Dispersion theory to connect $\eta \to \pi\pi\gamma$ to $\eta \to \gamma^*\gamma$

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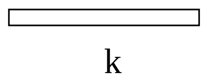
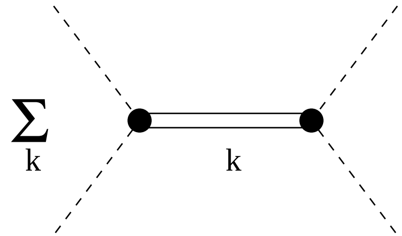
Modeling hadron physics

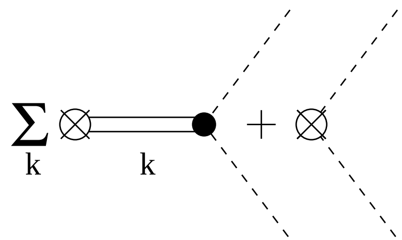
Standard treatment: sum of Breit-Wigners

Propagator: $iG_k(s) =$ 

$= i/(s - M_k^2 + iM_k\Gamma_k)$

Scattering: 

$= \sum_k ig_k^2 G_k(s)$

Production: 

$= \left(\sum_k ig_k G_k(s)\alpha_k\right) + i\beta$

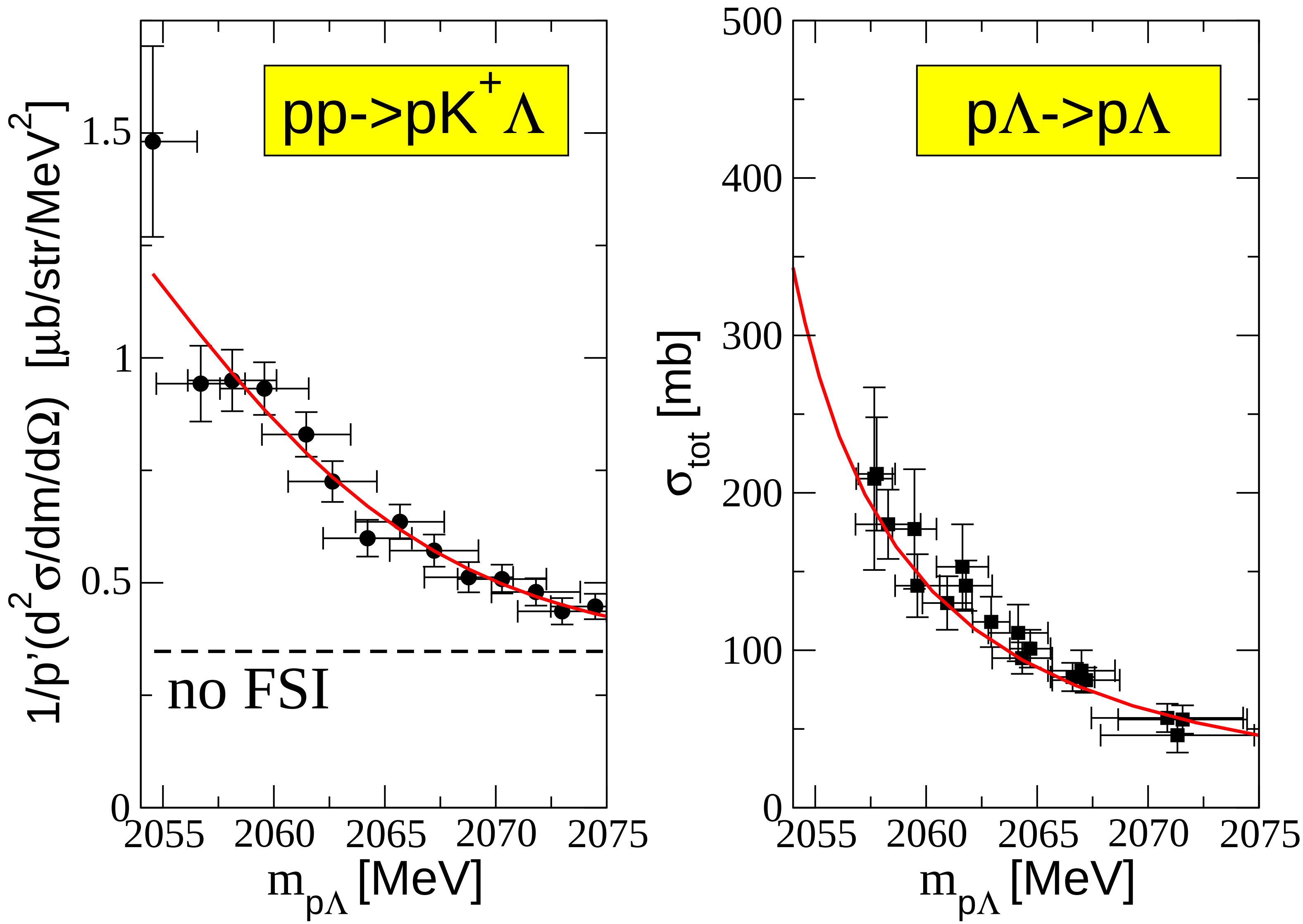
Problems:

$\rightarrow$ Wrong threshold behavior (cured by $\Gamma = \Gamma(s)$)

$\rightarrow$ Violates unitarity $\longrightarrow$ wrong phase motion

$\rightarrow$ Parameters reaction dependent
only pole positions and resides universal!

