pp Interaction at Very High Energies in Cosmic Ray Experiments

(A. K. Kohara, E. Ferreira, T. Kodama)
IF/UFRJ

International Symposium on Very High Energy Cosmic Ray Interactions, ISVHECRI
18-22 August 2014 - CERN

Outlook

- Description of pp elastic scattering
- b-space asymptotic behaviour
- p-air cross sections (Glauber)
- Conclusions

Elastic Scattering Amplitudes

pp differential cross section

A. Kendi Kohara, E. Ferreira and T. Kodama , *Eur. Phys. J. C* ,**73**, 2326 (2013).
A. K. Kohara , E. Ferreira and T. Kodama , *Phys. Rev. D* **87** , 054024 (2013).

$$\frac{d\sigma}{dt} = (\hbar c)^2 \; |T_R(s,t) + iT_I(s,t)|^2$$

Real and imaginary amplitudes , K = R , I

$$T_K^N(s,t) = \alpha_K(s)\mathrm{e}^{-\beta_K(s)|t|} + \lambda_K(s)\Psi_K(\gamma_K(s),t)$$

$$\Psi_K(\gamma_K(s),t) = 2\;\mathrm{e}^{\gamma_K}\left[\frac{\mathrm{e}^{-\gamma_K\sqrt{1+a_0|t|}}}{\sqrt{1+a_0|t|}} - \mathrm{e}^{\gamma_K}\;\frac{e^{-\gamma_K\sqrt{4+a_0|t|}}}{\sqrt{4+a_0|t|}}\right]$$

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with 4 parameters for each amplitude

quantities in forward scattering

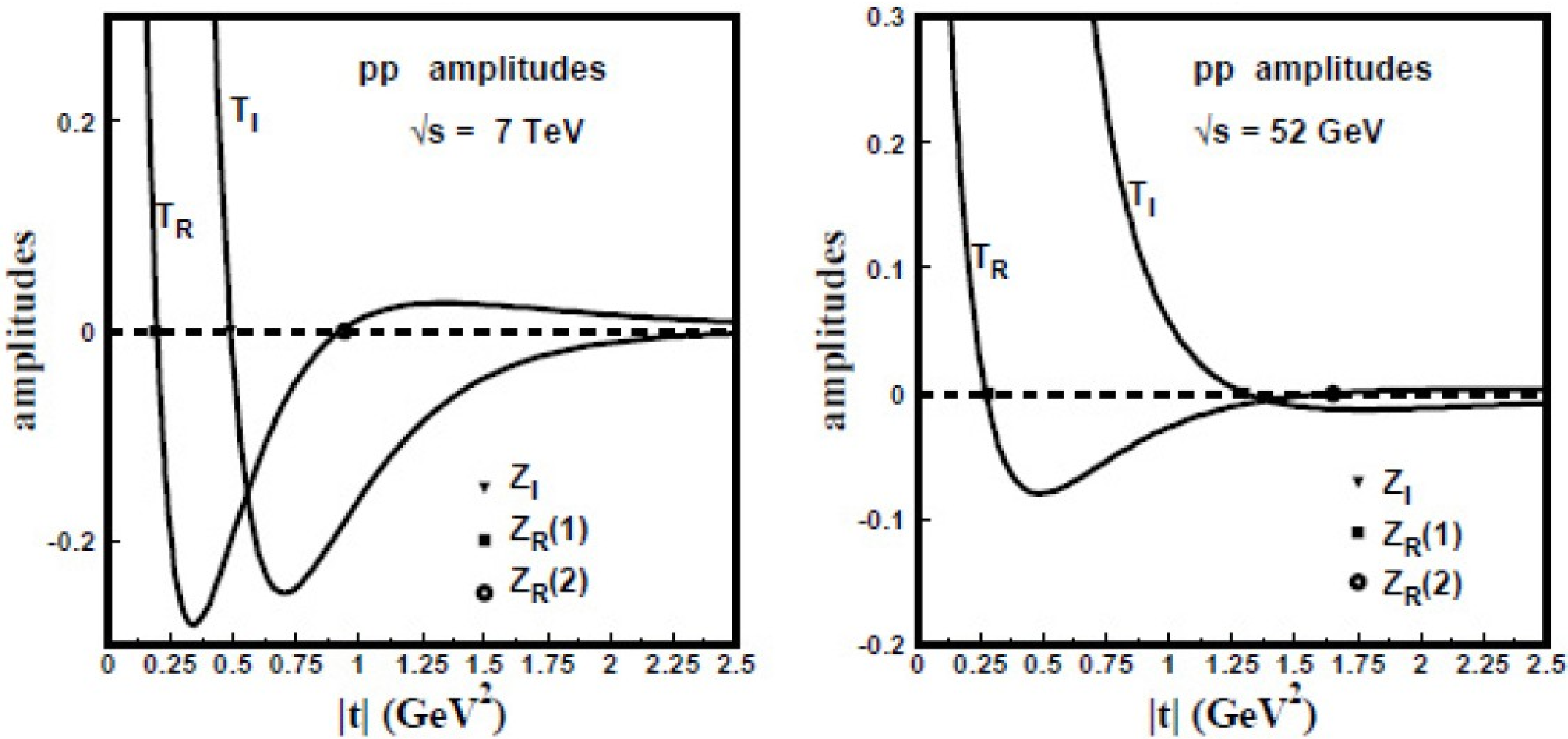
$$\sigma(s) = 4\sqrt{\pi}\,(\hbar c)^2\ \left(\alpha_I(s) + \lambda_I(s)\right)$$ Total cross section

$$\rho(s) = \frac{T_R^N(s,t=0)}{T_I^N(s,t=0)} = \frac{\alpha_R(s) + \lambda_R(s)}{\alpha_I(s) + \lambda_I(s)}$$ Real/Imaginary

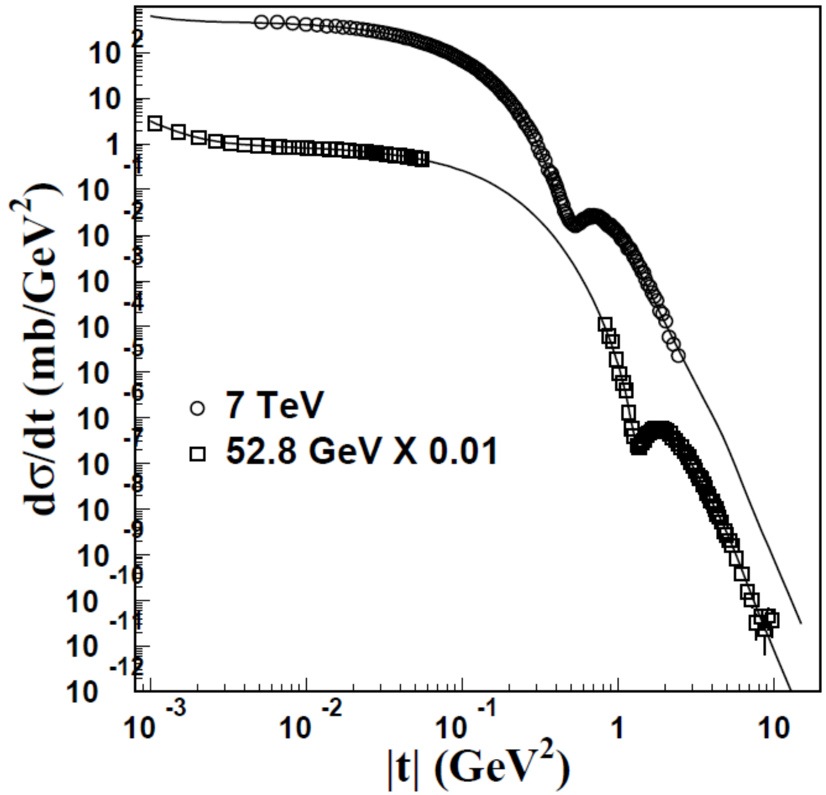
$$B_K(s) = \frac{2}{T_K^N(s,t)} \frac{dT_K^N(s,t)}{dt}\Big|_{t=0}$$ Real and Imaginary slopes

