Dispersion theory and chiral dynamics: from light- to heavy-meson decays

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Workshop on Light-Meson Dynamics

Mainz, February 10th 2014

Outline

Heavy-meson Dalitz plots — why, and how (not)?

Dispersion relations for three-body decays

- an ideal test case: $\phi \to 3\pi$ — Niecknig, BK, Schneider 2012
- vector-meson transition form factors — Schneider, BK, Niecknig 2012
- status report: $D^+ \to \pi^+\pi^+K^-$ — Niecknig, BK *in progress*

Heavy-meson semileptonic decays

- $B \to \pi\pi\ell\nu$ and $|V_{ub}|$ — Kang et al. 2013
- outlook: $D \to \pi K\ell\nu$ and πK phase shifts — Daub et al. *in progress*

Summary / Outlook

Heavy-meson Dalitz plots: hunting for CP violation

CP violation in partial widths $\Gamma(P \to f) \neq \Gamma(\bar{P} \to \bar{f})$

- at least two interfering decay amplitudes
- different weak (CKM) phases
- different strong (final-state-interaction) phases

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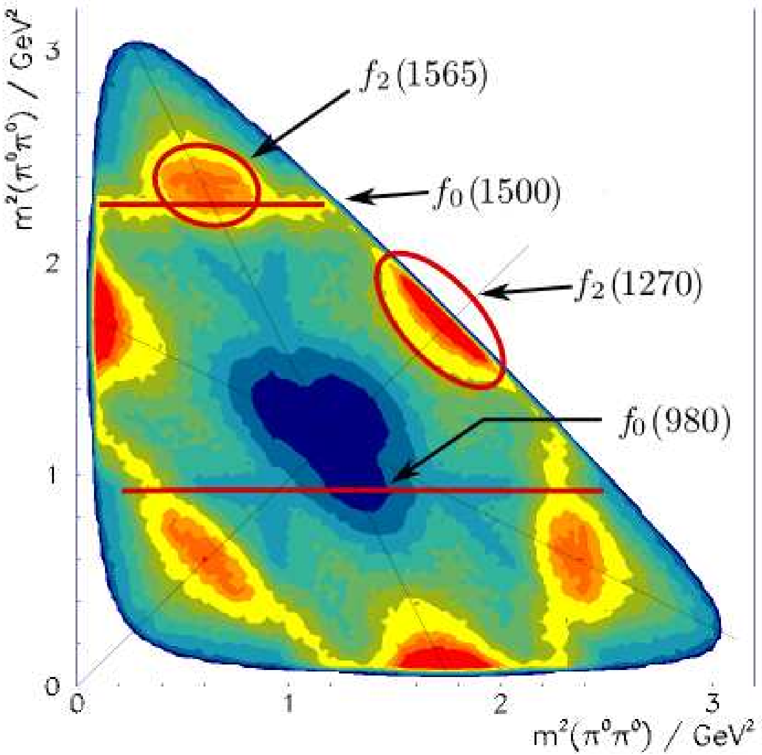
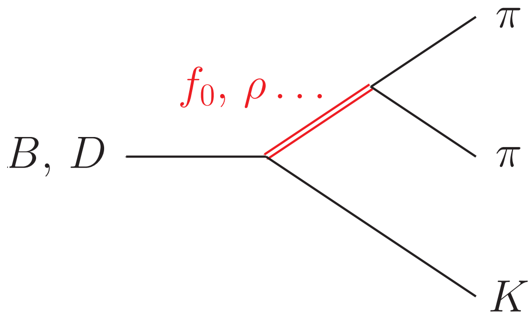
three-body decays: $D \to 3\pi,\ \pi\pi K$

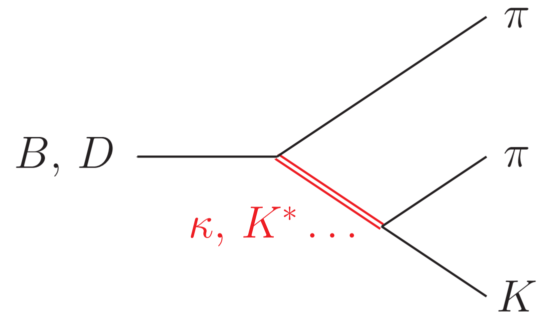
- Dalitz plot $\hat{=}$ density distribution in two kinematical variables
- resonances $\longrightarrow$ rapid phase variation enhances CP-violation in parts of the decay region

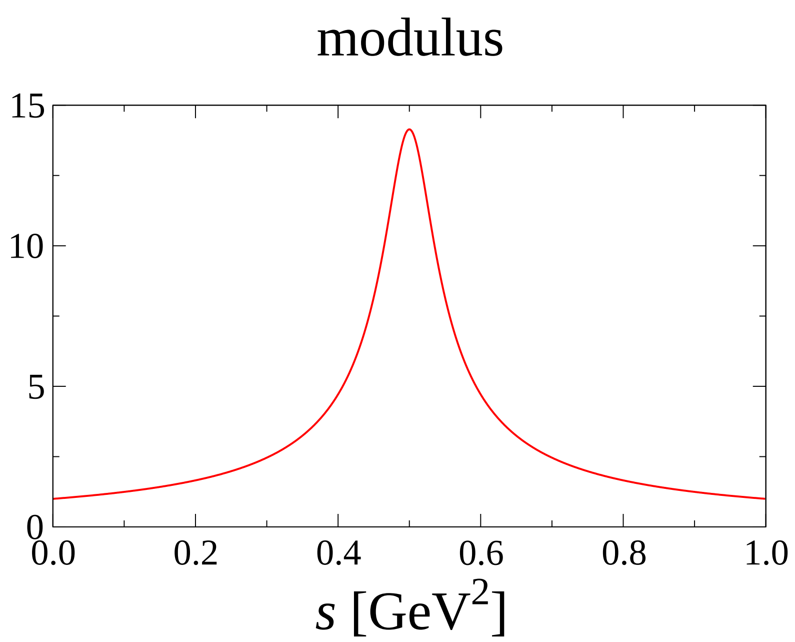


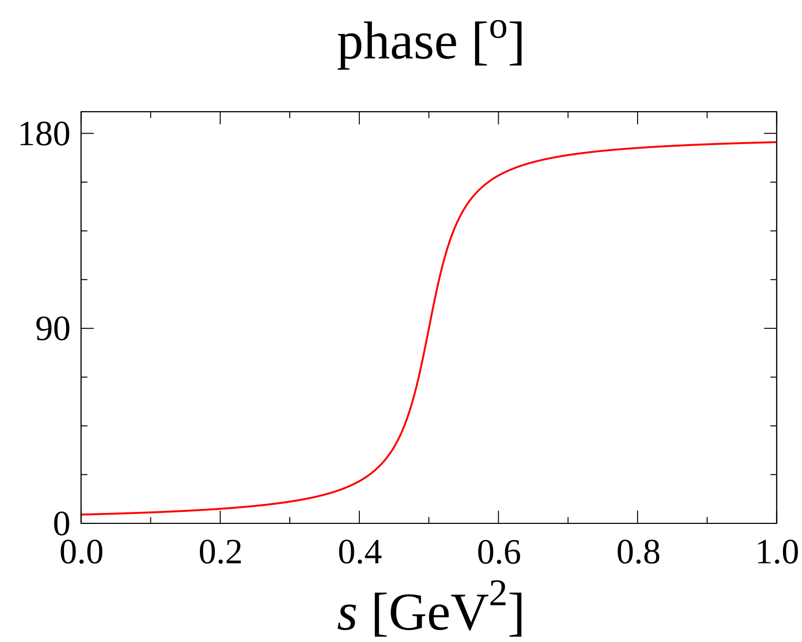
Amplitude analyses in Dalitz plots

The traditional picture: isobar model / Breit–Wigner resonances









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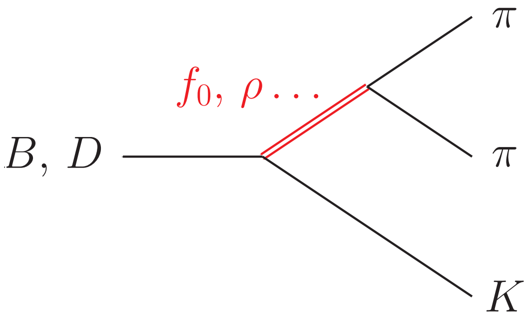
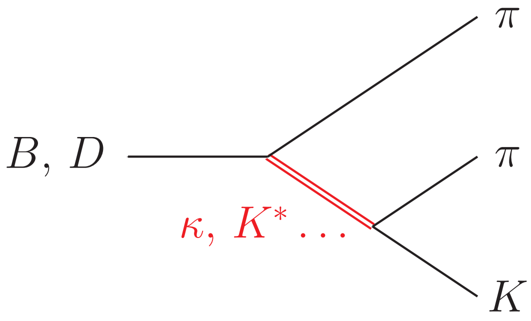
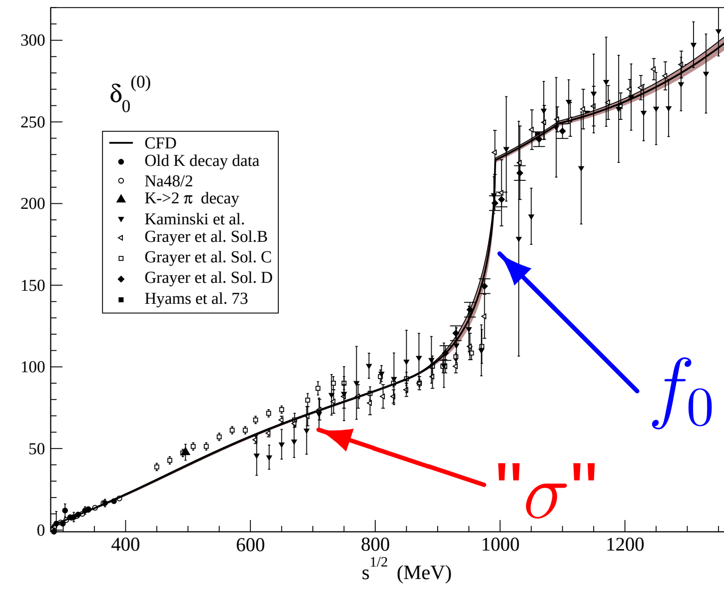




. . . so what's not to like?

- some resonances don't look like Breit–Wigners at all!

  $\longrightarrow$ use exact scattering phase shifts instead



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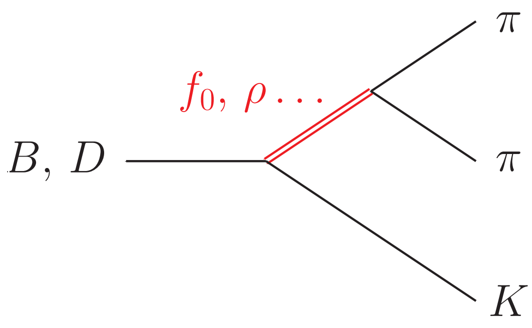

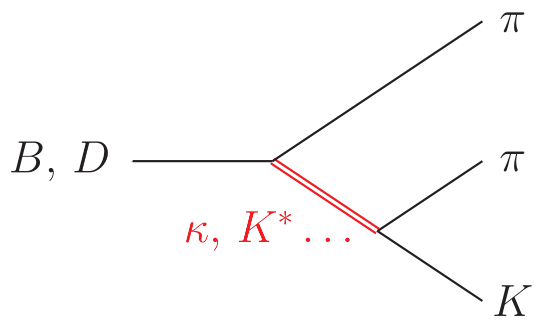

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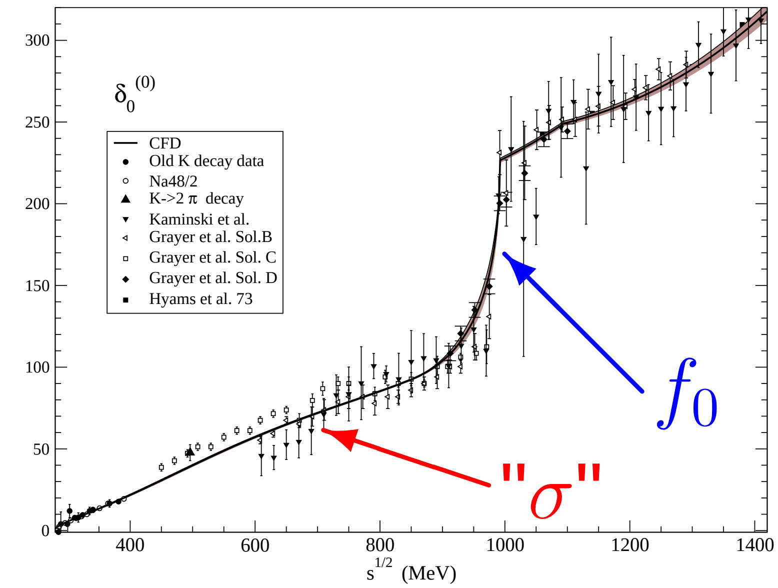
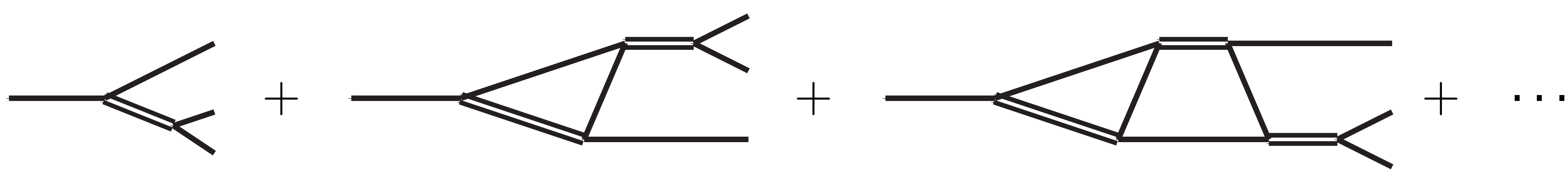
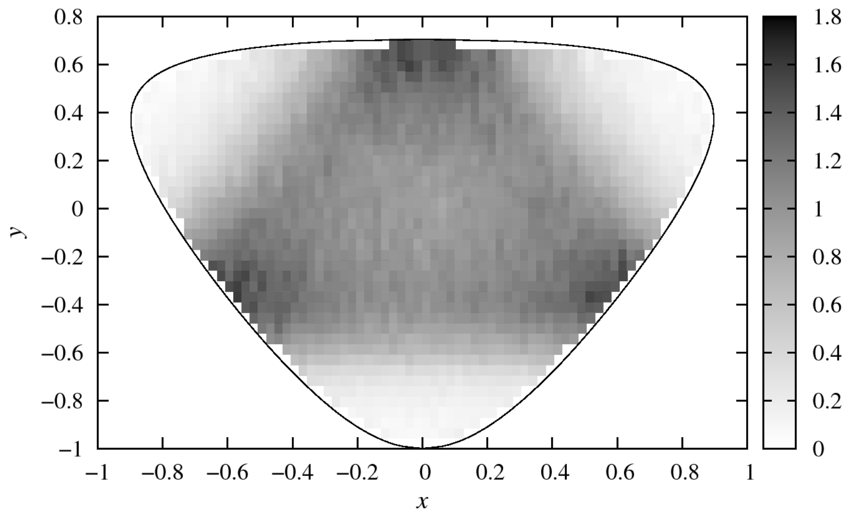
- 3-particle rescattering