Production vs. Scattering

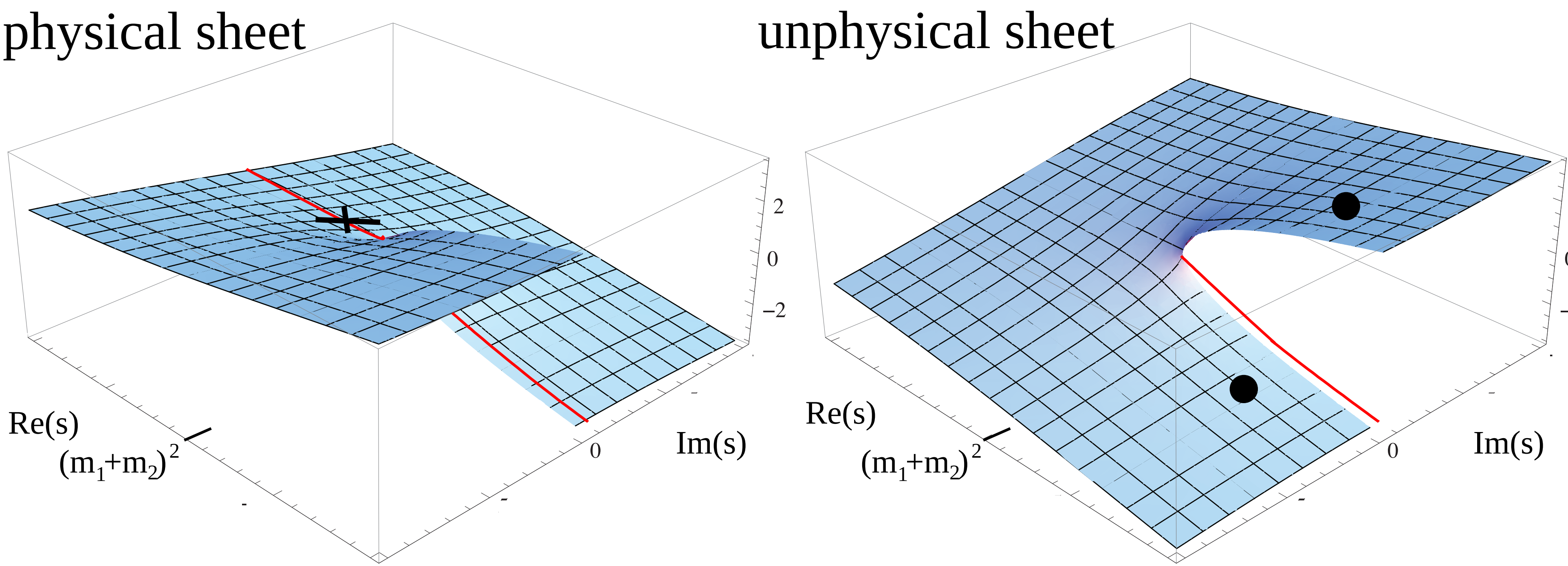


R. Siebert et al. (1994); G. Alexander et al. (1968)

In general: not identical, but related

Singularities

Every channel opening is accompanied with a branch point:



x = bound state; • = resonance pole

The strength of the resulting discontinuity is:

$$\mathrm{Disc}(F_i) = 2i \sum_k T^*_{ik} \sigma_k F_k \quad \rightarrow \text{Dispersion Integral(s)}$$

Imposing Unitarity

single channel:

$$\mathrm{Disc}(F)/2i = \mathrm{Im}(F) = e^{-i\delta(s)} \sin(\delta(s)) F(s)$$

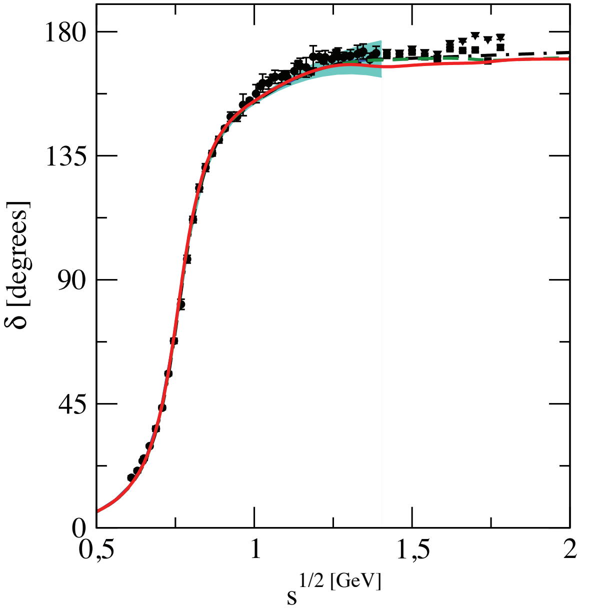
$\rightarrow$ Watson theorem: $F(s) = |F(s)| e^{i\delta(s)}$ and Omnès function