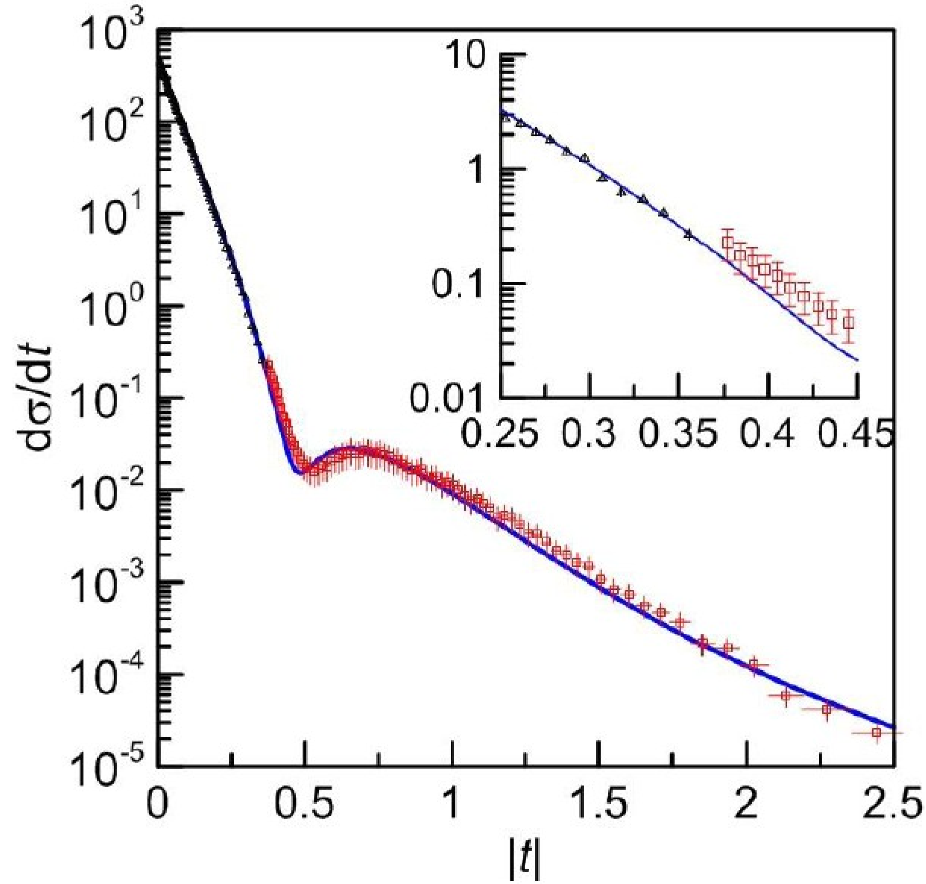
$$= \frac{1}{\alpha_K(s) + \lambda_K(s)}\left[\alpha_K(s)\beta_K(s) + \frac{1}{8}\lambda_K(s)a_0\Big(6\gamma_K(s) + 7\Big)\right]$$

Real and imaginary amplitudes together determined the dip/bump structure of the data.

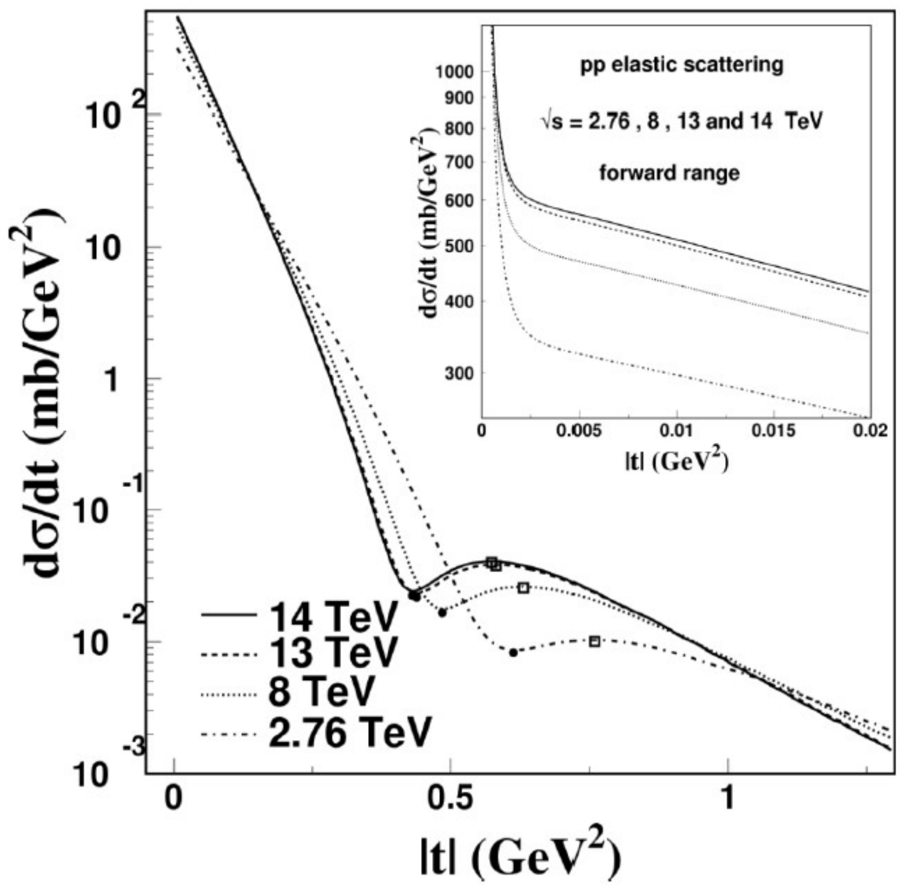


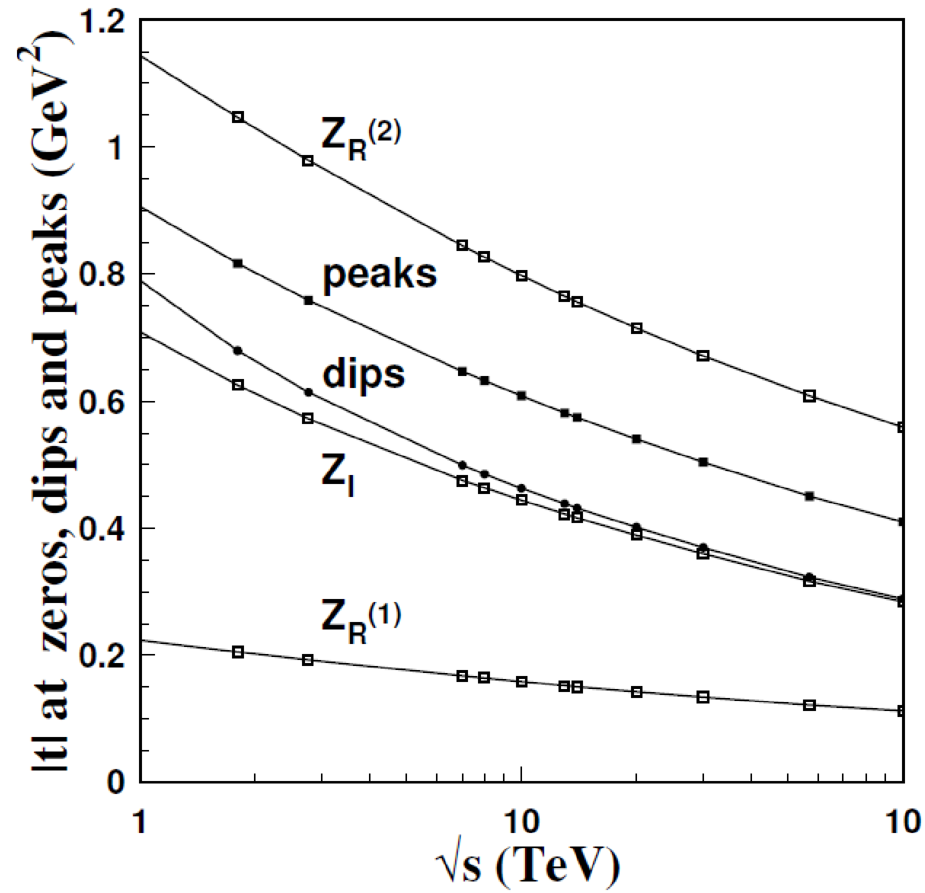
pp at 52 GeV (ISR-CERN)
and pp at 7 TeV (TOTEM-CERN)





