

η Transition Form Factor from space- and time-like data and applications

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Work in collaboration with
R. Escribano and P. Sanchez-Puertas
[Phys.Rev. D86 (2012) 094021, Phys.Rev. D89 (2014) 037303, ...]



Light Meson Dynamics
Mainz 10-12 February 2014



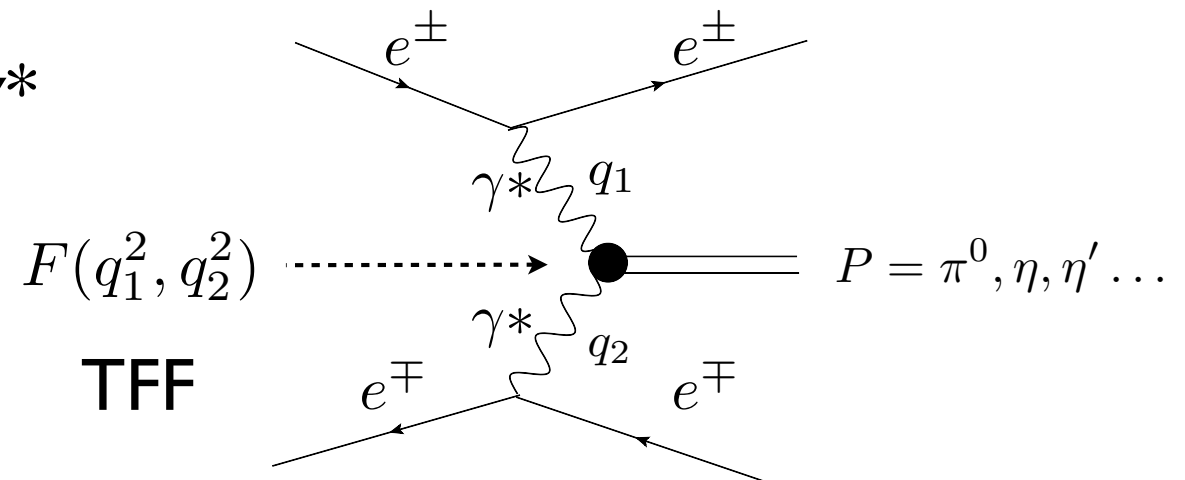
Outline

Pseudoscalar Transition Form Factors

- Pseudoscalar Transition Form Factors
- Parameterization using Rational Approximants
- Implications on
 - ▶ Low-energy parameters
 - ▶ η - η' mixing
 - ▶ Rare decays
- Outlook and Conclusions

Pseudoscalar Transition Form Factors

- Study of $ee \rightarrow ee\gamma^*\gamma^*$ with $\gamma^*\gamma^* \rightarrow \pi, \eta, \eta'$



- Meson Structure
 - Transition Form Factors (TFF) give access to Meson Distribution Amplitudes
- Precision Tests of the Standard Model
 - Relation to mixing parameters and muon anomaly $(g-2)_\mu$

How do we do that?

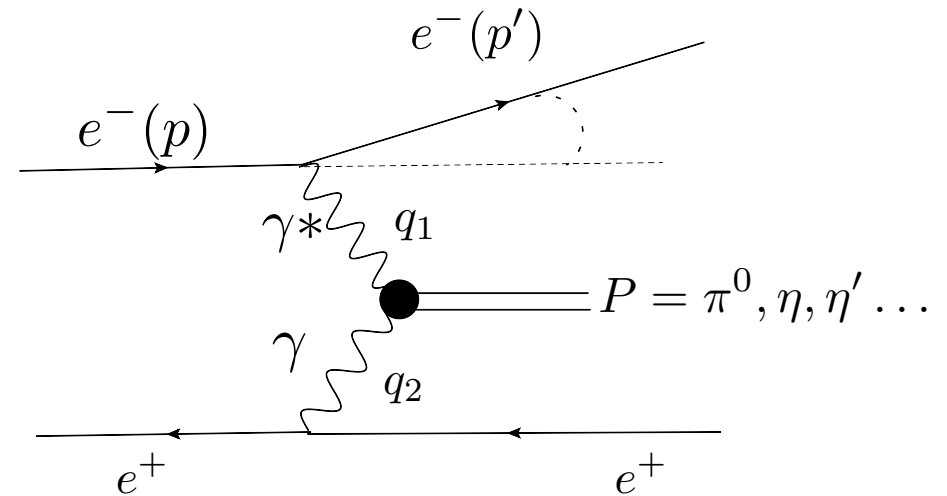
- Single Tag Method can access the Meson Transition Form Factor

Selection criteria

- 1 e^- detected
- 1 e^+ along beam axis
- Meson full reconstructed

Momentum transfer

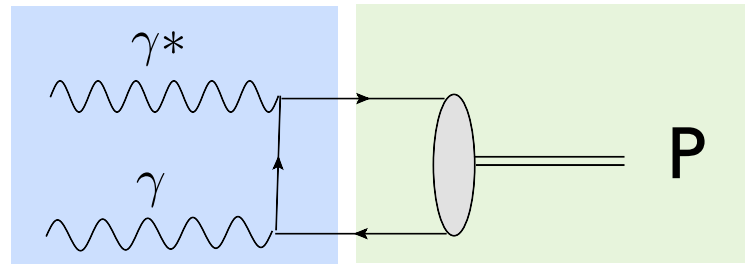
- tagged: $Q^2 = -q_1^2 = -(p - p')^2$
⇒ highly virtual photon
- untagged: $q^2 = -q_2^2 \sim 0 \text{ GeV}^2$
⇒ quasi-real photon



How do we do that?

Cross section for P production depends only on $F(q_1^2, q_2^2)$

With the Single Tag Method: $F(q_1^2, q_2^2) \rightarrow F(Q^2)$



$$F(Q^2) = \int T_H(x, Q^2) \Phi_P(x, \mu_F) dx$$

$$T_H(\gamma^* \gamma \rightarrow q\bar{q}) \quad \Phi_P(q\bar{q} \rightarrow P)$$

- μ_F is scale between soft and hard
- x -dependence of $\Phi_P(x, Q^2)$ not known but models
- Experimental data on $F(Q^2)$ is needed

convolution of perturbative and non-perturbative regimes

Our proposal use Padé Approximants

$$F_{P\gamma^*\gamma}(Q^2, 0) = a_0^P \left(1 + b_P \frac{Q^2}{m_P^2} + c_P \frac{Q^4}{m_P^4} + \dots \right)$$

$\Gamma_{P \rightarrow \gamma\gamma}$ slope curvature

Our proposal use Padé Approximants

$$F_{P\gamma^*\gamma}(Q^2, 0) = a_0^P \left(1 + b_P \frac{Q^2}{m_P^2} + c_P \frac{Q^4}{m_P^4} + \dots \right)$$

$\Gamma_{P \rightarrow \gamma\gamma}$ slope curvature

We have published space-like data for $Q^2 F_{P\gamma^*\gamma}(Q^2, 0)$

$$Q^2 F_{P\gamma^*\gamma}(Q^2, 0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P_M^N(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots + \mathcal{O}((Q^2)^{N+M+1})$$

Our proposal use Padé Approximants

$$F_{P\gamma^*\gamma}(Q^2, 0) = a_0^P \left(1 + b_P \frac{Q^2}{m_P^2} + c_P \frac{Q^4}{m_P^4} + \dots \right)$$

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$$Q^2 F_{P\gamma^*\gamma}(Q^2, 0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - a_1 Q^2}$$

Our proposal use Padé Approximants

$$F_{P\gamma^*\gamma}(Q^2, 0) = a_0^P \left(1 + b_P \frac{Q^2}{m_P^2} + c_P \frac{Q^4}{m_P^4} + \dots \right)$$

$\Gamma_{P \rightarrow \gamma\gamma}$ slope curvature

We have published space-like data for $Q^2 F_{P\gamma^*\gamma}(Q^2, 0)$

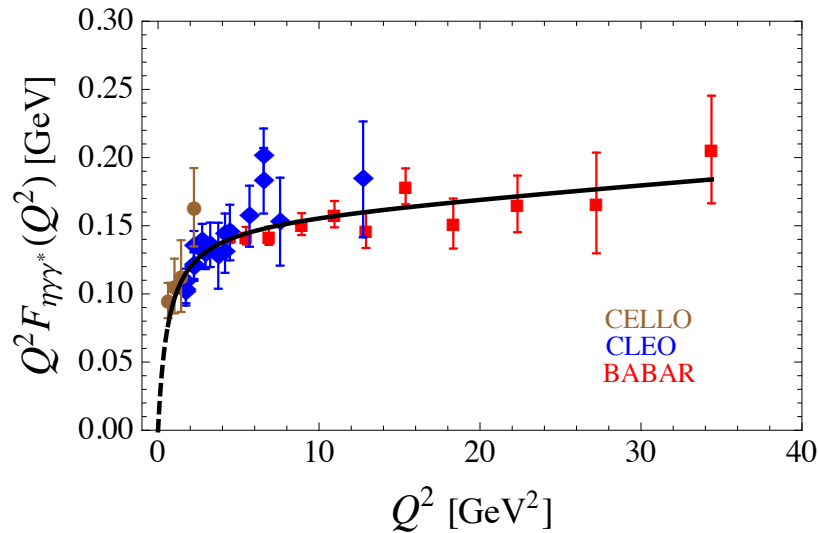
$$Q^2 F_{P\gamma^*\gamma}(Q^2, 0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - a_1 Q^2} \longrightarrow \begin{aligned} P_1^N(Q^2) &= P_1^1(Q^2), P_1^2(Q^2), P_1^3(Q^2), \dots \\ P_N^N(Q^2) &= P_1^1(Q^2), P_2^2(Q^2), P_3^3(Q^2), \dots \end{aligned}$$

η -TFF

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'11

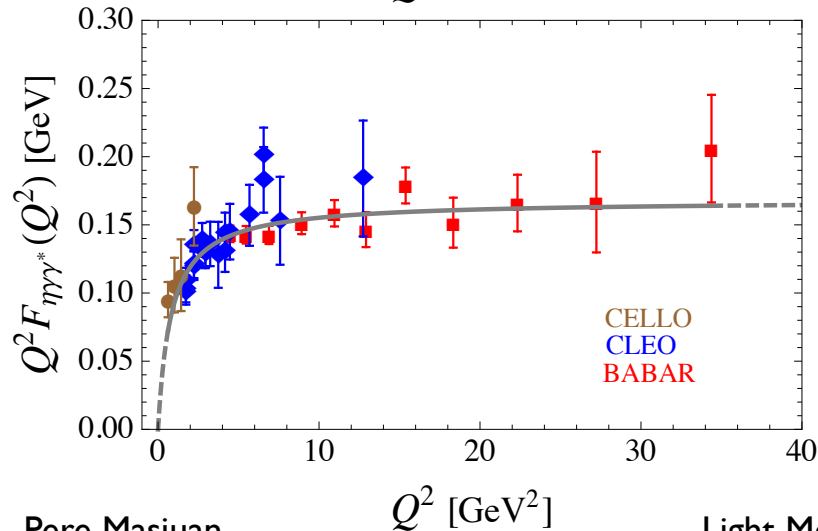
[R.Escribano, P.M., P. Sanchez-Puertas, '13]