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s-i\epsilon)}\right)$$

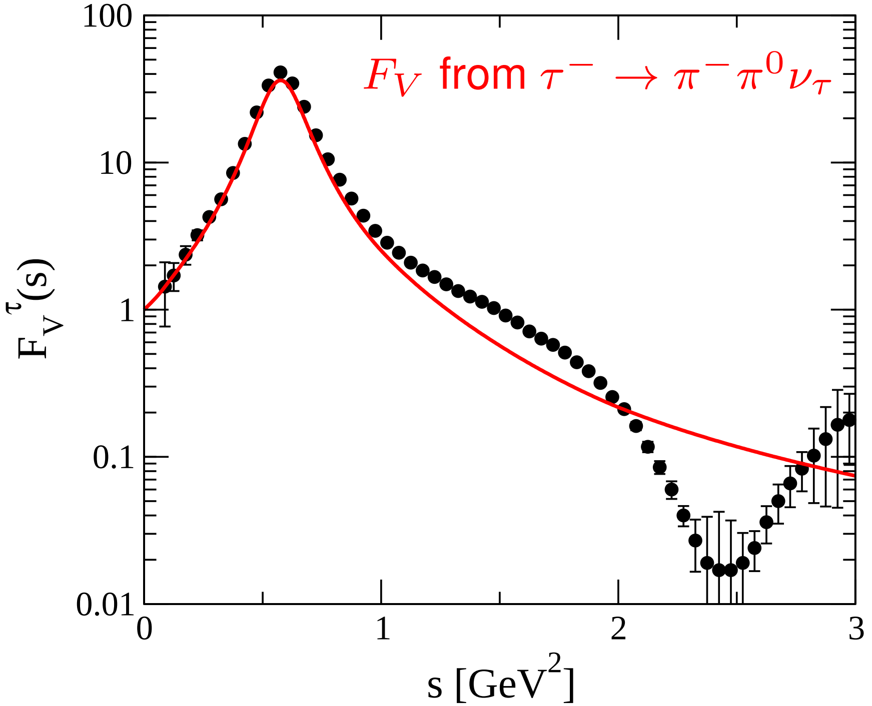
such that, for negligible left-hand cuts

$$\boxed{F(s) = P(s)\Omega(s)}$$

- $\rightarrow$ $\Omega(s)$ is universal and fixed in elastic regime
- $\rightarrow$ $P(s)$ reaction specific and contains e.g.
  - ▷ higher thresholds
  - ▷ inelastic resonances