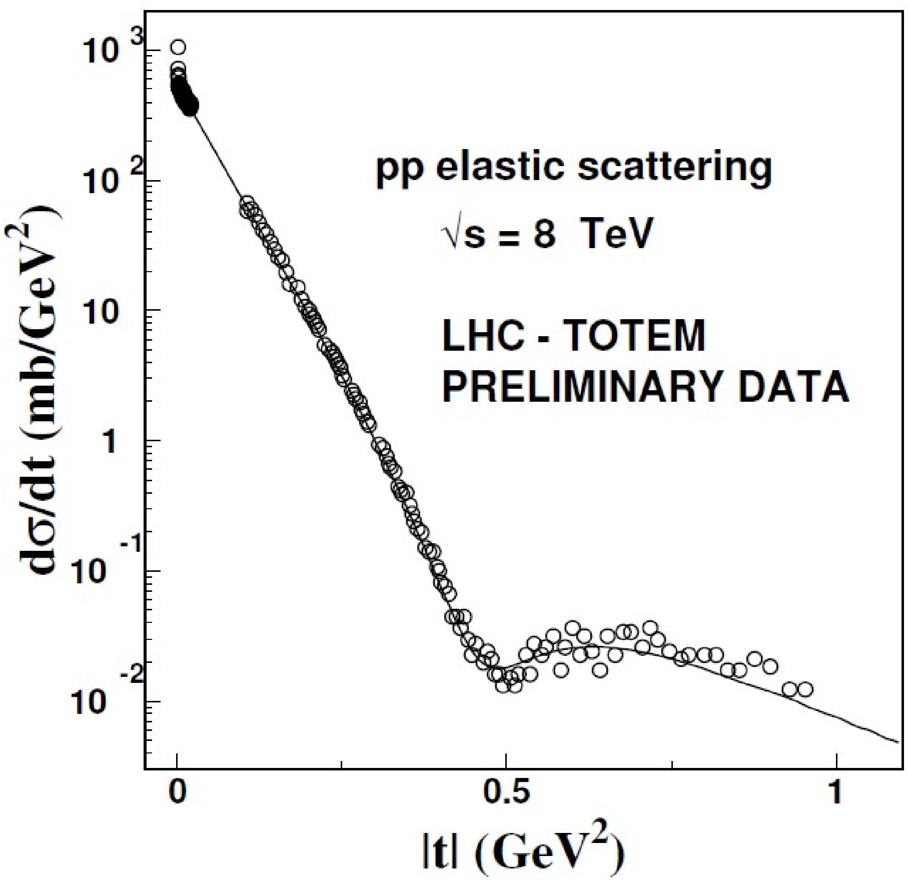
Predictions for LHC energies



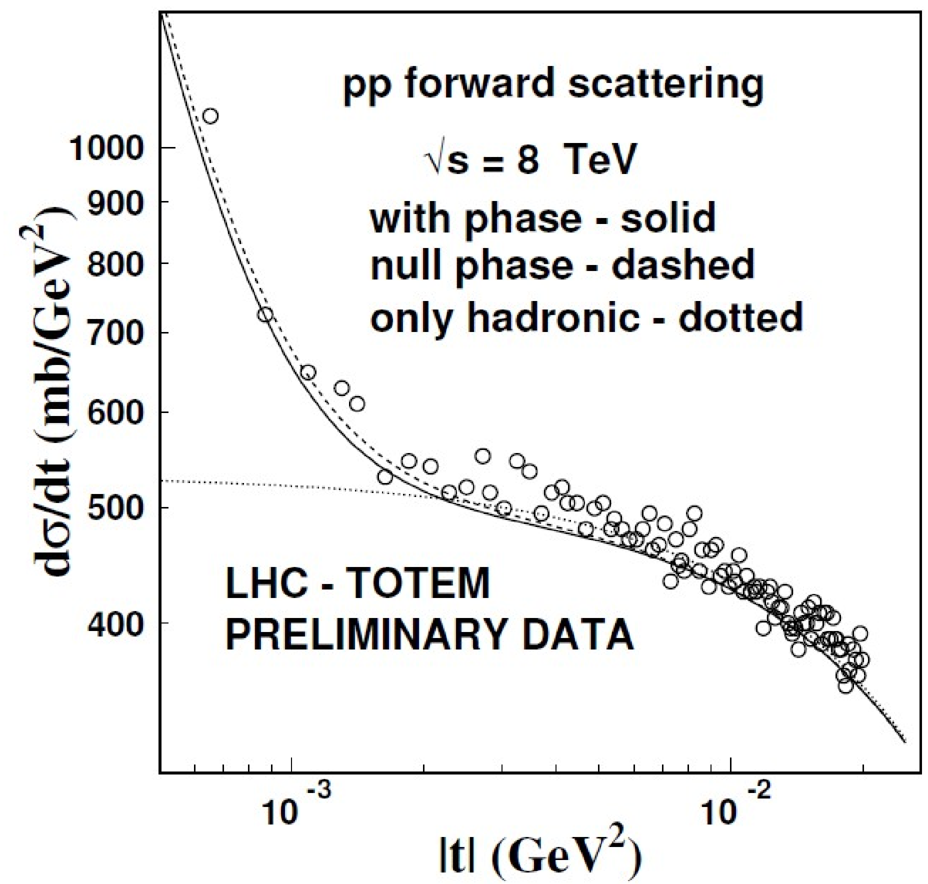


zeros, dips, peaks → $A + \dfrac{1}{a + b\log\sqrt{s} + c\log^2\sqrt{s}}$

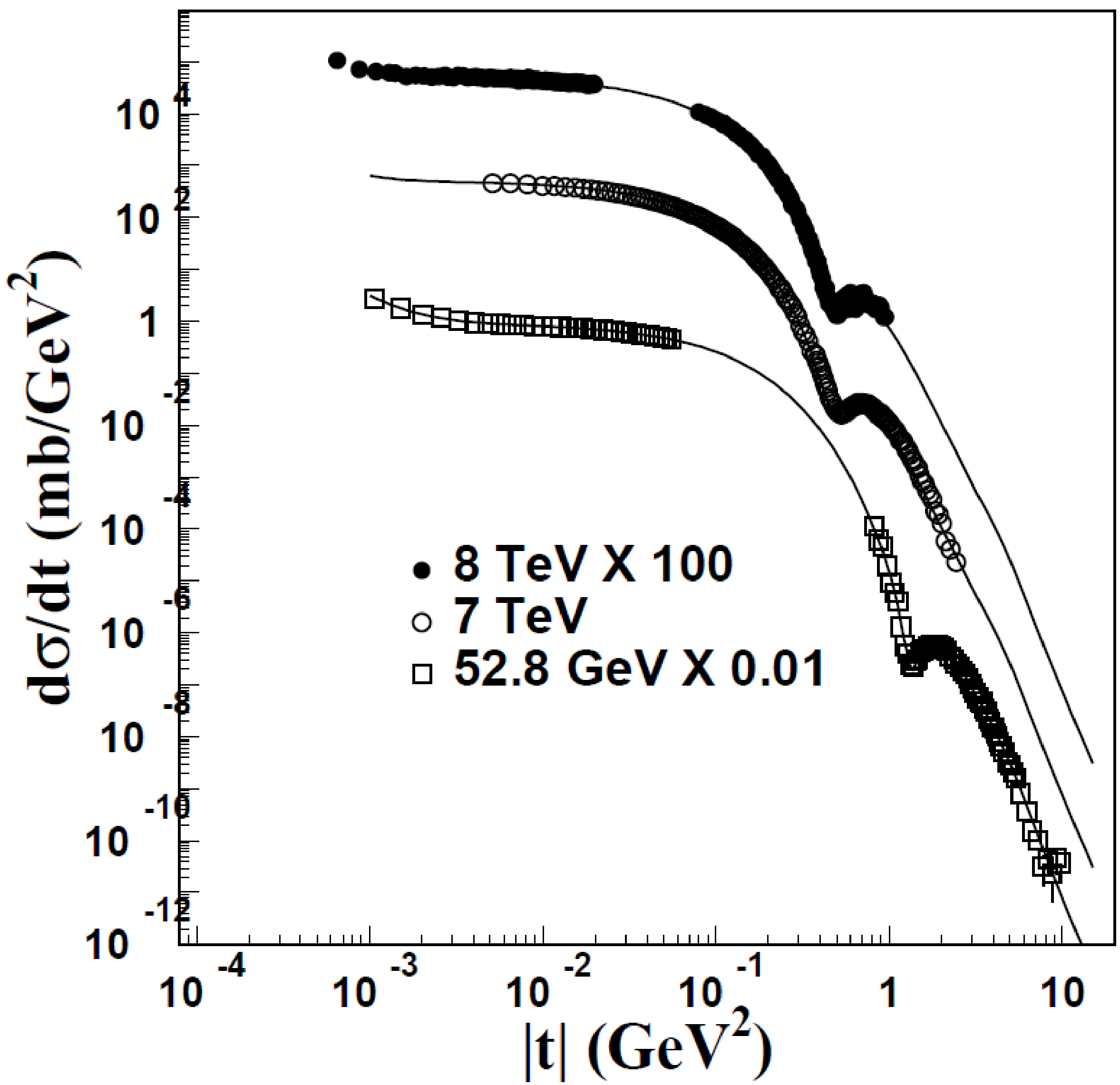
8 TeV case



pp elastic scattering
√s = 8 TeV
LHC - TOTEM
PRELIMINARY DATA
dσ/dt (mb/GeV²)
|t| (GeV²)



pp forward scattering
√s = 8 TeV
with phase - solid
null phase - dashed
only hadronic - dotted
LHC - TOTEM
PRELIMINARY DATA
dσ/dt (mb/GeV²)
|t| (GeV²)


dσ/dt (mb/GeV^2)
|t| (GeV^2)
8 TeV X 100
7 TeV
52.8 GeV X 0.01
10^4
10^2
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10

numbers of the model

$\sqrt{s}$ TeV	imaginary amplitude σ mb	B_I GeV^{-2}	α_I GeV^{-2}	β_I GeV^{-2}	real amplitude ρ	B_R GeV^{-2}	λ_R GeV^{-2}	β_R GeV^{-2}
1.8	77.21	17.17	11.8898	3.7175	0.1427	24.63	3.7566	1.2304
2.76	83.47	17.96	12.4689	3.8293	0.1431	26.08	4.0745	1.2959
7	98.65	19.77	13.7298	4.0745	0.1415	29.65	4.7667	1.4599
8	101.00	20.21	13.9107	4.1100	0.1411	30.21	4.8660	1.4858
13	109.93	21.35	14.5685	4.2409	0.1392	32.35	5.2271	1.5852
14	111.34	21.53	14.6689	4.26123	0.1389	32.68	5.2822	1.6011

$\sqrt{s}$ TeV	Z_I GeV^2	$Z_R(1)$ GeV^2	$Z_R(2)$ GeV^2	$\lvert t\rvert_{dip}$ GeV^2	$d\sigma/dt\rvert_{dip}$ mb/GeV^2	$\lvert t\rvert_{peak}$ GeV^2	$d\sigma/dt\rvert_{peak}$ mb/GeV^2	ratio R	σ_{inel} mb	σ_{el} mb	σ_{el}^I mb	σ_{el}^R mb	σ_{el}/σ
1.8	0.6250	0.2052	1.0464	0.6798	0.005832	0.8170	0.006627	1.1362	58.97	18.24	18.00	0.24	0.24
2.76	0.5723	0.1925	0.9788	0.6138	0.008248	0.7587	0.010080	1.2221	63.13	20.33	20.07	0.27	0.24
7	0.4757	0.1673	0.8445	0.4988	0.015339	0.6465	0.022841	1.4891	73.28	25.37	25.05	0.32	0.26
8	0.4635	0.1639	0.8267	0.4850	0.016571	0.6319	0.025466	1.5368	74.85	26.16	25.83	0.33	0.26
13	0.4225	0.1522	0.7654	0.4385	0.021558	0.5816	0.037378	1.7338	80.76	29.17	28.82	0.35	0.27
14	0.4166	0.1505	0.7565	0.4319	0.022397	0.5743	0.039593	1.7678	81.69	29.65	29.29	0.35	0.27

numbers of the model

INPUTS

$\sqrt{s}$ TeV	imaginary amplitude σ mb	B_I GeV^{-2}	α_I GeV^{-2}	β_I GeV^{-2}	real amplitude ρ	B_R GeV^{-2}	λ_R GeV^{-2}	β_R GeV^{-2}
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DERIVED QUANTITIES