A simple Dalitz plot: $\phi \to 3\pi$



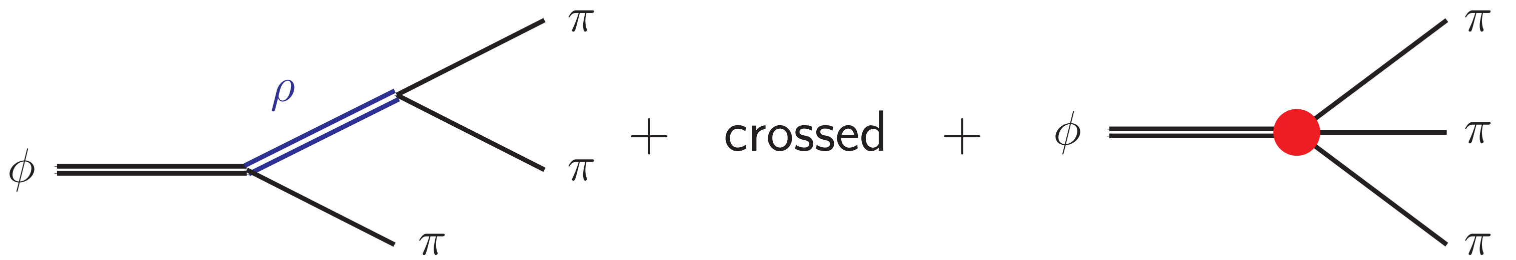
- 2×10^6 events in 1834 bins

  KLOE 2003

- analyzed in terms of:

  3 Breit–Wigners ($\rho^{\pm}$, ρ^0)

+ constant background term



Problem:

$\longrightarrow$ unitarity fixes Im/Re parts

$\longrightarrow$ adding a contact term destroys this relation

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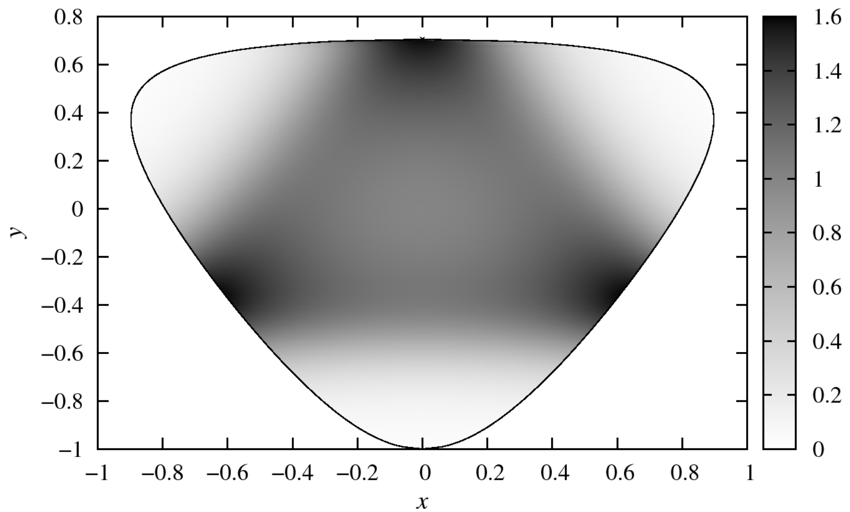

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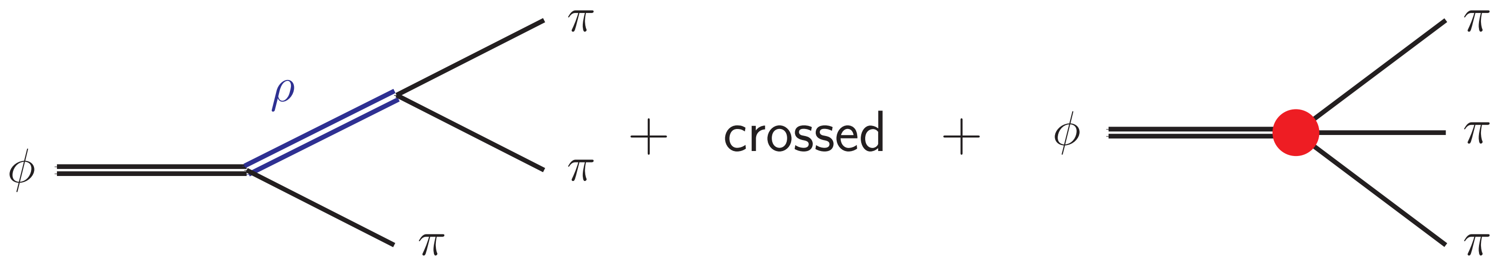


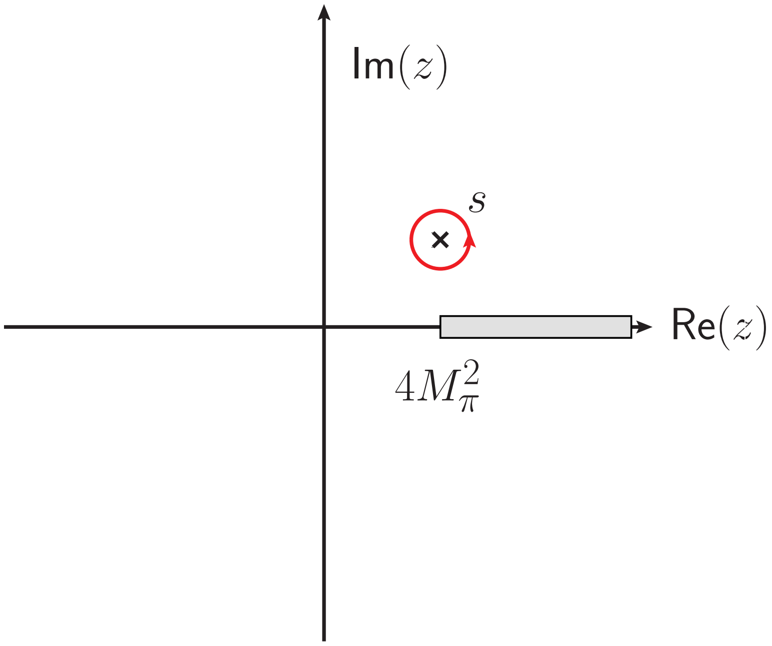
Problem:

$\longrightarrow$ unitarity fixes Im/Re parts

$\longrightarrow$ adding a contact term destroys this relation

$\longrightarrow$ reconcile data with dispersion relations? Niecknig, BK, Schneider 2012

Dispersion relations on one page



analyticity & Cauchy's theorem:

$$T(s) = \frac{1}{2\pi i} \oint_{\partial\Omega} \frac{T(z)dz}{z-s}$$

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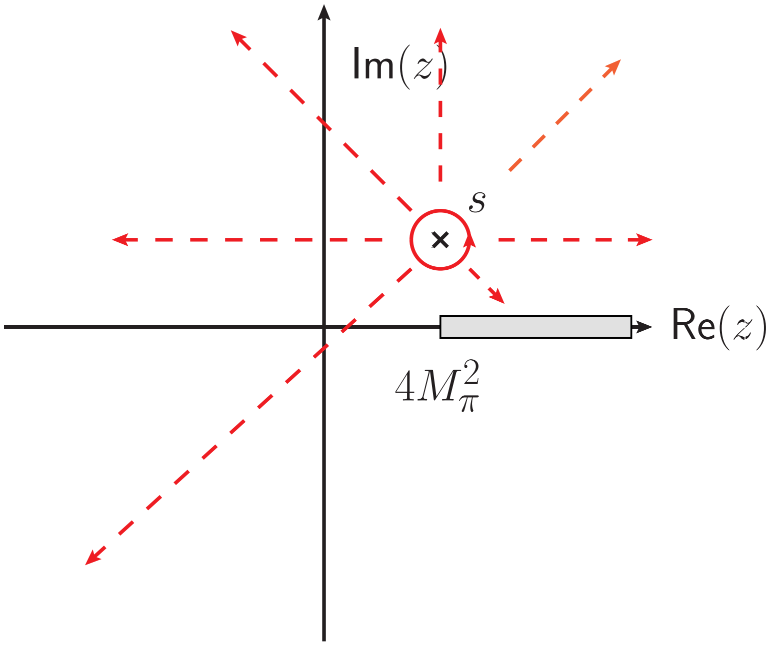

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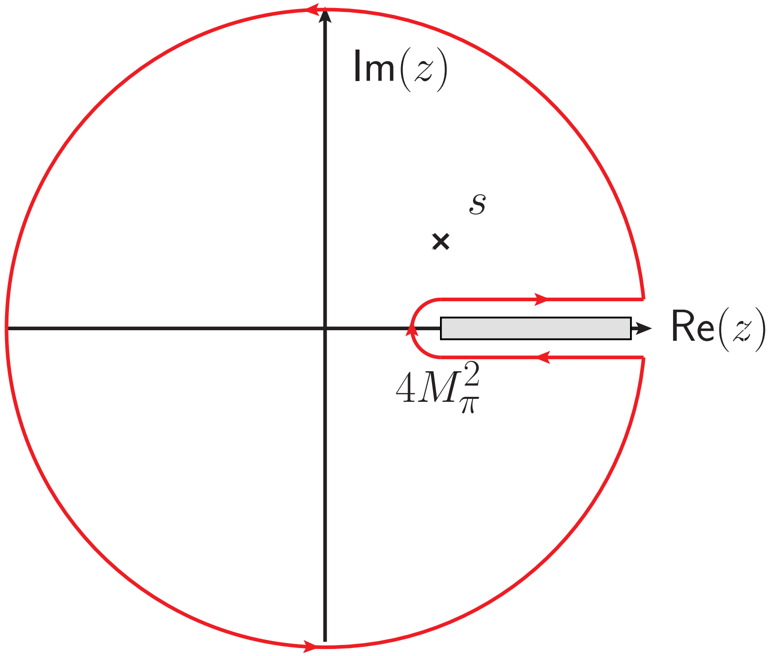

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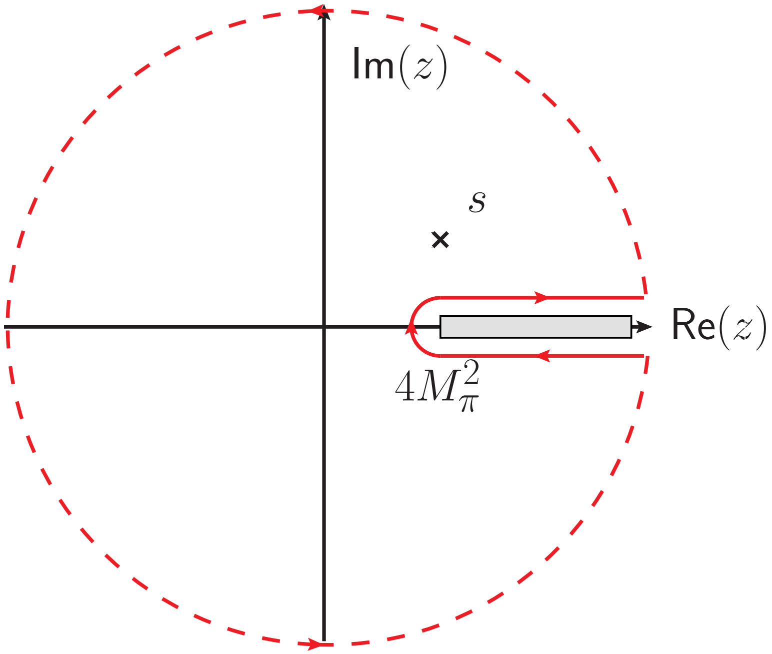

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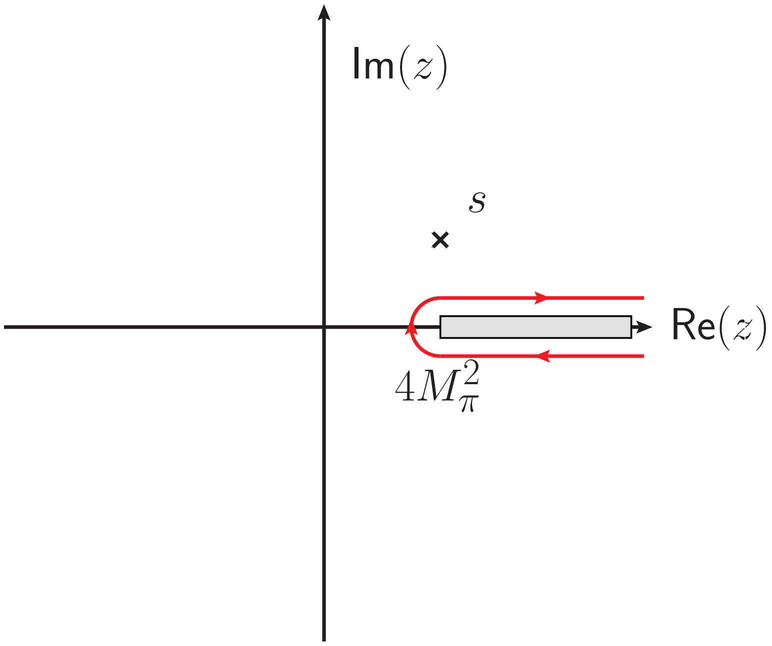

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$$\longrightarrow \frac{1}{2\pi i}\int_{4M_\pi^2}^{\infty}\frac{\operatorname{disc} T(z)dz}{z-s}$$

$$= \frac{1}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\operatorname{Im} T(z)dz}{z-s}$$

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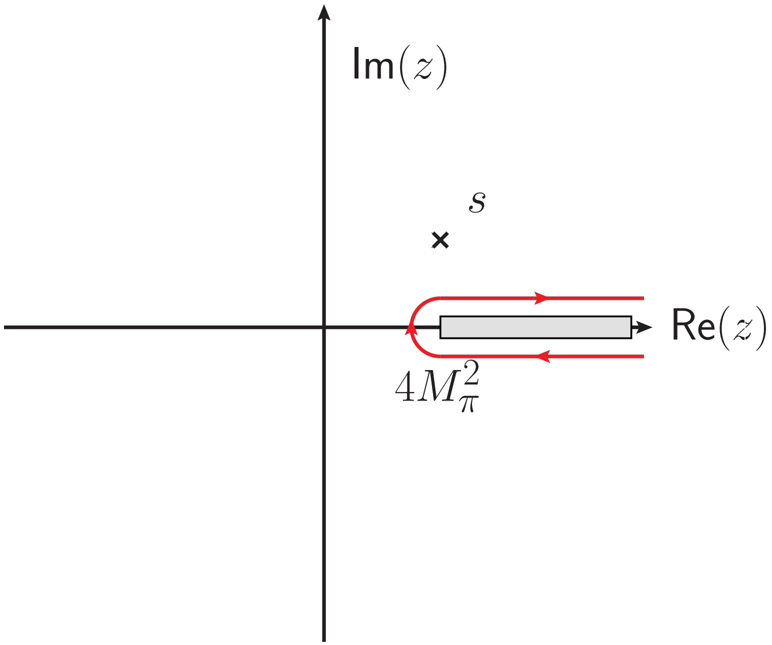

