THE ODD INTRINSIC PARITY SECTOR OF CHPT

Study of Strongly Interacting Matter
HadronPhysics

Johan Bijnens

Lund University

Vetenskapsrådet

bijnens@thep.lu.se
http://www.thep.lu.se/∼bijnens
http://www.thep.lu.se/∼bijnens/chpt.html

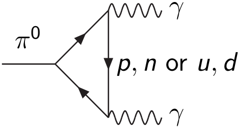
Light Meson Dynamics, Mainz 10-12 February 2014

Overview

Overview of the basics and comments about some topics
Much phenomenology is covered by other speakers

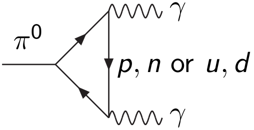
1. Wess-Zumino-Witten Lagrangian
2. Loops and Logarithms or LL1
3. Processes
4. $\pi^0 \to \gamma\gamma$
5. $\pi\gamma \to \pi\pi$
6. Kaons
7. $\gamma\gamma 3M$
8. C_i^{Wr}
9. Leading logarithms or LL2
10. Conclusions

Chiral Anomaly

- Problem: $\pi^0 \to \gamma\gamma$:
  - Veltman-Sutherland theorem: decay rate must be small (order p^6 in modern language)
  - (a) $\bar{q} i \gamma_5 q \pi^0$
  - (b) $\bar{q} \gamma^\mu \gamma_5 q \partial_\mu \pi^0$
  - Steinberger 1949: (a) gives the right answer (b) not
  - But from theory view (Ward identity) must give the same answer
- Solution:
  - 

  - Finite for (a), linearly divergent for (b)
- Adler-Bell-Jackiw-Bardeen anomaly (1969)
- Adler-Bardeen: anomaly is not renormalized

At the same time effective Lagrangians

- Weinberg 1968: current algebra and effective Lagrangians for $SO(4)/SO(3)$
- (Callan)-Coleman-Wess-Zumino: effective Lagrangians compatible with loop expansion in general and general G/H.
- Early review effective Lagrangians (tree level): Gasiorowicz-Geffen 1969
- But how to write an effective Lagrangian with chiral symmetry for the anomnaly?
- 

$\Longrightarrow \epsilon_{\mu\nu\alpha\beta}\pi^0 F^{\mu\nu} F^{\alpha\beta}$

Effective Lagrangian for anomaly

- $\epsilon_{\mu\nu\alpha\beta}\pi^0 F^{\mu\nu} F^{\alpha\beta}$
- Construct from $\left\{ \begin{array}{l} U \to g_R U g_L^\dagger \\ l_\mu \to g_L l_\mu g_L^\dagger - i\partial_\mu g_L g_L^\dagger \\ r_\mu \to g_R r_\mu g_R^\dagger - i\partial_\mu g_R g_R^\dagger \end{array} \right.$
- Does not go (even Witten didn't succeed), essentially $\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(\partial_\mu U \partial_\nu U^\dagger \partial_\alpha U \partial_\beta U^\dagger\right) = 0$
- Solution found by Wess-Zumino 1971
  - Noether's theorem $\delta\mathcal{L} \propto \partial_\mu j^\mu$
  - The anomaly actually gives $\partial_\mu j^\mu$
  - Integrate up from $U(z=0)=1$ to $U(z=1)=U=exp(i\sqrt{2}M/F)$
  - Ends up with the Lagrangian having an extra $\int_0^1 dz$
  - 5 dimensions: what is this?