$P_1^N(Q^2)$ up to $N=2$

$$\Gamma_{\eta \rightarrow \gamma\gamma}^{pred} = (0.41 \pm 0.18) keV$$

$$\Gamma_{\eta \rightarrow \gamma\gamma}^{PDG} = (0.518 \pm 0.018) keV$$

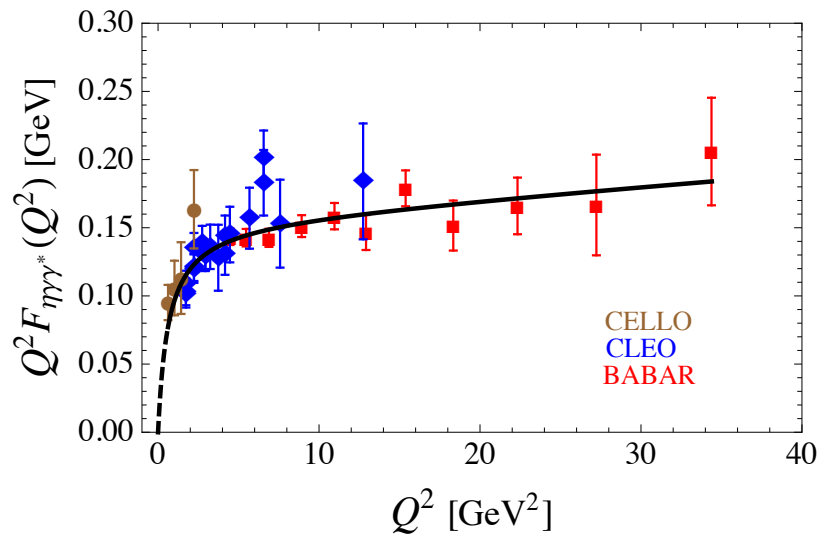


$P_N^N(Q^2)$ up to $N=1$

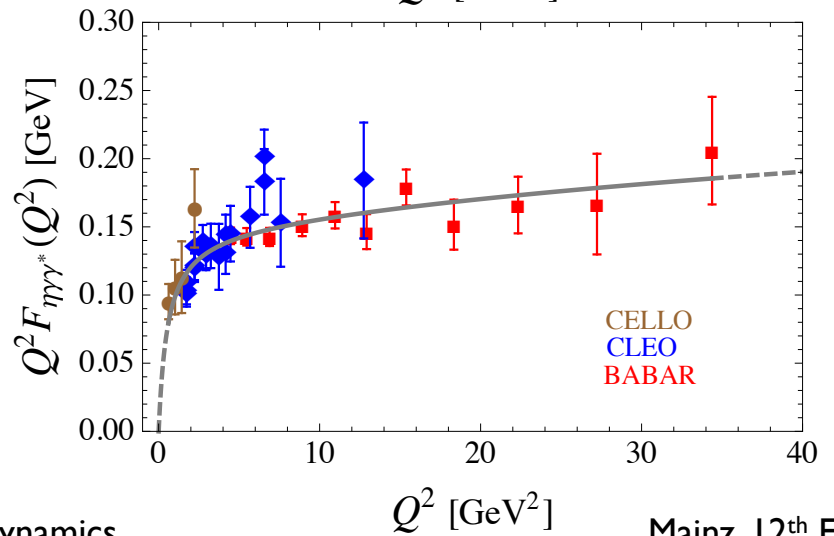
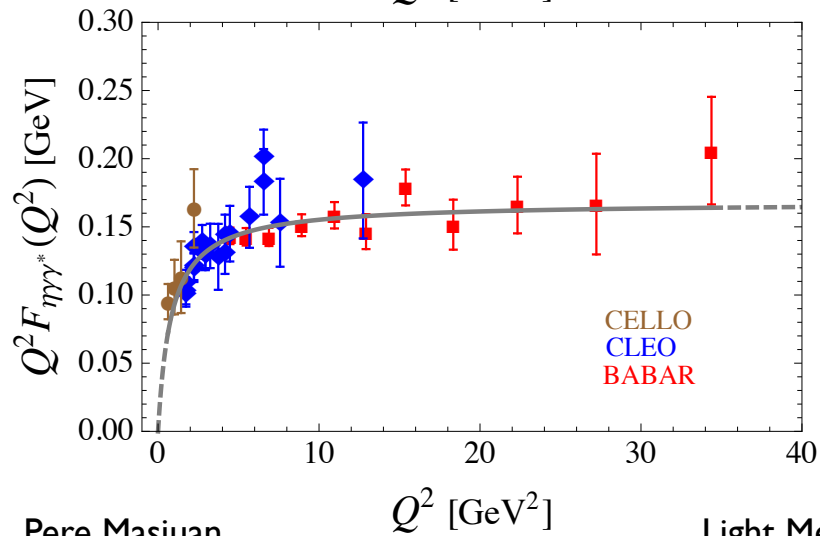
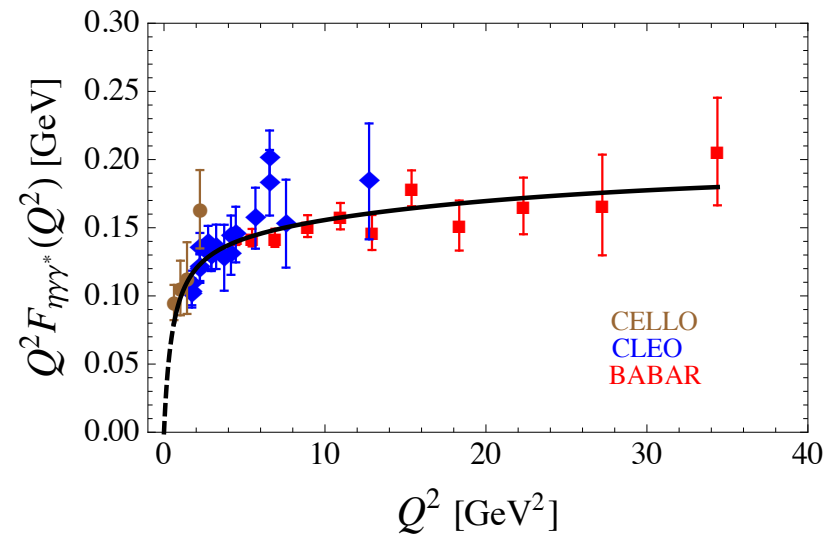
$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta\gamma^*}(Q^2, 0) = 0.17(6) GeV$$

η -TFF

$\Gamma_{\eta \rightarrow \gamma\gamma}$ not included



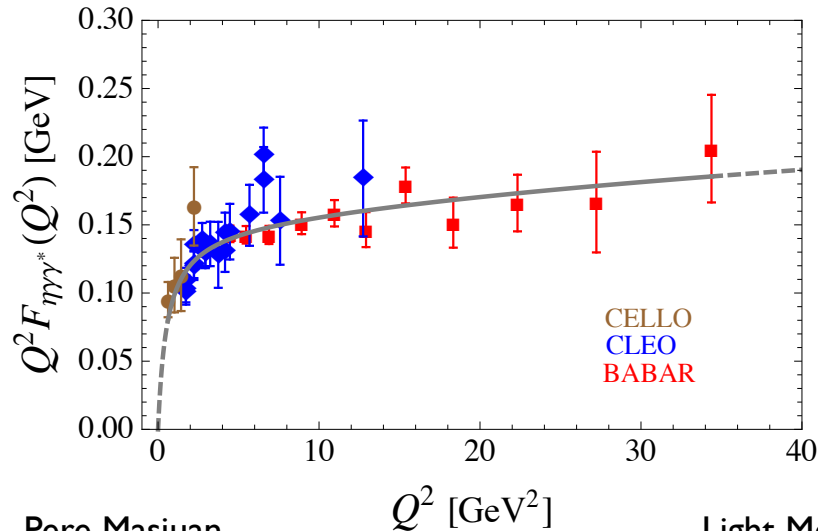
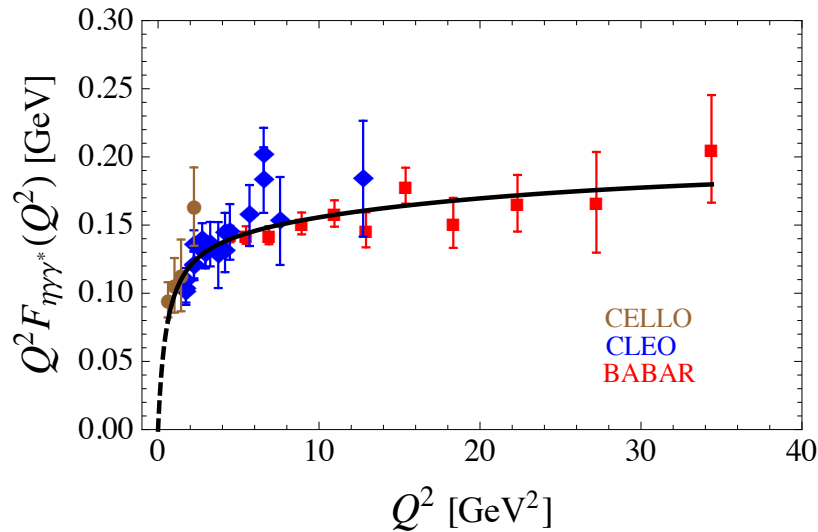
$\Gamma_{\eta \rightarrow \gamma\gamma}$ included