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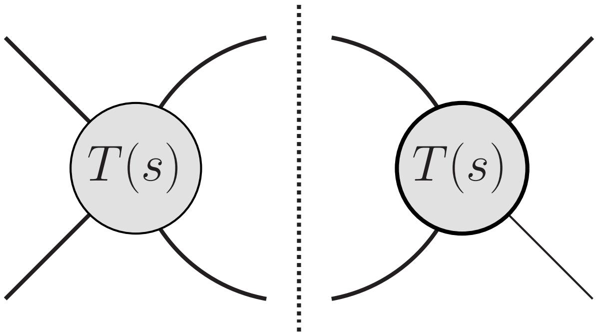
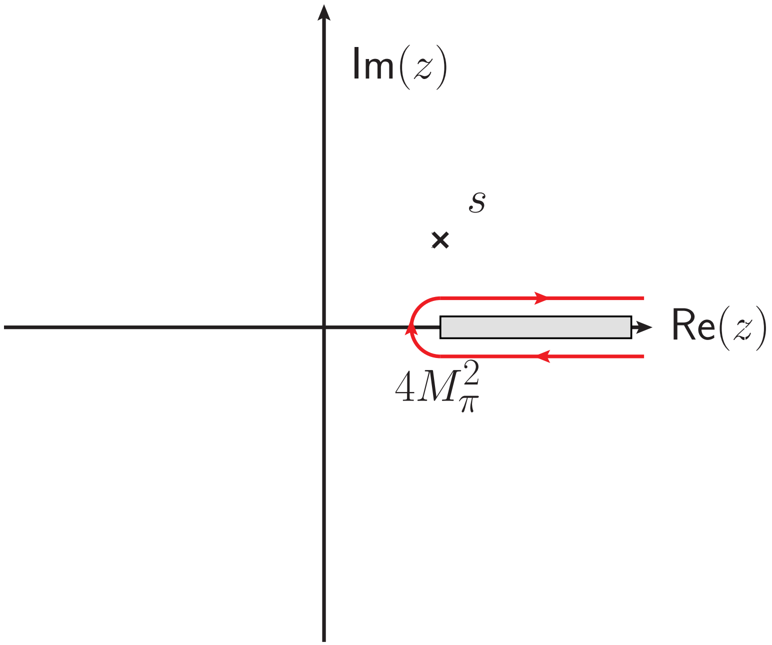


e.g. if $T(s)$ is a $\pi\pi$ partial wave $\longrightarrow$

$$\frac{\mathrm{disc}\, T(s)}{2i} = \mathrm{Im}\, T(s) = \frac{2q_\pi}{\sqrt{s}} \theta(s - 4M_\pi^2)|T(s)|^2$$

Dispersion relations on one page



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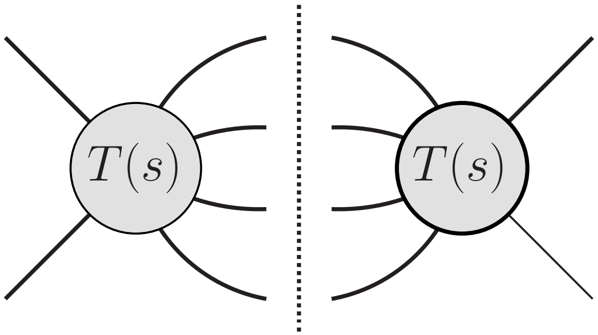
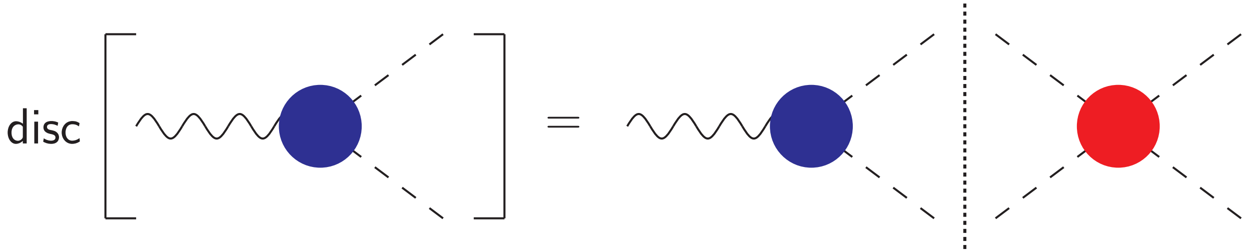


inelastic intermediate states ($K\bar{K}$, 4π) suppressed at low energies
$\longrightarrow$ will be neglected in the following

Two-body decays: form factors

- just two hadrons: form factor, e.g. $e^+e^- \to \pi^+\pi^-$ C. Hanhart's talk



$$\text{Im}\, F(s) \;\propto\; F(s) \times \text{phase space} \times T^*_{\pi\pi}(s)$$

$\longrightarrow$ final-state theorem: phase of $F(s)$ is scattering phase $\delta(s)$

Watson 1954

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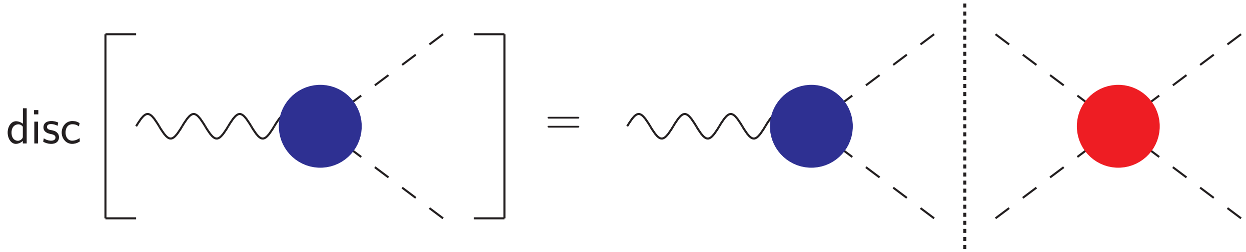

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Watson 1954

- dispersion relations allow to reconstruct form factor from imaginary part $\longrightarrow$ elastic scattering phase $\delta(s)$:

$$F(s) \;\propto\; \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s)}\right\}$$

Omnès 1958

- today: high-accuracy $\pi\pi$ (and πK) phase shifts available

Ananthanarayan et al. 2001, García-Martín et al. 2011; Büttiker et al. 2004

Three-body decays: $V \to 3\pi$

Decay amplitude can be decomposed into single-variable functions

$$\mathcal{M}(s,t,u) = i\epsilon_{\mu\nu\alpha\beta} n^{\mu} p^{\nu}_{\pi^+} p^{\alpha}_{\pi^-} p^{\beta}_{\pi^0} \left\{ \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u) \right\}$$

$\longrightarrow$ based on partial-wave expansion: only odd partial waves; neglect discontinuities/phases of F-waves and higher

Unitarity relation for $\mathcal{F}(s)$:

$$\text{disc}\, \mathcal{F}(s) = 2i \{ \underbrace{\mathcal{F}(s)}_{\text{right-hand cut}} + \underbrace{\hat{\mathcal{F}}(s)}_{\text{left-hand cut}} \} \times \theta(s - 4\, M_{\pi}^2) \times \sin\, \delta_1^1(s)\, e^{-i\delta_1^1(s)}$$

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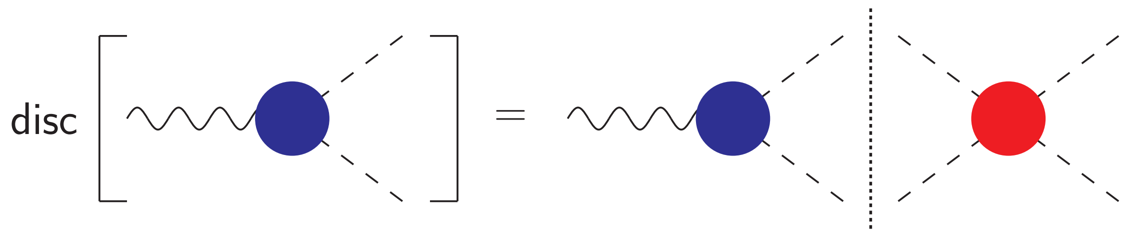
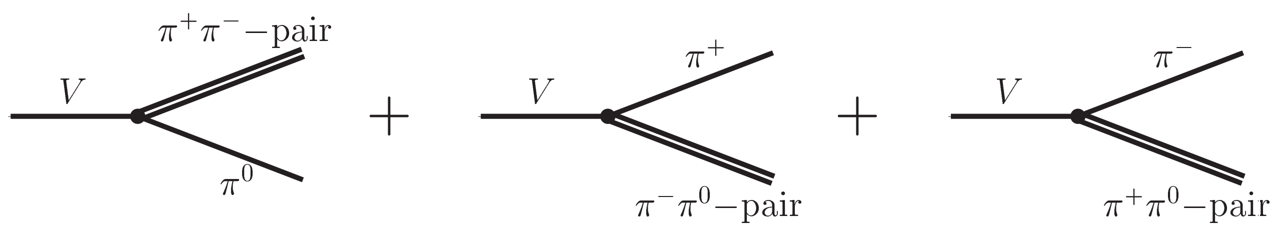


- right-hand cut only $\longrightarrow$ Omnès problem

$$\mathcal{F}(s) = a\,\Omega(s)\,, \qquad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta_1^1(s')}{s'-s}\right\}$$

$\longrightarrow$ amplitude given in terms of pion vector form factor

$\mathcal{F}(s,t,u) =$ 

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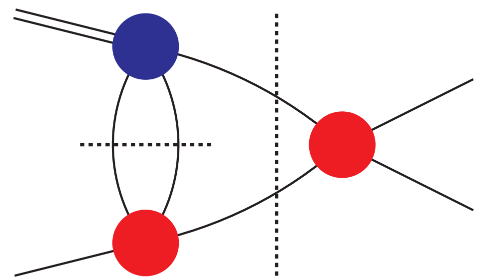
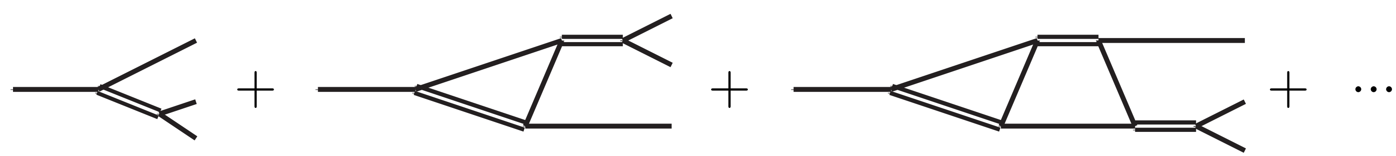


- inhomogeneities $\hat{\mathcal{F}}(s)$: angular averages over the $\mathcal{F}(t)$, $\mathcal{F}(u)$

$$\mathcal{F}(s) = a\,\Omega(s)\left\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right\}$$

$$\hat{\mathcal{F}}(s) = \frac{3}{2}\int_{-1}^{1} dz\,(1-z^2)\mathcal{F}\big(t(s,z)\big)$$

Anisovich, Leutwyler 1998

$\mathcal{F}(s) =$ 

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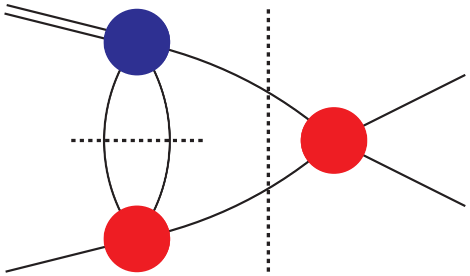

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$\longrightarrow$ crossed-channel scattering between s-, t-, and u-channel

Khuri, Treiman 1960

Subtraction constants

$$\mathcal{F}(s) = \Omega(s)\left\{a + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1(s')\hat{\mathcal{F}}(s')}{|\Omega_1(s')|(s'-s)}\right\}$$

- one subtraction $a \longrightarrow$ fix to partial width, Dalitz plot prediction

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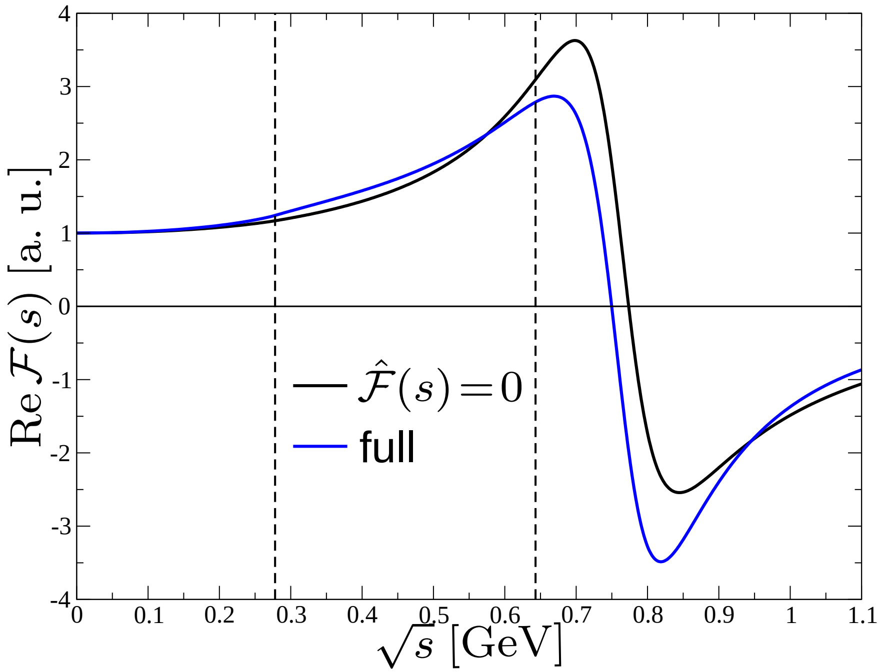
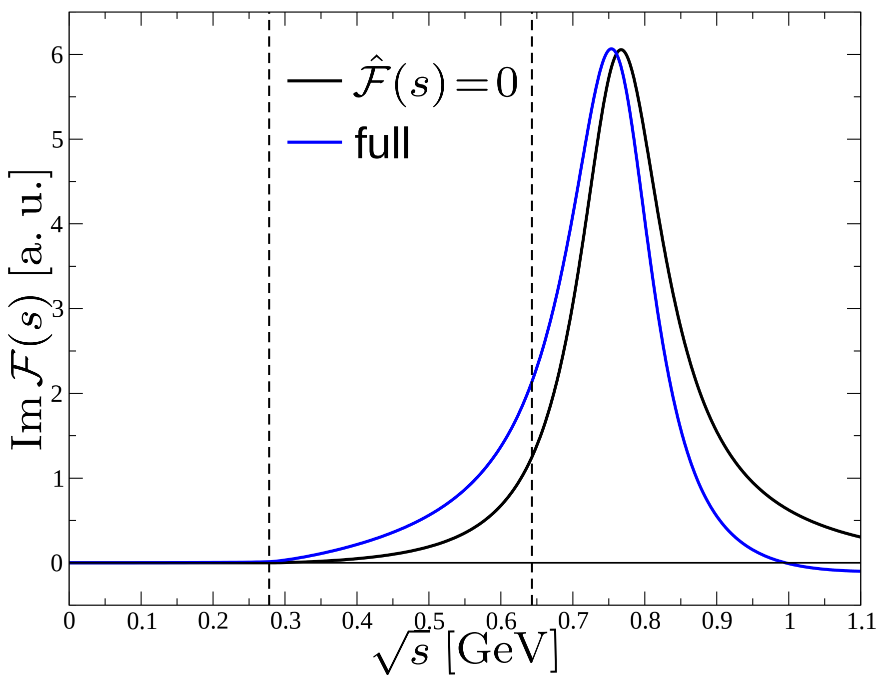
- one subtraction $a \longrightarrow$ fix to partial width, Dalitz plot prediction
- $\omega \to 3\pi$ vs. $\phi \to 3\pi$: crossed-channel effects depend on decay mass!