Wess-Zumino-Witten

- Witten 1983
- Does the road $U(z)$ from $U = 1$ to U matter?
- Not if you quantize the coefficient (N_c must be integer)
- The actual form of the Lagrangian is fixed by the topology of G/H
- $\int d^5x \epsilon^{ABCDE} \mathrm{tr} \left(U^\dagger \partial_A U \partial_B U^\dagger \partial_C U \partial_D U^\dagger \partial_E U \right) \neq 0$
- This object (and the anomaly) has a lot of funny mathematics with it

The Wess-Zumino-Witten Lagrangian

$$S[U,\ell,r]_{\mathrm{WZW}} = -\frac{iN_c}{240\pi^2}\int d\sigma^{ijklm}\left\langle \Sigma^L_i\Sigma^L_j\Sigma^L_k\Sigma^L_l\Sigma^L_m\right\rangle$$

$$-\frac{iN_c}{48\pi^2}\int d^4x\,\varepsilon_{\mu\nu\alpha\beta}\left(W(U,\ell,r)^{\mu\nu\alpha\beta} - W(\mathbf{1},\ell,r)^{\mu\nu\alpha\beta}\right),$$

$$W(U,\ell,r)_{\mu\nu\alpha\beta} = \Big\langle U\ell_\mu\ell_\nu\ell_\alpha U^\dagger r_\beta + \frac{1}{4}U\ell_\mu U^\dagger r_\nu U\ell_\alpha U^\dagger r_\beta$$

$$+ iU\partial_\mu\ell_\nu\ell_\alpha U^\dagger r_\beta + i\partial_\mu r_\nu U\ell_\alpha U^\dagger r_\beta - i\Sigma^L_\mu\ell_\nu U^\dagger r_\alpha U\ell_\beta$$

$$+ \Sigma^L_\mu U^\dagger\partial_\nu r_\alpha U\ell_\beta - \Sigma^L_\mu\Sigma^L_\nu U^\dagger r_\alpha U\ell_\beta + \Sigma^L_\mu\ell_\nu\partial_\alpha\ell_\beta + \Sigma^L_\mu\partial_\nu\ell_\alpha\ell_\beta$$

$$- i\Sigma^L_\mu\ell_\nu\ell_\alpha\ell_\beta + \frac{1}{2}\Sigma^L_\mu\ell_\nu\Sigma^L_\alpha\ell_\beta - i\Sigma^L_\mu\Sigma^L_\nu\Sigma^L_\alpha\ell_\beta\Big\rangle - (L\leftrightarrow R),$$

with $\Sigma^L_\mu = U^\dagger\partial_\mu U\,,\qquad \Sigma^R_\mu = U\partial_\mu U^\dagger\,,$
$(L\leftrightarrow R)$: $U\leftrightarrow U^\dagger$, $\ell_\mu\leftrightarrow r_\mu$ and $\Sigma^L_\mu\leftrightarrow\Sigma^R_\mu$

Some comments

- This is the left-right symmetric form of the anomaly
- The 5-dimensional part is a total derivative so only depends on the boundary, expand U in M, term by term is explicitly integrable
- Witten actually got $W(U, l, r)$ a bit wrong

Odd intrinsic parity

- Parity: $P(t,\vec{x}) = (t,-\vec{x})$
  - Scalar: $P(\phi(t,\vec{x})) = \phi(t,-\vec{x})$
  - Pseudo-scalar: $P(\Phi(t,\vec{x})) = -\Phi(t,-\vec{x})$
- Intrinsic parity: $IP(t,\vec{x}) = (t,+\vec{x})$
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- So it's parity without the spatial part
- Unintended consequence: P in $\mathcal{L}$ implies IP if no $\epsilon_{\mu\nu\alpha\beta}$ present: only vertices with even number of pseudo-scalars
- Lorentz-invariance always has an even number of spatial indices without $\epsilon_{\mu\nu\alpha\beta}$
- Pentangle anomaly: $\pi^+\pi^-\pi^0K^+K^-$ vertex exist
- Anomaly requires $\epsilon_{\mu\nu\alpha\beta}$, but no Lagrangian at order p^4, so back at earlier discussion

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Loops and Logarithms

- Chiral logarithms: Early 1970s Pagels, Langacker, Ecker,... Review: Pagels, Phys.Rept. 16 (1975) 219
- ChPT Weinberg 1979, Gasser-Leutwyler 1984,1985
- Chiral logs in $\pi^0, \eta \to \gamma\gamma$

  Donoghue, Holstein, Lin, Phys.Rev.Lett. 55 (1985) 2766

  Logs nicely go into F_π and F_η
- Is this Adler-Bardeen in action?
- No: Corrections found in $\pi^0, \eta \to \gamma\gamma^*$