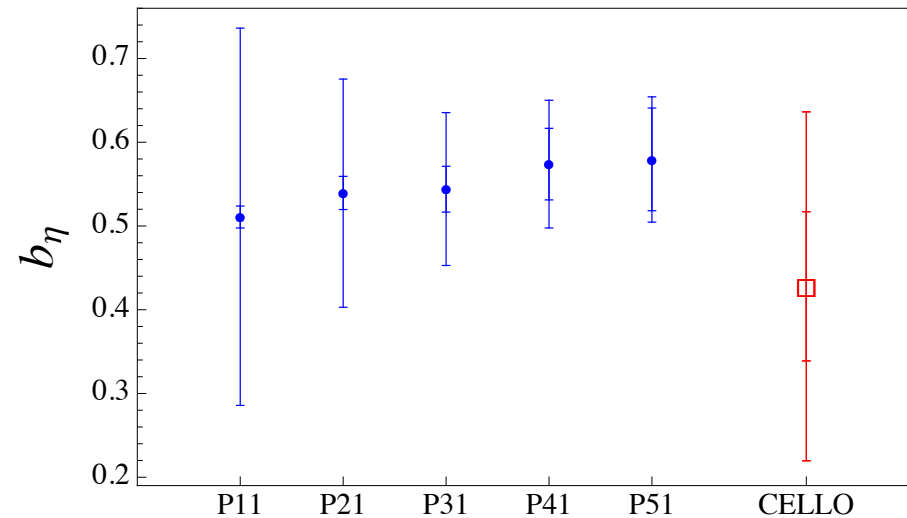
η -TFF

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'11 + $\Gamma_{\eta \rightarrow \gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, '13]



$P_1^N(Q^2)$ up to N=5



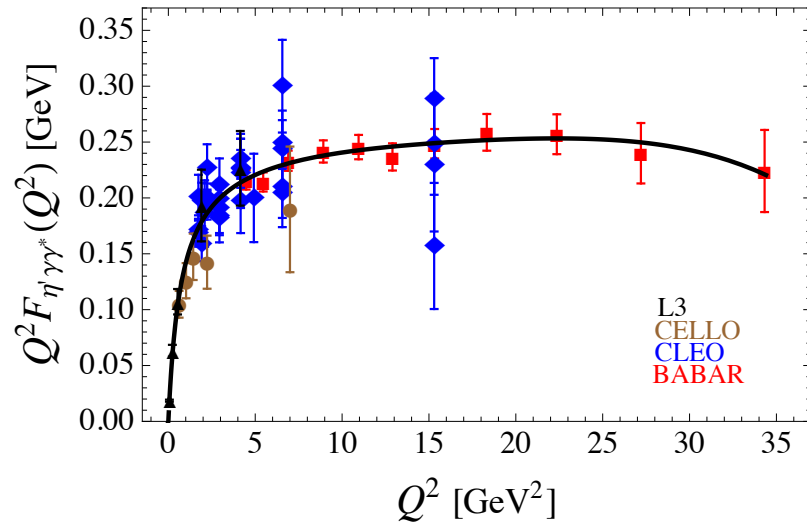
$P_N^N(Q^2)$ up to N=2

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta\gamma^*}(Q^2, 0) = 0.160(24) \text{ GeV}$$

η' -TFF

Fit to Space-like data: CELLO'91, CLEO'98, L3'98, BABAR'11

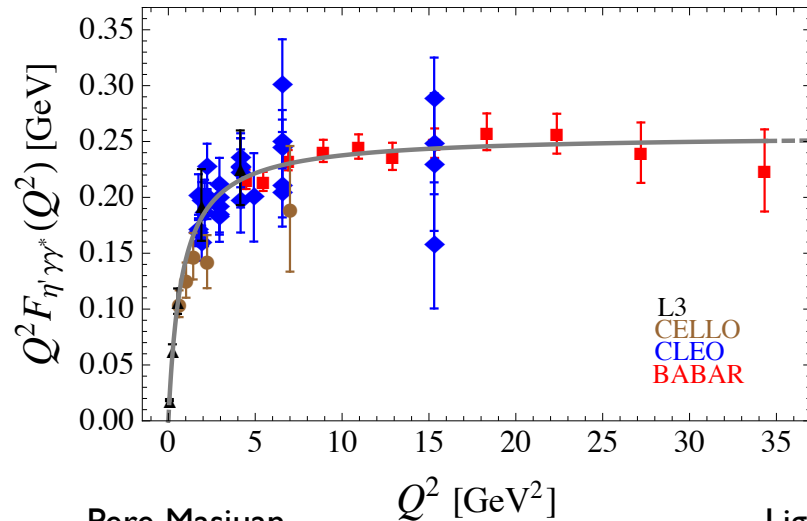
[R.Escribano, P.M., P. Sanchez-Puertas, '13]



$P_1^N(Q^2)$ up to $N=5$

$$\Gamma_{\eta' \rightarrow \gamma \gamma}^{pred} = (4.21 \pm 0.43) \text{keV}$$

$$\Gamma_{\eta' \rightarrow \gamma \gamma}^{PDG} = (4.34 \pm 0.14) \text{keV}$$

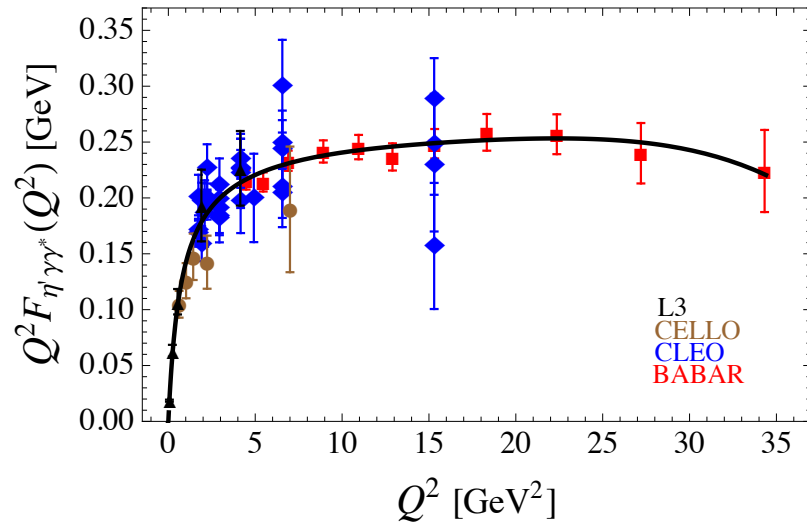


$P_N^N(Q^2)$ up to $N=1$

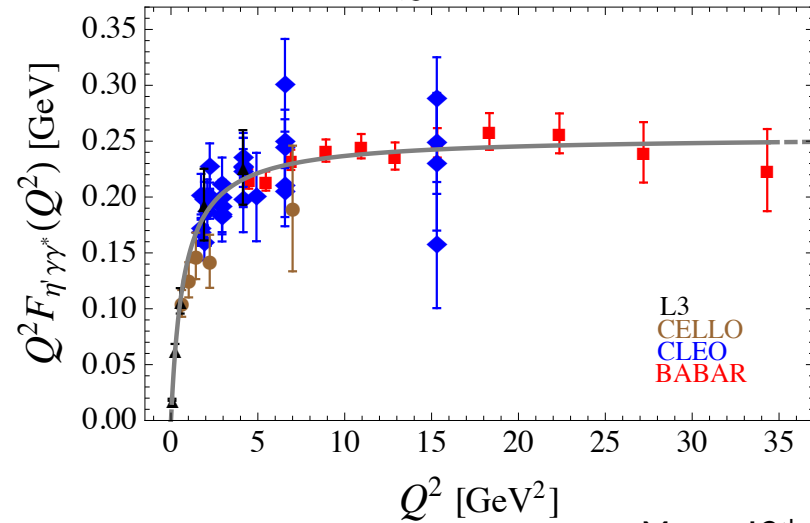
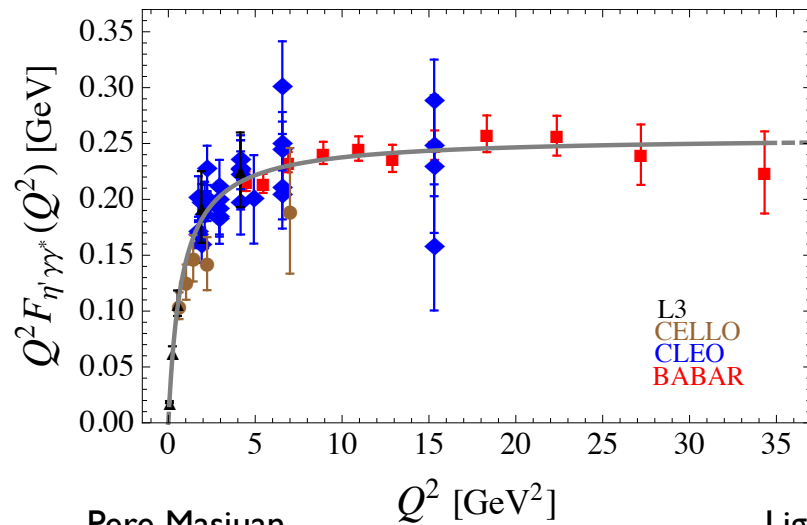
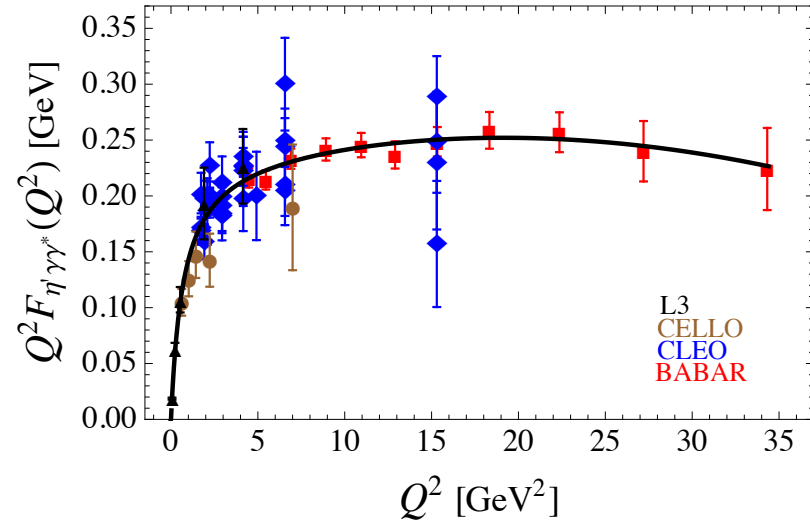
$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta' \gamma \gamma^*}(Q^2, 0) = 0.256(4) \text{GeV}$$