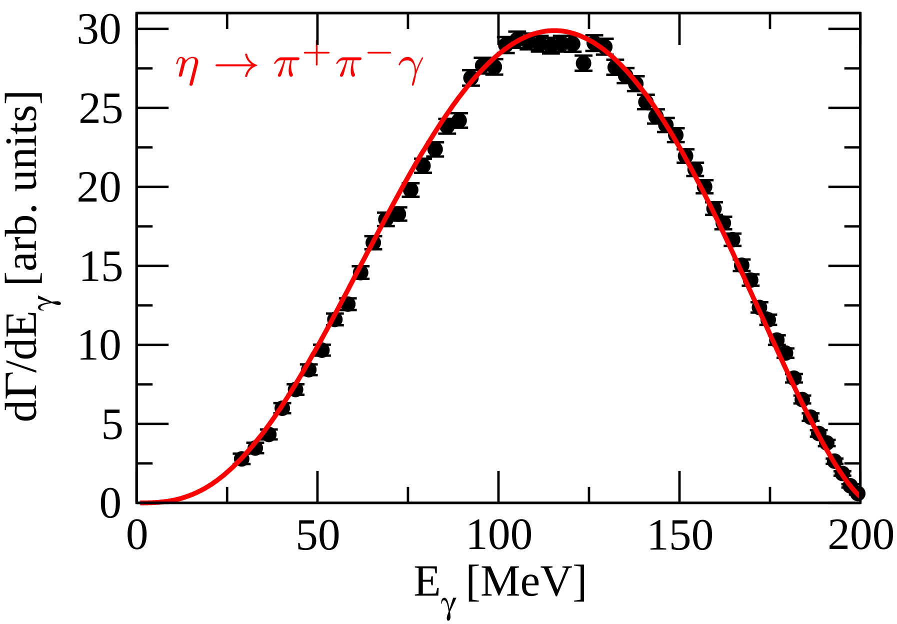


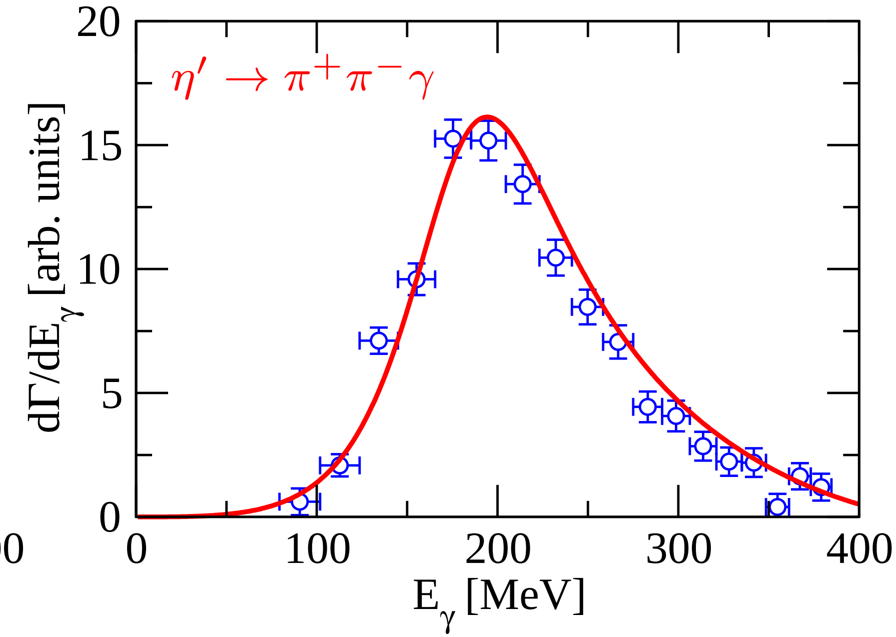
Universality of FSI



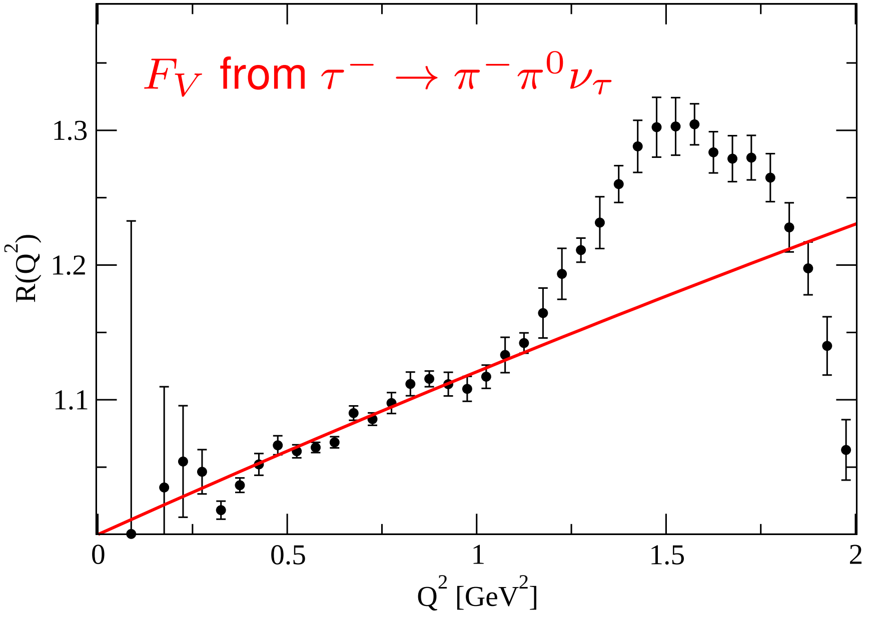
red lines: p-wave Omnes
$\times$ kinematic factors

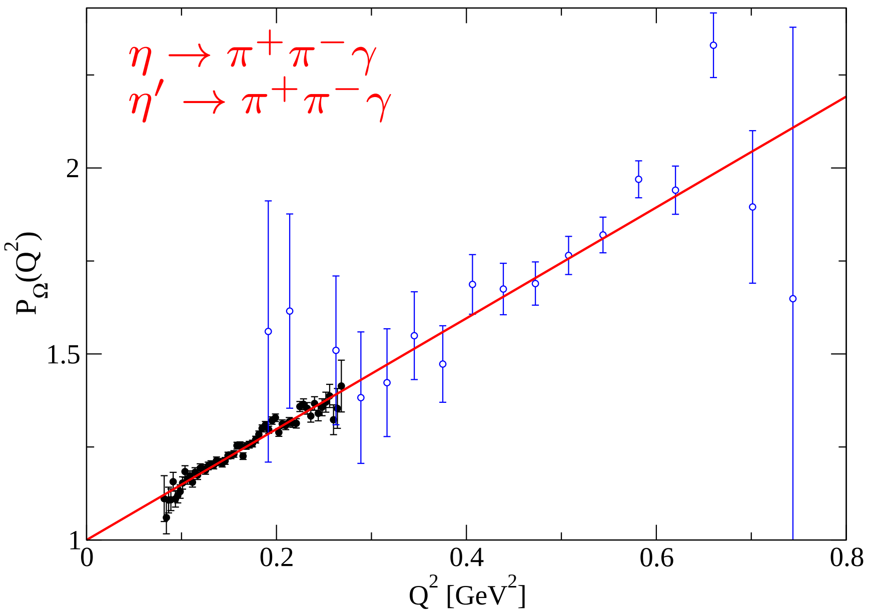
- bulk described properly
- there are deviations
  $\longrightarrow P(s)$ not constant!