  Donoghue, Wyler, Nucl.Phys. B316 (1989) 289

  JB, Bramon, Cornet, Phys.Rev.Lett. 61 (1988) 1453
- Why: The WZW term gives the anomaly, these corrections are not anomnalous, they obey chiral symmetry fully
- Loop corrections are chiral invariant, not trivial since WZW has the anomaly

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Odd intrinsic parity

- Infinities calculated

  JB, Bramon, Cornet, Z.Phys. C46 (1990) 599

  Issler, SLAC-PUB 4943

  Akhourih, Alfakih, Ann. Phys. (NY) 210 (1991) 81
- Terms classsified (last two before wrong)

  JB, Girlanda, Talavera, Eur.Phys.J. C23 (2002) 539 [hep-ph/0110400]

  Ebertshauser,Fearing,Scherer,Phys.Rev.D65(2002)054033[hep-ph/0110261]
- Electromagnetic terms (two-flavour)

  Ananthanarayan, Moussallam, JHEP 0205 (2002) 052 [hep-ph/0205232]

Odd intrinsic parity

Loops and $\epsilon_{\mu\nu\alpha\beta}$ in dimensional regularization

- Can use a fourdimensional method like higher derivative regularization
- Main reason it works: $\epsilon_{\mu\nu\alpha\beta}$ is always an overall factor and it multiplies something already made finite
- One option: Define $$\epsilon_{\mu\nu\alpha\beta} \equiv \frac{1}{4i}\mathrm{tr}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta\gamma_5$$

  with a proper definition of the *d*-dimensional gamma matrices.
- This leads to our naive factorized way of doing it with a fully antisymmetric $\epsilon_{\mu\nu\alpha\beta}$
- But beware when products of two $\epsilon_{\mu\nu\alpha\beta}$, but only relevant at p^8.

List of processes

- $\pi^0 \to \gamma\gamma$
- $\eta \to \gamma\gamma$
- With one photon off-shell/single Dalitz
- With two photons off-shell/double dalitz
- $\pi \to \ell\nu\gamma$
- $K \to \ell\nu\gamma$
- $K \to \pi\pi\ell\nu$
- $\pi\gamma \to \pi\pi$
- $\eta \to \pi\pi\gamma$ and $\eta \to \pi^+\pi^-e^+e^-$
- $\gamma\gamma \to 3\pi$
- $\eta \to \pi\pi\gamma\gamma$

$\pi^0 \to \gamma\gamma$

- PRIMEX: $\Gamma(\pi^0 \to \gamma\gamma) = 7.82 \pm 0.14 \pm 0.17\ \mathrm{eV}$
  Phys.Rev.Lett. 106 (2011) 162303 [arXiv:1009.1681]
- CERN: $\Gamma(\pi^0 \to \gamma\gamma) = 7.25 \pm 0.18 \pm 0.14\ \mathrm{eV}$
  Atherton, et al., Phys. Lett. B **158** (1985) 81
- Older Primakoff experiments
- Crystal Ball $e^+e^- \to e^+e^-\pi^0$
  $\Gamma(\pi^0 \to \gamma\gamma) = 7.7 \pm 0.5 \pm 0.5\ \mathrm{eV}$
- Review experiment and theory:
  Bernstein, Holstein, Rev.Mod.Phys. 85 (2013) 49 [arXiv:1112.4809]
- Anomaly: $\Gamma = \dfrac{\alpha^2 m_\pi^3}{64\pi^3 F_\pi^2} = 7.76\ \mathrm{eV}$