Niecknig, BK, Schneider 2012

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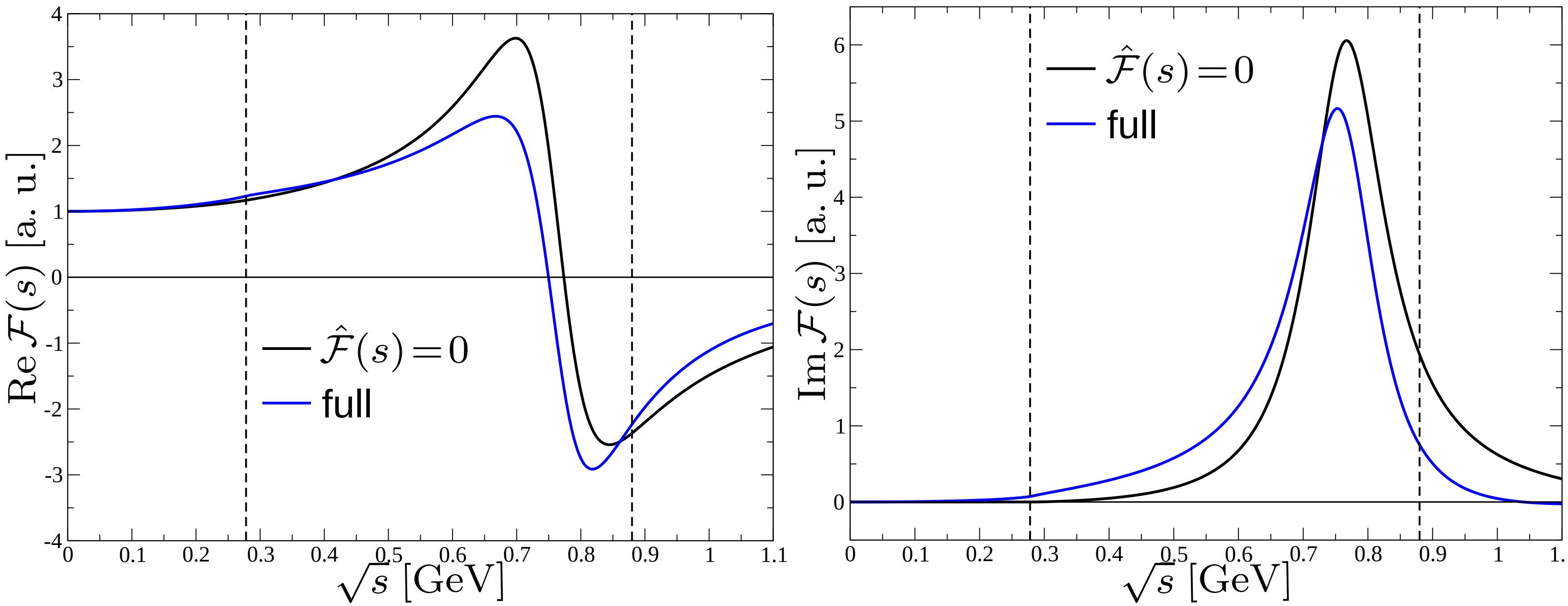

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Subtraction constants

$$\mathcal{F}(s) = \Omega(s)\left\{a + b\,s + \frac{s^2}{\pi}\int_{4M_\pi^2}^\infty \frac{ds'}{s'^2}\frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right\}$$

- number of necessary subtractions guessed from high-energy behaviour $\longrightarrow$ not very reliable
- increase precision by oversubtraction:
  - ▷ suppress high-energies / inelastic effects more efficiently
  - ▷ gain some flexibility in the Dalitz plot description (one more complex constant b)
- important observation: $\mathcal{F}(s)$ linear in $a,\, b$

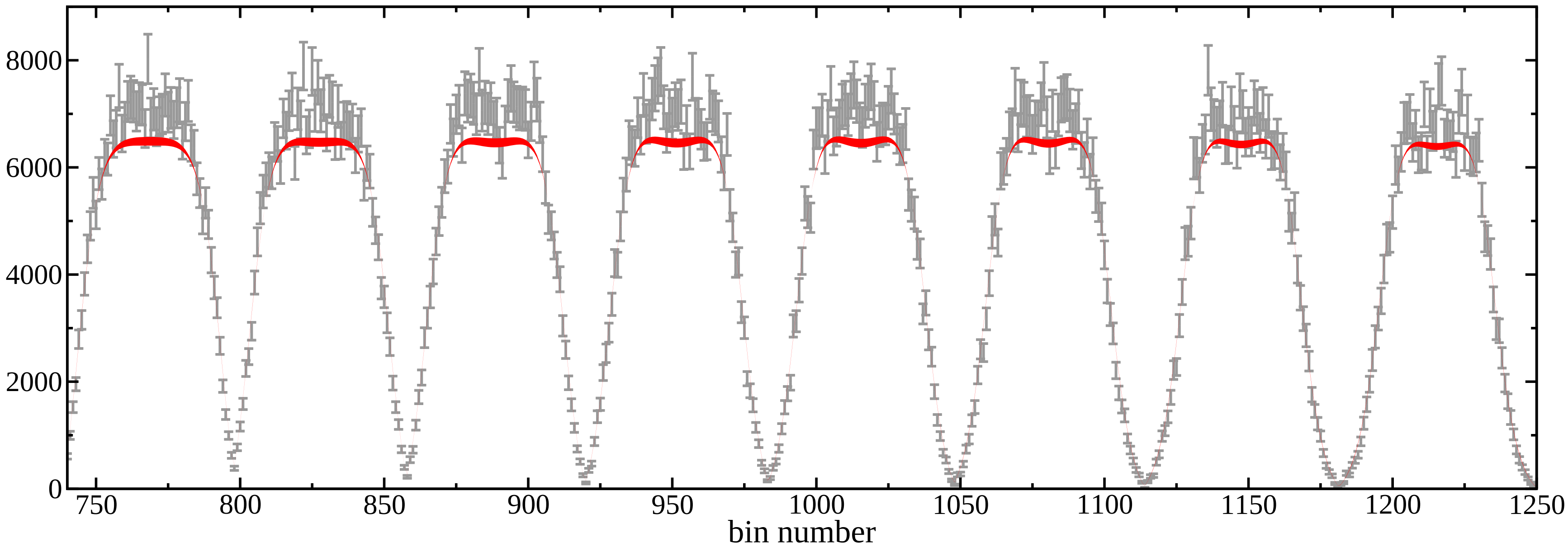
$$\mathcal{F}(s) = a\,\mathcal{F}_a(s) + b\,\mathcal{F}_b(s)$$

$\longrightarrow$ basis functions $\mathcal{F}_{a,b}(s)$ calculated independently of $a,\, b$

$\longrightarrow$ subtraction constants $a,\, b \,\hat{=}$ experimental fit parameters

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012



χ^2/ndof	$1.7\ldots2.1$

$\longrightarrow$ pairwise interaction only (with correct $\pi\pi$ scattering phase)

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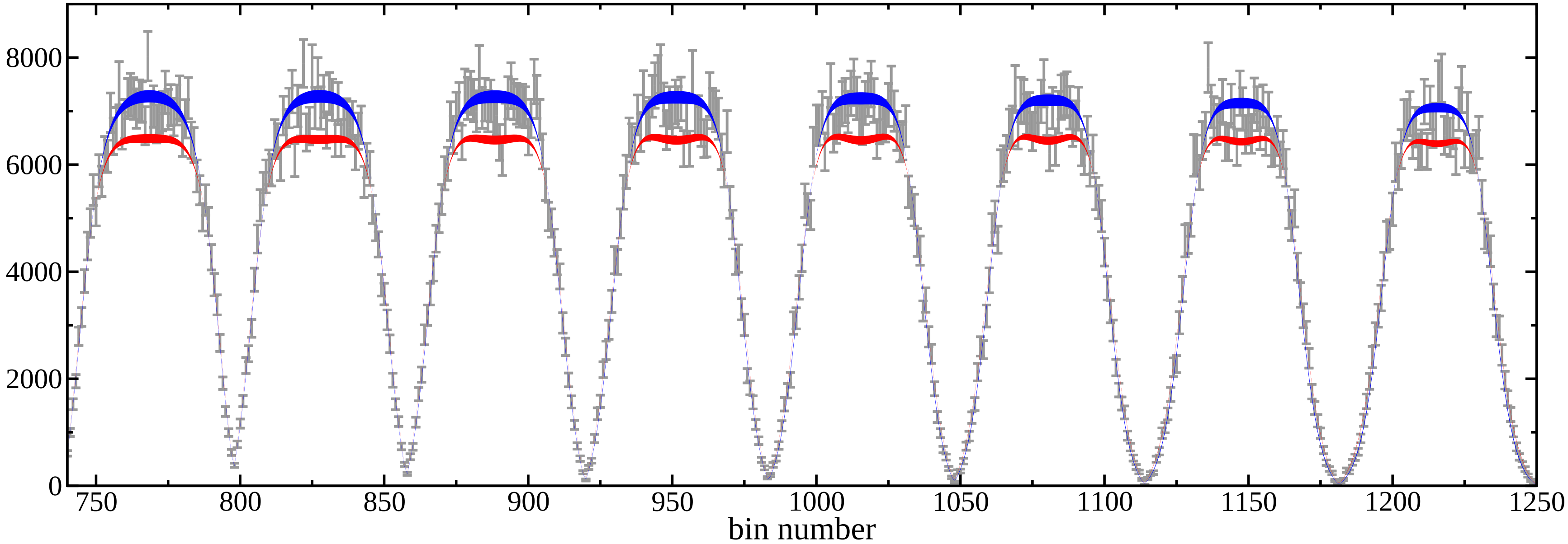
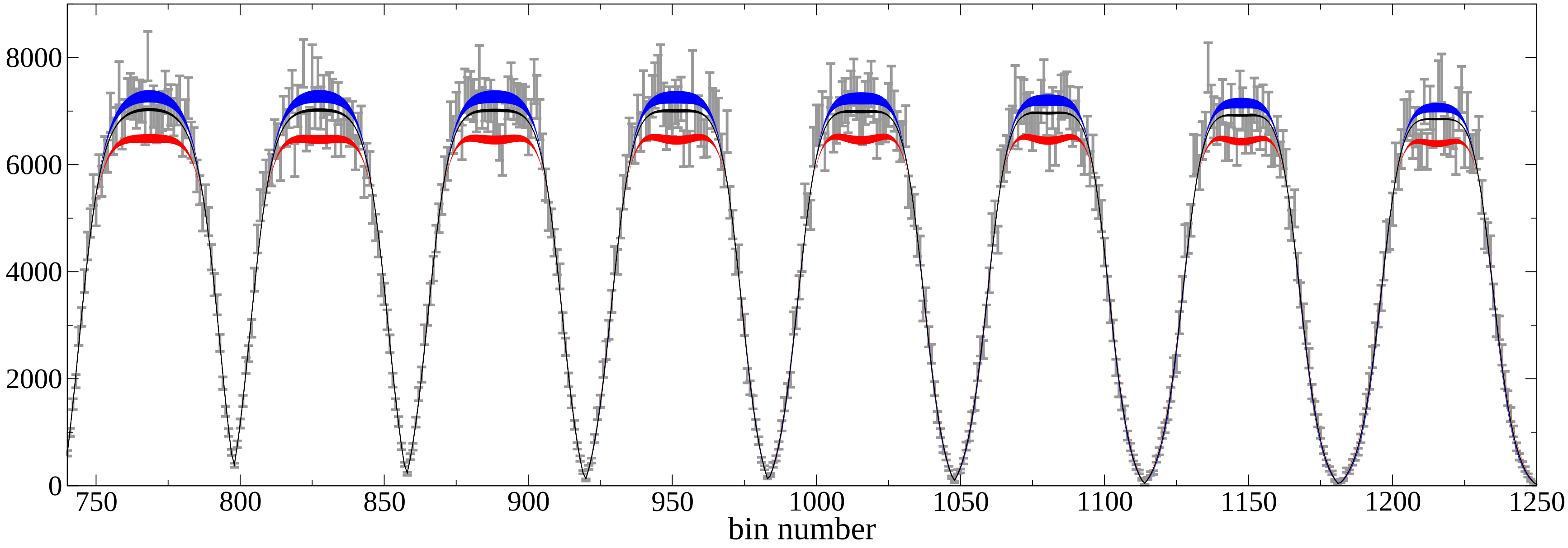


χ^2/ndof	$1.7\ldots2.1$	$1.2\ldots1.5$

$\longrightarrow$ full 3-particle rescattering, only overall normalization adjustable

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012



χ^2/ndof	$1.7 \ldots 2.1$	$1.2 \ldots 1.5$	1.0

$\longrightarrow$ full 3-particle rescattering, 2 adjustable parameters (additional "subtraction constant" to suppress inelastic effects)

Experimental comparison to $\phi \to 3\pi$

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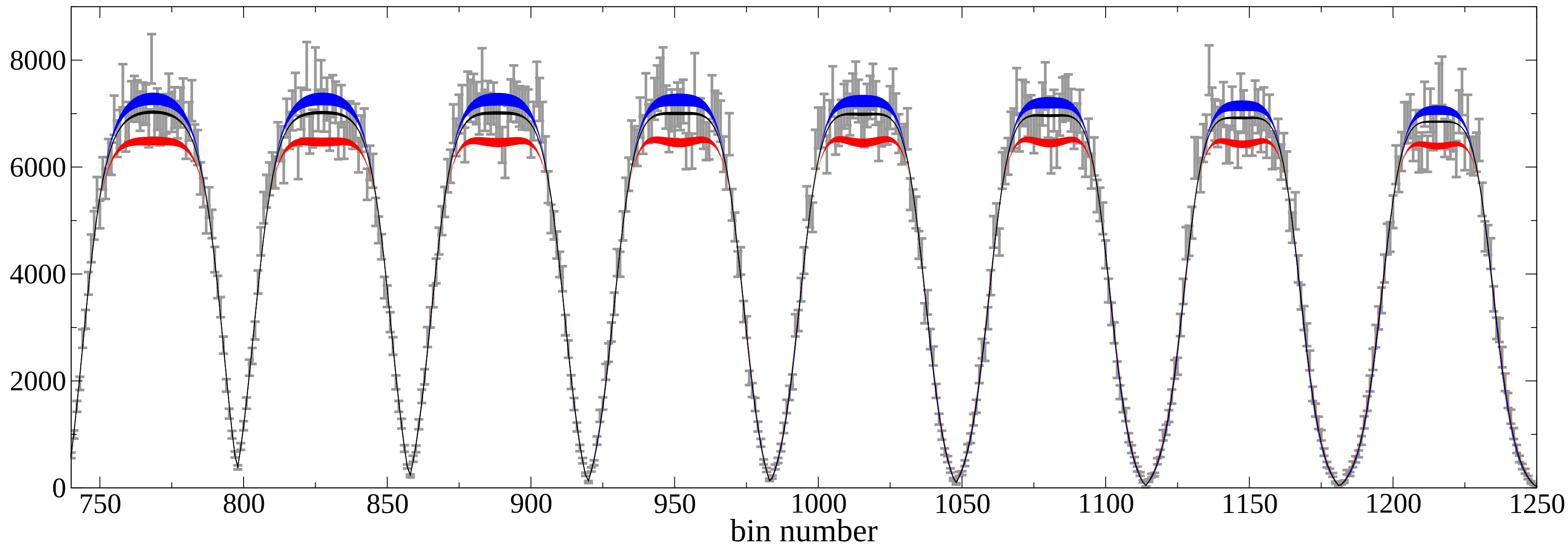
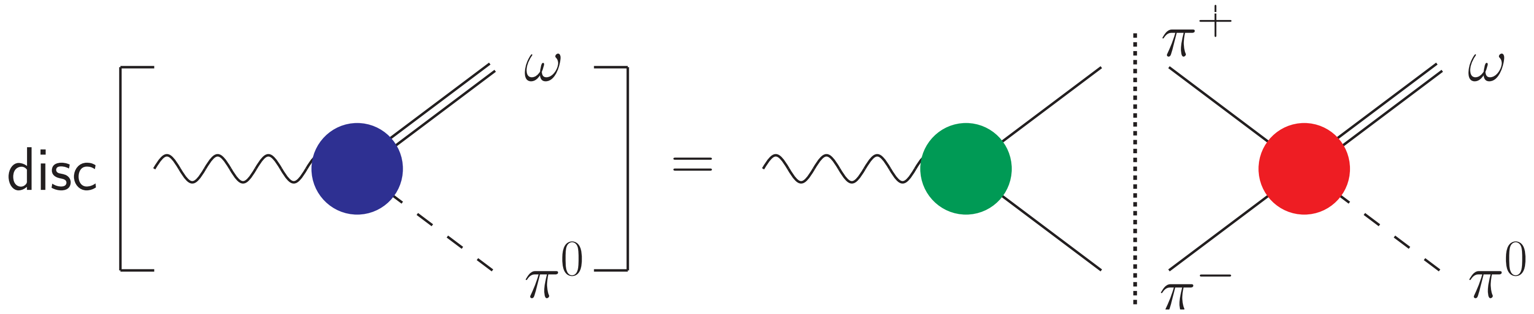


