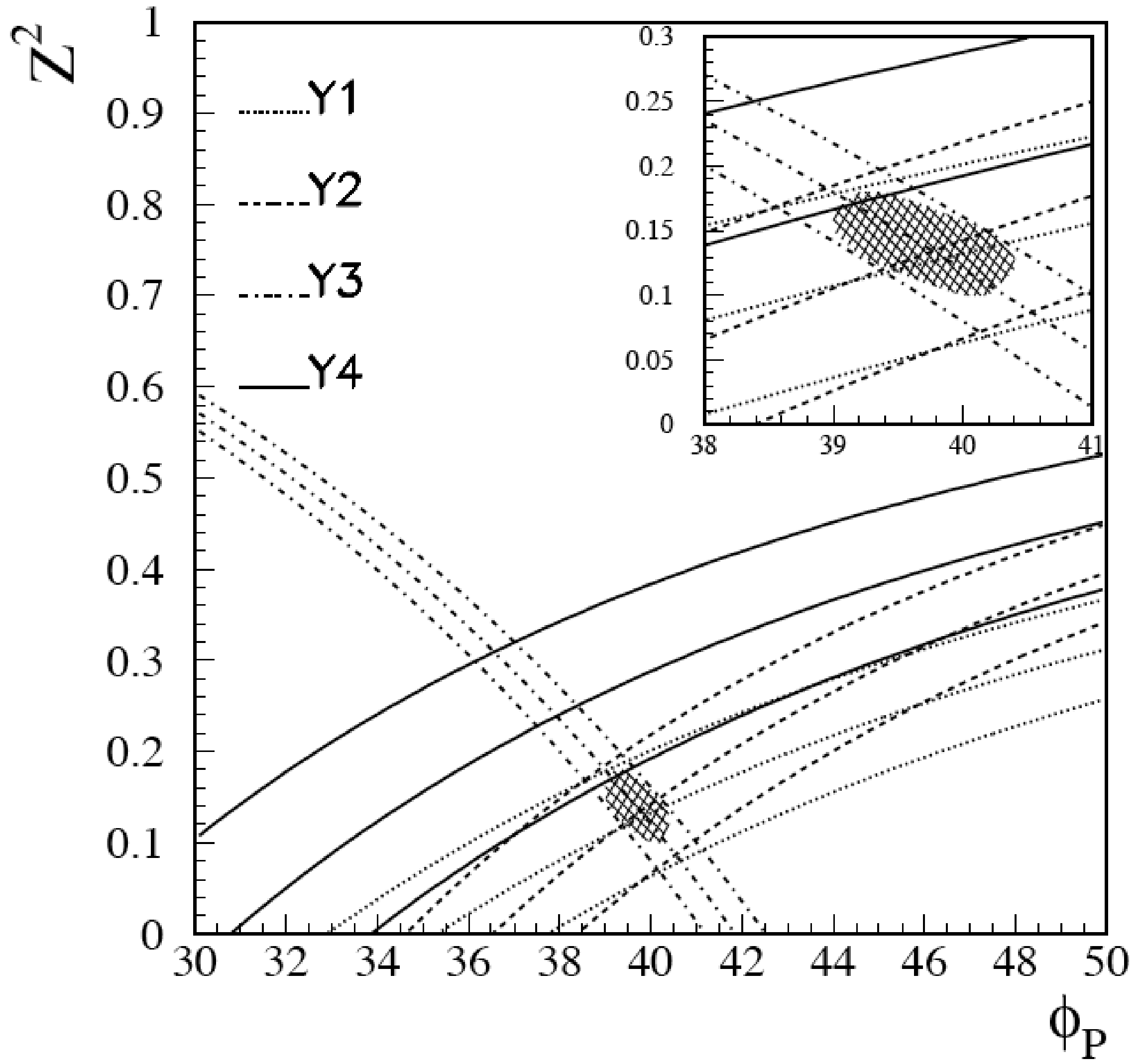
- *Motivation*

KLOE Collaboration, Phys. Lett. B648 (2007) 267



$$\phi_P = (39.7 \pm 0.7)^\circ$$

$$Z^2_{\eta'} = 0.14 \pm 0.04$$



Y1=η'→γγ/π⁰→γγ

Y2=η'→ργ/ω→π⁰γ

Y3=ϕ→η'γ/ϕ→ηγ

Y4=η'→ωγ/ω→π⁰γ

• $V \to P\gamma$ analysis: a model for $VP\gamma$ M1 transitions

We will work in a conventional quark model context: P and V are simple quark-antiquark S-wave bound states

⇒ all these hadrons are thus extended objects with characteristics spatial extensions fixed by their respective P and V wave functions

SU(2) limit ⇒ identical spatial extension within each isomultiplet

SU(3) broken ⇒ constituent quark masses with $m_s > m$ and different spatial extensions for each isomultiplet

Ingredients of the model:

i) a $VP\gamma$ magnetic dipole transition proceeding via quark or antiquark spin flip amplitude $\propto \mu_q = e_q/2m_q$

ii) spin-flip $V \to P$ conversion amplitude corrected by the relative overlap between the P and V wave functions

iii) OZI-rule reduces considerably the possible transitions and overlaps

U(1)$_A$ anomaly →

$$C_\pi \equiv \langle \pi | \omega_q \rangle = \langle \pi | \rho \rangle \qquad C_K \equiv \langle K | K^* \rangle$$

$$C_q \equiv \langle \eta_q | \omega_q \rangle = \langle \eta_q | \rho \rangle \qquad C_s \equiv \langle \eta_s | \phi_s \rangle$$

• *A model for VPγ M1 transitions*

Amplitudes:

$$g_{\rho^0\pi^0\gamma} = g_{\rho^+\pi^+\gamma} = \tfrac{1}{3} g \ , \quad g_{\omega\pi\gamma} = g \cos\phi_V \ , \quad g_{\phi\pi\gamma} = g \sin\phi_V \ ,$$

$$g_{K^{*0}K^0\gamma} = -\tfrac{1}{3} g \, z_K \left(1 + \tfrac{\bar{m}}{m_s}\right) \ , \quad g_{K^{*+}K^+\gamma} = \tfrac{1}{3} g \, z_K \left(2 - \tfrac{\bar{m}}{m_s}\right) \ ,$$

$$g_{\rho\eta\gamma} = g \, z_q \, X_\eta \ , \quad g_{\rho\eta'\gamma} = g \, z_q \, X_{\eta'} \ ,$$

$$g_{\omega\eta\gamma} = \tfrac{1}{3} g \left(z_q \, X_\eta \cos\phi_V + 2 \tfrac{\bar{m}}{m_s} z_s \, Y_\eta \sin\phi_V \right) \ ,$$

$$g_{\omega\eta'\gamma} = \tfrac{1}{3} g \left(z_q \, X_{\eta'} \cos\phi_V + 2 \tfrac{\bar{m}}{m_s} z_s \, Y_{\eta'} \sin\phi_V \right) \ ,$$

$$g_{\phi\eta\gamma} = \tfrac{1}{3} g \left(z_q \, X_\eta \sin\phi_V - 2 \tfrac{\bar{m}}{m_s} z_s \, Y_\eta \cos\phi_V \right) \ ,$$

$$g_{\phi\eta'\gamma} = \tfrac{1}{3} g \left(z_q \, X_{\eta'} \sin\phi_V - 2 \tfrac{\bar{m}}{m_s} z_s \, Y_{\eta'} \cos\phi_V \right) \ ,$$

with $g_{\omega\pi\gamma} = g \, \cos\phi_V = e \, C_\pi \cos\phi_V / \bar{m}$

and $z_q \equiv C_q / C_\pi \ , \quad z_s \equiv C_s / C_\pi \ , \quad z_K \equiv C_K / C_\pi$

$$\Gamma(V \to P\gamma) = \frac{1}{3} \frac{g^2_{VP\gamma}}{4\pi} |\mathbf{p}_\gamma|^3 = \frac{1}{3} \Gamma(P \to V\gamma)$$

• *Data fitting*

R. E. and J. Nadal, JHEP 05 (2007) 6

Three possibilities:

i) $Z_\eta = Z_{\eta'} = 0$ → gluonium not allowed for η or η'

ii) $Z_\eta = 0$ → gluonium allowed only for η'

iii) $Z_{\eta'} = 0$ → gluonium allowed only for η

i) assuming $Z_\eta = Z_{\eta'} = 0$ from the beginning, we get from $\chi^2/\text{d.o.f.} = 14.0/7$ to

$$g = 0.72 \pm 0.01\ \text{GeV}^{-1}\ , \quad \phi_P = (41.5 \pm 1.2)^\circ, \quad \phi_V = (3.2 \pm 0.1)^\circ\ ,$$
$$\frac{m_s}{\bar{m}} = 1.24 \pm 0.07\ , \quad z_K = 0.89 \pm 0.03\ , \quad z_q = 0.86 \pm 0.03\ , \quad z_s = 0.78 \pm 0.05\ .$$

$\chi^2/\text{d.o.f.} = 4.4/5$

ii) assuming $Z_\eta = 0$ from the beginning, we get

$$g = 0.72 \pm 0.01\ \text{GeV}^{-1}\ , \quad \frac{m_s}{\bar{m}} = 1.24 \pm 0.07\ , \quad \phi_V = (3.2 \pm 0.1)^\circ\ ,$$
$$\phi_P = (41.4 \pm 1.3)^\circ, \quad |\phi_{\eta' G}| = (12 \pm 13)^\circ,$$
$$z_K = 0.89 \pm 0.03\ , \quad z_q = 0.86 \pm 0.03\ , \quad z_s = 0.79 \pm 0.05\ ,$$