$\pi^0 \to \gamma\gamma$

- $m_u - m_d$: largest part is π^0-η-η' mixing: 4.5%increase

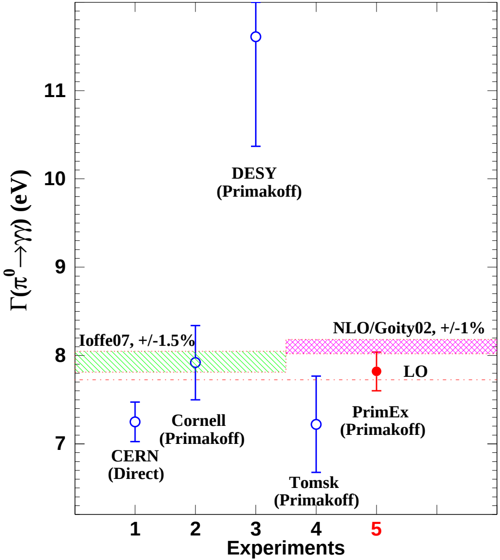
  Goity, Bernstein, Holstein,Phys.Rev. D66 (2002) 076014 [hep-ph/0206007]

  Ananthanarayan, Moussallam, JHEP **0205** (2002) 052 [hep-ph/0205232]
- e^2: mainly F_{π^+} versus F_{π^0}: small A&M
- m_π^2 almost a guess: small A&M
- NNLO logarithm and contributions

  Kampf, Moussallam, Phys. Rev. D **79** (2009) 076005 [arXiv:0901.4688]
- even higher logs: very small

  JB, Kampf, Lanz, Nucl. Phys. B **860** (2012) 245 [arXiv:1201.2608]
- Lattice: compatible but not competitive (yet)
- GBH: $\Gamma(\pi^0 \to \gamma\gamma) = 8.13 \pm 0.08\ \mathrm{eV}$
- AM : $\Gamma(\pi^0 \to \gamma\gamma) = 8.06 \pm 0.02 \pm 0.06\ \mathrm{eV}$
- KM : $\Gamma(\pi^0 \to \gamma\gamma) = 8.09 \pm 0.11\ \mathrm{eV}$

$\pi^0 \to \gamma\gamma$



Phys.Rev.Lett. 106(2011)162303 [arXiv:1009.1681]

compatible with LO and NLO

$\pi^0, \eta \leftrightarrow \gamma^*\gamma^*$

- One-loop calculation JB, Bramon, Cornet, Phys.Rev.Lett. 61 (1988) 1453; Z.Phys. C46 (1990) 599
- Aspects covered (I hope): Hanhart, Escribano, Masjuan
- Parts of the two-loop calculation exists JB, Kampf

$\pi\gamma \to \pi\pi$

- $\pi^-(p_1)\gamma(k) \to \pi^-(p_2)\pi^0(p_0)$
- Amplitude: $A = F^{3\pi}(s,t,u)\epsilon_{\mu\nu\alpha\beta}\varepsilon^\mu(k)p_1^\nu p_2^\alpha p_0^\beta$
- Chiral anomaly prediction: $F^{3\pi} = \dfrac{e}{4\pi^2 F_\pi^3} = 9.8\ \mathrm{GeV}^{-3}$
- Experiment
  - Serpukhov $F^{3\pi} = 12.9 \pm 0.9 \pm 0.5\ \mathrm{GeV}^{-3}$
  - $\pi^- e^- \to \pi^0\pi^- e^-$
    $F^{3\pi} = ((9.9 \pm 1.1)\ \mathrm{or}\ (9.6 \pm 1.1))\ \mathrm{GeV}^{-3}$
    Giller et al., Eur.Phys.J. A25 (2005) 229 [hep-ph/0503207]
- But note $F^{3\pi}(s,t,u)$ is definitely not constant

$\pi\gamma \to \pi\pi$

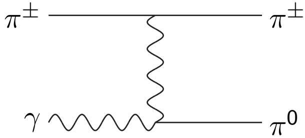
- One-loop calculation

  JB,Bramon, Cornet, Phys. Lett. B **237** (1990) 488

  Result: increase of 7-12% over Serpukhov phase space
- Electromagnetic corrections can be large

  Ametller, Knecht, Talavera, Phys. Rev. D **64** (2001) 094009 [hep-ph/0107127]