η' -TFF

$\Gamma_{\eta' \rightarrow \gamma\gamma}$ not included



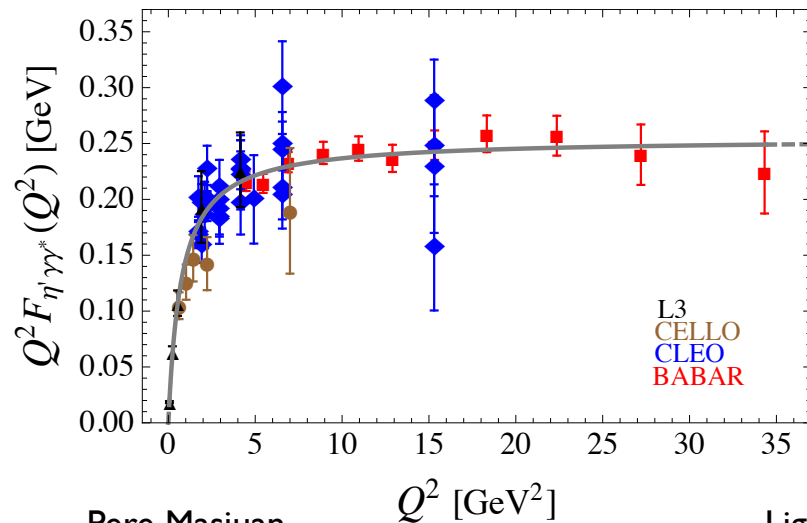
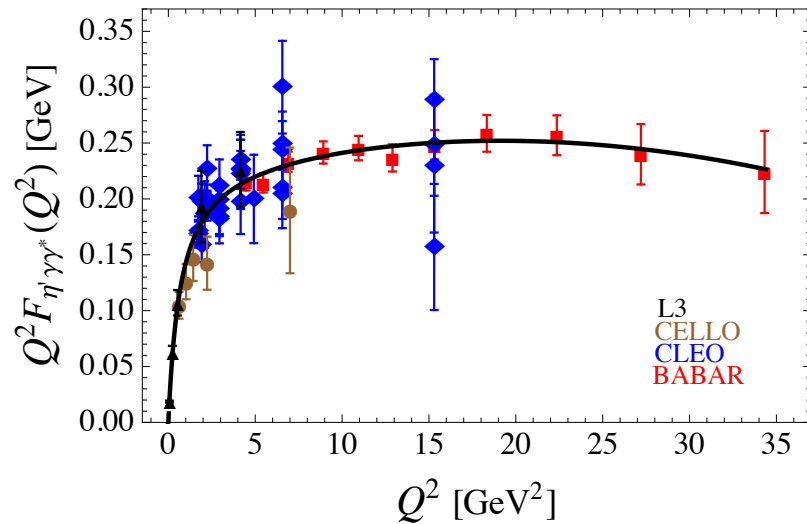
$\Gamma_{\eta' \rightarrow \gamma\gamma}$ included



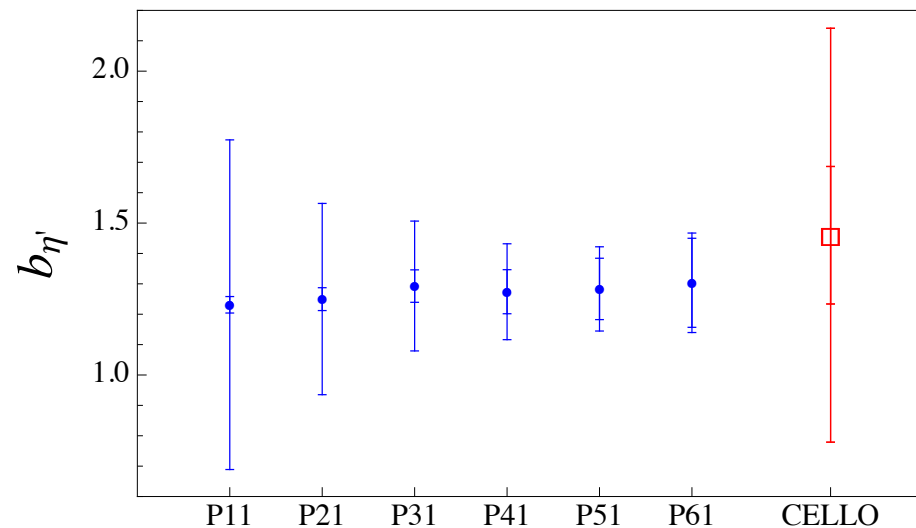
η' -TFF

Fit to Space-like data: CELLO'91, CLEO'98, L3'98, BABAR'11 + $\Gamma_{\eta' \rightarrow \gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, '13]



$P_1^N(Q^2)$ up to N=6



$P_N^N(Q^2)$ up to N=1

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta' \gamma^* \gamma}(Q^2, 0) = 0.254(4) \text{ GeV}$$

η - η' mixing

η - η' mixing in the flavor basis

$$\begin{pmatrix} f_{\eta}^q & f_{\eta}^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

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From the TFFs we can determine f_q, f_s, ϕ

$$\Gamma_{\eta \rightarrow \gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta}^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_{\eta}^q \frac{10}{3} + f_{\eta}^s \frac{2\sqrt{2}}{3},$$

$$\Gamma_{\eta' \rightarrow \gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3}.$$

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$$\Gamma_{\eta' \rightarrow \gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

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$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3}.$$

η - η' mixing

η - η' mixing in the flavor basis

$$\begin{pmatrix} f_{\eta}^q & f_{\eta}^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

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$$\Gamma_{\eta \rightarrow \gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta}^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2$$

$$\Gamma_{\eta' \rightarrow \gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_{\eta}^q \frac{10}{3} + f_{\eta}^s \frac{2\sqrt{2}}{3},$$

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3}.$$

[R.Escribano, P.M., P. Sanchez-Puertas, '13]

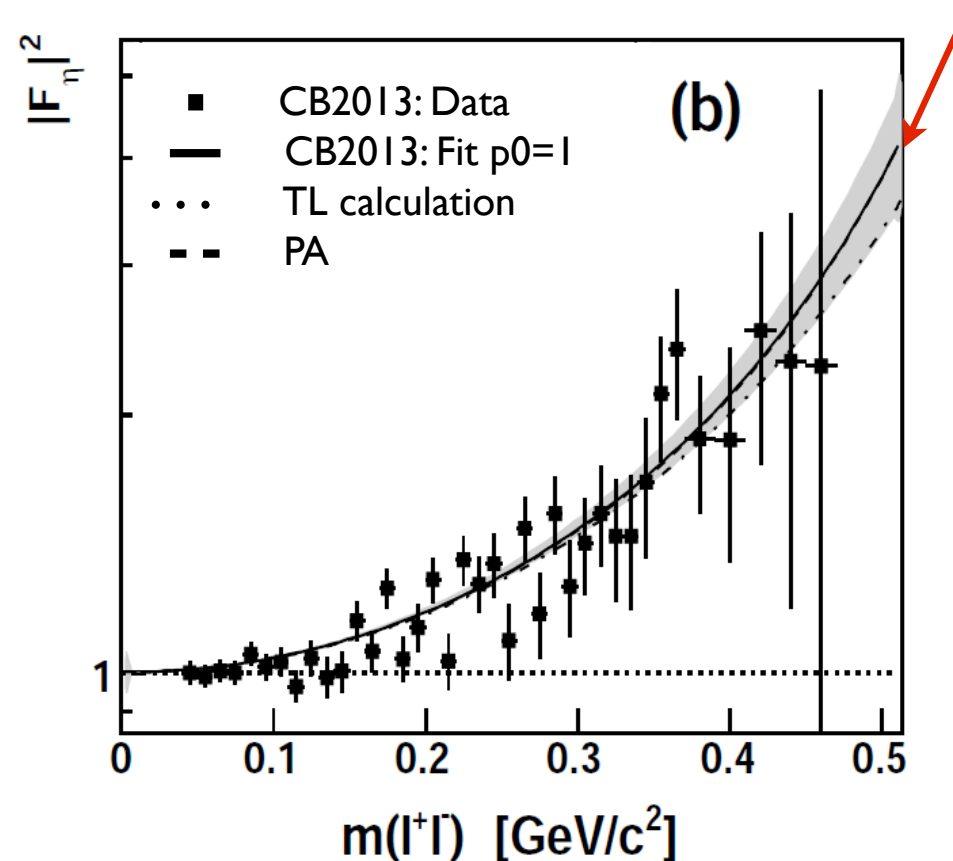
$$f_q = 1.065(13) f_{\pi}, \quad f_s = 1.53(22) f_{\pi}, \quad \phi = 40.2(1.5)^{\circ}$$

Update of Frere-Escribano '05 with PDG12 using 9 inputs

$$f_q = 1.07(1) f_{\pi}, \quad f_s = 1.63(2) f_{\pi}, \quad \phi = 40.4(0.3)^{\circ}$$

Pseudoscalar Transition Form Factors

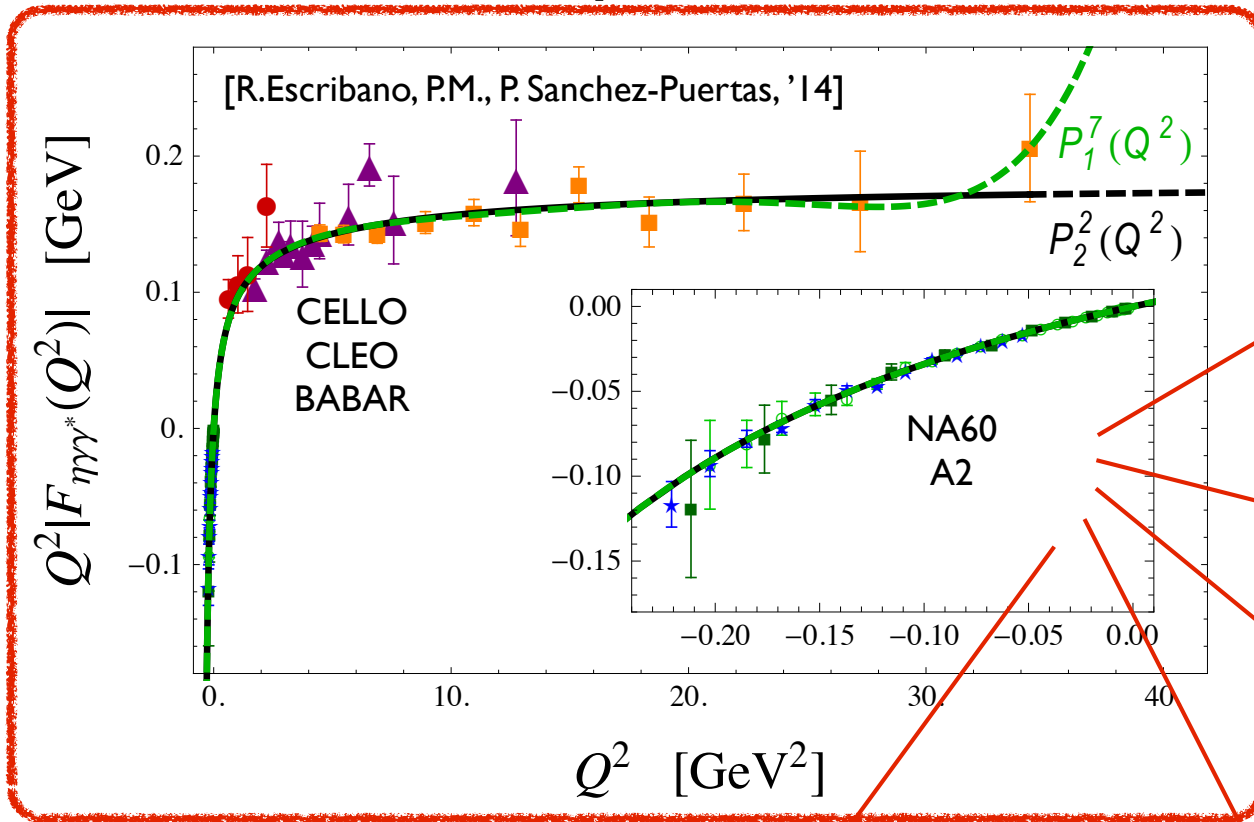
- Study Dalitz decays
 $\eta \rightarrow \gamma^* \gamma \rightarrow e^+ e^- \gamma$
- Meson Structure
 - Transition Form Factors give access to Meson Distribution Amplitudes
- Precision Tests of the Standard Model
 - Relation to mixing parameters and muon anomaly $(g-2)_\mu$