$\chi^2/\text{d.o.f.} = 4.2/4$

→ Accepting the absence of gluonium for the η meson, the gluonic content of the η' wave function amounts to $|\phi_{\eta' G}| = (12 \pm 13)^\circ$ or $(Z_{\eta'})^2 = 0.04 \pm 0.09$ and the η-η' mixing angle is found to be $\phi_P = (41.4 \pm 1.3)^\circ$

• *Data fitting*

iii) assuming $Z_{\eta'}=0$ from the beginning, we get

$$g = 0.72 \pm 0.01 \text{ GeV}^{-1}\,, \quad \frac{m_s}{\bar{m}} = 1.24 \pm 0.07\,, \quad \phi_V = (3.2 \pm 0.1)^\circ\,,$$
$$\phi_P = (41.5 \pm 1.3)^\circ\,, \quad |\phi_{\eta G}| \simeq 0^\circ\,,$$
$$z_q = 0.86 \pm 0.04\,, \quad z_s = 0.78 \pm 0.06\,, \quad z_K = 0.89 \pm 0.03\,,$$

χ^2/d.o.f.=4.4/4

Accepting the absence of gluonium for the η' meson, the gluonic content of the η wave function amounts to $|\phi_{\eta G}|\simeq 0^\circ$ or $(Z_\eta)^2=0.00\pm0.12$ and the η-η' mixing angle is found to be $\phi_P=(41.5\pm1.3)^\circ$

The current experimental data on VPγ transitions indicate within our model a negligible gluonic content for the η and η' mesons

Using the latest experimental data on (ρ,ω,ϕ)→ηγ (SND) and ϕ→η'γ (KLOE), we get

$$\phi_P = (42.7 \pm 0.7)^\circ, \quad z_q = 0.83 \pm 0.03\,, \quad z_s = 0.79 \pm 0.05\,,$$

χ^2/d.o.f.=4.0/5

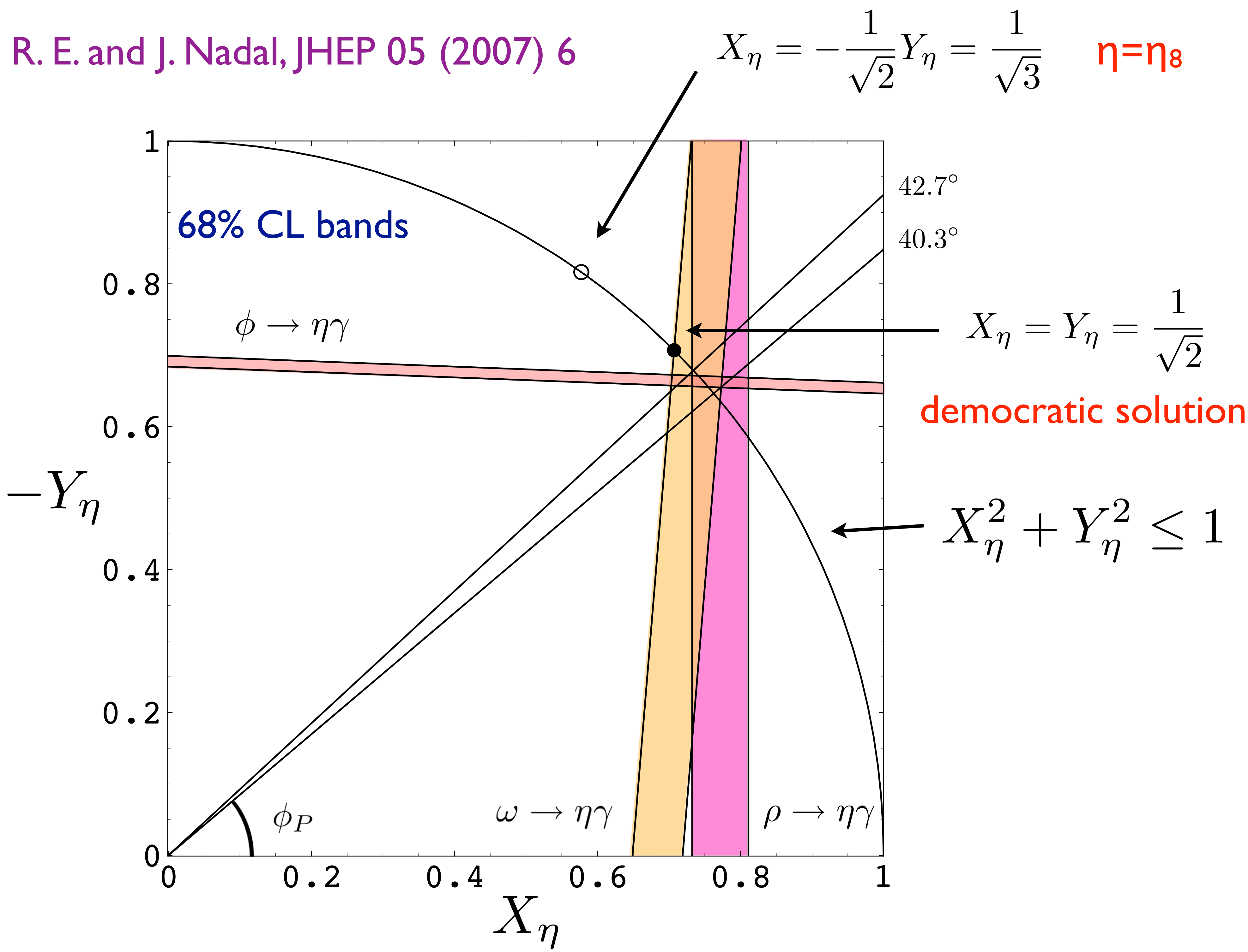
$$\phi_P = (42.6\pm1.1)^\circ, \quad |\phi_{\eta' G}| = (5\pm21)^\circ, \quad z_q = 0.83\pm0.03\,, \quad z_s = 0.79\pm0.05\,,$$

χ^2/d.o.f.=4.0/4

confirmation of the null gluonic content of the η and η' wave functions

• *Results*

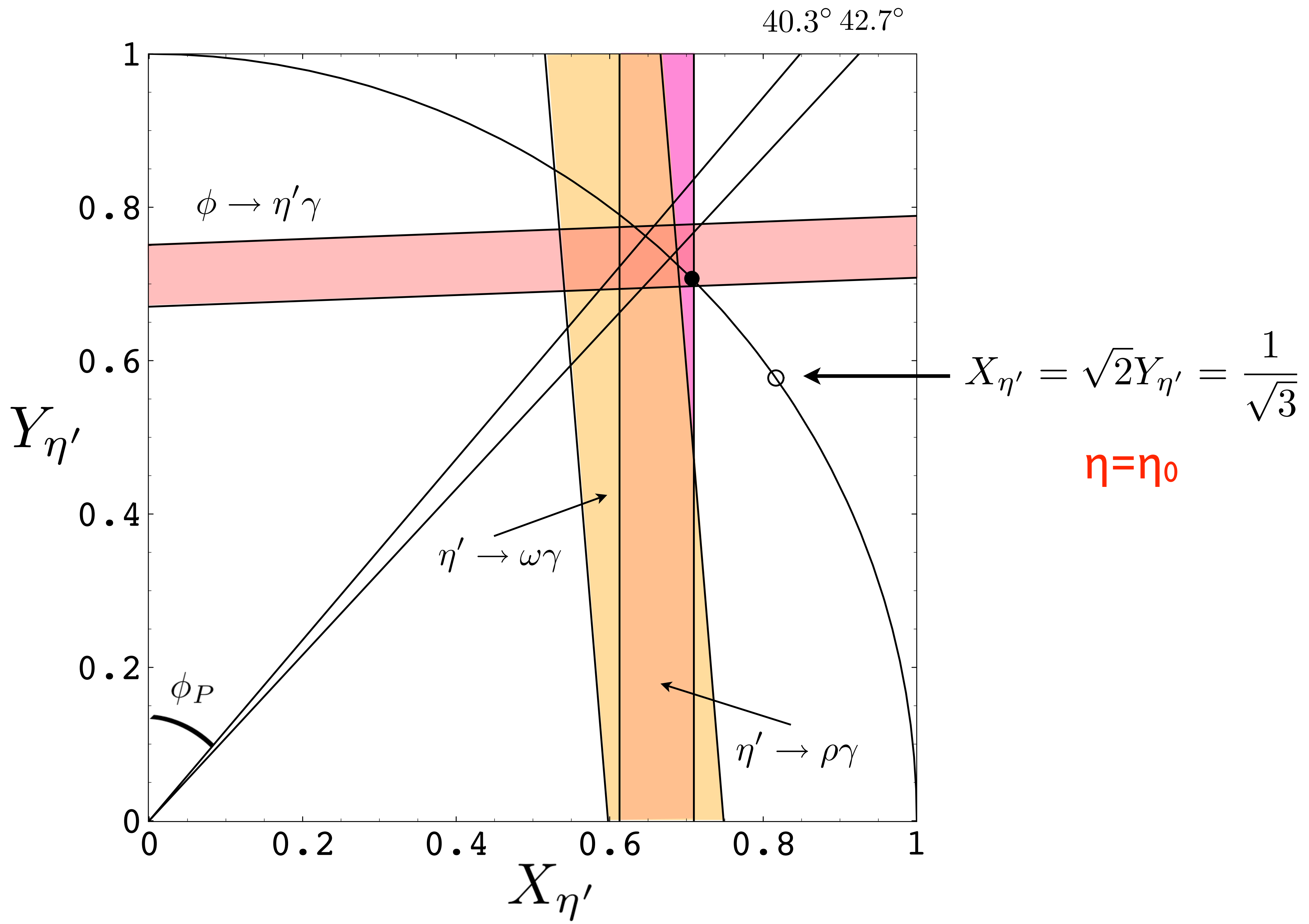
R. E. and J. Nadal, JHEP 05 (2007) 6



✔ importance of $\phi \to \eta\gamma$

✔ importance of the slopes (ϕ_V)