$\sqrt{s}$ TeV	Z_I GeV^2	$Z_R(1)$ GeV^2	$Z_R(2)$ GeV^2	$\|t\|_{dip}$ GeV^2	$d\sigma/dt\|_{dip}$ mb/GeV^2	$\|t\|_{peak}$ GeV^2	$d\sigma/dt\|_{peak}$ mb/GeV^2	ratio R	σ_{inel} mb	σ_{el} mb	σ_{el}^I mb	σ_{el}^R mb	σ_{el}/σ
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pp inputs

A. K. Kohara , E. Ferreira and T. Kodama
arXiv:1408.1599v1 [hep-ph] 7 Aug 2014

energy dependence of the inputs in forward scattering

Forward differential cross section

$$\frac{d\sigma}{dt} = \pi(\hbar c)^2 \left\{ \left[\frac{\rho\sigma}{4\pi(\hbar c)^2} e^{B_R t/2}\right]^2 + \left[\frac{\sigma}{4\pi(\hbar c)^2} e^{B_I t/2}\right]^2 \right\}$$

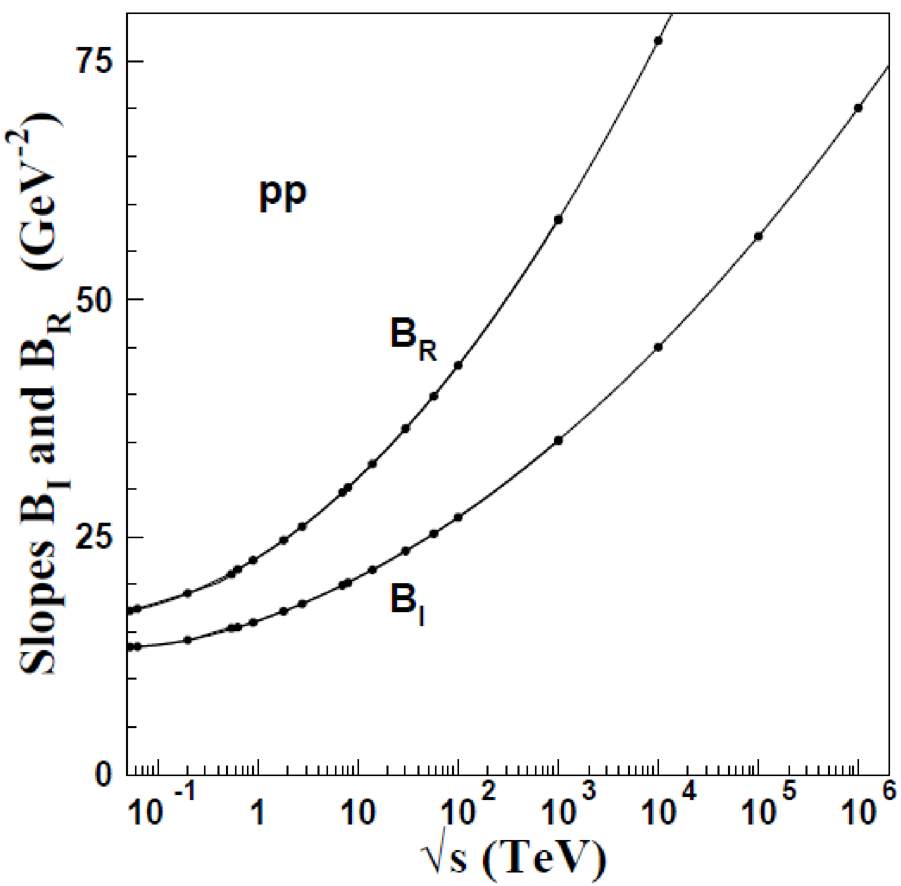
$$\sigma(s) = 69.3286 + 12.6800 \log\sqrt{s} + 1.2273 \log^2\sqrt{s}$$

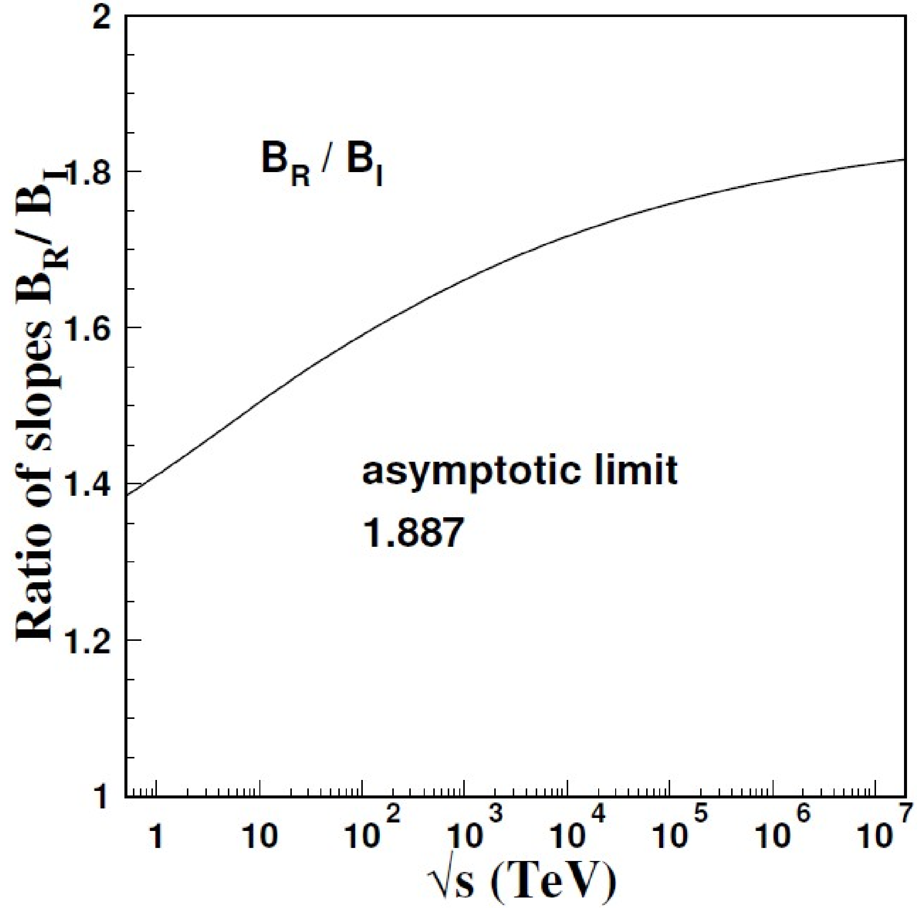
$$B_I(s) = 15.7848 + 1.75795 \log\sqrt{s} + 0.149067 \log^2\sqrt{s}$$

$$\rho(s) = \frac{3.528018 + 0.7856088 \log\sqrt{s}}{25.11358 + 4.59321 \log\sqrt{s} + 0.444594 \log^2\sqrt{s}}$$

$$B_R(s) = 22.8365 + 2.86093 \log\sqrt{s} + 0.329886 \log^2\sqrt{s}$$

Slopes of real and imaginary amplitudes





b-space (geometric space)

Fourier transform of the amplitudes

$$i\sqrt{\pi}\,\left(1 - e^{i\chi(s,\vec{b})}\right) \equiv \widetilde{T}(s,\vec{b}) = \widetilde{T}_R(s,\vec{b}) + i\widetilde{T}_I(s,\vec{b})$$

in eikonal formalism → $\chi(s,\vec{b}) = \chi_R(s,\vec{b}) + i\chi_I(s,\vec{b})$

with

$$\widetilde{T}_K(s,\vec{b}) = \frac{\alpha_K}{2\beta_K} e^{-b^2/4\beta_K} + \lambda_K \widetilde{\psi}_K(s,b)$$

$$\widetilde{\psi}_K(s,b) = \frac{2e^{\gamma_K - \sqrt{\gamma_K^2 + b^2/a_0}}}{a_0\sqrt{\gamma_K^2 + b^2/a_0}}\left[1 - e^{\gamma_K - \sqrt{\gamma_K^2 + b^2/a_0}}\right]$$

unitarity conditions

$$\frac{\widetilde{T}_R^2}{\pi} \leq e^{-2\chi_I(s,\vec{b})} \leq 1 \qquad \text{or} \qquad 0 \leq \chi_I \leq -\frac{1}{2}\log(\widetilde{T}_R^2/\pi)$$

satisfied by our solutions

In this space the cross sections are written

$$\sigma_{\rm el}(s) = \frac{(\hbar c)^2}{\pi} \int d^2\vec{b}\, |\widetilde{T}(s,\vec{b})|^2 \equiv \int d^2\vec{b}\, \frac{d\widetilde{\sigma}_{\rm el}(s,\vec{b})}{d^2\vec{b}}$$

$$\sigma(s) = \frac{2}{\sqrt{\pi}}(\hbar c)^2 \int d^2\vec{b}\, \widetilde{T}_I(s,\vec{b}) \equiv \int d^2\vec{b}\, \frac{d\widetilde{\sigma}_{\rm tot}(s,\vec{b})}{d^2\vec{b}}$$

$$\sigma_{\rm inel} = (\hbar c)^2 \int d^2\vec{b} \left(\frac{2}{\sqrt{\pi}} \widetilde{T}_I(s,\vec{b}) - \frac{1}{\pi} |\widetilde{T}(s,\vec{b})|^2 \right) \equiv \int d^2\vec{b}\, \frac{d\widetilde{\sigma}_{\rm inel}(s,\vec{b})}{d^2\vec{b}}$$