(see talk of M. Unverzagt)

η -TFF

space-like and time-like data



Low-energy parameters
up to the third derivative!

$$b_\eta, c_\eta, d_\eta$$

mixing parameters
of the η - η' system

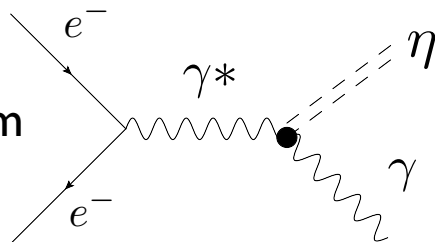
$$f_q, f_s, \phi$$

Rare η decays

$$\Gamma_{\eta \rightarrow e^+e^-}, \Gamma_{\eta \rightarrow \mu^+\mu^-}$$

beautiful synergy experiment - theory

η continuum
on charmonium
region



η -TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]
+ Time-like data [NA60'09, A2'11, A2'13]

$P_1^N(Q^2)$ up to N=7

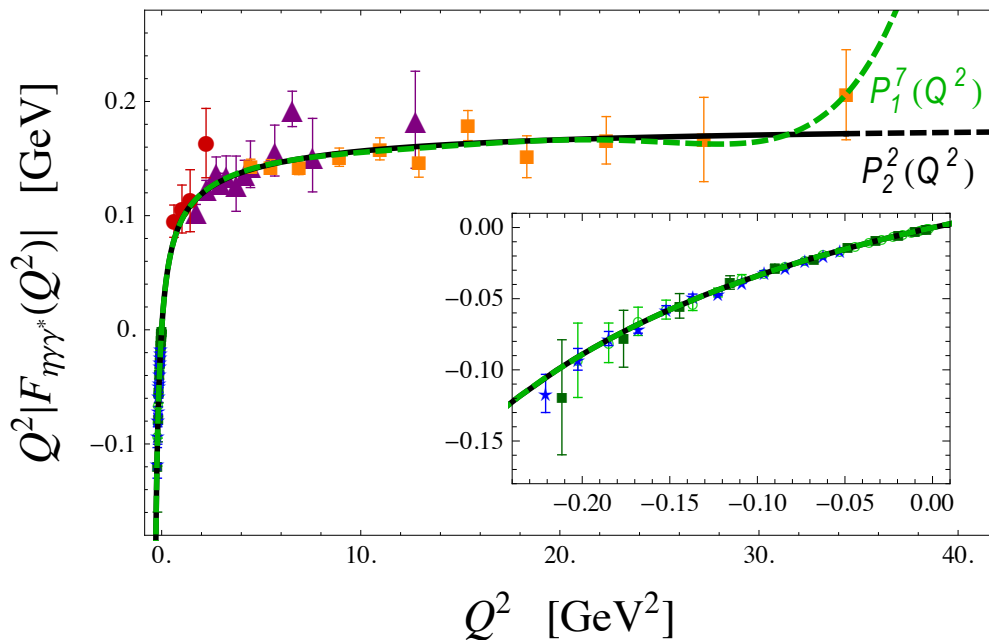
$$\Gamma_{\eta \rightarrow \gamma\gamma}^{pred} = (0.42 \pm 0.10) keV$$

$$\Gamma_{\eta \rightarrow \gamma\gamma}^{PDG} = (0.518 \pm 0.018) keV$$

$P_N^N(Q^2)$ up to N=2

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2, 0) = 0.169(14) GeV$$

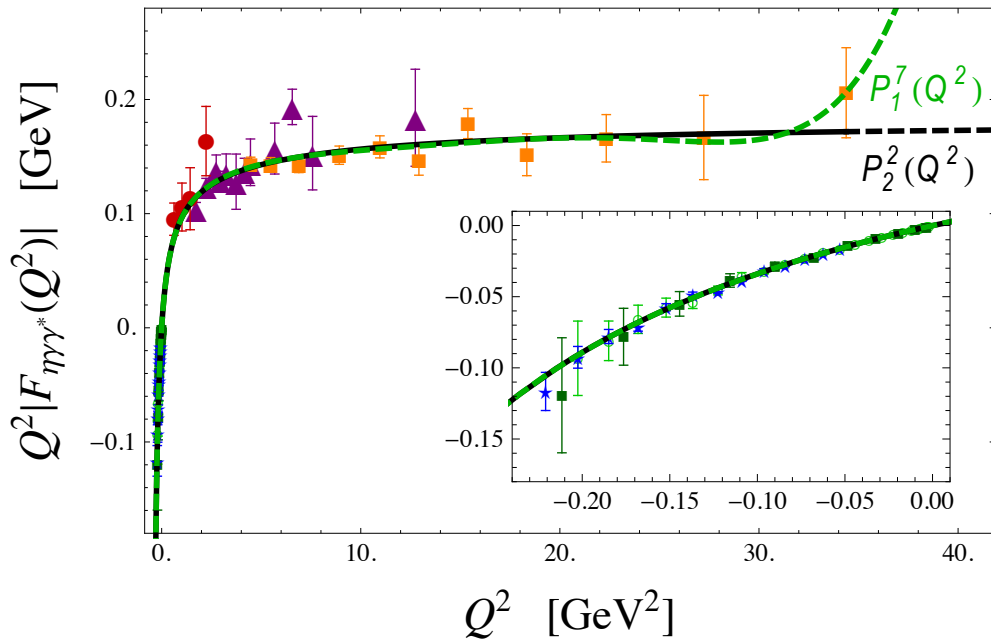
[R.Escribano, P.M., P. Sanchez-Puertas, '14]



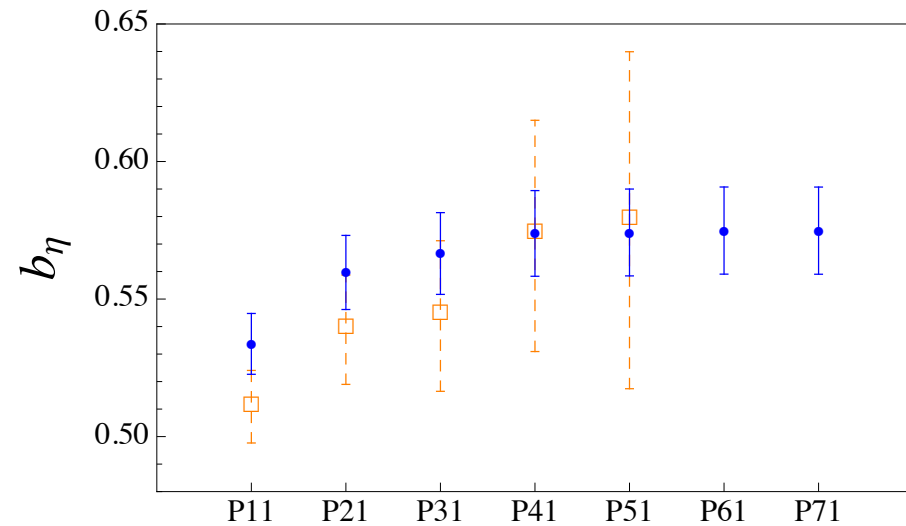
η -TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11] + $\Gamma_{\eta \rightarrow \gamma\gamma}$
 + Time-like data [NA60'09, A2'11, A2'13]

[R.Escribano, P.M., P. Sanchez-Puertas, '14]