- *Results*

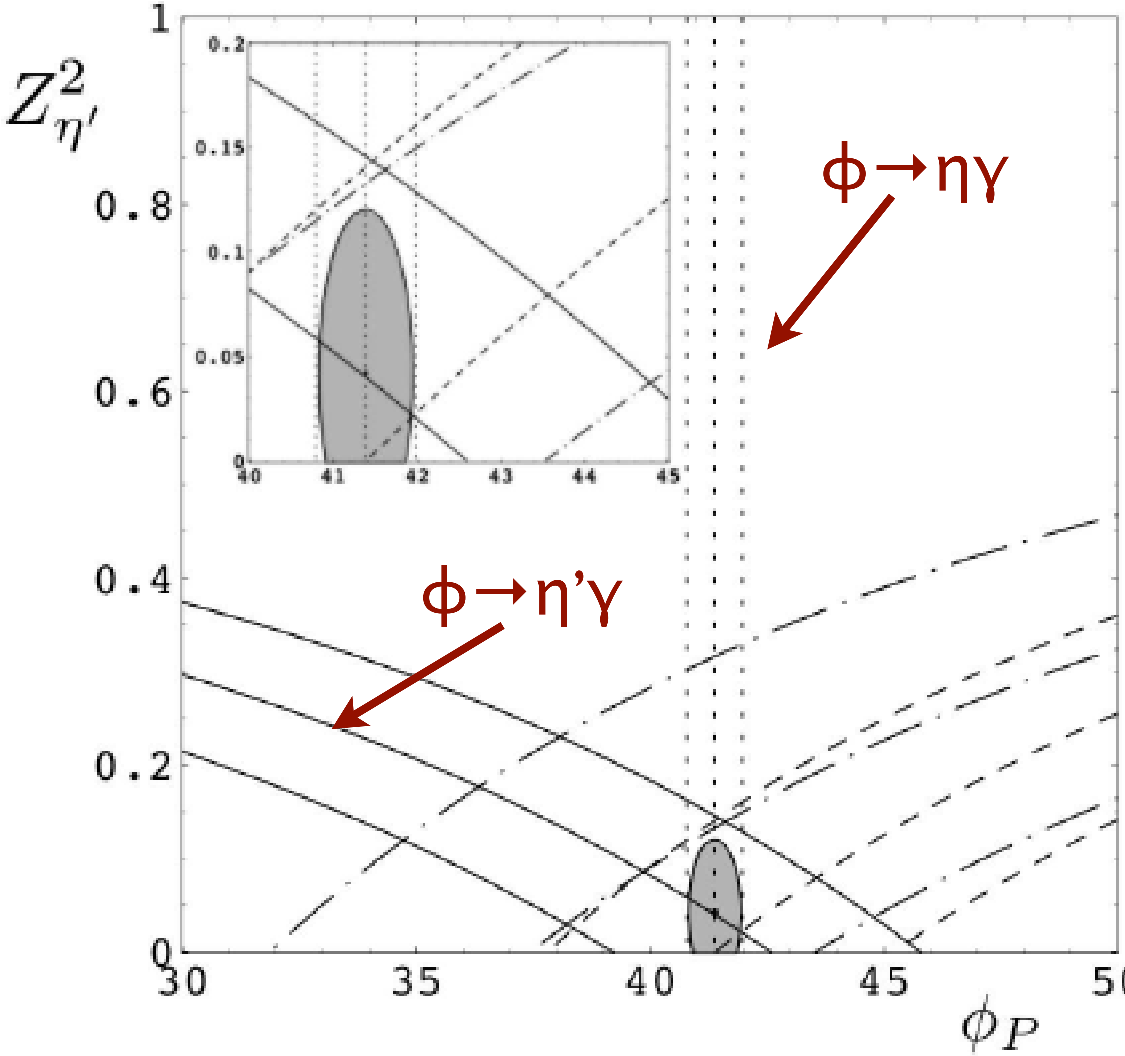


✔ importance of constraining even more $\phi \to \eta'\gamma$

More refined data for this channel will contribute decisively to clarify this issue

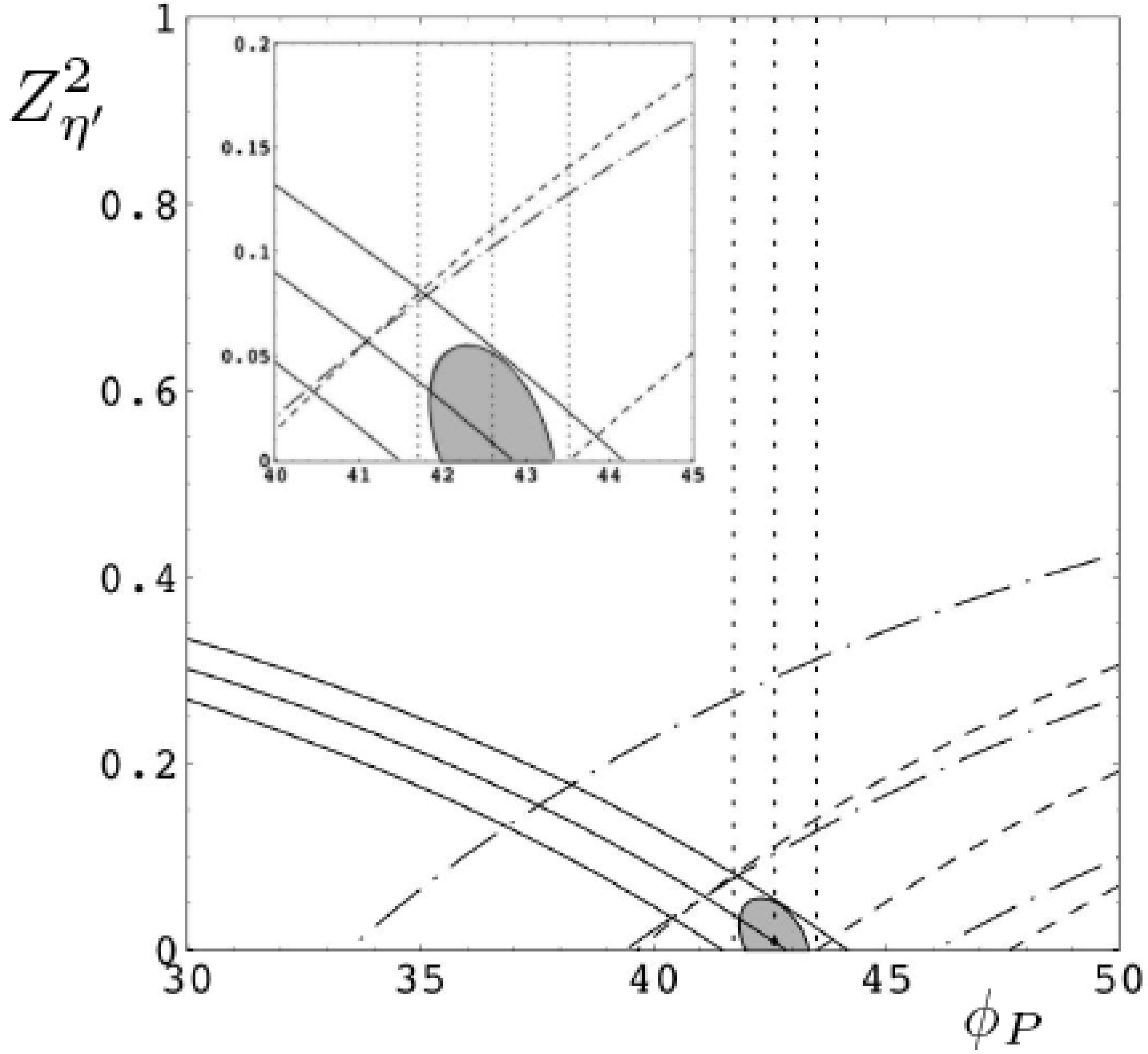
• *Results*

PDG'06 data



$$(\phi_P, Z^2_{\eta'}) = (41.4^\circ, 0.04)$$

latest data



$$(\phi_P, Z^2_{\eta'}) = (42.6^\circ, 0.01)$$

• *Summary of the $V \to P\gamma$ analysis and conclusions*

We have performed a phenomenological analysis of radiative $V \to P\gamma$ and $P \to V\gamma$ decays with the purpose of determining the gluon content of the η and η' mesons

1) The current experimental data on $VP\gamma$ transitions indicate within our model a negligible gluonic content for the η and η' mesons,

$$Z_\eta^2 = 0.00 \pm 0.12 \quad \text{and} \quad Z_{\eta'}^2 = 0.04 \pm 0.09$$

2) Accepting the absence of gluonium for the η meson, the gluonic content of the η' wave function amounts to $|\phi_{\eta' G}|=(12\pm13)^\circ$ or $(Z_{\eta'})^2=0.04\pm0.09$ and the η-η' mixing angle is found to be $\phi_P=(41.4\pm1.3)^\circ$