Universality of FSI





We had: $F(Q^2) = P(Q^2)\Omega(Q^2)$

We find for all 3 cases

- $P(Q^2)$ linear for $Q^2 < 1$ GeV^2
- deviations in F_V by ρ' & ρ''

$$P(Q^2) = A_0(1 + \alpha Q^2)$$

$\rightarrow$ α reaction specific

$\rightarrow$ $\alpha[\eta] = \alpha[\eta']$ understood

1-loop ChPT + large N_c

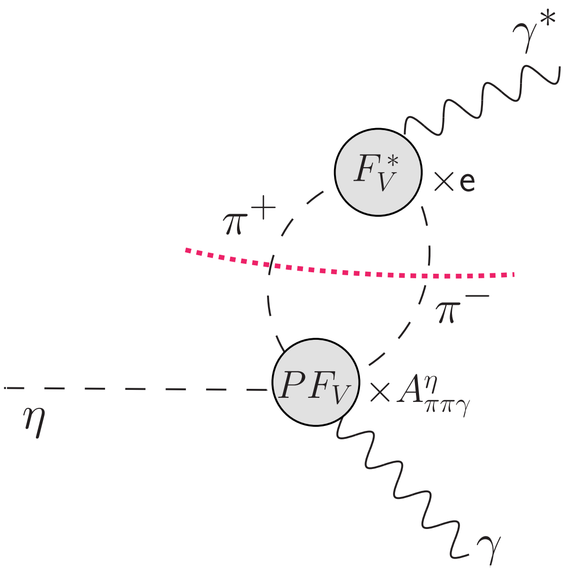
$\rightarrow$ $\alpha = 1.3 \pm 0.1$ GeV^{-2} understood

box vs. triangle anomaly + large N_c

F. Stollenwerk et al., PLB **707**, 184 (2012)

Going to $\eta \to \gamma\gamma^*$

Allows for parameter free prediction for isovector part of slope



$$F_{\eta\gamma^\star\gamma}(Q^2,0) \equiv 1 + \Delta F^{(I=1)}_{\eta\gamma^\star\gamma}(Q^2,0) + \Delta F^{(I=0)}_{\eta\gamma^\star\gamma}(Q^2,0)$$

$$\Delta F^{(I=1)}_{\eta\gamma^\star\gamma} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2}\right) \int_{4m_\pi^2}^{\infty} ds' \, \sigma_\pi(s')^3 \, P(s') \, \frac{|F_V(s')|^2}{s' - Q^2 - i\epsilon}$$

$\Delta F^{(I=0)}_{\eta\gamma^\star\gamma}$ needs to be modeled

We thus demonstrated:

- $\eta \to \gamma\gamma^*$ and $\eta \to \pi^+\pi^-\gamma$ closely connected
- FSI ($\Omega(Q^2)$) universal; driven by ρ-pole and residues
- $P(Q^2)$ (reaction dependent!) also modifies $\eta \to \gamma^*\gamma$

C. H. et al. Eur.Phys.J. C73 (2013) 2668.

The isoscalar contribution

Modeled using VMD (okay, since resonances are narrow)

$$\Delta F^{(I=0)}_{\eta\gamma^\star\gamma}(Q^2,0) = \frac{w_{\eta\omega\gamma}\,Q^2}{m_\omega^2 - Q^2 - i m_\omega \Gamma_\omega} + \frac{w_{\eta\phi\gamma}\,Q^2}{m_\phi^2 - Q^2 - i m_\phi \Gamma_\phi}\,.$$

using 'data' (isoscalar contribution isolated in exp. analysis) via

$$\sigma(e^+e^- \to \eta\gamma)\Big|^{\text{isoscalar}}_{s=m_V^2} = \frac{12\pi\,\mathrm{BR}(\mathrm{V}\to\eta\gamma)\,\mathrm{BR}(\mathrm{V}\to \mathrm{e}^+\mathrm{e}^-)}{m_V^2}\,,$$

we find

$w_{\eta\omega\gamma} \approx (0.78 \pm 0.04)\times 1/8$ and $w_{\eta\phi\gamma} \approx (0.75 \pm 0.03)\times(-2/8)$

where $1/8$ and $-2/8$ are the standard VMD values.