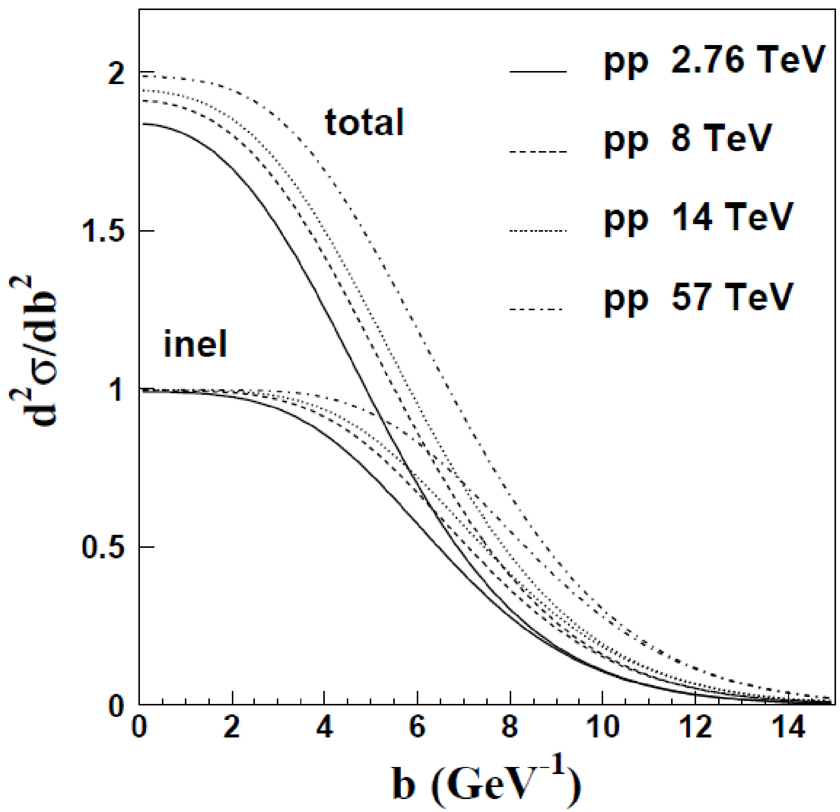
differential cross sections in terms of eikonal functions

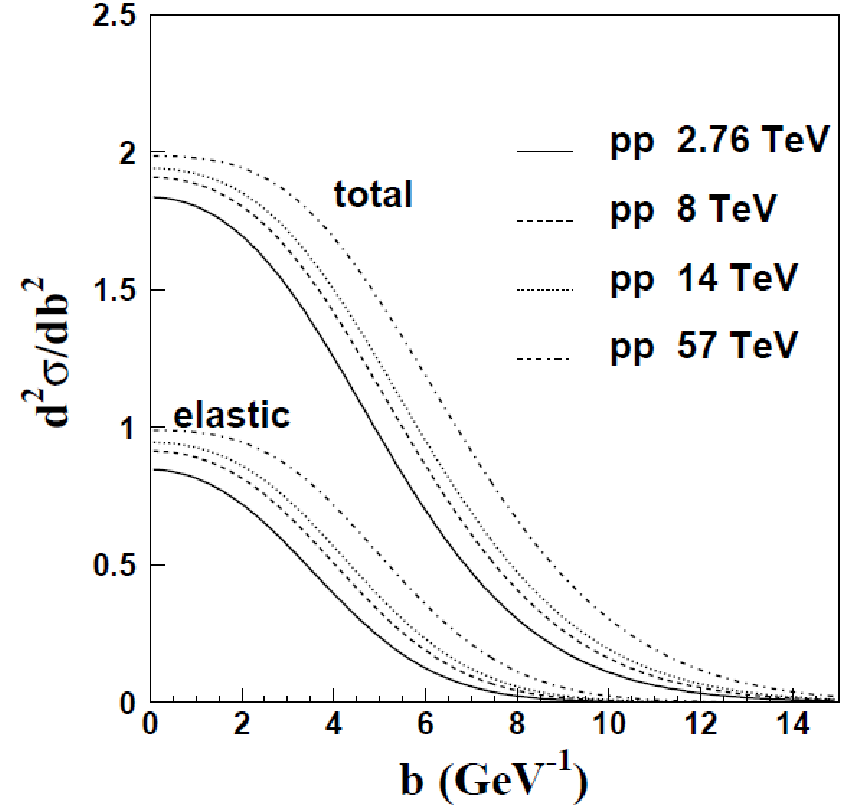
$$\frac{d\widetilde{\sigma}_{\rm el}(s,\vec{b})}{d^2\vec{b}} = 1 - 2\cos\chi_R e^{-\chi_I} + e^{-2\chi_I}$$

$$\frac{d\widetilde{\sigma}(s,\vec{b})}{d^2\vec{b}} = 2\left(1 - \cos\chi_R e^{-\chi_I}\right)$$

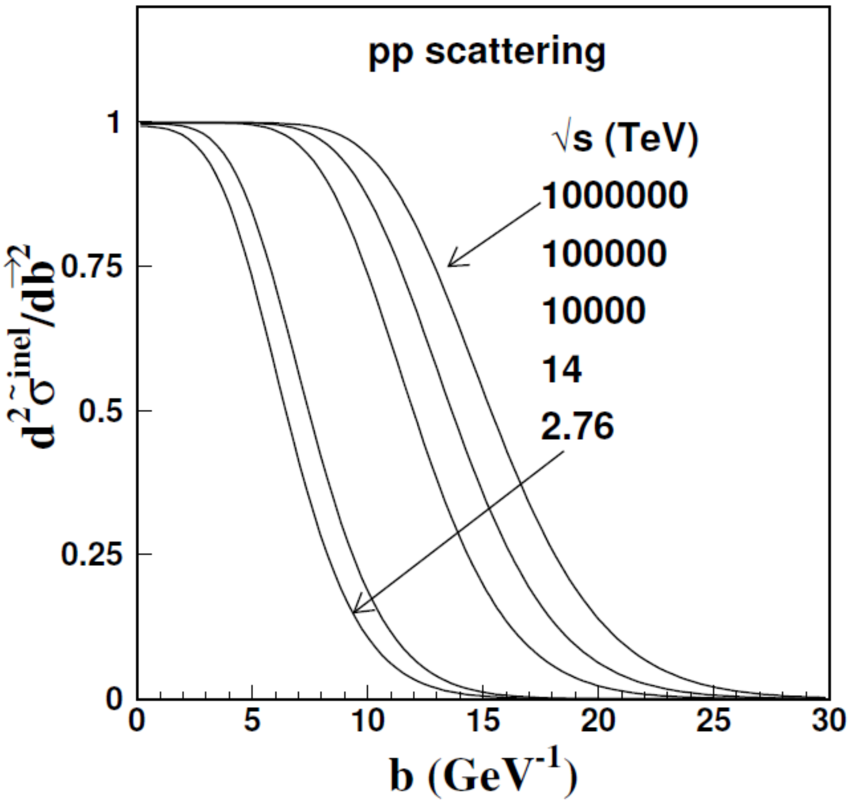
$$\frac{d\widetilde{\sigma}_{\rm inel}(s,\vec{b})}{d^2\vec{b}} = 1 - e^{-2\chi_I}$$