χ^2/ndof	$1.7 \ldots 2.1$	$1.2 \ldots 1.5$	1.0

- perfect fit respecting analyticity and unitarity possible
- contact term emulates neglected rescattering effects
- no need for "background" — inseparable from "resonance"

Transition form factors $\omega(\phi) \to \pi^0 \ell^+ \ell^-$

- dispersion relations link hadronic to radiative decays:



- ω transition form factor related to

pion vector form factor $\times$ $\omega \to 3\pi$ decay amplitude

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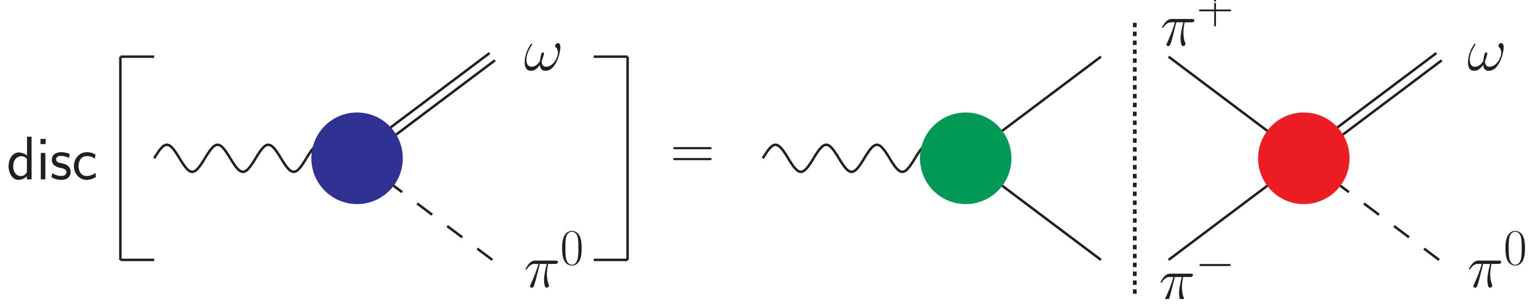

$$f_{\omega\pi^0}(s) = f_{\omega\pi^0}(0) + \frac{s}{12\pi^2} \int_{4M_\pi^2}^{\infty} ds' \frac{q_\pi^3(s') F_\pi^{V*}(s') f_1(s')}{s'^{3/2}(s'-s)}$$ Köpp 1974

- $f_1(s) = f_1^{\omega\to 3\pi}(s) = \mathcal{F}(s) + \hat{\mathcal{F}}(s)$ P-wave projection of $\mathcal{F}(s,t,u)$

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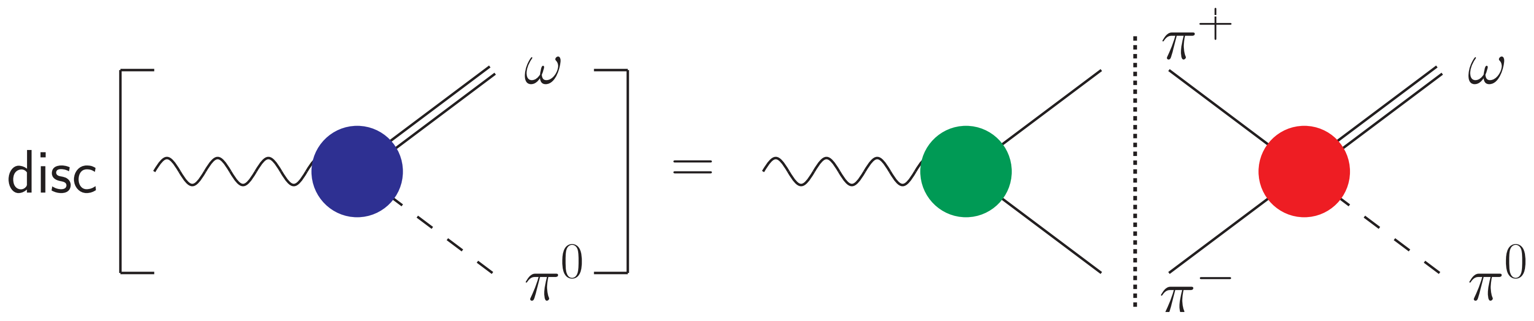

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- sum rule for $\omega \to \pi^0\gamma \longrightarrow$ saturated at 90–95%

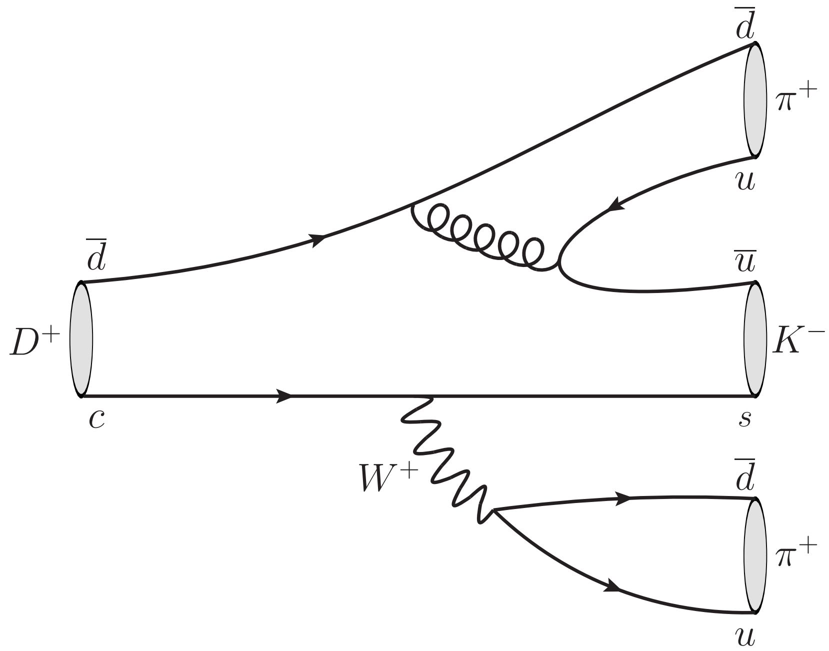
$$f_{\omega\pi^0}(0) = \frac{1}{12\pi^2}\int_{4M_\pi^2}^{\infty} ds' \frac{q_\pi^3(s')}{s'^{3/2}} F_\pi^{V*}(s') f_1(s') \,, \quad \Gamma_{\omega\to\pi^0\gamma} \propto |f_{V\pi^0}(0)|^2$$

Schneider, BK, Niecknig 2012

- comparison to $\omega \to \pi^0\mu^+\mu^-$ mysterious NA60 2009, 2011

  experiments for $\phi \to \pi^0\ell^+\ell^-$ would be highly welcome KLOE?

Heavier decays: $D^+ \to \pi^+\pi^+K^-$



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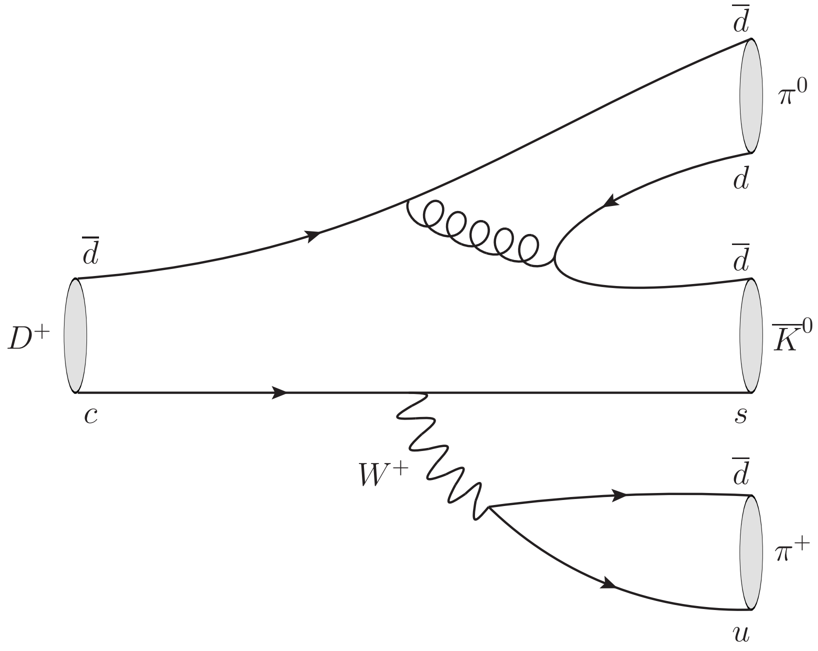

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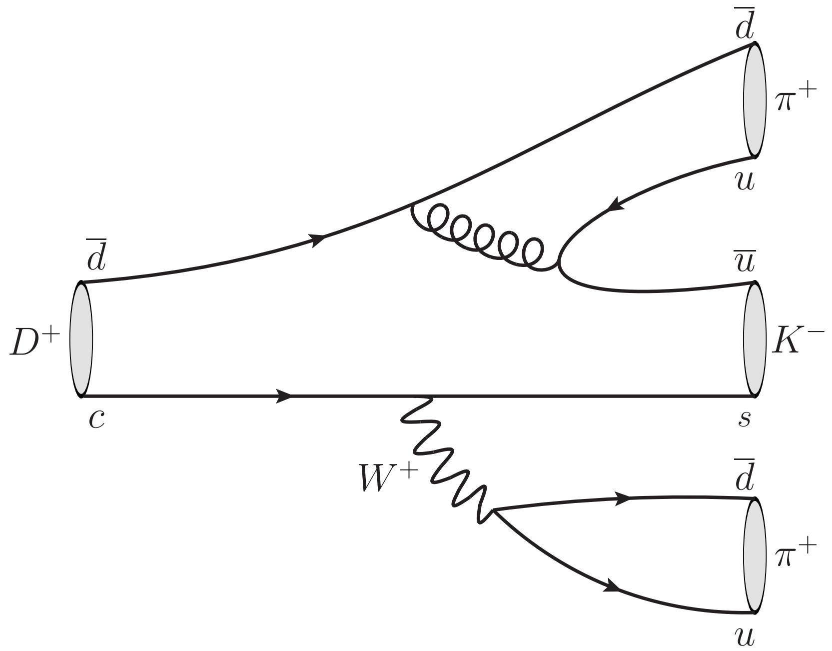

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Partial waves:

pion–pion	$S^2_{\pi\pi}$	$P^1_{\pi\pi}$
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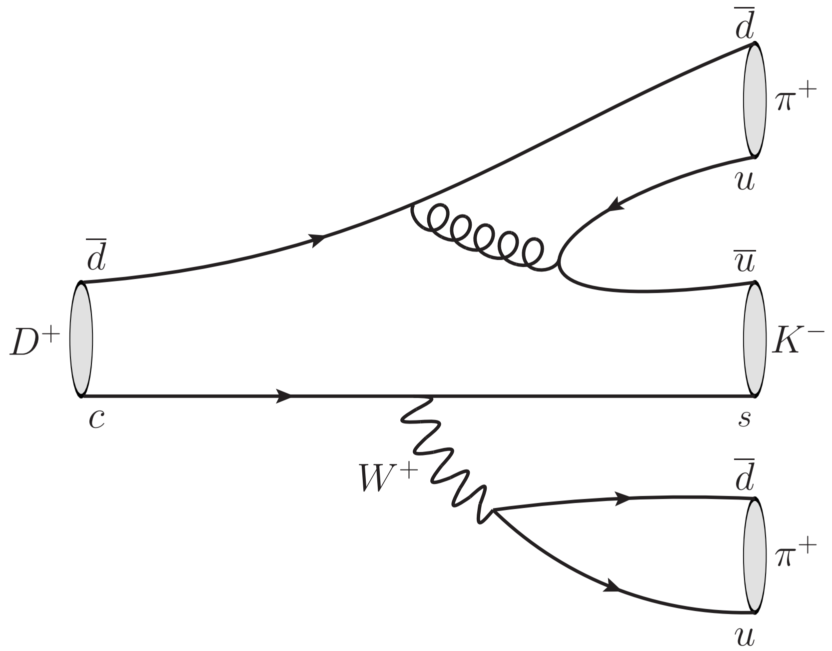

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$\longrightarrow$ exotic partial waves, weak, repulsive

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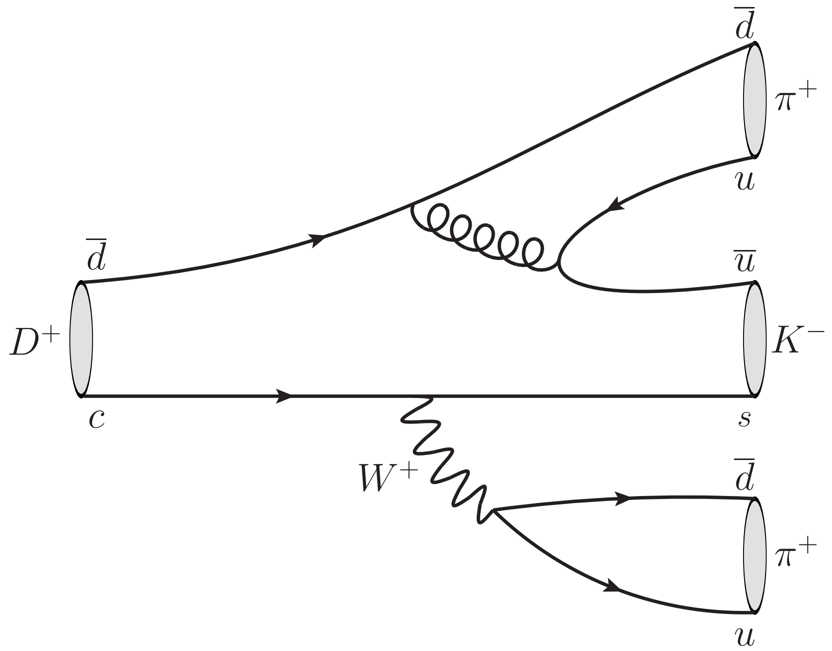