Isoscalar contribution small for η ($\sim$ 2%)

Input

$$\Delta F_{\eta\gamma^\star\gamma}^{(I=1)} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2}\right)\int_{4m_\pi^2}^{\infty} ds'\,\sigma_\pi(s')^3\;P(s')\;\frac{|F_V(s')|^2}{s'-Q^2-i\epsilon}$$

$\rightarrow$ $P(s') = (1+\alpha\; s')$ with

$\alpha = (1.32 \pm 0.13)$ GeV^{-2} from KLOE — Babusci et al., PLB718(2013)910

$\rightarrow$ $F_V(s')$ from recent parametrization — C.H., PLB715(2012)170

- $\triangleright$ At low energies $\Omega(s')$
- $\triangleright$ At higher energies: inelastic channels/ρ'/ρ''

$\rightarrow$ $\kappa_\eta \equiv eA^\eta_{\pi\pi\gamma} f_\pi^2 / A^\eta_{\gamma\gamma}$ from data — Review of Particle Properties (PDG)

Uncertainties

$$\Delta F^{(I=1)}_{\eta\gamma^\star\gamma} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2}\right) \int_{4m_\pi^2}^{\Lambda^2} ds'\, \sigma_\pi(s')^3\, P(s')\, \frac{|F_V(s')|^2}{s' - Q^2 - i\epsilon}$$

→ Uncertainties in input (especially α)

→ Isoscalar piece: here we needed model (for phases)

→ Large s' behavior of $P(s')$ and $F_V(s')$:

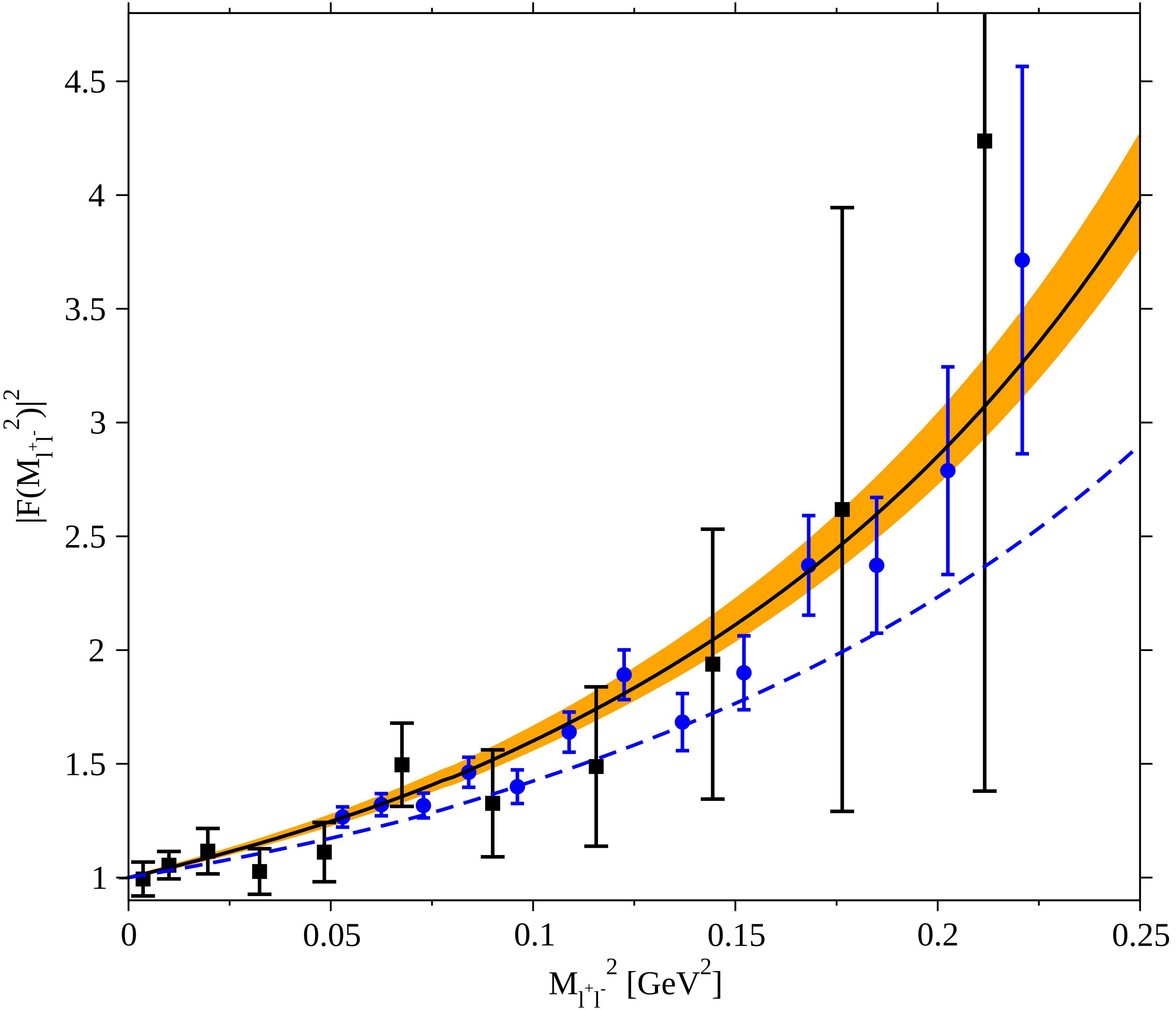
FSI in $e^+e^- \to \pi\pi$ and $\gamma\eta \to \pi\pi$ different in inelastic regime (e.g. diff. couplings of ρ' and ρ'') $\to \Lambda = m_{\eta'} \ldots 2$ GeV