  Large part is from

  Effect in total a little larger than the NLO Chiral corrections

$\pi\gamma \to \pi\pi$

- Dispersive/vector models (two examples only)
  Hoferichter, Kubis, Sakkas, Phys. Rev. D **86** (2012) 116009
  [arXiv:1210.6793]
  Holstein, Phys.Rev. D53 (1996) 4099 [hep-ph/9512338]
- Leading logs in higher loops: small
  JB, Kampf, Lanz, Nucl. Phys. B **860** (2012) 245 [arXiv:1201.2608]
  $F^{3\pi} = (9.8-0.3+0.04+0.02+0.006+0.001+\cdots)\,\mathrm{GeV}^{-3}$.
- Conclusion: reasonable agreement but would like a better measurement
- $e^+e^- \to 3\pi$ near threshold already close to limit of ChPT

Kaons and other weak decays

- $\pi \to e\nu\gamma$
- $K \to \ell\nu\gamma$
- $K \to \pi\pi\ell\nu$
- General feature: more than one form-factor, some anomalous, some not
- One-loop calculation Ametller, Bijnens, Bramon, Cornet ,Phys.Lett. B303 (1993) 140-146 [hep-ph/9302219]
- Typically very good agreement with data
- SIGN of the anomaly checked
- review: Bijnens, PoS KAON (2008) 027 [arXiv:0707.0419]

Oddities

- $\gamma\gamma \to \pi\pi\pi$
  - Leading order

    Adler, Lee, Treiman, Zee, Phys.Rev. D4 (1971) 3497

    Bos, Lin, Shih, Phys.Lett. B337 (1994) 152 [hep-ph/9407216]
  - One-loop

    Talavera, Ametller, Bijnens, Bramon and Cornet, Phys. Lett. B **376** (1996) 186 [hep-ph/9512296]
- $\eta \to \gamma\gamma\pi\pi$
  - Leading order

    Knöchlein, Scherer, Drechsel, Phys. Rev. D **53** (1996) 3634 [hep-ph/9601252]
  - One-loop Ametller, Bijnens, Bramon, Talavera, Phys. Lett. B **400** (1997) 370 [hep-ph/9702302]
  - Main interest: in some corners loops dominate for the neutral case in $\eta \to \pi^0\pi^0\gamma\gamma$
- Allow for nontrivial checks of the vector meson Lagrangians

Values of the C_i^{Wr}

- Values of the C_i^W from experiment, no single published analysis exist
- Two partial but unpublished are:
  - Olof Strandberg hep-ph/0302064 (magister thesis Lund)
  - Christian Hacker, PhD thesis Mainz 2008
- Theoretical estimates: there are very many
  - Our original work used HLS JB, Bramon, Cornet, Z.Phys. C46 (1990) 599
  - The most comprehensive resonance saturation Kampf, Novotny, Phys. Rev. D **84** (2011) 014036 [arXiv:1104.3137]
  - Chiral quark model JB, Nucl. Phys. B367 (1991) 709
  - SDE Jiang, Wang, Phys.Rev. D81 (2010) 094037, [arXiv:1001.0315]
  - But many more papers exist
  - A Caveat: (After Ruiz-Arriola had problems with ENJL) JB, Prades, Phys. Lett. B320 (1994) 130 [hep-ph/9310355] Model has to to keep the anomaly as it is (cut-off finite vs infinite)

Leading Logarithms

- Take a quantity with a single scale: $F(M)$
- The dependence on the scale in field theory is typically logarithmic
- $L = \log(\mu/M)$
- $F = F_0 + F_1^1 L + F_0^1 + F_2^2 L^2 + F_1^2 L + F_0^2 + F_3^3 L^3 + \cdots$
- Leading Logarithms: The terms $F_m^m L^m$

The F_m^m can be more easily calculated than the full result

- $\mu\,(dF/d\mu) \equiv 0$
- Ultraviolet divergences in Quantum Field Theory are always local

Renormalizable theories

- Loop expansion $\equiv \alpha$ expansion
- $F = \alpha + f_1^1\alpha^2 L + f_0^1\alpha^2 + f_2^2\alpha^3 L^2 + f_1^2\alpha^3 L + f_0^2\alpha^3 + f_3^3\alpha^4 L^3 + \cdots$
- f_i^j are pure numbers
- $\mu\dfrac{d\alpha}{d\mu} = \beta_0\alpha^2 + \beta_1\alpha^3 + \cdots$
- $\mu\dfrac{dF}{d\mu} = 0 \Longrightarrow \boxed{\beta_0 = -f_1^1 = f_2^2 = -f_3^3 = \cdots}$
- Relies on the α the same in all orders
- In effective field theories: different Lagrangian at each order
- The recursive argument does not work