3) The use of these different overlapping parameters (a specific feature of our analysis) is shown to be of primary importance in order to reach a good agreement

4) The latest experimental data on $(\rho,\omega,\phi) \to \eta\gamma$ and $\phi \to \eta'\gamma$ decays confirm the null gluonic content of the η and η' wave functions

5) More refined experimental data, particularly for the $\phi \to \eta'\gamma$ channel, will contribute decisively to clarify this issue

• *Reason for the discrepancy?*

KLOE Collaboration, JHEP 07 (2009) 105

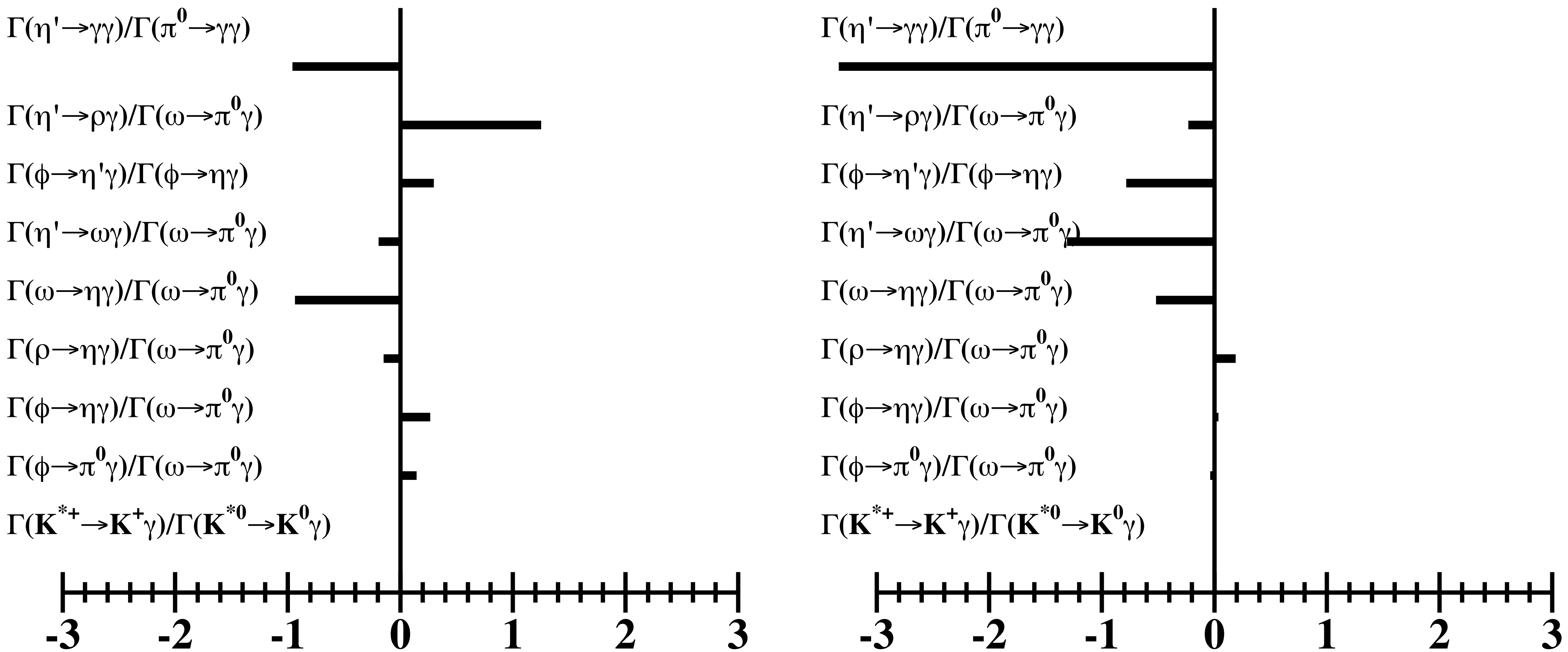


Figure 1. Pulls of the fit shown in table 2, *left:* Z_G free, *right:* $Z_G = 0$ (fixed).

- *Reason for the discrepancy?*

KLOE Collaboration, JHEP 07 (2009) 105

	Fit with PDG-2006	Fit of ref. [4]
χ^2/ndf (CL)	1.8/2 (41%)	4.2/4 (38%)
Z_G^2	0.03 ± 0.06	0.04 ± 0.09
ψ_G	$(10 \pm 10)^\circ$	$(12 \pm 13)^\circ$
ψ_P	$(41.6 \pm 0.8)^\circ$	$(41.4 \pm 1.3)^\circ$
Z_q	0.85 ± 0.03	0.86 ± 0.03
Z_s	0.78 ± 0.05	0.79 ± 0.05
ψ_V	$(3.16 \pm 0.10)^\circ$	$(3.2 \pm 0.1)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

Table 3. Comparison among the fit results without the $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ measurements and the results of ref. [4]. PDG-2006 data [9] have been used in both fits.

$$X_{\eta'} = \sin\psi_P \cos\psi_G, \quad Y_{\eta'} = \cos\psi_P \cos\psi_G$$

$$\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{1}{9}\left(\frac{m_{\eta'}}{m_{\pi^0}}\right)^3 \left(5\frac{f_\pi}{f_q}\cos\psi_G \sin\psi_P + \sqrt{2}\frac{f_\pi}{f_s}\cos\psi_G \cos\psi_P\right)^2$$

$$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = 3\frac{Z_q^2}{\cos^2(\psi_V)}\left(\frac{m_{\eta'}^2 - m_\rho^2}{m_\omega^2 - m_\pi^2} \cdot \frac{m_\omega}{m_{\eta'}}\right)^3 X_{\eta'}^2$$

$$\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{3}\left(\frac{m_{\eta'}^2 - m_\omega^2}{m_\omega^2 - m_\pi^2} \cdot \frac{m_\omega}{m_{\eta'}}\right)^3 \left[Z_q X_{\eta'} + 2\frac{\bar{m}}{m_s} Z_s \cdot \tan\psi_V Y_{\eta'}\right]^2.$$

- *Reason for the discrepancy?*

KLOE Collaboration, JHEP 07 (2009) 105

	Z_G free	$Z_G = 0$ fixed
χ^2/ndf (CL)	7.9/3 (5%)	15/4 (5×10^{-3})
Z_G^2	0.097 ± 0.037	0 fixed
ψ_P	$(41.0 \pm 0.7)^\circ$	$(41.7 \pm 0.5)^\circ$
Z_q	0.86 ± 0.02	0.86 ± 0.02
Z_s	0.79 ± 0.05	0.78 ± 0.05
ψ_V	$(3.17 \pm 0.09)^\circ$	$(3.19 \pm 0.09)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

Table 4. Fit results using the PDG-2008 data.

	Z_G free	$Z_G = 0$ fixed
χ^2/ndf (CL)	4.6/3 (20%)	14.7/4 (0.5%)
Z_G^2	0.115 ± 0.036	0
ψ_P	$(40.4 \pm 0.6)^\circ$	$(41.4 \pm 0.5)^\circ$
Z_q	0.936 ± 0.025	0.927 ± 0.023
Z_s	0.83 ± 0.05	0.82 ± 0.05
ψ_V	$(3.32 \pm 0.09)^\circ$	$(3.34 \pm 0.09)^\circ$
$m_s/\bar{m}$	1.24 ± 0.07	1.24 ± 0.07

Table 6. Fit results using PDG-2008 inputs, BR($\omega \to \eta\gamma$) from PDG direct measurement average and the KLOE BR($\omega \to \pi^0\gamma$) and R_ϕ. The equations (4.1) have been used for the f_q/f_π and f_s/f_π parameters.