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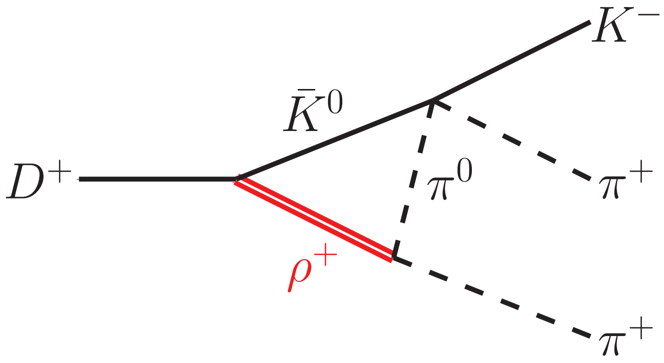

$\longrightarrow$ $\pi\pi$ P-wave only couples indirectly via $D^+ \to \pi^+\pi^0\bar{K}^0$

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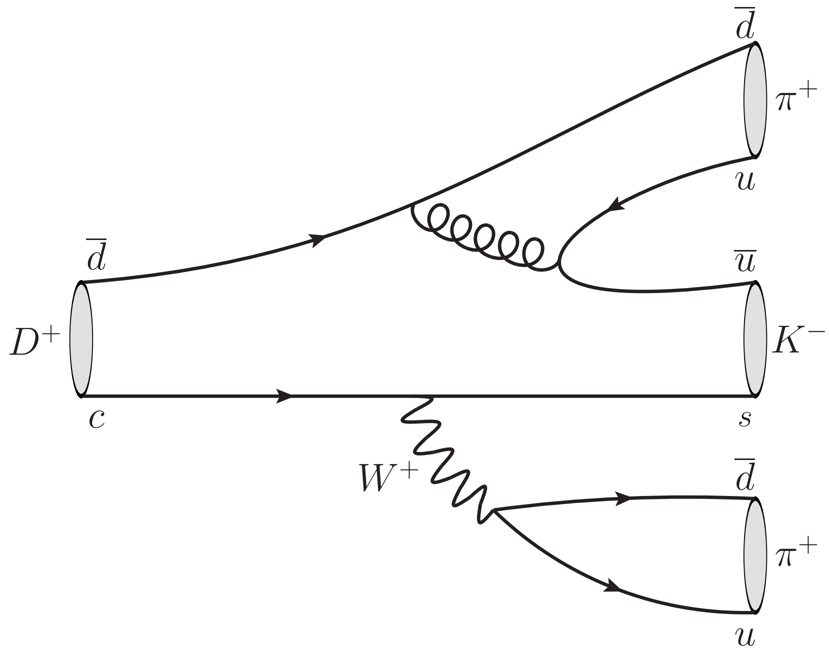

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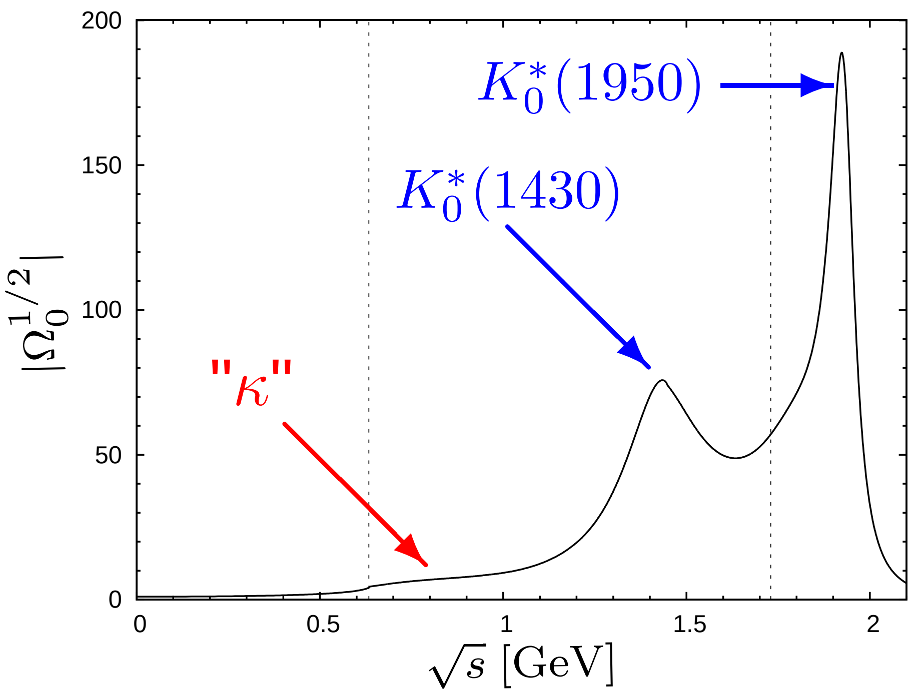

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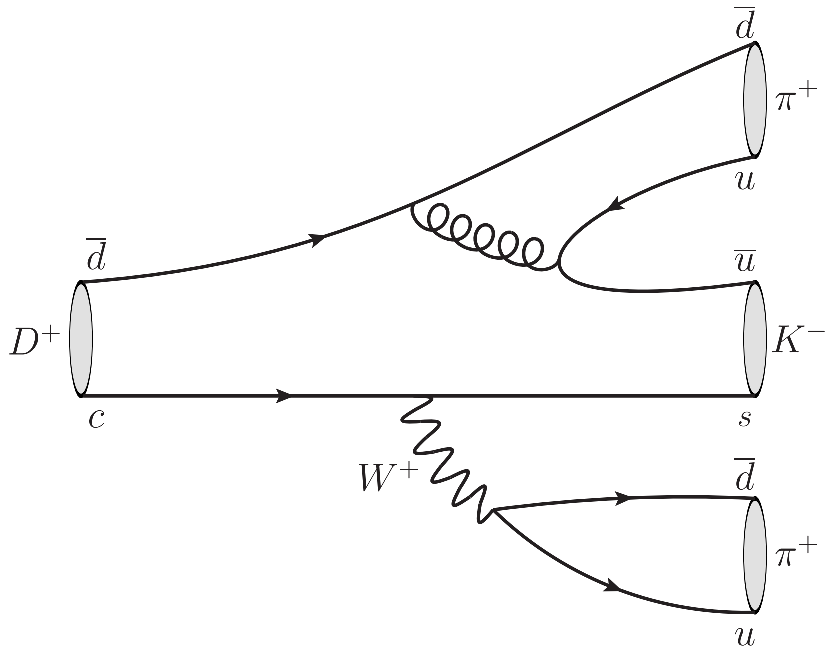

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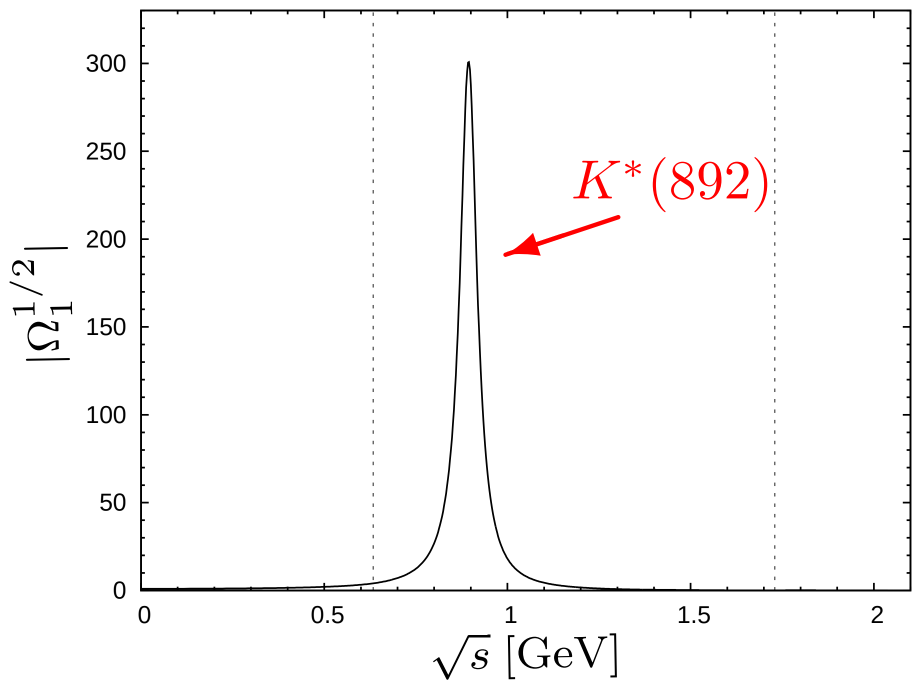

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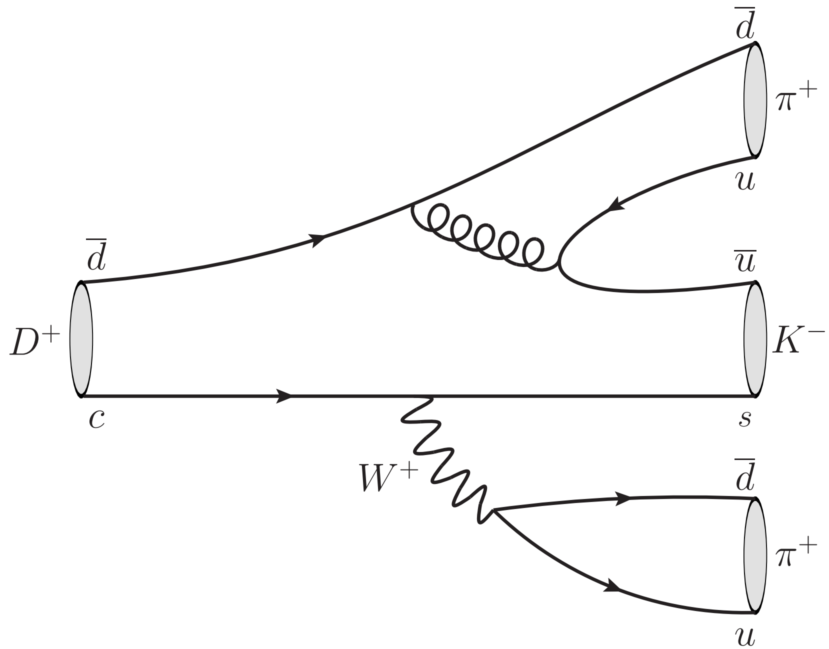

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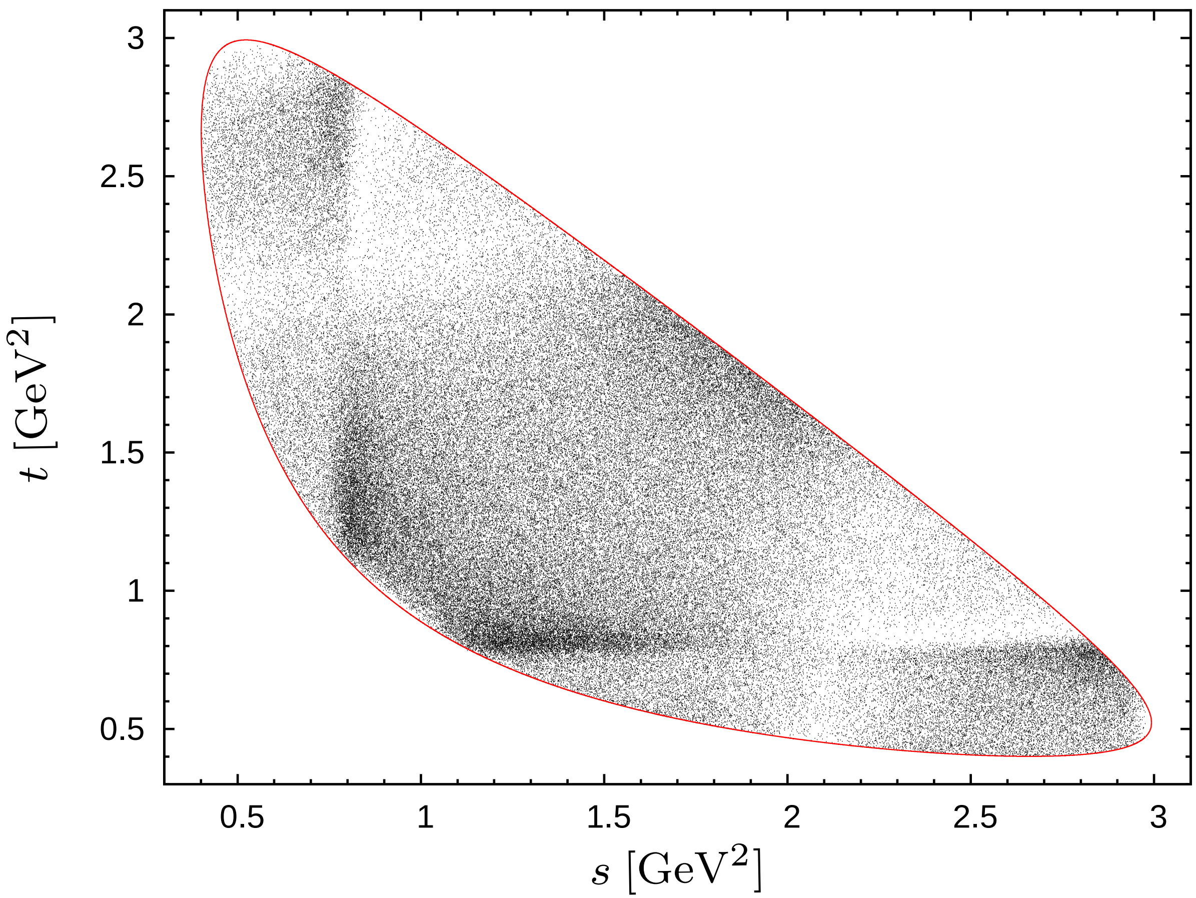
Partial waves:

pion–pion	$S^2_{\pi\pi}$	2	$P^1_{\pi\pi}$	$2 \to 0$
pion–kaon	$S^{1/2}_{\pi K}$	4	$P^{1/2}_{\pi K}$	1
	$S^{3/2}_{\pi K}$	$2 \to 0$	$P^{3/2}_{\pi K}$	0

number of subtraction constants $\longrightarrow$ using $s + t + u =$ const.