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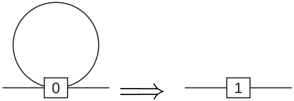
Weinberg's argument

- Weinberg, Physica A96 (1979) 327
- Two-loop leading logarithms can be calculated using only one-loop: Weinberg consistency conditions
- Proof at all orders:
  - using β-functions: Büchler, Colangelo, hep-ph/0309049
  - Proof with diagrams: JB, Carloni, arXiv:0909.5086
- Proof relies on
  - μ: dimensional regularization scale
  - $d = 4 - w$
  - at n-loop order $(\hbar^n)$ must cancel:
    - $1/w^n$, $\log \mu/w^{n-1}, \dots, \log^{n-1} \mu/w$
    - This allows for relations between diagrams
    - All needed for $\log^n \mu$ coefficient can be calculated from one-loop diagrams

Mass to $\hbar^2$

- $\hbar^1$: 

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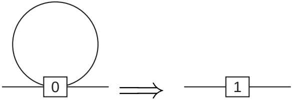
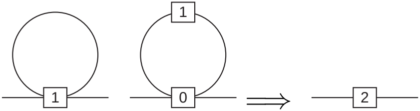
- $\hbar^1$: 

- $\hbar^2$: 

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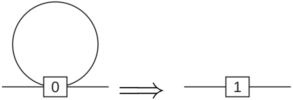
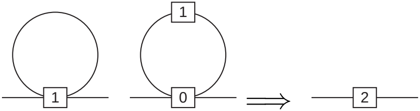
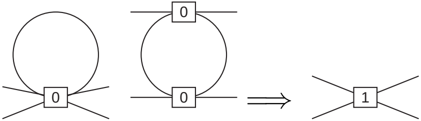


- $\hbar^2$:



- but also needs $\hbar^1$:



General

- Calculate the divergence
- rewrite it in terms of a local Lagrangian
  - Luckily: symmetry kept: we know result will be symmetrical, hence do not need to explicitly rewrite the Lagrangians in a nice form
  - Luckily: we do not need to go to a minimal Lagrangian
  - So everything can be computerized
- We keep all terms to have all 1PI (one particle irreducible) diagrams finite

Same game for anomalous sector

JB, Kampf, Lanz, Nucl. Phys. B **860** (2012) 245 [arXiv:1201.2608]

$\pi\gamma \to \pi\pi$

$$\tilde{k} = k^2/m_\pi^2 \;\; \tilde{\Delta}_n = (s^n + t^n + u^n)/m_\pi^{2n},$$
$$L_{\mathcal{M}} = m_\pi^2/(16\pi^2)\log(\mu^2/\mathcal{M}^2)$$

$$\begin{aligned}
f^{LL}(s,t,u) = {} & 1 + L_{\mathcal{M}}\frac{1}{6}\left(3+\tilde{k}^2\right) + L_{\mathcal{M}}^2\frac{1}{72}(\tilde{k}^2-3)(\tilde{k}^2+33) \\
& + L_{\mathcal{M}}^3\frac{1}{1296}\left(90\tilde{\Delta}_3 - 640\tilde{\Delta}_2 - 8157 + 2105\tilde{k}^2 + 81\tilde{k}^4 + \tilde{k}^6\right) + L_{\mathcal{M}}^4\frac{1}{155520}\Big[-1532\tilde{\Delta}_4 \\
& + \tilde{\Delta}_3(88538 + 1890\tilde{k}^2) - \tilde{\Delta}_2(577760 + 12240\tilde{k}^2 + 540\tilde{k}^4) - 2433375 + 1296190\tilde{k}^2 \\
& + 57430\tilde{k}^4 + 480\tilde{k}^6 + 185\tilde{k}^8\Big] + L_{\mathcal{M}}^5\frac{1}{326592000}\Big[\tilde{\Delta}_5(13252156) \\
& - \tilde{\Delta}_4(160744570 + 518350\tilde{k}^2) + \tilde{\Delta}_3(1465187530 + 39593272\tilde{k}^2 + 247260\tilde{k}^4) \\
& - \tilde{\Delta}_2(6756522937 + 257781206\tilde{k}^2 + 11188776\tilde{k}^4 - 9160\tilde{k}^6) - 6498695163 \\
& + 12675091794\tilde{k}^2 + 801259373\tilde{k}^4 + 4780240\tilde{k}^6 + 2948600\tilde{k}^8 - 1832\tilde{k}^{10}\Big].
\end{aligned}$$