• *Reason for the discrepancy?*

KLOE Collaboration, JHEP 07 (2009) 105

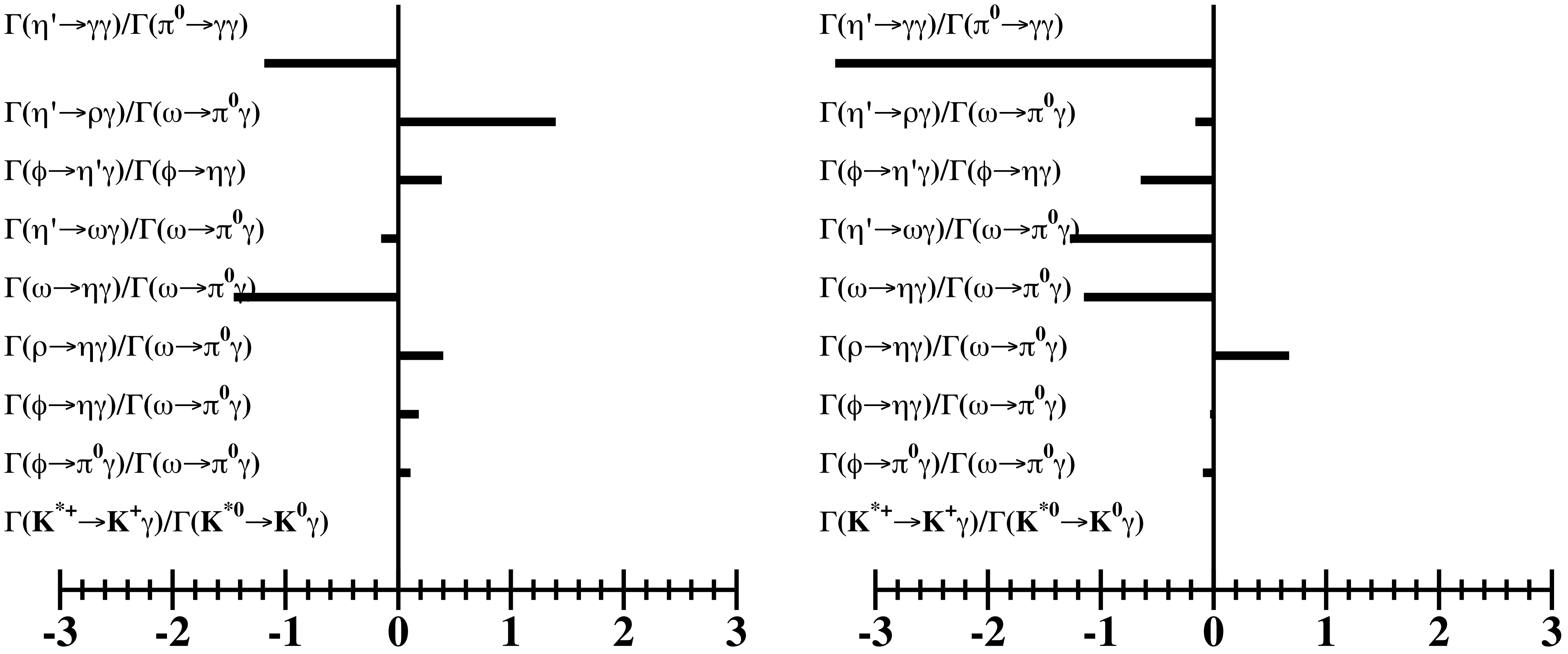


Figure 2. Pulls of the fit using PDG-2008 data, *left:* Z_G free, *right:* $Z_G = 0$ (fixed).

• *Mixing parameters from the η and η' transition form factors*

Purpose:

To present an analysis of the η and η' transition form factors in the space-like region at low and intermediate energies in a model-independent way through the use of rational approximants

Motivations:

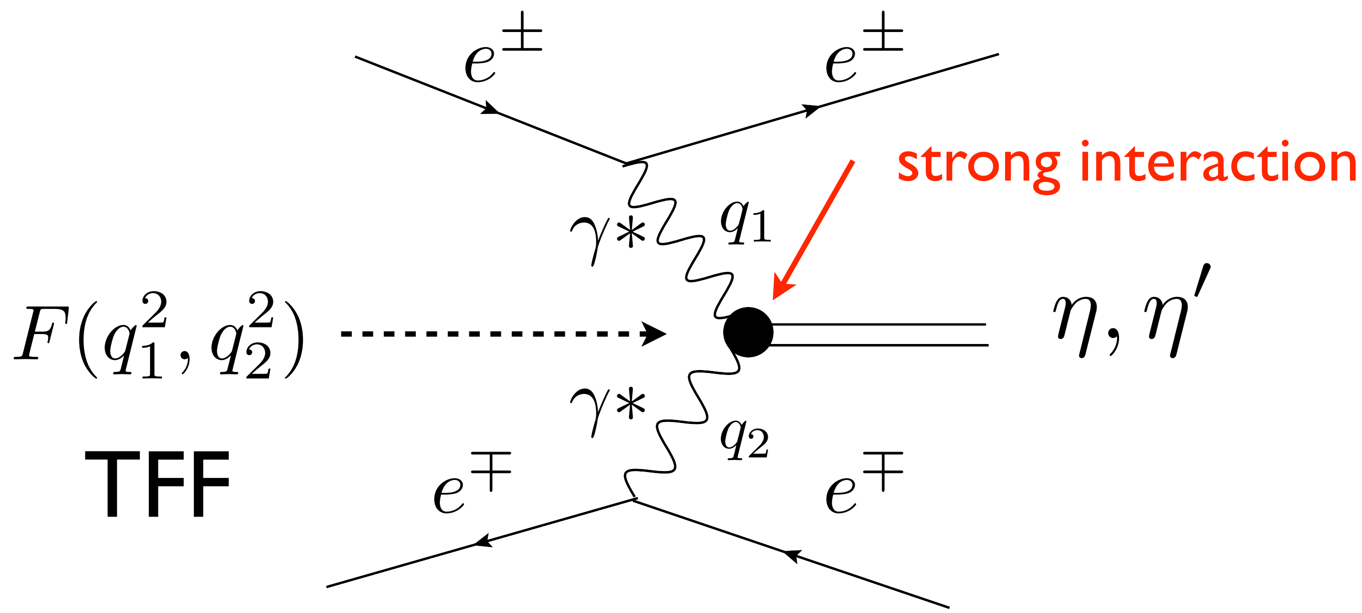
- To extract the slope and curvature parameters of the TFFs as well as their values at zero and infinity from experimental data

- To discuss the impact of these results on the mixing parameters of the η and η' system

In collab. with P. Masjuan and P. Sánchez-Puertas (Mainz)

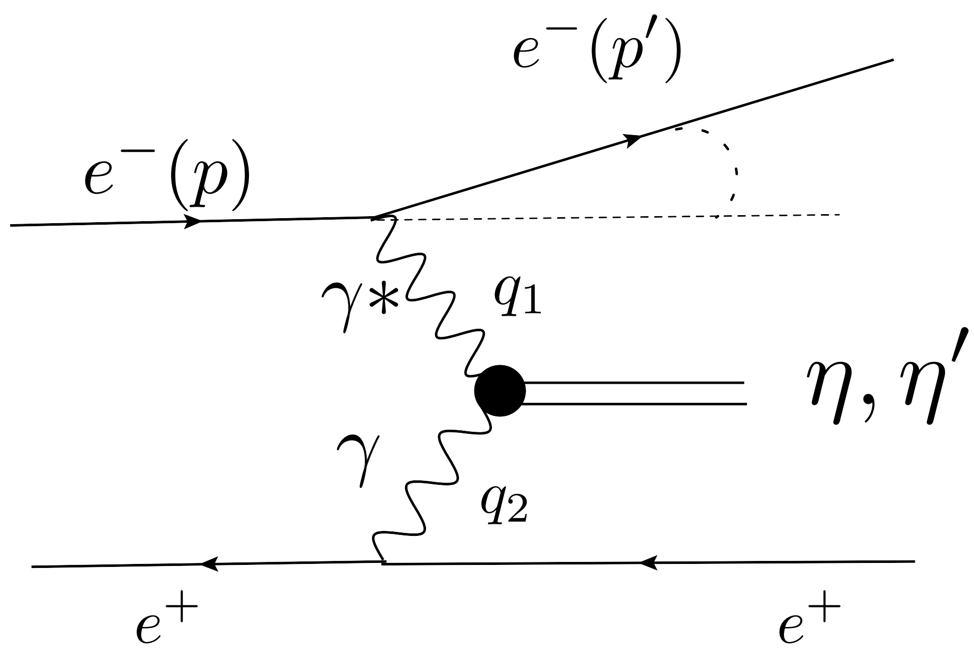
arXiv:1307.2061 [hep-ph]

• *Pseudoscalar transition form factors*



not exp. accesible

Single Tag Method



$$F_{P\gamma^*\gamma}(Q^2) \equiv F_{P\gamma^*\gamma^*}(-Q^2, 0)$$

Momentum transfer

- highly virtual photon ⇒ tagged
- quasi-real photon ⇒ untagged

Selection criteria

- 1 e^- detected
- 1 e^+ along beam axis
- Meson full reconstructed

• *Pseudoscalar transition form factors*

@ low-momentum transfer:

slope (related to charge radius)

$$F_{P\gamma^*\gamma}(Q^2) = F_{P\gamma\gamma}(0)\left(1 - b_P \frac{Q^2}{m_P^2} + c_P \frac{Q^4}{m_P^4} + \cdots\right)$$

curvature

$$|F_{P\gamma\gamma}(0)|^2 = \frac{64\pi}{(4\pi\alpha)^2}\frac{\Gamma(P\to\gamma\gamma)}{m_P^3}$$

exp. decay width

or

$$F_{\pi^0\gamma\gamma}(0) = 1/(4\pi^2 F_\pi)$$

axial anomaly

(not for η and η')

@ large-momentum transfer:

$$F(Q^2) = \int T_H(x, Q^2)\Phi_P(x, \mu_F)\mathrm{d}x$$

@ lowest order in pQCD

$$Q^2F(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 \frac{dx}{x}\phi_\pi(x, Q^2) + O(\alpha_s) + O\left(\frac{\Lambda_{QCD}^2}{Q^2}\right),$$

$T_H(\gamma^*\gamma \to q\bar{q})$ $\quad$ $\Phi_P(q\bar{q} \to P)$

convolution of perturbative and non-perturbative regimes

$$Q^2F(Q^2) = \sqrt{2}f_\pi$$

- *Padé Approximants*

$$Q^2 F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P^N_M(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0 Q^2 + a_1 Q^4 + a_2 + Q^6 + \cdots + \mathcal{O}((Q^2)^{N+M+1})$$

→ simple, systematic and model-independent parametrization of experimental data in the whole energy range (better convergence)

Fitting method: use of different sequences of PAs

- *How many sequences?*
  depends on the analytic structure of the exact function
- *How many elements per sequence?*
  limited by exp. data points and statistical errors