Monotonic behaviour of differential cross sections





...and this regular behaviour continues to asymptotic energies



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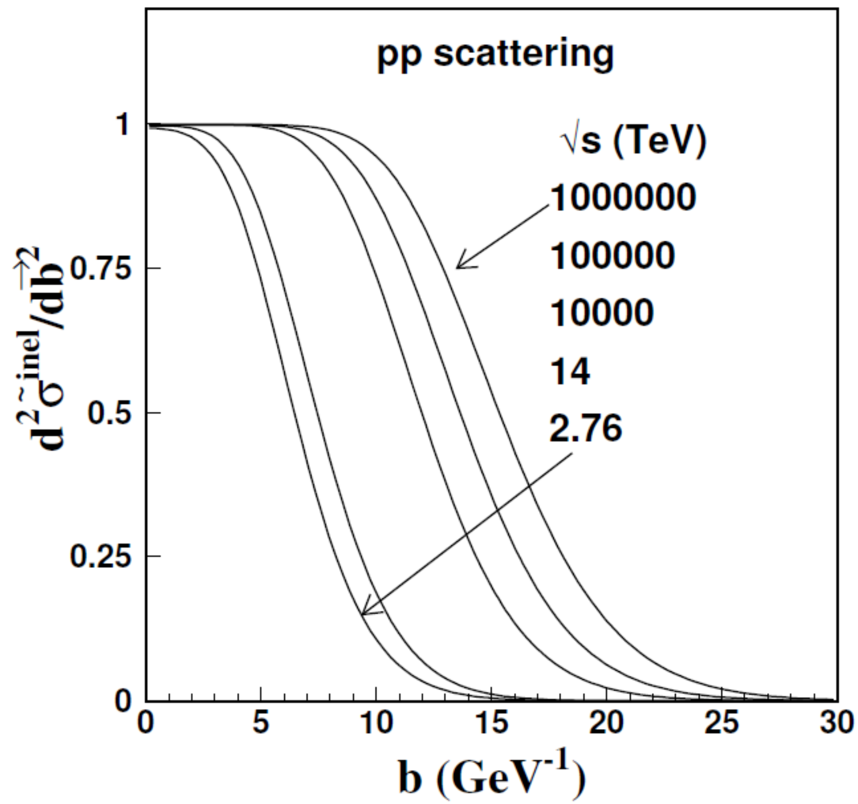
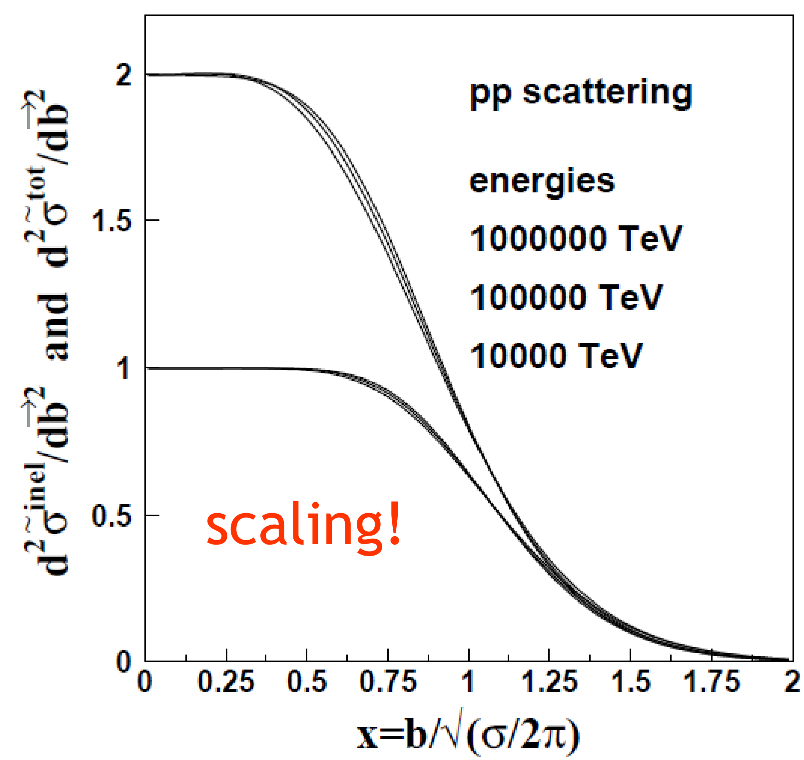


with a decreasing range

Observe a scaling variable

$$x = b/\sqrt{\sigma\left(\sqrt{s}\right)/2\pi}$$

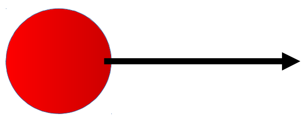


This property determines asymptotic behaviours

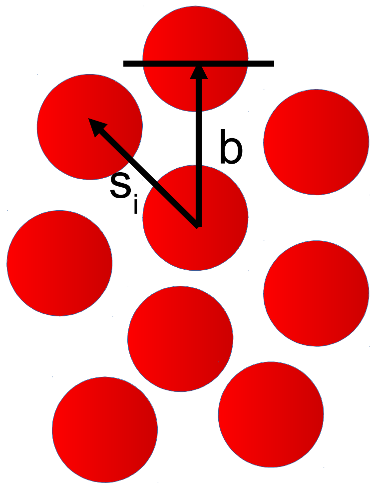
COSMIC RAY MEASUREMENTS

using our pp input, calculate p-air cross sections

p-air cross sections measurements in EAS (extended air showers)

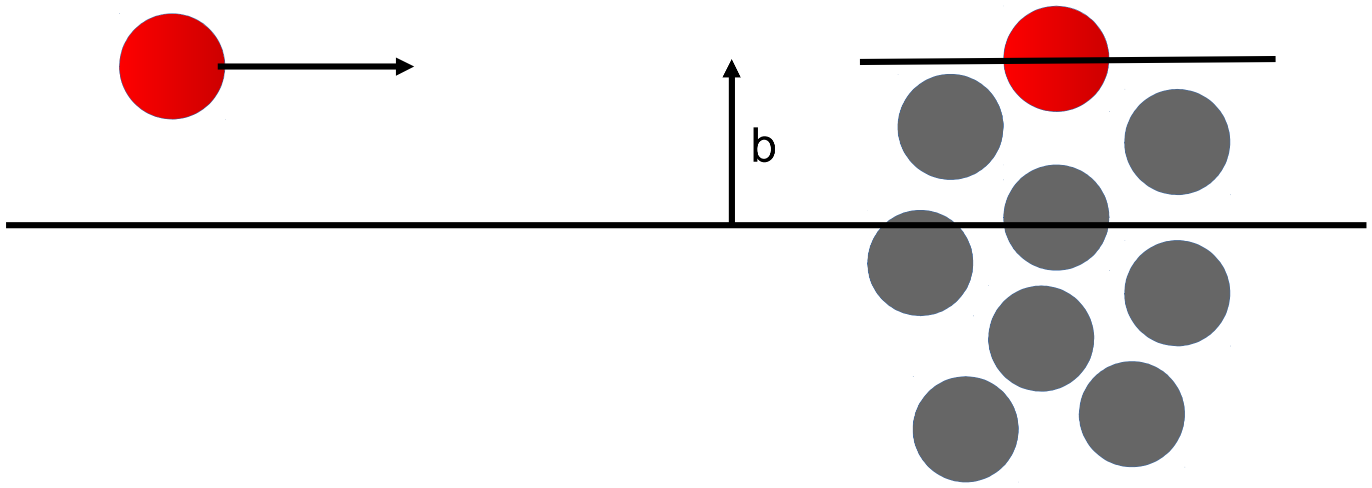


proton from cosmic ray



atom of atmosphere

...considering the nucleus composed by uncorrelated nucleons distributed according a wave function



This scattering is like one against everybody

Glauber framework

R. Engel and R. Ulrich , Internal Pierre Auger Note
GAP-2012, March 2012

forward amplitudes for pp elastic scattering

$$\widehat{T}_{\rm pp}(s,\vec{b}) = \widehat{T}_R(s,\vec{b}) + i\widehat{T}_I(s,\vec{b})$$
$$= \frac{\sigma^{\rm tot}_{\rm pp}}{4\pi(\hbar c)^2}\left[\frac{\rho}{B_R}e^{-\frac{b^2}{2B_R}} + i\frac{1}{B_I}e^{-\frac{b^2}{2B_I}}\right]$$

In terms of eikonal functions

S-matrix in b space

$$-\,i\;\widehat{T}_{\rm pp}(s,\vec{b}) \;=\; 1 - e^{i\chi_{\rm pp}(s,\vec{b})} \;\equiv\; \Gamma_{\rm pp}(s,\vec{b})$$

Optical theorem

$$\sigma^{\rm tot}_{\rm pp}(s) \;=\; 2\;(\hbar c)^2\;\Re\int d^2\vec{b}\;\Gamma_{\rm pp}(s,\vec{b})$$

Analogous optical theorem for p-Air

$$\sigma^{\rm tot}_{\rm pA}(s) \;=\; 2\;(\hbar c)^2\;\Re\int d^2\vec{b}\;\Gamma_{\rm pA}(s,\vec{b})$$

Glauber method introduces the p-A amplitude for A independent nucleons

$$\Gamma_{\rm pA}(s,\vec{b},\vec{s}_1,...,\vec{s}_A) = 1 - \prod_{j=1}^{A}\left[1 - \Gamma_{\rm pp}(s,|\vec{b}-\vec{s}_j|)\right]$$

We want to compute the production cross section defined by

$$\sigma^{\rm prod}_{\rm p-air} = \sigma^{\rm tot}_{\rm p-air} - (\sigma^{\rm el}_{\rm p-air} + \sigma^{\rm q-el}_{\rm p-air})$$

with

Nuclear density

$$\sigma^{\rm el}_{\rm pA} + \sigma^{\rm q-el}_{\rm pA} = (\hbar c)^2 \int d^2\vec{b} \int \left| 1 - \prod_{j=1}^{A} \left[1 - \Gamma_{\rm pp}(s, |\vec{b} - \vec{s_j}|) \right] \right|^2 \prod_{k=1}^{A} \rho_k(\vec{r_k})\, d^3\vec{r_k}$$