$$F^{3\pi} = (9.8 - 0.3 + 0.04 + 0.02 + 0.006 + 0.001 + \cdots)\ \mathrm{GeV}^{-3}.$$

$\pi^0 \to \gamma^*\gamma^*$

- JB, Kampf, Lanz, Nucl. Phys. B **860** (2012) 245 [arXiv:1201.2608]
- $F_{\pi\gamma\gamma}(k_1^2,k_2^2) = \frac{e^2}{4\pi^2 F_\pi} F_\gamma(k_1^2) F_\gamma(k_2^2) F_{\gamma\gamma}(k_1^2,k_2^2)\hat{F}$
- $F_\gamma(k_i^2) = F_{\gamma\gamma}(k_1^2,k_2^2)$=1 at $k_i^2 = 0$.
- $\hat{F}$ correction to $\pi^0 \to \gamma\gamma$
- $F_\gamma(k_1)$ the form factor
- $F_{\gamma\gamma}(k_1^2,k_2^2)$ the nonfactorizable part for both off-shell
- Done to six loops

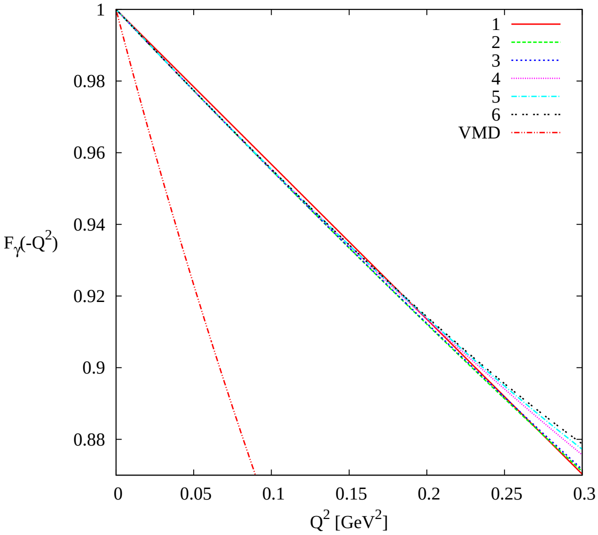
$$\hat{F} = 1 - 1/6\, L_{\mathcal{M}}^2 + 5/6\, L_{\mathcal{M}}^3 + 56147/7776\, L_{\mathcal{M}}^4 \\ + 446502199/11664000\, L_{\mathcal{M}}^5 + 65694012997/367416000\, L_{\mathcal{M}}^6$$

$$\hat{F} = 1 + 0 - 0.000372 + 0.000088 + 0.000036 + 9\cdot10^{-6} + 2\cdot10^{-7}$$