• *Application to η and η'TFFs*

asymptotic behaviour

To use the P[N,1](Q^2) and P[N,N](Q^2) sequences of PAs

single resonance dominance

η TFF

η' TFF

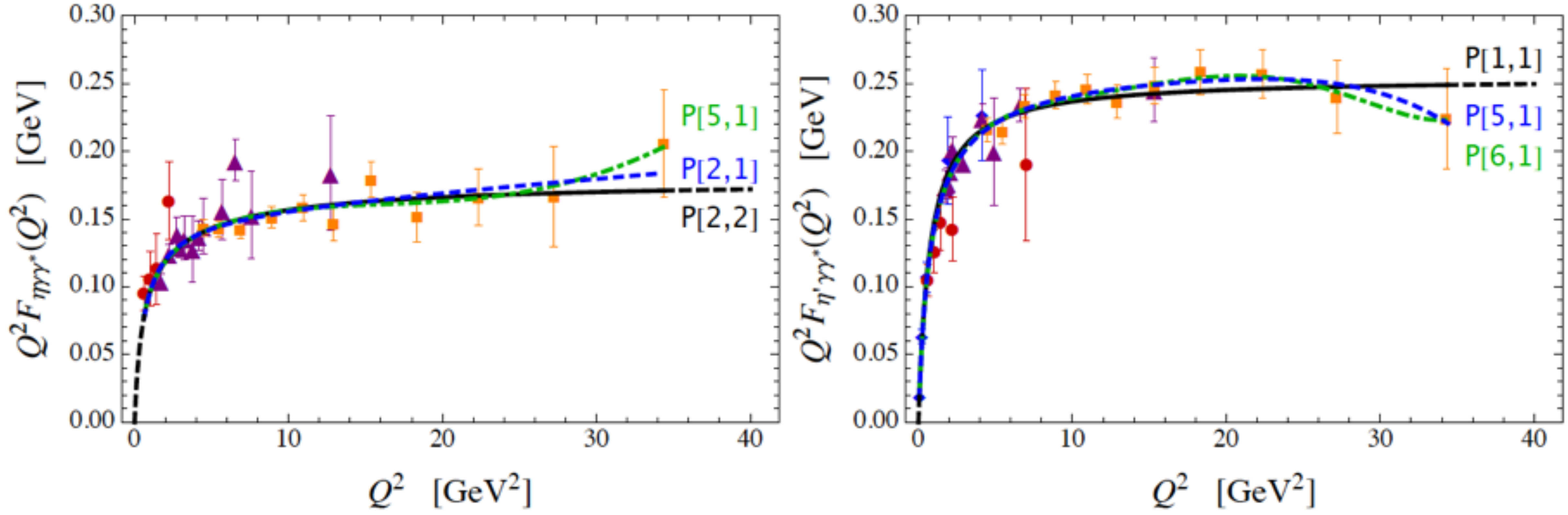


FIG. 1. η- and η'-TFFs best fits (left and right panels reps.). Blue dashed line shows our best $P_1^L(Q^2)$ when the two-photon partial decay width is *not* included in our set of data to be fitted. When the two-photon partial decay width *is* included, dark-green dot-dashed line shows our best $P_1^L(Q^2)$, and black solid line shows our best $P_N^N(Q^2)$. Black dashed lines are the extrapolation of such approximant at $Q^2 = 0$ and at $Q^2 \to \infty$. Data points are from CELLO (red circles) [28], CLEO (purple triangles) [36], L3 (blue diamonds) [31], and *BABAR* (orange squares) [30] Collaborations. See main text for details.

- *Results*

Slope and curvature:

$$b_\eta = 0.596(48)_{stat}(33)_{sys}$$
$$c_\eta = 0.362(66)_{stat}(76)_{sys} \times 10^{-3}$$

$$b_{\eta'} = 1.37(16)_{stat}(8)_{sys}$$
$$c_{\eta'} = 1.94(52)_{stat}(41)_{sys} \times 10^{-3}$$

Comparison with other results:

$$F_{P\gamma^*\gamma}(Q^2) = \frac{F_{P\gamma\gamma}(0)}{1 + Q^2/\Lambda_P^2}$$

ChPT: b_η=0.51, $b_{\eta'}$=1.47

VMD: b_η=0.53, $b_{\eta'}$=1.33

cQL: b_η=0.51, $b_{\eta'}$=1.30

BL: b_η=0.36, $b_{\eta'}$=2.11

$$\mathcal{F}_{\gamma^*\gamma R}(Q^2) \sim \frac{1}{4\pi^2 f_R} \frac{1}{1+(Q^2/8\pi^2 f_R^2)}$$

CELLO: b_η=0.428(89), $b_{\eta'}$=1.46(23)

CLEO: b_η=0.501(38), $b_{\eta'}$=1.24(8)

Lepton-G: b_η=0.57(12), $b_{\eta'}$=1.6(4)

NA60: b_η=0.585(51)

MAMI: b_η=0.58(11), WASA: b_η=0.68(26)

Disp: b_η=0.61(+0.07)(-0.03), $b_{\eta'}$=1.45(+0.17)(-0.12)

$\eta,\eta' \to \gamma^*\gamma$

- *Results*

η,η'→γγ decay widths (TFFs @ $Q^2=0$):

$$\Gamma^{pred}_{\eta\to\gamma\gamma} = (0.41 \pm 0.18) keV \qquad \Gamma^{pred}_{\eta'\to\gamma\gamma} = (4.21 \pm 0.43) keV$$

$$\Gamma^{PDG}_{\eta\to\gamma\gamma} = (0.51 \pm 0.03) keV \qquad \Gamma^{PDG}_{\eta'\to\gamma\gamma} = (4.34 \pm 0.14) keV$$

Asymptotic values (TFFs @ $Q^2\to\infty$):

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 0.164(21)\ \mathrm{GeV}$$

← *disagrees* with BABAR @112 GeV (time-like)

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 0.254(4)\ \mathrm{GeV}$$

← *agrees* with BABAR @112 GeV (time-like)

→ determination of η-η' mixing parameters

- *Impact on η-η' mixing parameters*

Quark-flavour basis:

large-N_c limit: $\phi_q = \phi_s \equiv \phi$

$$\begin{pmatrix} F_\eta^q & F_\eta^s \\ F_{\eta'}^q & F_{\eta'}^s \end{pmatrix} = \begin{pmatrix} F_q \cos\phi_q & -F_s \sin\phi_s \\ F_q \sin\phi_s & F_s \cos\phi_s \end{pmatrix}$$

pseudoscalar decay constants

Decay widths:

$$F_{\eta\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q F_{\eta'}^s - \hat{c}_s F_{\eta'}^q}{F_{\eta'}^s F_\eta^q - F_{\eta'}^q F_\eta^s}\right) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\cos\phi - \frac{\hat{c}_s}{F_s}\sin\phi\right)$$

$$F_{\eta'\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q F_\eta^s - \hat{c}_s F_\eta^q}{F_\eta^s F_{\eta'}^q - F_\eta^q F_{\eta'}^s}\right) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\sin\phi + \frac{\hat{c}_s}{F_s}\cos\phi\right)$$

Asymptotic expressions:

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_\eta^q + \hat{c}_s F_\eta^s) = 2(\hat{c}_q F_q \cos\phi - \hat{c}_s F_s \sin\phi)$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_{\eta'}^q + \hat{c}_s F_{\eta'}^s) = 2(\hat{c}_q F_q \sin\phi + \hat{c}_s F_s \cos\phi)$$

- *Impact on η-η' mixing parameters*

Results:

η,η'→γγ not included

$$F_q/F_\pi = 1.21(7),\ F_s/F_\pi = 1.5(2) \text{ and } \phi = 45(3)^\circ$$

η,η'→γγ included

$$F_q/F_\pi = 1.07(1)\ , \quad F_s/F_\pi = 1.53(23)\ ,$$

$$\phi = 40.2(1.6)^\circ\ ,$$

η' TFF used

$$F_q/F_\pi = 1.01(2)\ , \quad F_s/F_\pi = 0.95(4)\ ,$$

$$\phi = 33.2(0.7)^\circ\ ,$$

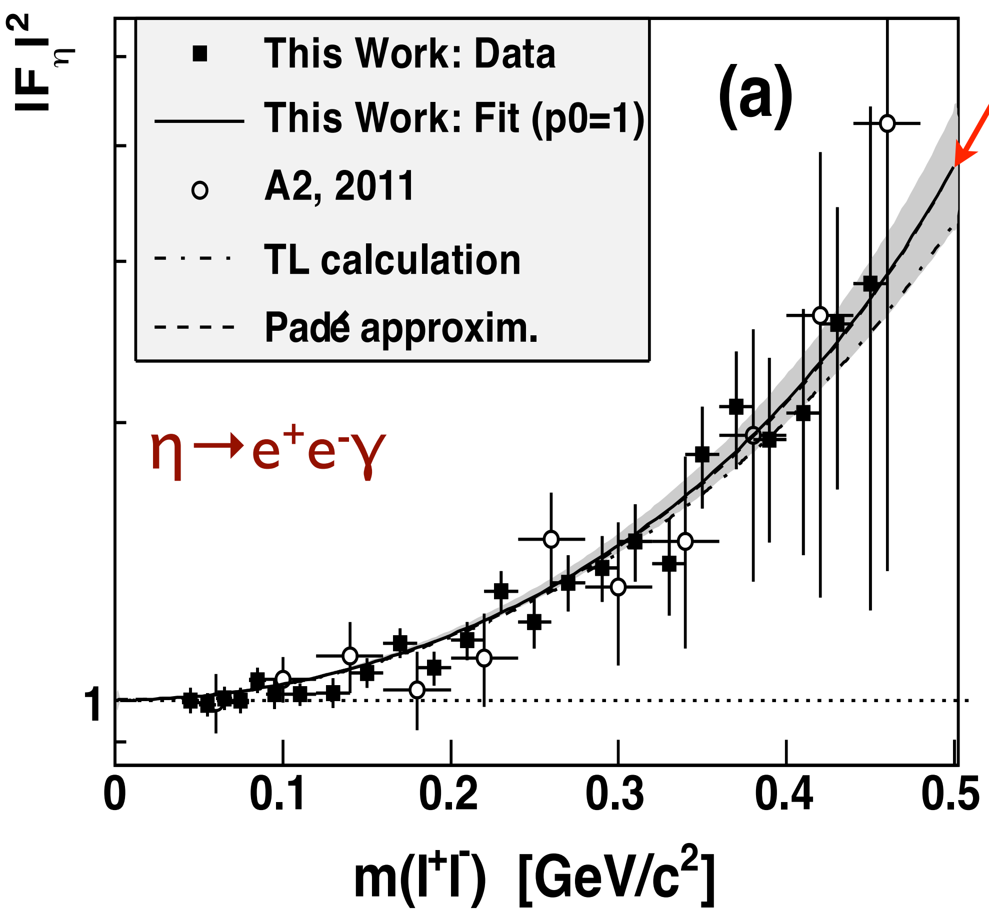
to compare with:

$$F_q/F_\pi = 1.07(1),\ F_s/F_\pi = 1.63(3) \text{ and } \phi = 39.6(0.4)^\circ$$

Update'13 of R. Escribano and J.M.-Frère, JHEP0506 (2005) 029

- *Further applications of this method*

Analysis of time-like processes ($\eta,\eta' \to l^+l^-\gamma$)



Our *prediction* is *behind* the experimental fit!

PREVIOUS RESULTS
$b_\eta = 0.596(48)(33)$,
$c_\eta = 0.362(66)(76)$
Asymptotics $= 0.164(21)$ GeV

Adding MAMI data to our fit

UPDATED RESULTS
(PRELIMINARY RESULTS)
$b_\eta = 0.588(27)(25)$,
$c_\eta = 0.357(38)(61)$
Asymptotics $= 0.174(15)$ GeV

M. Unverzagt et al. (A2 Coll. @MAMI), arXiv:1309.5648 [hep-ex]

Analysis of π^0, η and η' contributions to HLbL of $(g-2)_\mu$

- *Summary and Conclusions*

We have analyzed the experimental data on the η and η' TFF at low and intermediate energies with a model independent approach based on Padé approximants (extending the analysis for the π^0-TFF) P. Masjuan, PRD 86 (2012) 094021

We have obtained accurate values of the corresponding slope and curvature parameters as well as the values of the TFFs at zero and infinity

We have quantified the impact of these results on the η and η' mixing parameters

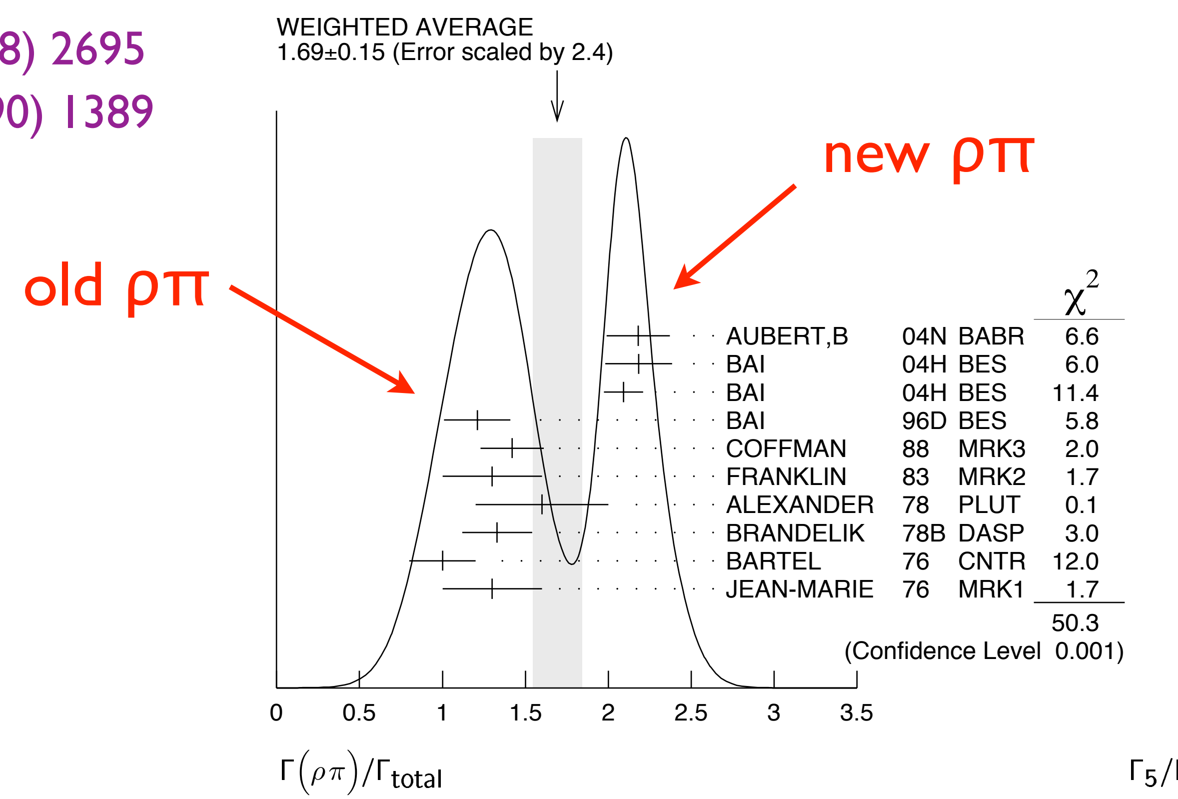
We have foreseen further applications of the method of Padé approximants (time-like processes, muon g-2)

More experimental data would be desirable (BELLE?) to further improve this method

• $J/\psi \rightarrow VP$ analysis

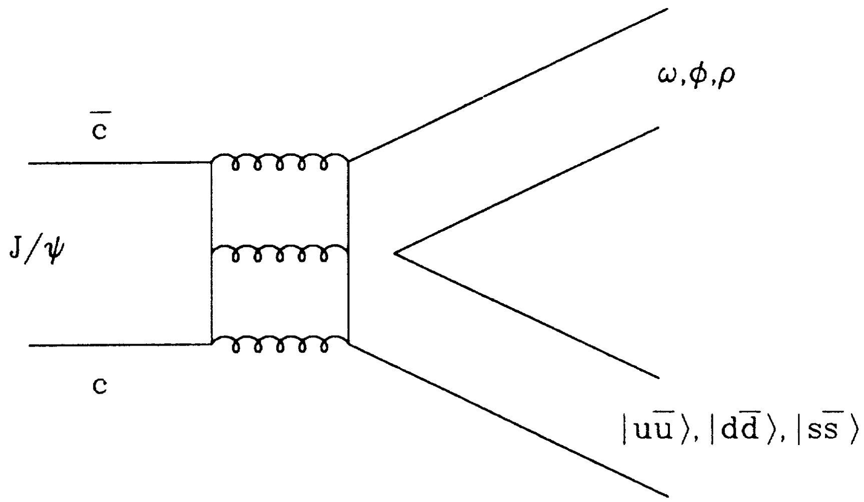
$BR \times 10^{-3}$	PDG'97*	PDG'08	
$\rho\pi$	12.8 ± 1.0	16.9±1.5	BABAR Coll., Phys. Rev. D70 (04) 072004 BES Coll., Phys. Rev. D70 (04) 012005
$K^{*+}K^{-}$ + c.c.	5.0 ± 0.4	=	
$K^{*0}\bar{K}^{0}$ + c.c.	4.2 ± 0.4	=	
$\omega\eta$	1.58 ± 0.16	1.74±0.20	BABAR Coll., Phys. Rev. D73 (06) 052003
$\omega\eta'$	0.167 ± 0.025	0.182±0.021	BES Coll., Phys. Rev. D73 (06) 052007
$\phi\eta$	0.65 ± 0.07	0.75±0.08	BES Coll., Phys. Rev. D71 (05) 032003
$\phi\eta'$	0.33 ± 0.04	0.40±0.07	
$\rho\eta$	0.193 ± 0.023	=	
$\rho\eta'$	0.105 ± 0.018	=	
$\omega\pi^0$	0.42 ± 0.06	0.45±0.05	BES Coll., Phys. Rev. D73 (06) 052007
$\phi\pi^0$	< 0.0068	<0.0064 C.L. 90%	BES Coll., Phys. Rev. D71 (05) 032003