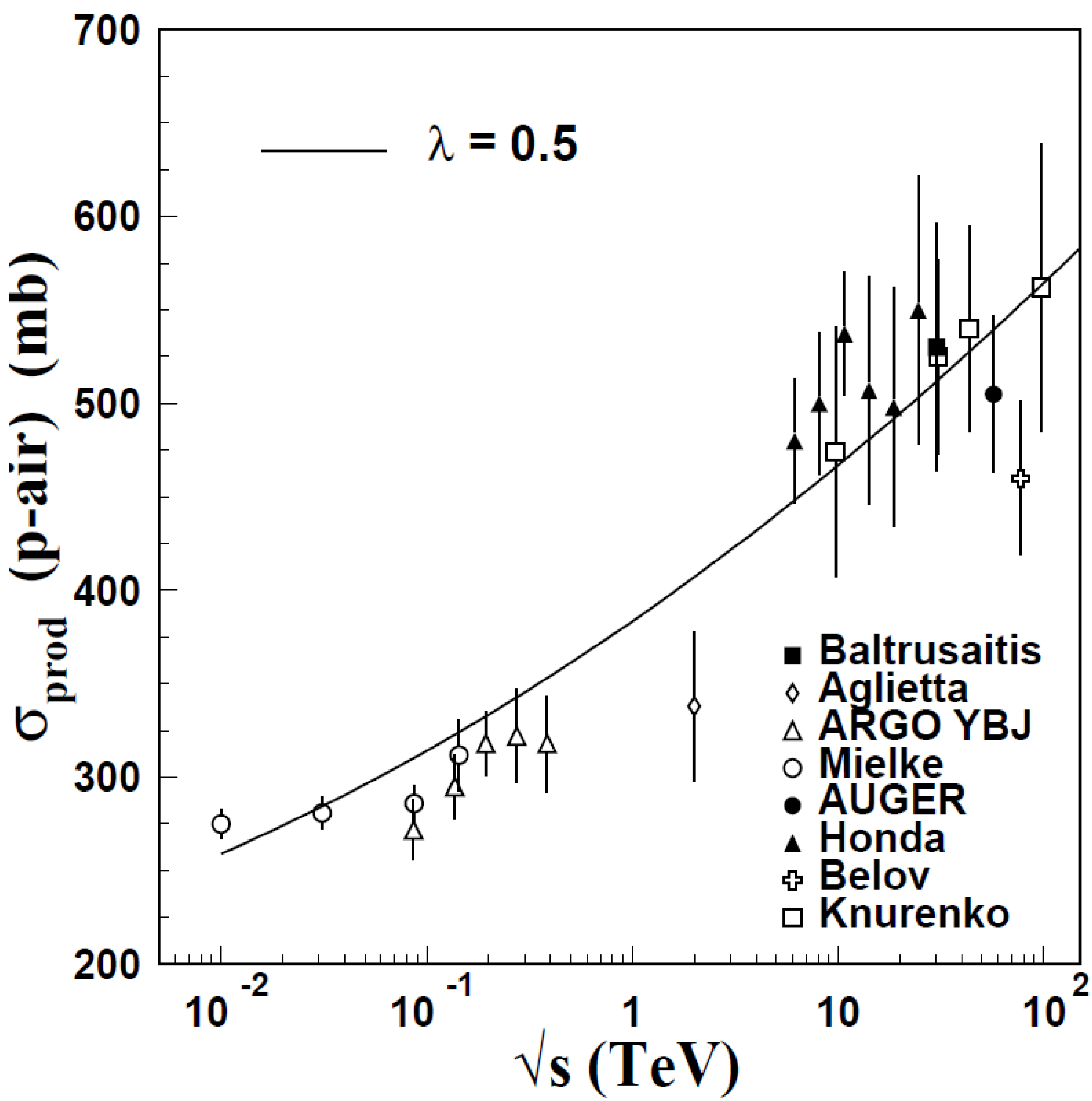
We also test the effects of the diffractive intermediate states according to the Good Walker framework (with a parameter λ) that modifies the p-air amplitude to

$$\Gamma_{\rm pA}(s, \vec{b}, \vec{s}_1, ..., \vec{s}_A) = 1 - \frac{1}{2} \prod_{j=1}^{A} \left[1 - (1+\lambda)\Gamma_{\rm pp}(\vec{b} - \vec{s}_j) \right] - \frac{1}{2} \prod_{j=1}^{A} \left[1 - (1-\lambda)\Gamma_{\rm pp}(\vec{b} - \vec{s}_j) \right]$$

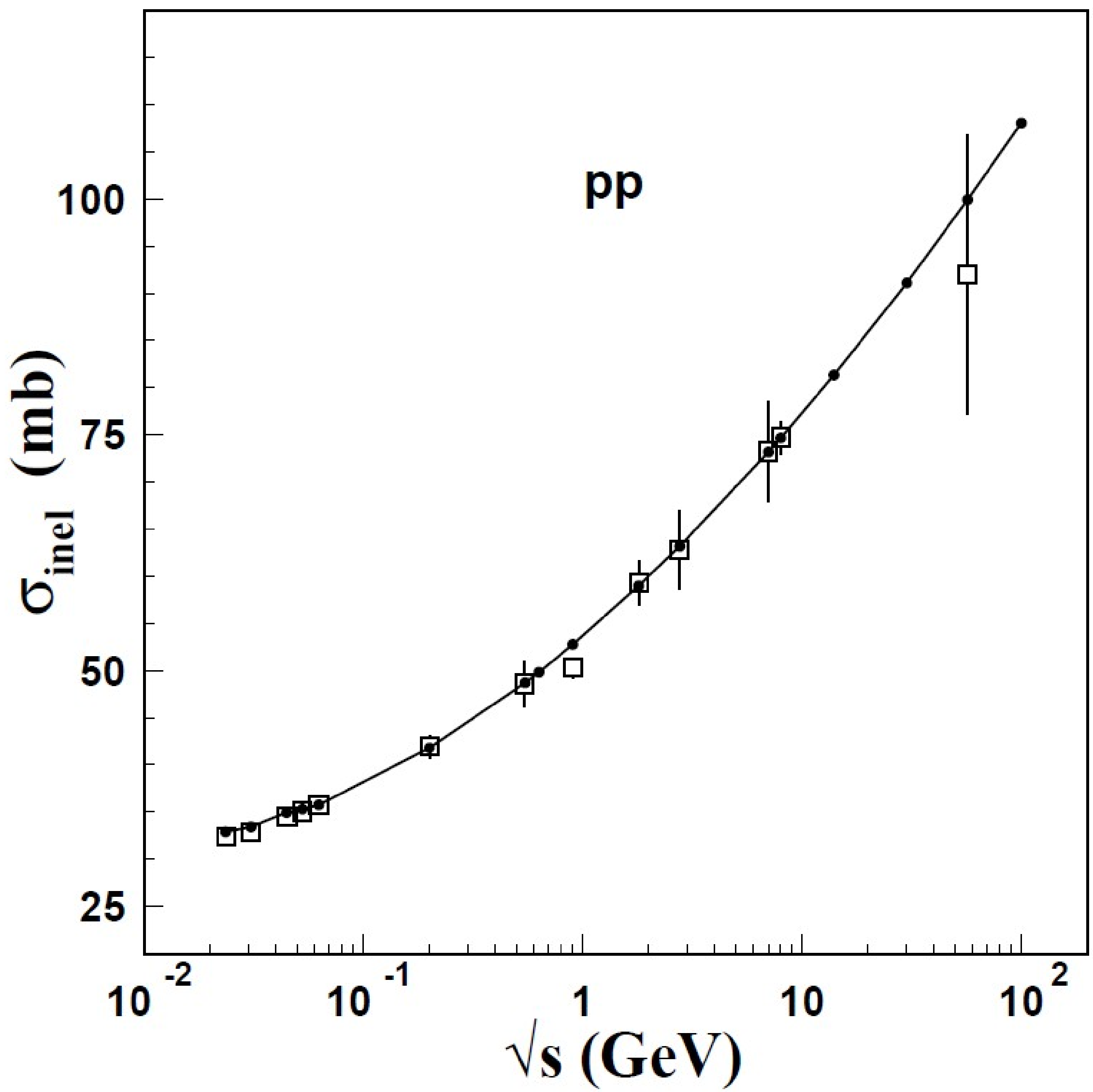
From the energy dependences in our input we obtain the parametrization for p-air production cross section with powers of $\log\sqrt{s}$.

$$\sigma^{\rm prod}_{\rm p-air}(s) = 383.474 + 33.158 \log\sqrt{s} + 1.3363 \log^2\sqrt{s}$$