$P_1^N(Q^2)$ up to N=7



$P_N^N(Q^2)$ up to N=2

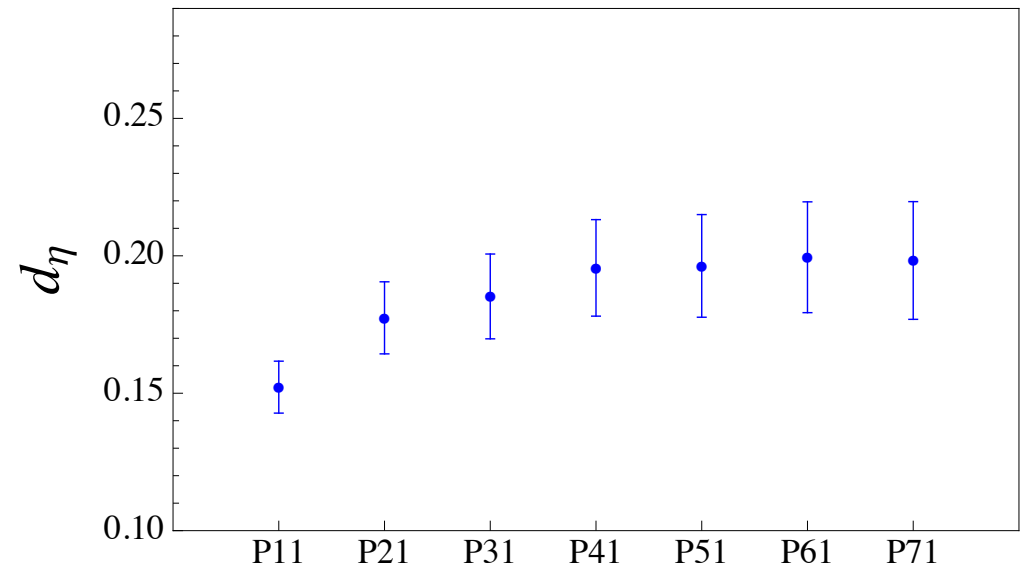
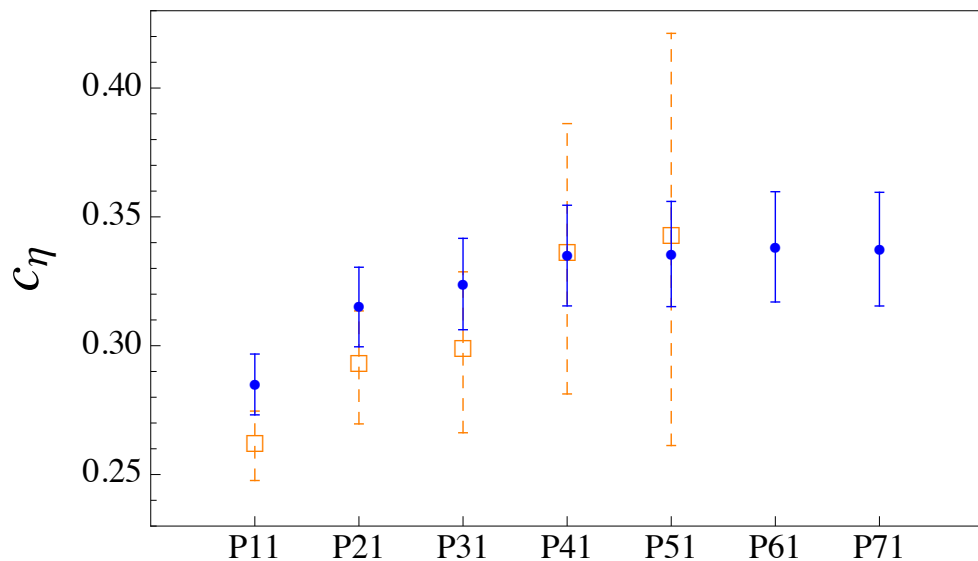
$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2, 0) = 0.177(15) \text{ GeV}$$

η -TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11] + $\Gamma_{\eta \rightarrow \gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

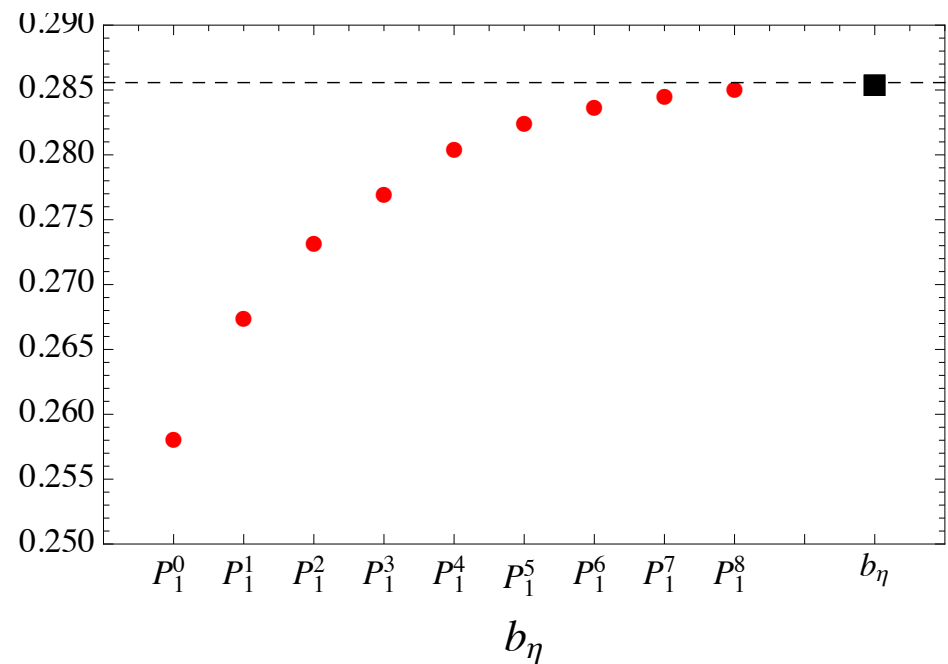
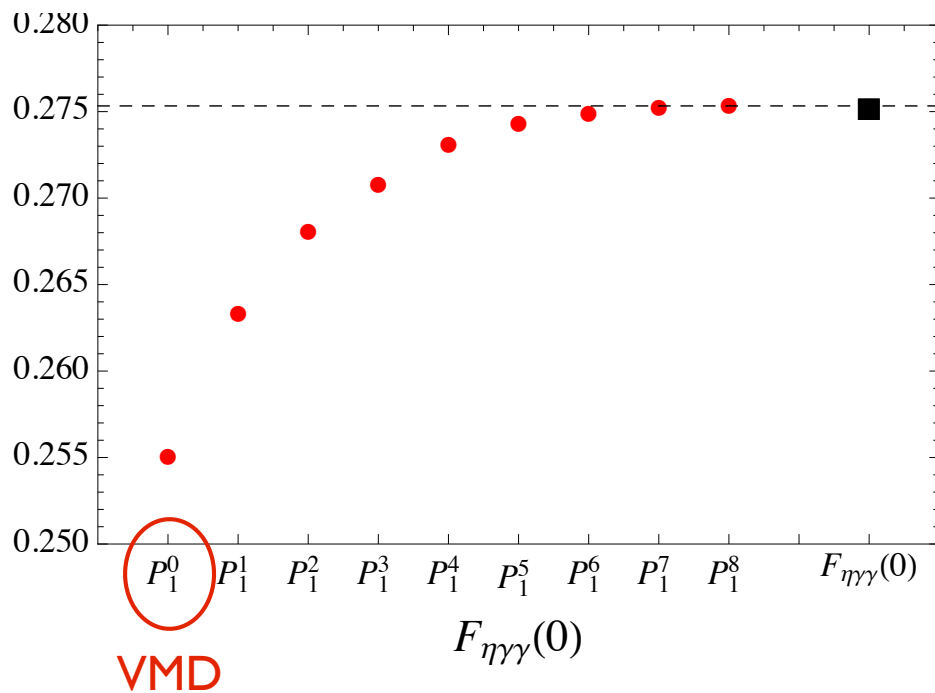
$P_1^N(Q^2)$ up to N=7

[R.Escribano, P.M., P. Sanchez-Puertas, '14]



A word on systematics

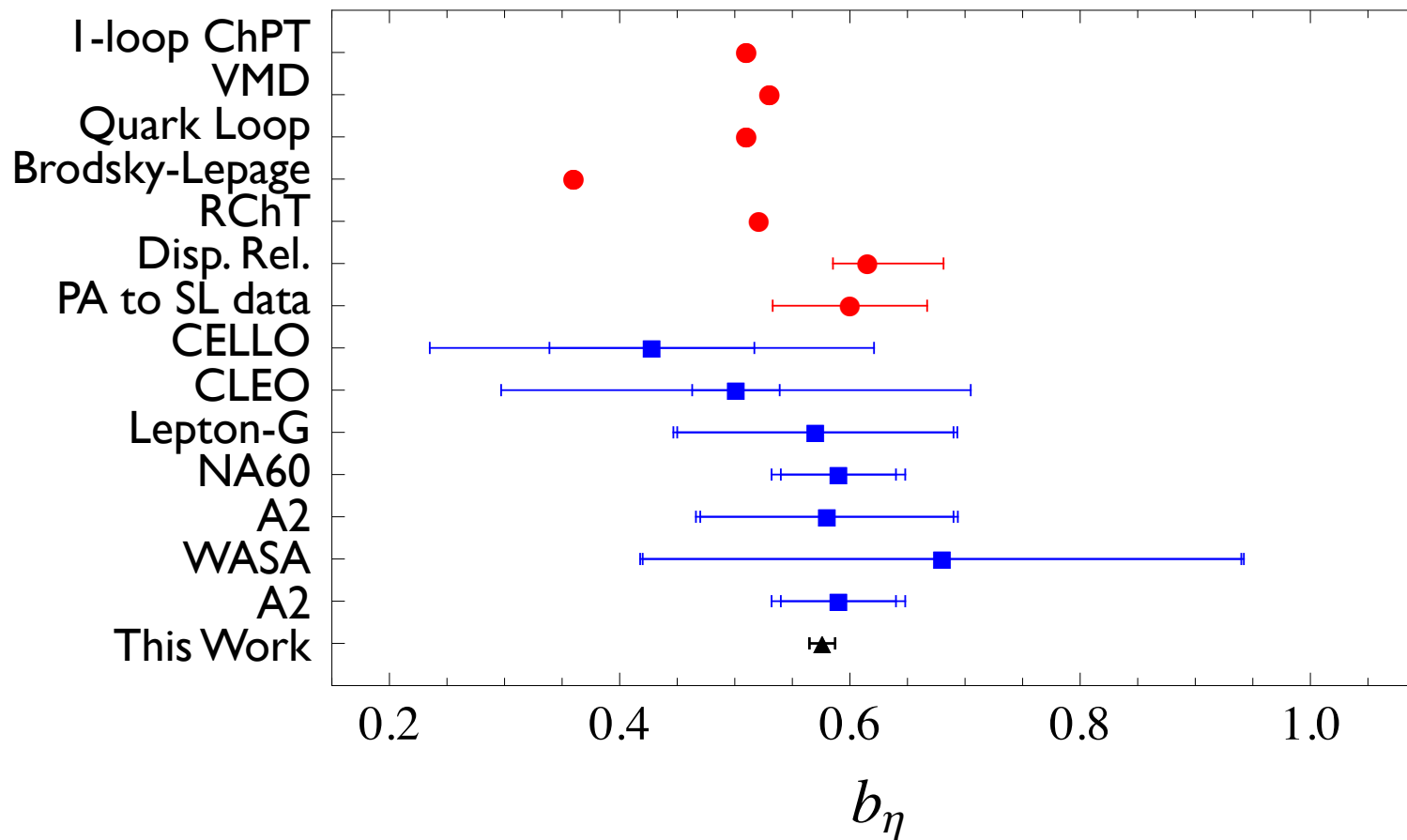
- Consider a model for η TFF
- Generate a pseudodata set emulating the physical situation (SL+TL)
- Build up your PA sequence
- Fit and compare