*MARK III Coll., Phys. Rev. D38 (88) 2695
DM2 Coll., Phys. Rev. D41 (90) 1389

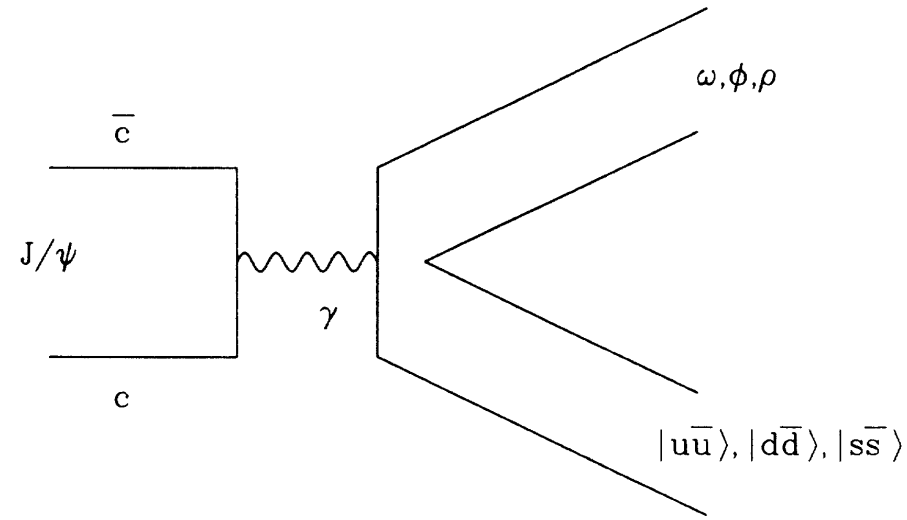


• *A model for J/ψ→VP transitions*

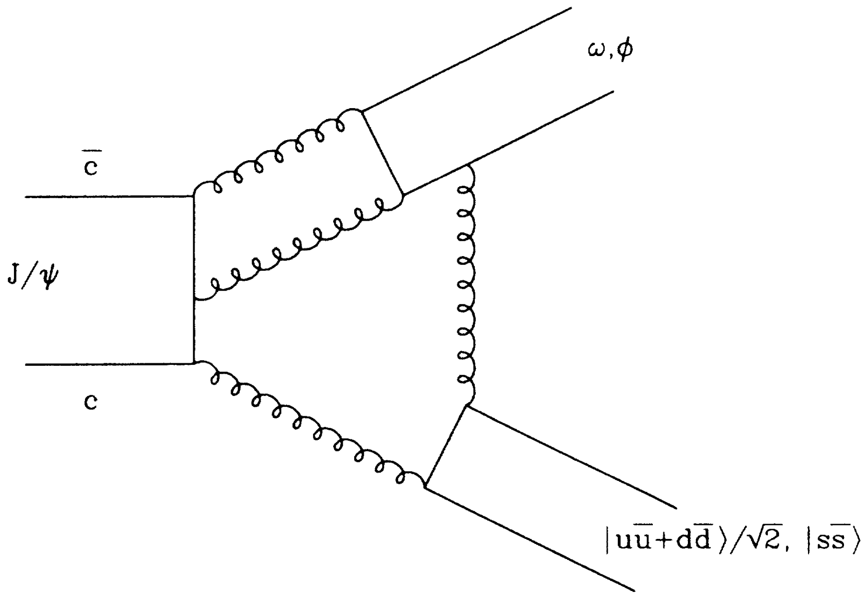
Amplitudes:



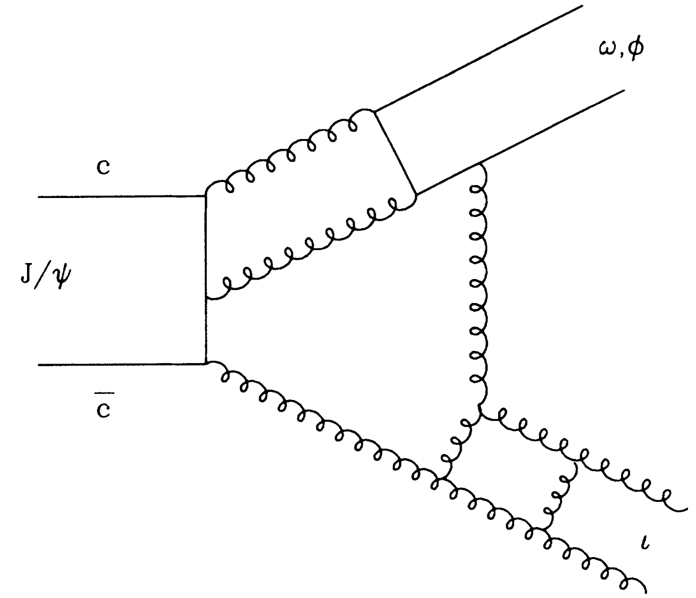
strong singly disconnected (SOZI) ≡ g



electromagnetic singly disconnected (eSOZI) ≡ e



strong doubly disconnected (DOZI) ≡ rg



DOZI for J/ψ→V+Glueball ≡ r’g

• *A model for $J/\psi \to VP$ transitions*

Amplitudes:

TABLE VIII. General parametrization of amplitudes for $J/\psi \to P+V$.

Process	Amplitude
$\rho^+\pi^-, \rho^0\pi^0, \rho^-\pi^+$	$g+e$
$K^{*+}K^-, K^{*-}K^+$	$g(1-s)+e(1+s_e)$
$K^{*0}\bar{K}^0, \bar{K}^{*0}K^0$	$g(1-s)-e(2-s_e)$
$\omega\eta$	$(g+e)X_\eta+\sqrt{2}rg[\sqrt{2}X_\eta+(1-s_p)Y_\eta]+\sqrt{2}r'gZ_\eta$
$\omega\eta'$	$(g+e)X_{\eta'}+\sqrt{2}rg[\sqrt{2}X_{\eta'}+(1-s_p)Y_{\eta'}]+\sqrt{2}r'gZ_{\eta'}$
$\phi\eta$	$[g(1-2s)-2e(1-s_e)]Y_\eta+rg(1-s_v)[\sqrt{2}X_\eta+(1-s_p)Y_\eta]+r'g(1-s_v)Z_\eta$
$\phi\eta'$	$[g(1-2s)-2e(1-s_e)]Y_{\eta'}+rg(1-s_v)[\sqrt{2}X_{\eta'}+(1-s_p)Y_{\eta'}]+r'g(1-s_v)Z_{\eta'}$
$\rho^0\eta$	$3eX_\eta$
$\rho^0\eta'$	$3eX_{\eta'}$
$\omega\pi^0$	$3e$
$\phi\pi^0$	0

A. Seiden *et al.*, Phys. Rev. D38 (1988) 824

s, s_e, s_p and s_v are SU(3)-breaking parameters

Simplifications of our analysis:

i) second order SU(3)-breaking contributions s_p and s_v are neglected

ii) $x \equiv 1-s_e=m/m_s$ with $m_s/m=1.24\pm0.07$ and $\phi_V=(3.2\pm0.1)^\circ$

iii) $Z_\eta=0$ from $V\to P\gamma$ and $P\to V\gamma$ decays
R. E. and J. Nadal, JHEP 05 (2007) 6

- ***Results***

R. E., Eur. Phys. J. C65 (2010) 467

a) gluonium not allowed for η' → $Z_{\eta'}=0$

i) $x=1$ and $\phi_V=0^\circ$ → χ^2/d.o.f.=3.4/4 with $\phi_P=(40.2\pm2.4)^\circ$

ii) $x=0.81\pm0.05$ and $\phi_V=(3.2\pm0.1)^\circ$ → χ^2/d.o.f.=4.2/4 with $\phi_P=(40.5\pm2.4)^\circ$

with $s=(29\pm3)\%$ and $|r|=(37\pm1)\%$ in i)

b) gluonium allowed for η' → $Z_{\eta'}\neq0$

i) $x=1$ and $\phi_V=0^\circ$ → χ^2/d.o.f.=1.9/2 with $\phi_P=(45.0\pm4.3)^\circ$ and $(Z_{\eta'})^2=0.30+0.15-0.38$

ii) as before → χ^2/d.o.f.=3.0/2 with $\phi_P=(44.5\pm4.3)^\circ$ and $(Z_{\eta'})^2=0.28+0.16-0.44$

with $s=(27\pm3)\%$, $|r|=(36\pm8)\%$ and $|r'|=(12\pm22)\%$ in i)

Remarks:

- the effect of second order SU(3)-breaking contributions s_P and s_V is negligible
- the same fits with the pion modes removed are slightly better
- the same fits with the old data are worse, χ^2/d.o.f.=7.3/4 vs. χ^2/d.o.f.=3.4/4 for instance

- *Summary of the $J/\psi \to VP$ analysis and conclusions*

We have performed an updated phenomenological analysis of an accurate and exhaustive set of $J/\psi \to VP$ decays with the purpose of determining the quark and gluon content of the η and η' mesons

1) The current experimental data on $J/\psi \to VP$ decays are described in terms of one mixing angle in a consistent way

2) Accepting the absence of gluonium for the η' meson, the η-η' mixing angle is found to be $\phi_P=(40.2\pm2.4)^\circ$ or $\theta_P=(-14.5\pm2.4)^\circ$, in agreement with recent phenomenological estimates

Text

3) The values found for $(Z_{\eta'})^2=0.30+0.15-0.38$ or $\phi_{\eta' G}=(33+10-24)^\circ$ suggest within the model some small gluonic component of the η'

4) The inclusion of the vector mixing angle (not included in previous analyses) is irrelevant

5) The recent values of $BR(J/\psi \to \rho\pi)$ by BABAR and BES Coll. are crucial in order to get a consistent description of data