check: $A^{\eta\,(I=1)}_{\gamma\gamma} = \left(eA^\eta_{\pi\pi\gamma}/96\pi^2\right) \int_{4m_\pi^2}^{\Lambda^2} ds'\, \sigma_\pi(s')^3\, P(s')\, |F_V(s')|^2$

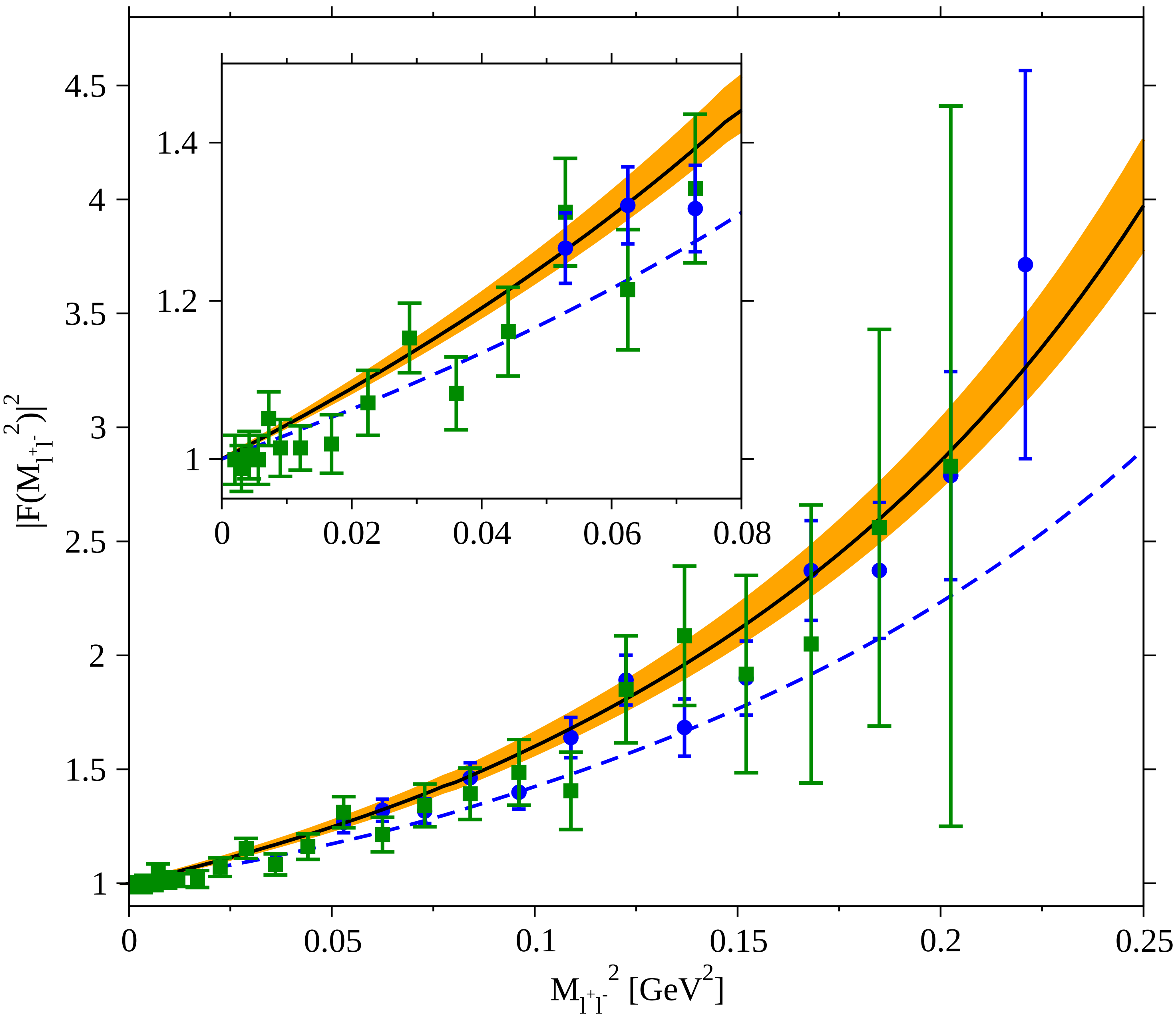
overestimates exp. by (7 ± 5)% for $\Lambda = 1$ GeV