$F_\gamma(k^2)$

$$
\begin{aligned}
F_\gamma(k^2) = {} & 1 + L_{\mathcal{M}}(1/6\,\tilde{k}^2) + L_{\mathcal{M}}^2(5/24\,\tilde{k}^2 + 1/72\,\tilde{k}^4) \\
& + L_{\mathcal{M}}^3(71/432\,\tilde{k}^2 + 1/24\,\tilde{k}^4 + 1/1296\,\tilde{k}^6) \\
& + L_{\mathcal{M}}^4(-24353/31104\,\tilde{k}^2 + 4873/10368\,\tilde{k}^4 - 2357/31104\,\tilde{k}^6 \\
& + 145/31104\,\tilde{k}^8) + L_{\mathcal{M}}^5(-548440741/81648000\,\tilde{k}^2 + 9793363/3024000\,\tilde{k}^4 \\
& - 32952389/54432000\,\tilde{k}^6 + 487493/13608000\,\tilde{k}^8 - 2069/10886400\,\tilde{k}^{10}) \\
& + L_{\mathcal{M}}^6(-3519465627493/102876480000\,\tilde{k}^2 \\
& + 3560724235307/205752960000\,\tilde{k}^4 - 1524042680197/411505920000\,\tilde{k}^6 \\
& + 4741599089/11757312000\,\tilde{k}^8 - 510932327/13716864000\,\tilde{k}^{10} \\
& + 1775869/914457600\,\tilde{k}^{12})
\end{aligned}
$$

$F_\gamma(k^2)$



1
2
3
4
5
6
VMD
1
0.98
0.96
0.94
0.92
0.9
0.88
$F_\gamma(-Q^2)$
0
0.05
0.1
0.15
0.2
0.25
0.3
Q^2 [GeV^2]

$F_{\gamma\gamma}(k_1^2, k_2^2)$

$$
\begin{aligned}
F_{\gamma\gamma}(k_1^2, k_2^2) = {} & 1 + {\color{red}0} + {\color{red}0} + L_{\mathcal{M}}^3 \tilde{k}_1^2 \tilde{k}_2^2 \frac{1}{72} \\
& + L_{\mathcal{M}}^4 \tilde{k}_1^2 \tilde{k}_2^2 \big[- 203/7776 + 29/10368\, (\tilde{k}_1^2 + \tilde{k}_2^2) \\
& + 1/216\, (\tilde{k}_1^4 + \tilde{k}_2^4) - 1/144\, \tilde{k}_1^2 \tilde{k}_2^2 \big] + L_{\mathcal{M}}^5 \tilde{k}_1^2 \tilde{k}_2^2 \big[- 5983633/10206000 \\
& + 46103/1632960\, (\tilde{k}_1^2 + \tilde{k}_2^2) + 372113/11664000\, (\tilde{k}_1^4 + \tilde{k}_2^4) \\
& - 211/5443200\, (\tilde{k}_1^6 + \tilde{k}_2^6) - 394157/9072000\, \tilde{k}_1^2 \tilde{k}_2^2 - 4/25515\, \tilde{k}_1^2 \tilde{k}_2^2 (\tilde{k}_1^2 + \tilde{k}_2^2) \big] \\
& + L_{\mathcal{M}}^6 \tilde{k}_1^2 \tilde{k}_2^2 \big[- 1072421939773/205752960000 \\
& + 1444445383/6531840000\, (\tilde{k}_1^2 + \tilde{k}_2^2) + 10840553807/102876480000\, (\tilde{k}_1^4 + \tilde{k}_2^4) \\
& + 282016297/205752960000\, (\tilde{k}_1^6 + \tilde{k}_2^6) + 6157391/4115059200\, (\tilde{k}_1^8 + \tilde{k}_2^8) \\
& - 3852620057/29393280000\, \tilde{k}_1^2 \tilde{k}_2^2 - 154739/58320000\, \tilde{k}_1^2 \tilde{k}_2^2 (\tilde{k}_1^2 + \tilde{k}_2^2) \\
& - 75041473/20575296000\, \tilde{k}_1^2 \tilde{k}_2^2 (\tilde{k}_1^4 + \tilde{k}_2^4) + 174329/35721000\, \tilde{k}_1^4 \tilde{k}_2^4 \big] \,.
\end{aligned}
$$

- 0 at one-loop expected
- 0 at two-loop not expected
- Three loop coefficient quite small
- LL give a small nonfactorizable part

Conclusions

- Odd intrinsic parity ChPT basically done at one-loop
- A number of funny theoretical observations exist in this sector: the anomaly is always good for surprises
- Two-loop and higher orders: some results exist
- No indication that ChPT doesn't work
- Overall good agreement with experiment
- Further precision tests always welcome