$\longrightarrow$ 7 altogether

Dalitz plot $D^+ \to \pi^+\pi^+K^-$



CLEO 2008

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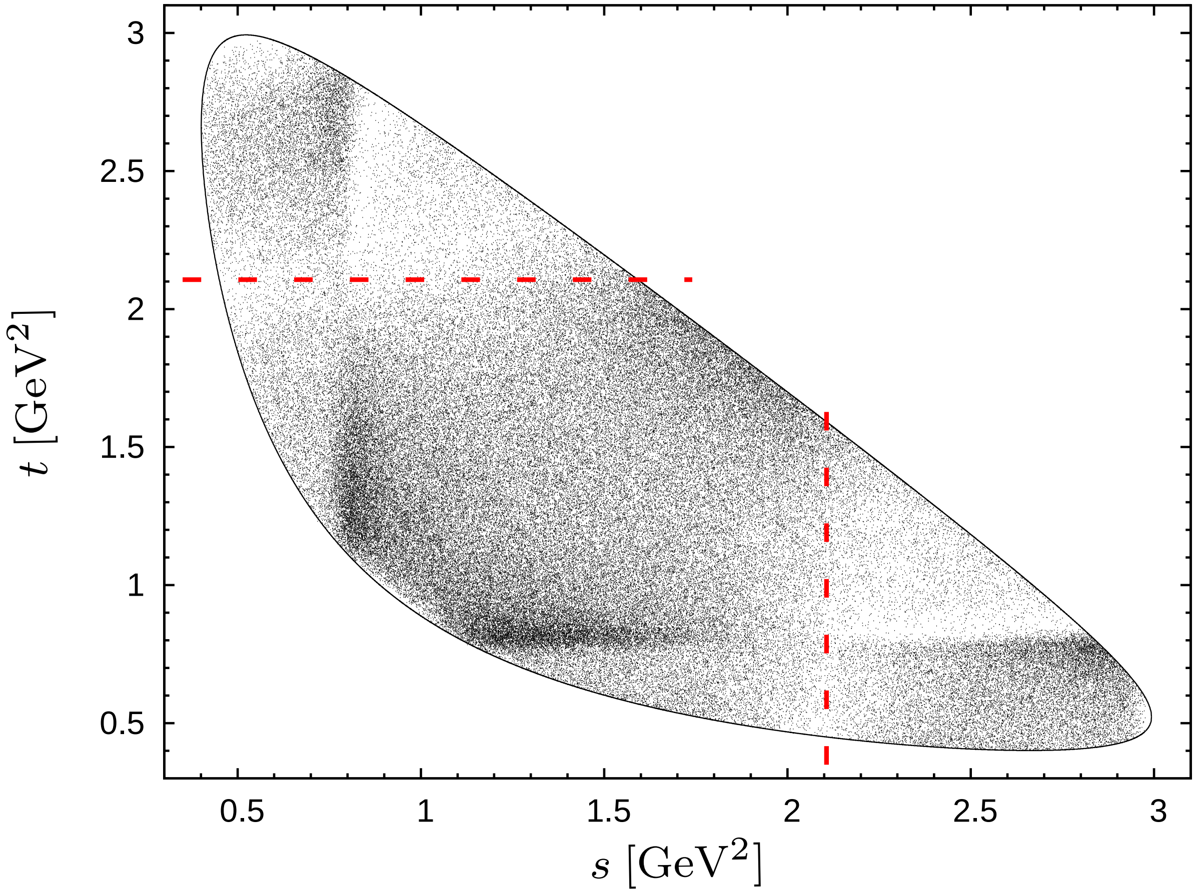
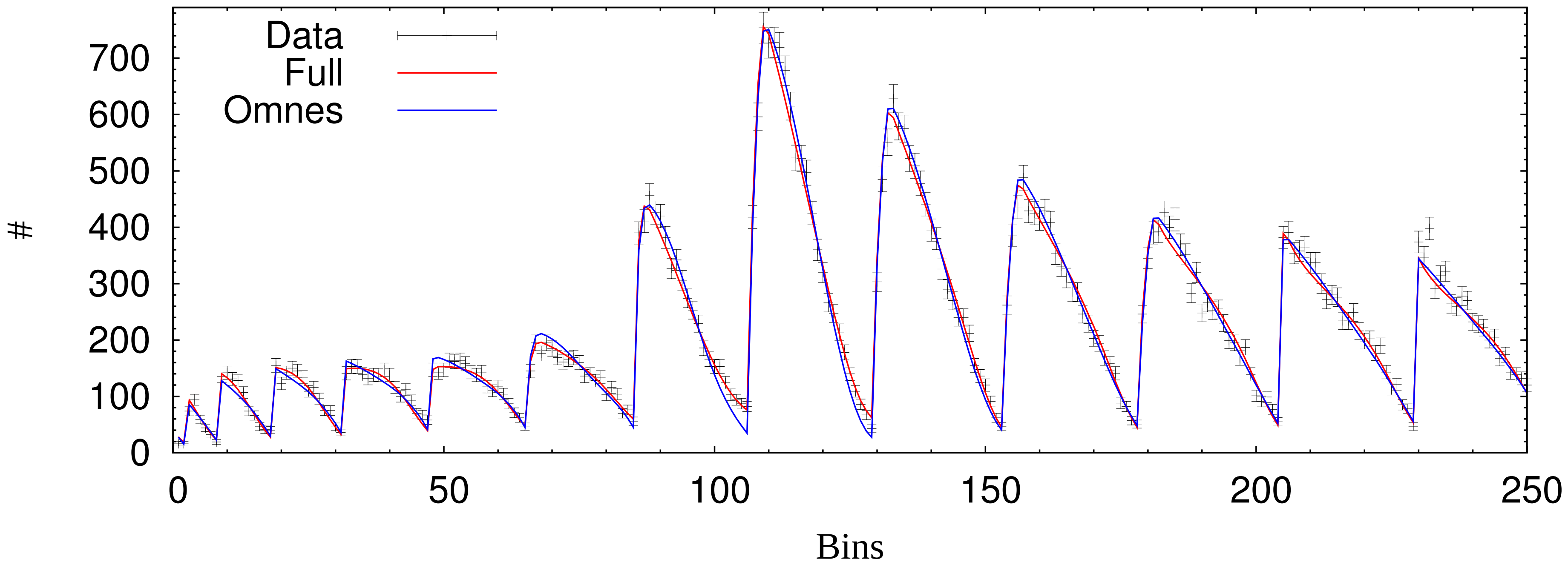


CLEO 2008

Breakdown of the elastic approximation?

- phase space limit: $\sqrt{s}, \sqrt{t} \leq M_D - M_\pi \approx 1.73\,\mathrm{GeV}$
- onset of inelasticity in $S^{1/2}_{\pi K}$ for $\sqrt{s} \geq M_{\eta'} + M_K \approx 1.45\,\mathrm{GeV}$
  $\longrightarrow$ fit deteriorates beyond; exclude this region for now

(Slices through) Dalitz plot $D^+ \to \pi^+\pi^+K^-$



- Omnès fit: $\chi^2/\mathrm{ndof} \approx 1.42$

  ("isobar model" + non-resonant background waves)

- full dispersive solution: $\chi^2/\mathrm{ndof} \approx 1.11$

  $\longrightarrow$ visible improvement similar to $\phi \to 3\pi$

- full fit in terms of 7 complex subtraction constants
  (–1 phase, –1 overall normalisation)

Niecknig, BK *in progress*

Heavy semileptonics: $B \to \pi\pi\ell\nu$

- $B \to$ "ρ" $\ell\nu$ exclusive decay to access $|V_{ub}|$
- "ρ" spectral function? S-wave background?
  $\longrightarrow$ control both by using dispersion relations for $\pi\pi$ rescattering

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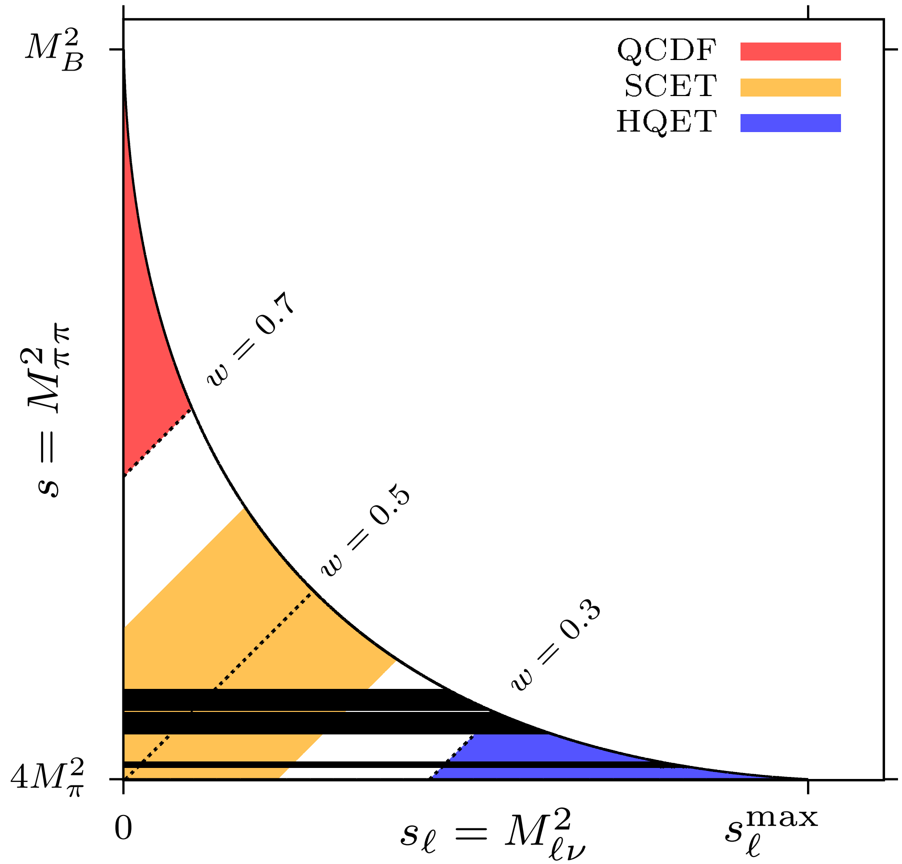
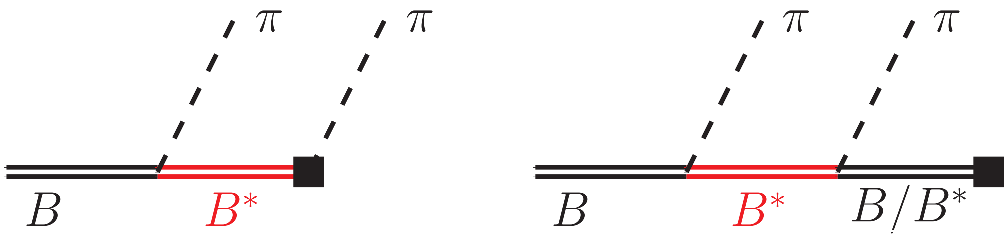


Faller et al. 2013

- idea: match to heavy-meson ChPT at large s_ℓ, soft-pion limit

Heavy-meson ChPT and dispersive representation



- heavy-meson ChPT: similtaneous chiral and $1/m_B$ expansion
- at leading order: $B \to \pi\pi\ell\nu$ given by B^* pole terms
  determined by $g_{B^*B\pi}$ and f_B Burdman, Donoghue 1992; Wise et al. 1992

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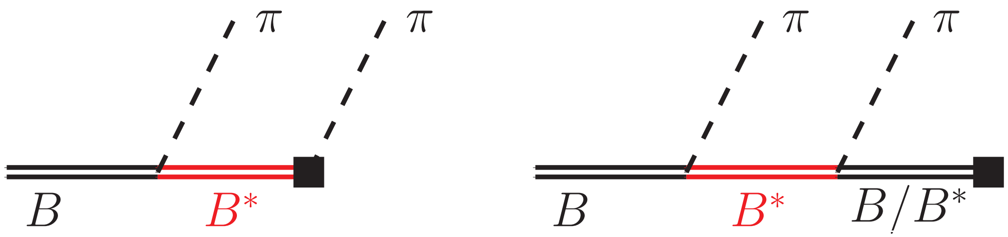

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- dispersive representation for partial waves at fixed s_ℓ:

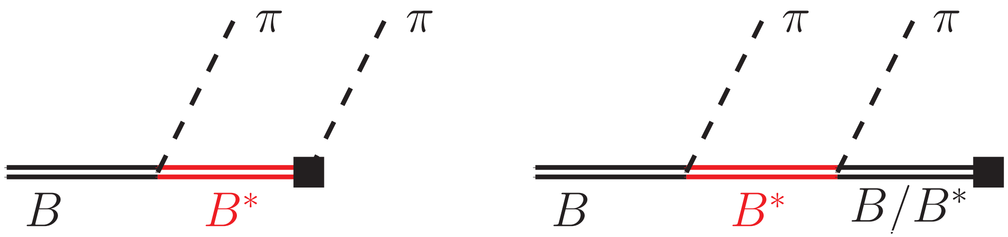
$$f(s) = \hat{M}(s) + \Omega(s)\left\{a_0 + a_1\, s + \frac{s^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\sin\delta(s')\hat{M}(s')}{|\Omega(s')|(s'-s)}\right\}$$

$\hat{M}(s)$: partial-wave-projected B^*-pole terms Kang et al. 2013

$\longrightarrow$ left-hand cut / square-root singularity at $s=0$

$\longrightarrow$ match $a_{0,1}$ at $s=0$ (and to high-energy asymptotics)

Heavy-meson ChPT and dispersive representation



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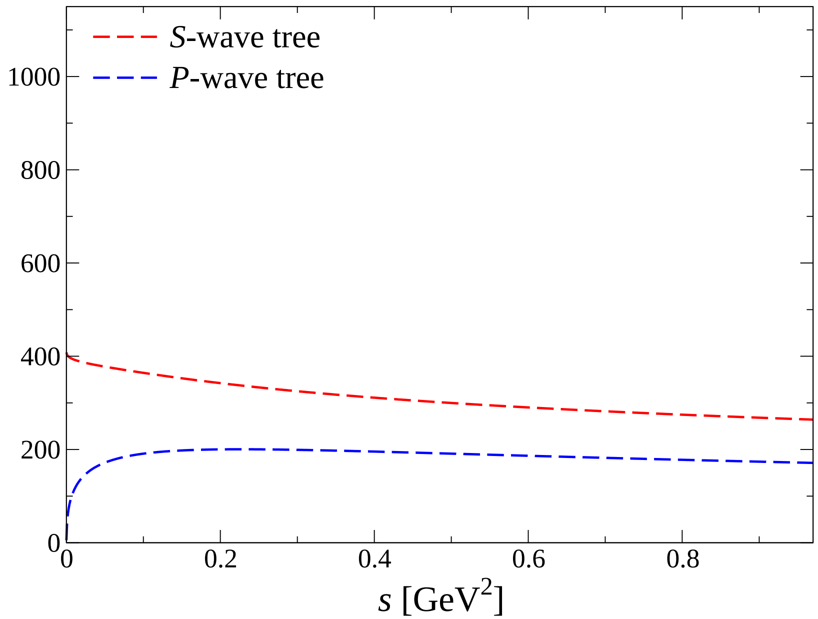

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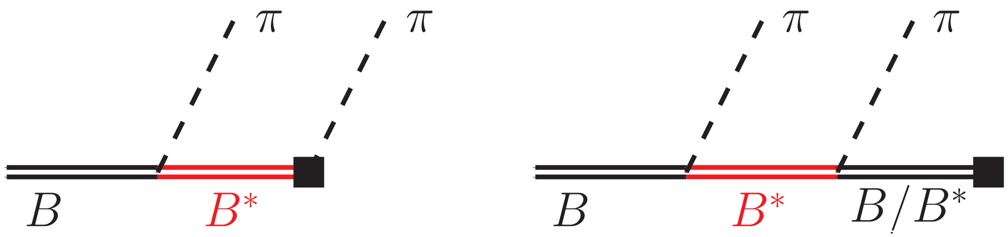

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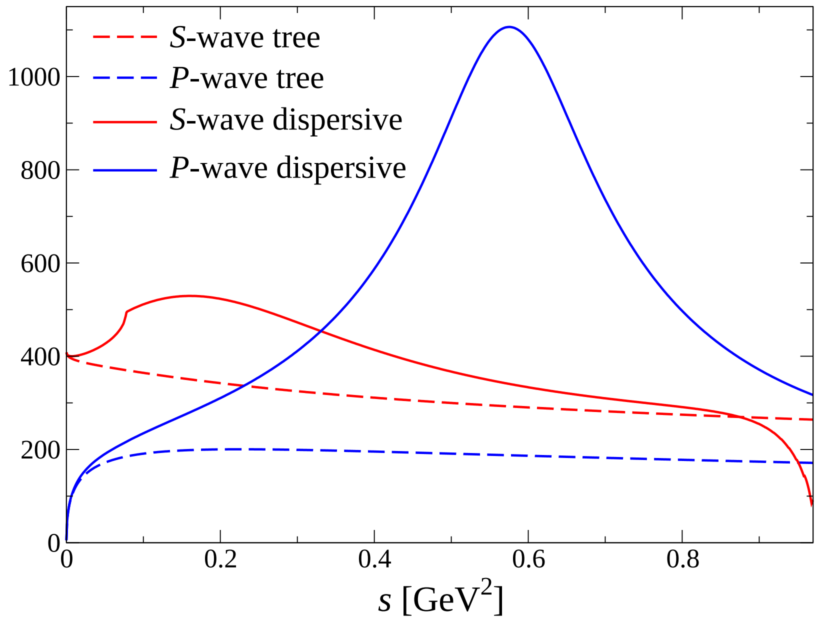