S.P. Schneider et al., PRD86(2012)054013; M. Hoferichter et al., PRD86(2012)116009

Results

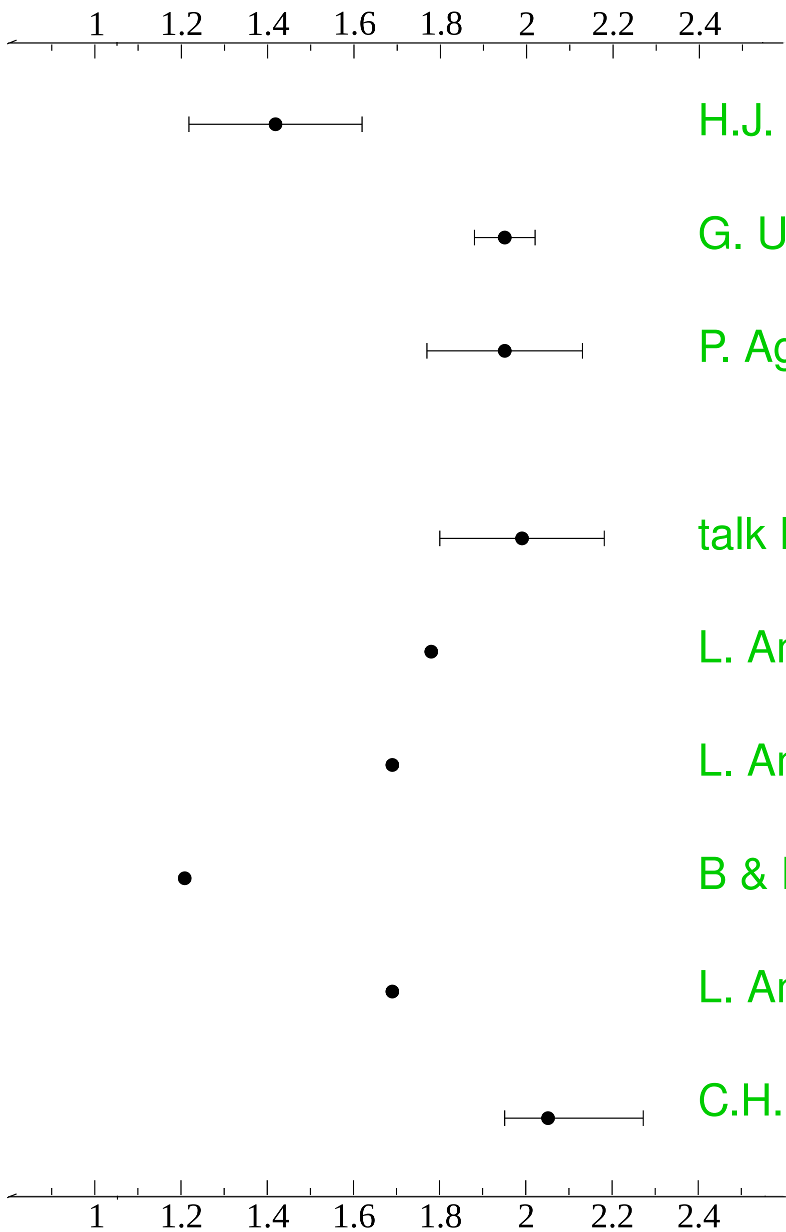


Results



4.5
4
3.5
3
2.5
2
1.5
1
0
0.05
0.1
0.15
0.2
0.25
1.4
1.2
1
0
0.02
0.04
0.06
0.08
|F(M$_{l^+l^-}$^{2})|^2
M$_{l^+l^-}$^{2} [GeV^2]

Slope



Recent experiments

$e^+e^- \to e^+e^-\eta$ (CELLO)	H.J. Behrend et al., ZPC49(1991)401
$\eta \to \mu^+\mu^-\gamma$ (NA60)	G. Usai et al., NPA855(2011)189
$\eta \to e^+e^-\gamma$ (A2)	P. Aguar-Bartolome et al., 1309.5648

Theory

Padé approx. fit to data	talk P. Masjuan; Wed.
VMD	L. Ametller et al. NPB228(1983)301
Quark loop	L. Ametller et al. NPB228(1983)301
Brodsky-Lepage	B & L, PRD34(1981)1808
1-loop ChPT	L. Ametller et al. NPB228(1983)301
Dispersion integral	C.H. et al., EPJC73(2013)2668

Summary

→ $\eta \to \pi\pi\gamma$ and $\eta \to \gamma^*\gamma$ are characterized by two scales

- ▷ A universal one ($\sim m_\rho$) from $\pi\pi$–interaction
- ▷ A reaction specific one (from the decay dynamics)

→ The latter for $\eta \to \pi\pi\gamma$ and $\eta \to \gamma^*\gamma$ are related but not equal

→ The method outlined guarantees

- ▷ Correct pole positions
- ▷ Correct phase motion

Precondition for CP studies

Thanks a lot for your attention