η -TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11] + $\Gamma_{\eta \rightarrow \gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

[R.Escribano, P.M., P. Sanchez-Puertas, '14]



η - η' mixing

η - η' mixing in the flavor basis

$$\begin{pmatrix} f_{\eta}^q & f_{\eta}^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

From the TFFs we can determine f_q, f_s, ϕ

$$\Gamma_{\eta \rightarrow \gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta}^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2$$

$$\Gamma_{\eta' \rightarrow \gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_{\eta}^q \frac{10}{3} + f_{\eta}^s \frac{2\sqrt{2}}{3},$$

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3}.$$

η - η' mixing

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$$\Gamma_{\eta' \rightarrow \gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_{\eta}^q \frac{10}{3} + f_{\eta}^s \frac{2\sqrt{2}}{3},$$

$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3}.$$

[R.Escribano, P.M., P. Sanchez-Puertas, '14]

$$f_q = 1.07(1)f_{\pi}, \quad f_s = 1.39(14)f_{\pi}, \quad \phi = 39.3(1.3)^{\circ}$$

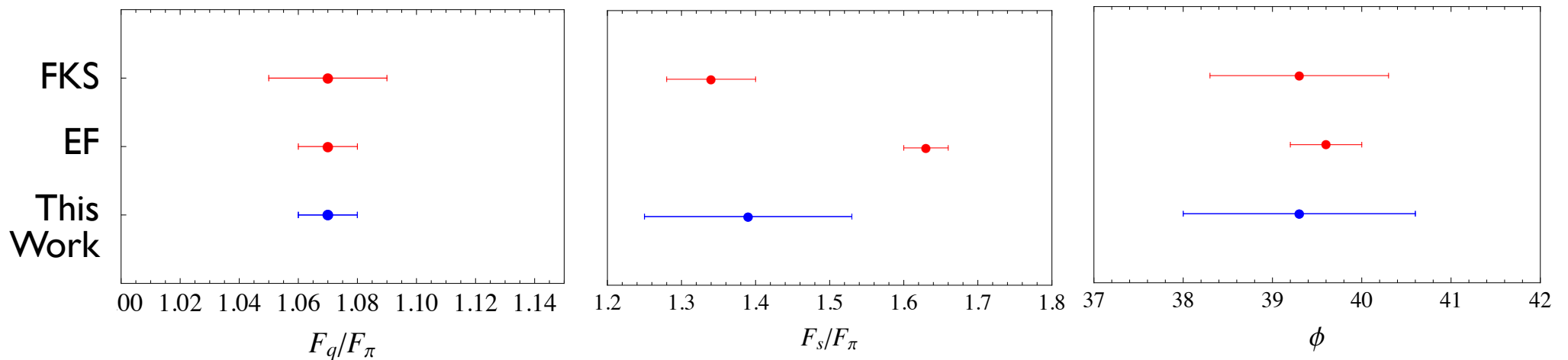
Update of Frere-Escribano '05 with PDG12 using 9 inputs

$$f_q = 1.07(1)f_{\pi}, \quad f_s = 1.63(2)f_{\pi}, \quad \phi = 40.4(0.3)^{\circ}$$

η - η' mixing

η - η' mixing in the flavor basis

From the TFFs we can determine F_q, F_s, ϕ



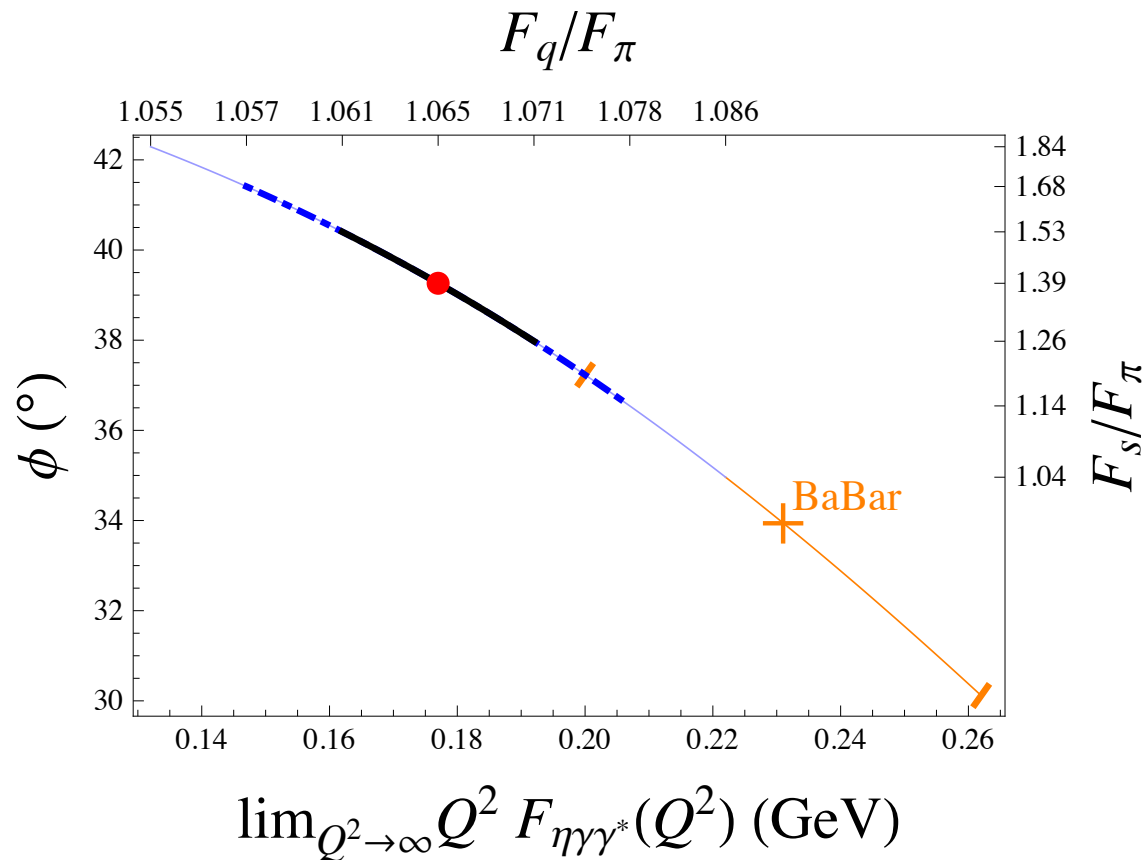
FKS: Feldmann, Kroll, Stech, PLB 449, 339, (1999)

EF: Escribano, Frere, JHEP 0506, 029 (2005) updated in Escribano, P.M, Sanchez-Puertas, 2013.

η - η' mixing

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η - η' mixing

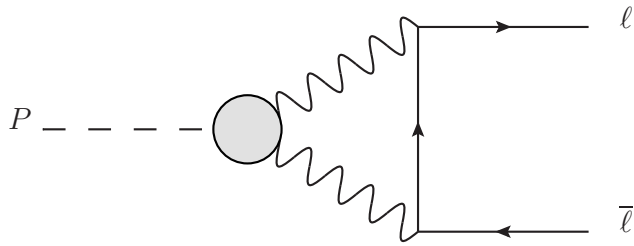
From the TFFs we can determine F_q, F_s, ϕ

and the $VP\gamma$ and J/Ψ decays used in FKS and EF as inputs

(using $F_{\pi^0} = 131.5 \pm 1.4$ MeV instead of $F_{\pi^-} = 92.21 \pm 0.14$ MeV)

	Our predictions	Experimental determinations
$g_{\rho\eta\gamma}$	1.46(3)	1.58(5)
$g_{\rho\eta'\gamma}$	1.20(4)	1.32(3)
$g_{\omega\eta\gamma}$	0.56(2)	0.45(2)
$g_{\omega\eta'\gamma}$	0.55(2)	0.43(2)
$g_{\phi\eta\gamma}$	-0.78(8)	-0.69(1)
$g_{\phi\eta'\gamma}$	0.88(10)	0.72(1)
$\frac{J/\Psi \rightarrow \eta'\gamma}{J/\Psi \rightarrow \eta\gamma}$	5.09(47)	4.67(20)

Dissection of $\eta \rightarrow l^+ l^-$



$$\frac{BR(P \rightarrow \bar{l}l)}{BR(P \rightarrow \gamma\gamma)} = 2 \left(\frac{\alpha m_\ell}{\pi m_P} \right)^2 \beta_\ell(m_P^2) |\mathcal{A}(m_P^2)|^2$$

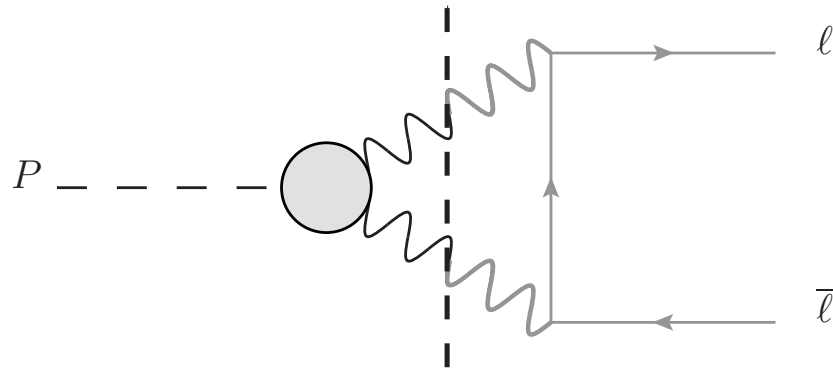
The only unknown $\mathcal{A}(m_P^2)$ from loop calculation where the TFF enters.

$$\mathcal{A}(q^2) = \frac{2i}{\pi^2} \int d^4 k \frac{q^2 k^2 - (k \cdot q)^2}{k^2 (k - q)^2 ((p - k)^2 - m_\ell^2)} \frac{F_{P\gamma^*\gamma^*}(k^2, (q - k)^2)}{F_{P\gamma\gamma}(0, 0)}$$

Dissection of $\eta \rightarrow l^+ l^-$

As model independent as possible:

Cutkosky rules provides the imaginary part



Dissection of $\eta \rightarrow l^+ l^-$

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Cutkosky rules provides the imaginary part

$$\text{Im}\mathcal{A}(q^2) = \frac{\pi}{2\beta_I(q^2)} \ln \left(\frac{1 - \beta_I(q^2)}{1 + \beta_I(q^2)} \right); \quad \beta_I(q^2) = \sqrt{1 - \frac{4m_l^2}{q^2}}$$