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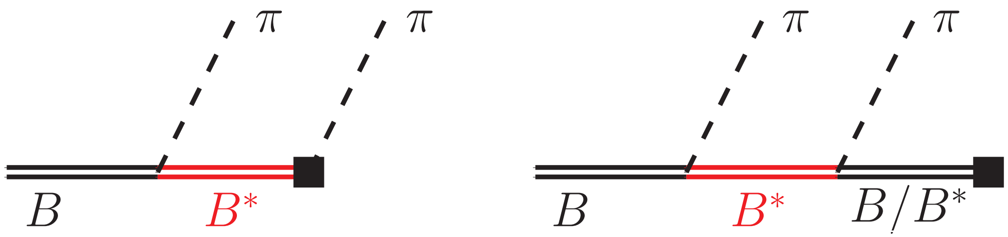

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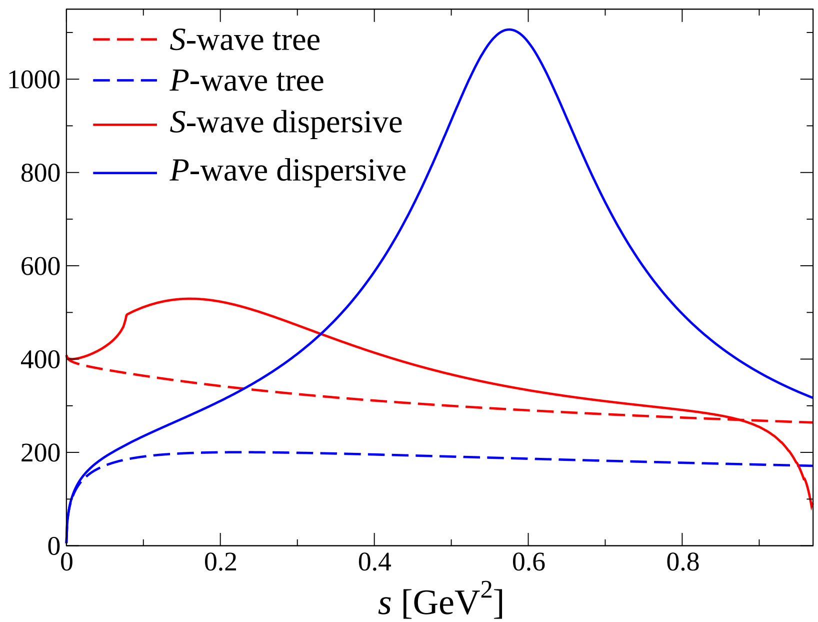

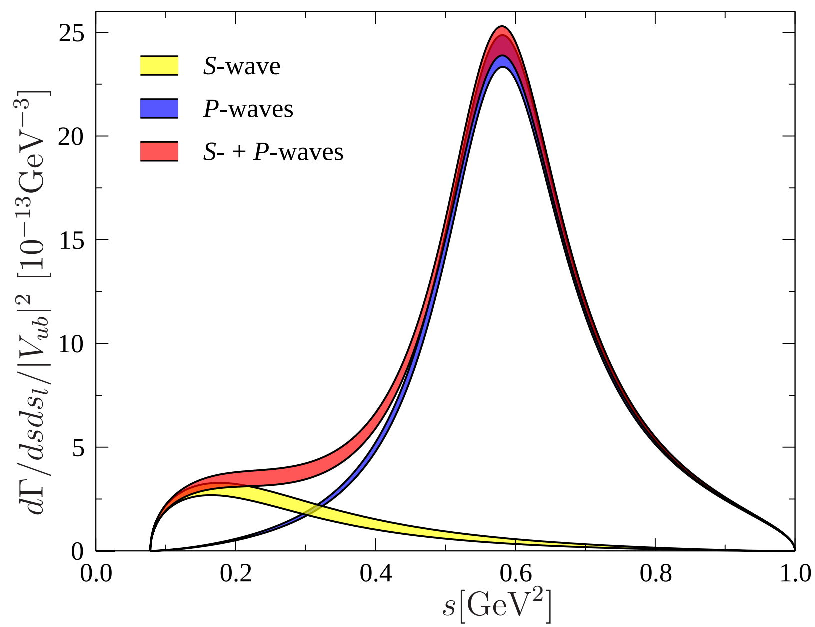

Kang et al. 2013

Heavy semileptonics: $D \to \pi K \ell \nu$

Pais–Treiman method

- $K^\pm \to \pi^+\pi^- e^\pm \nu$ (K_{e4}): model-independent method to access $\pi\pi$ scattering phases $\delta_0^0 - \delta_1^1$ E865 2001, NA48/2 2008
- $D \to \pi K \ell \nu$: analogous measurement of πK scattering phases $\delta_0^{1/2} - \delta_1^{1/2}$

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Heavy-meson ChPT + dispersion relations

- $1/m_c \gg 1/m_b$, $M_K^2 \gg M_\pi^2 \longrightarrow$ convergence worse than $B_{\ell 4}$
- aim: provide reasonable parametrisations of moduli of decay form factors Daub, Descotes-Genon, BK... *in progress*
- dependence on dilepton invariant mass s_ℓ still given by heavy-meson ChPT: D_s, D_s^* pole terms
  - ▷ how to extend this to lower s_ℓ?
  - ▷ use soft-pion (soft-kaon) theorems to relate to $D \to K(\pi)\ell\nu$

Summary / ▷ Outlook

Dalitz plot analyses

- rigorous using modern phase shift input
- allow to understand ad-hoc "background"
- ideal demonstration case: $\phi \to 3\pi$ (elastic, one partial wave)
- work in progress: $D \to \pi\pi K$
  - ▷ combination of inelastic effects with Khuri–Treiman method
  - ▷ extraction of πK phase shifts from three-body decays?

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Dispersive link hadronic $\leftrightarrow$ radiative decays

- ▷ towards a dispersive analysis of $\pi^0 \to \gamma^*\gamma^*$ S. Leupold's talk

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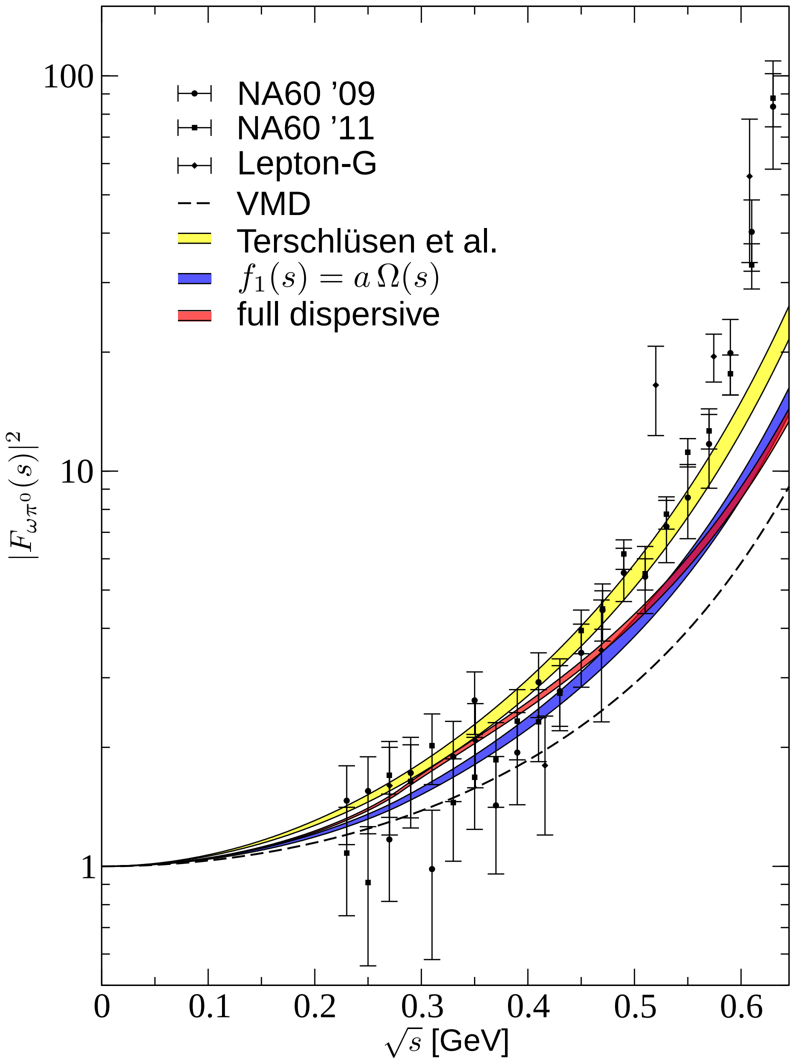
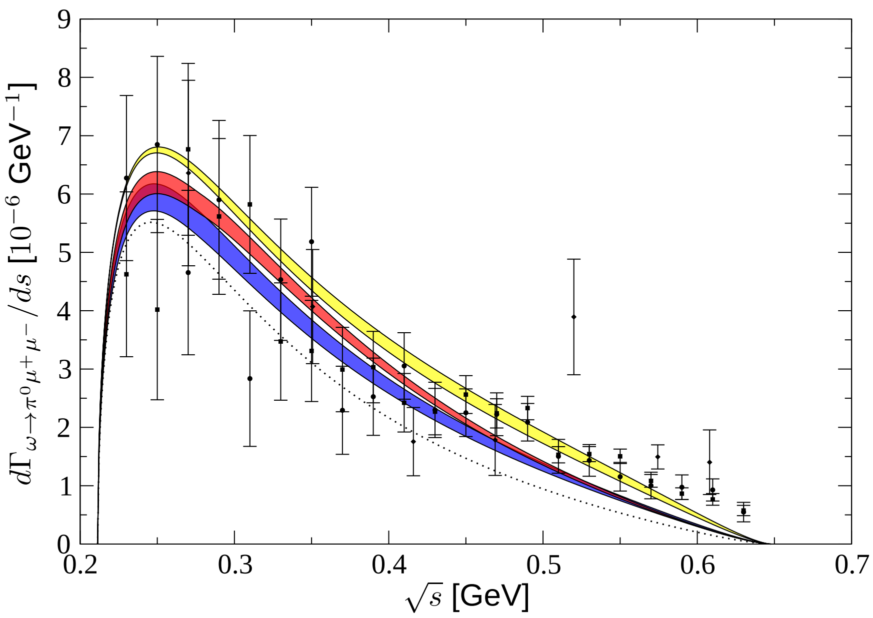
Heavy semileptonics

- dispersion theory $\longrightarrow$ good form-factor parametrisations
  - ▷ $B \to \pi\pi\ell\nu \quad \longrightarrow \quad |V_{ub}|$
  - ▷ $D \to \pi K\ell\nu \quad \longrightarrow \quad \pi K$ scattering phases

Spares

Numerical results: $\omega \to \pi^0 \mu^+ \mu^-$

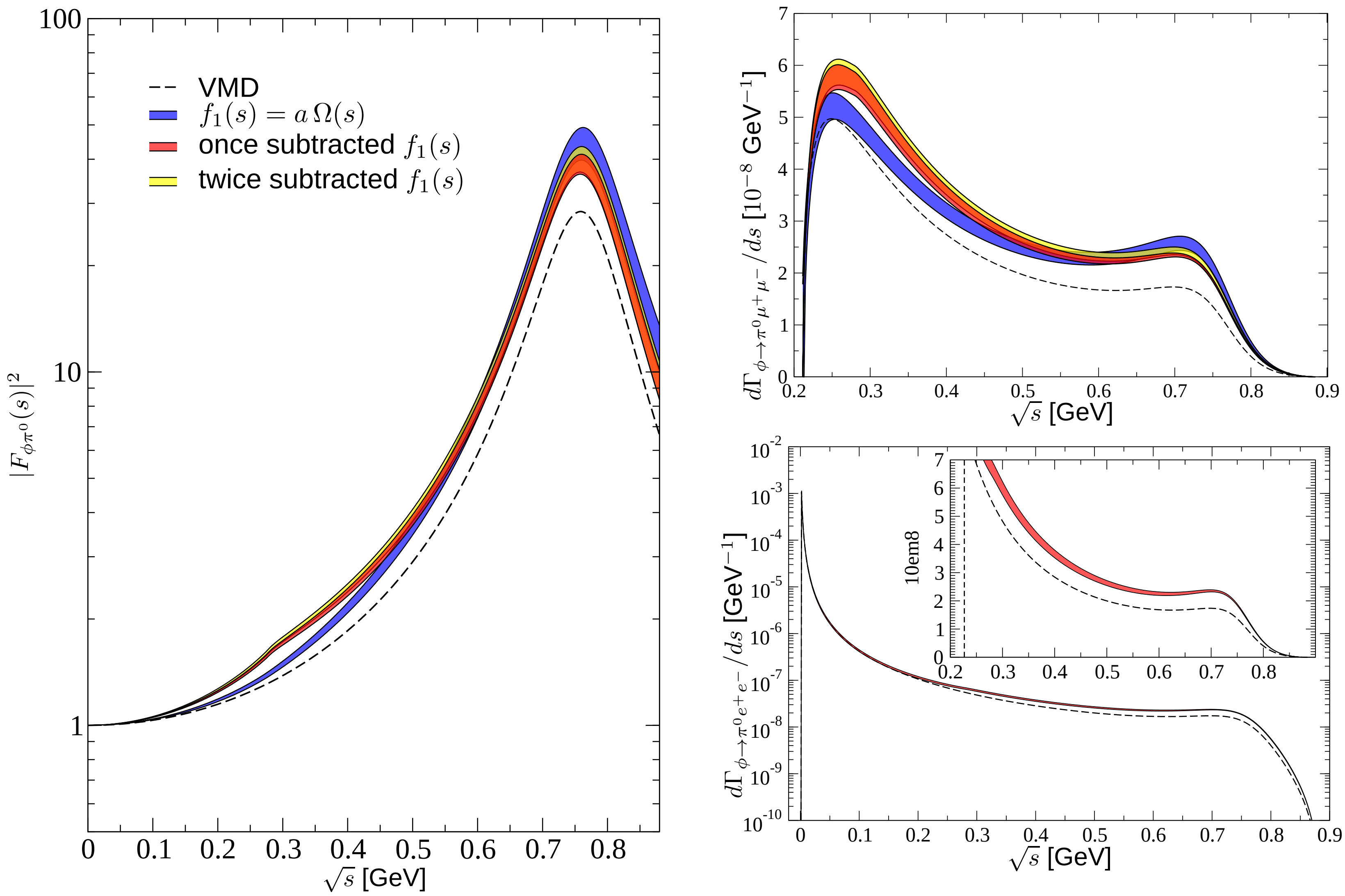




- unable to account for steep rise in data (from heavy-ion collisions) NA60 2009, 2011
- more "exclusive" data?!
- $\omega \to 3\pi$ Dalitz plot?

KLOE, WASA-at-COSY, CLAS?

Numerical results: $\phi \to \pi^0 \ell^+ \ell^-$



- measurement would be extremely helpful: ρ in physical region!
- partial-wave amplitude backed up by experiment