$q^2 = m_P^2$

Dissection of $\eta \rightarrow l^+ l^-$

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$q^2 = m_P^2$

Use dispersion relations to get the real part

$$\text{Re}(\mathcal{A}(q^2)) = \mathcal{A}(0) + \frac{1}{\beta_I(q^2)} \left(\frac{\pi^2}{12} + \frac{1}{4} \ln^2 \left(\frac{1 - \beta_I(q^2)}{1 + \beta_I(q^2)} \right) + \text{Li}_2 \left(\frac{1 - \beta_I(q^2)}{1 + \beta_I(q^2)} \right) \right)$$

Dissection of $\eta \rightarrow l^+l^-$

PDG value dominated by the KTeV measurement

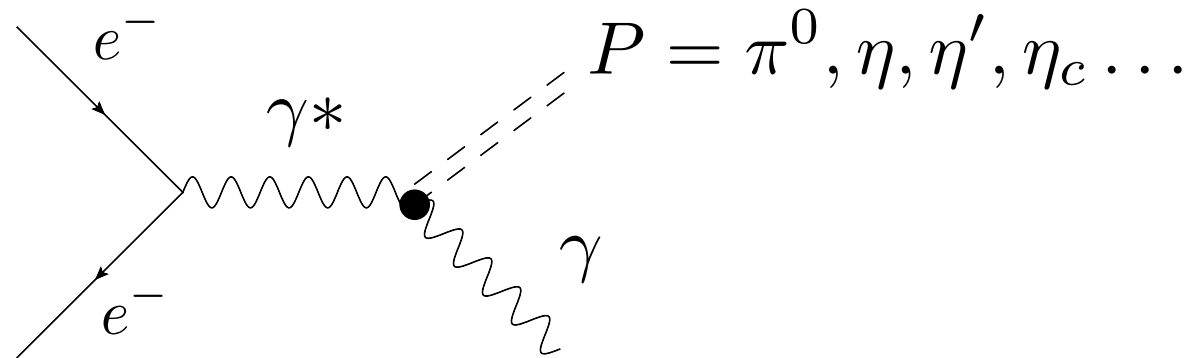
$$\frac{BR(P \rightarrow \bar{l}l)}{BR(P \rightarrow \gamma\gamma)} = 2 \left(\frac{\alpha m_\ell}{\pi m_P} \right)^2 \beta_\ell(m_P^2) |\mathcal{A}(m_P^2)|^2 = 5.8(8) \cdot 10^{-6} \quad (\mu^+\mu^-)$$
$$\leq 5.6 \cdot 10^{-6} \quad (e^+e^-)$$

$$\text{Unitary Bound for the } \mu\mu \text{ case} = 4.37 \cdot 10^{-6}$$

$$\text{SM calculations with } m_\eta^2/\Lambda^2 \sim 0 = 4.99 \cdot 10^{-6}$$

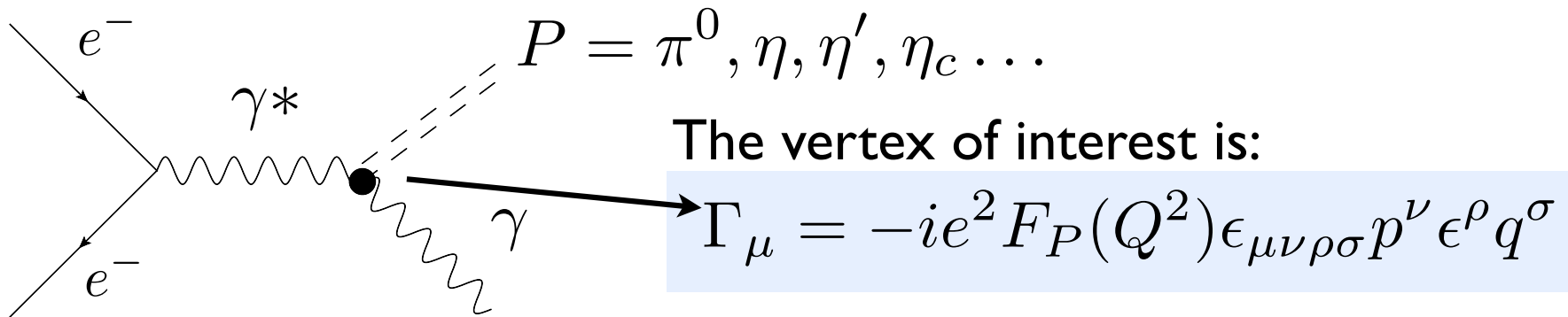
$$\text{Our result from SL+TL (full result)} = 4.51(2) \cdot 10^{-6}$$

Time-like TFF: prediction



- Asymptotic limits in time-like and space-like FFs are expected to be close, is important to measure this time-like FF because:
 - the charmonium region is between the perturbative and non-perturbative regimes of the π -, η -, and η' -TFF
 - background for charmonium decays

Time-like TFF: prediction



Differential cross section:

$$\frac{d\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma P)}{d(\cos\theta)} = \frac{\pi^2 \alpha^3}{4} (F_{P\gamma^*\gamma}(s, 0))^2 \left(1 - \frac{M_P^2}{s}\right)^3 (1 + \cos^2\theta)$$

Integrating with respect to $\cos\theta$

$$\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \gamma P) = \frac{2\pi^2 \alpha^3}{3} (F_{P\gamma^*\gamma}(s, 0))^2 \left(1 - \frac{M_P^2}{s}\right)^3$$

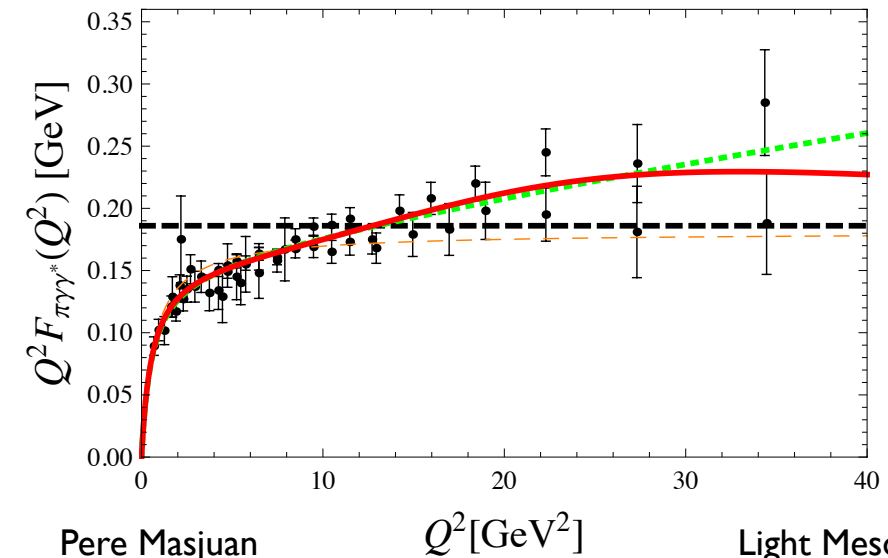
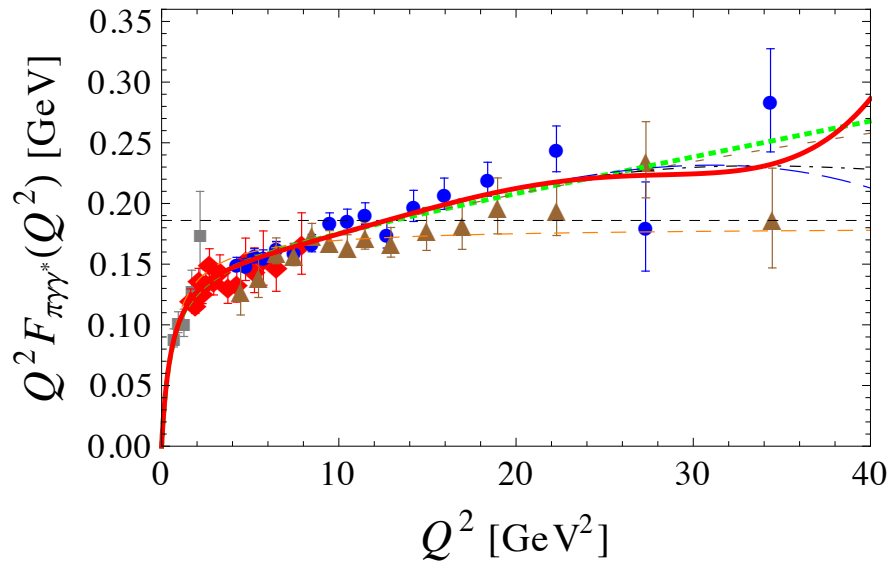
Conclusions

- Transition Form Factors are a good laboratory to study meson properties
- Need for a model independent approach: we use Padé App.
- Padé Approximants' method is easy, systematic and can be improved upon by including new data
- Considering Space-like and time-like data
 - provides very accurate LECs and asymptotic limits
 - provides insight in mixing scheme and meson structure
 - predicts $V\rho\gamma$, J/ψ , rare decays, continuum...
 - beautiful synergy experiment - theory

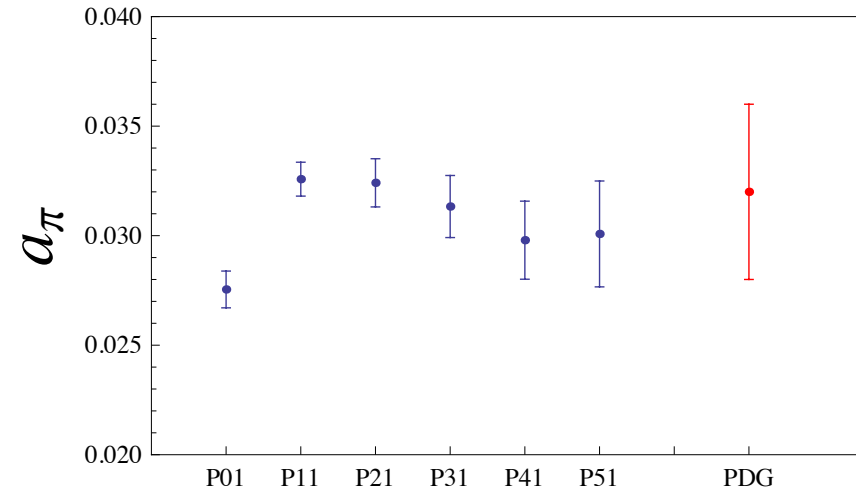
Thank you!

π^0 -TFF

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'09 and Belle'12



$P_1^N(Q^2)$ up to N=5 [P.M., '12]



$P_N^N(Q^2)$ up to N=3