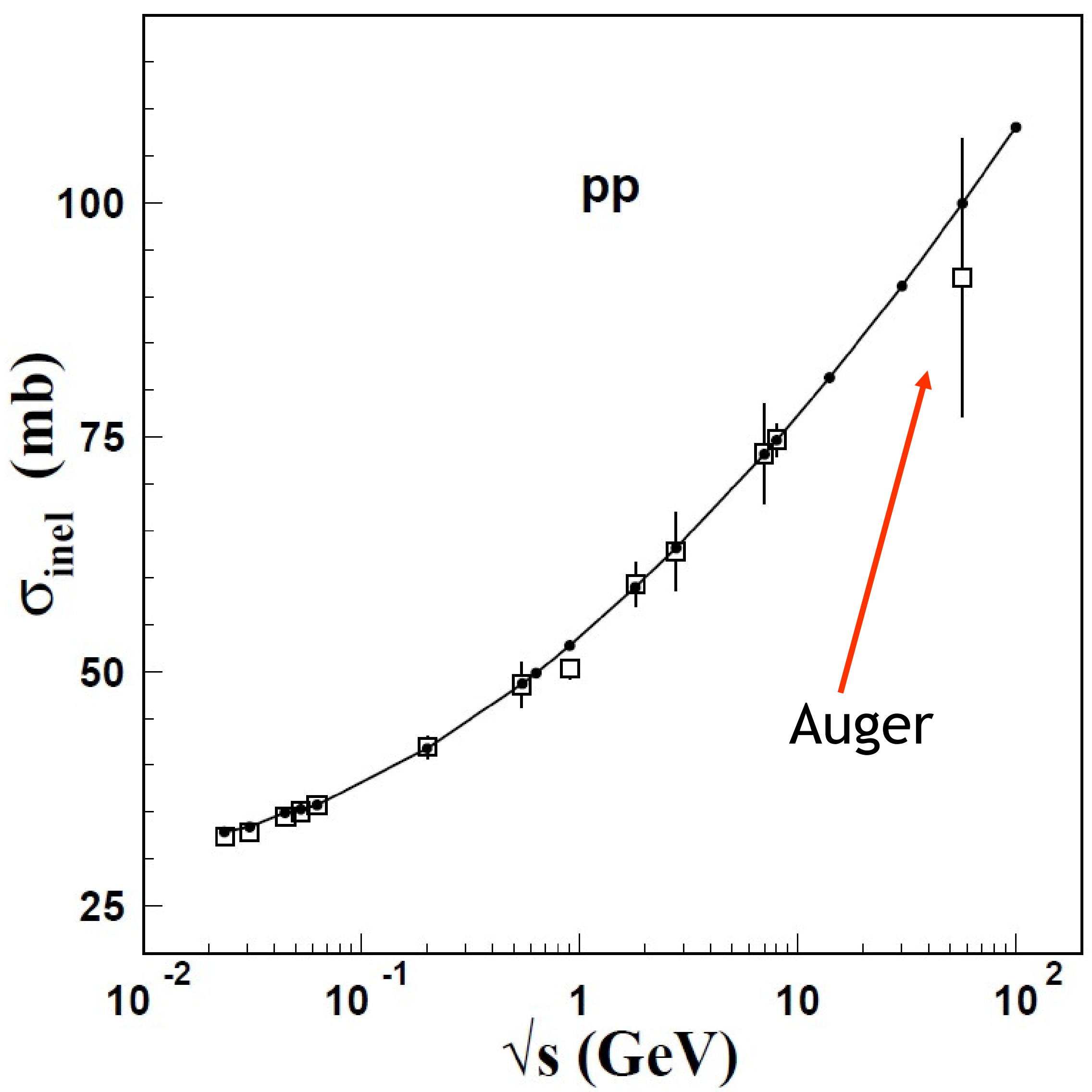
Comparison with p-air cosmic ray measurements



pp inelastic cross section



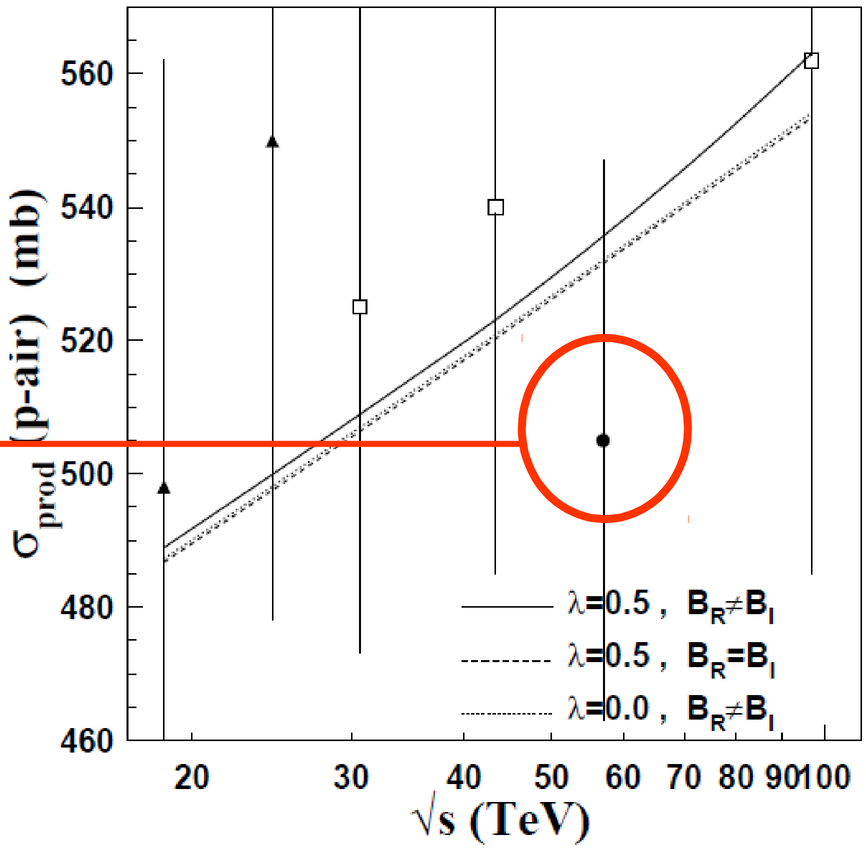
pp inelastic cross section



Effects of Good Walker diffractive states and B_R slope

$\sqrt{s} = 57$ TeV

λ	B_I	B_R	$\sigma^{\rm prod}_{\rm p-air}$
0.5	25.329	39.796	539.225
0.5	25.329	25.329	536.617
0.0	25.329	39.796	537.547
0.0	25.329	25.329	537.333



p-air in b-space

p-air elastic scattering amplitude

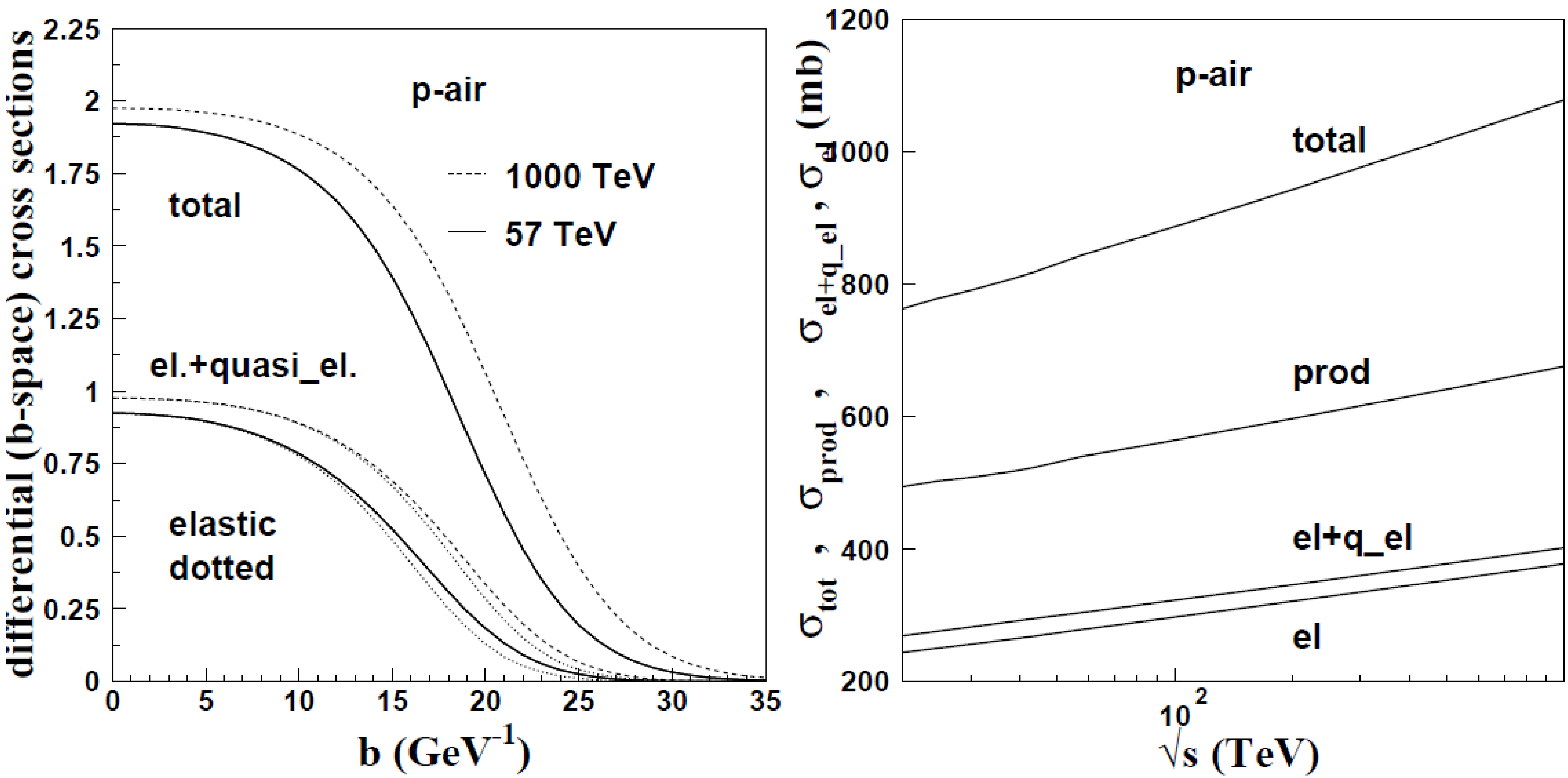
$$-i\widehat{T}_{\rm pA}(\vec{b}) = 1 - e^{i\chi_{pA}} \simeq 1 - \left\langle \prod_{j=1}^{A} e^{i\chi_{pN_j}} \right\rangle = 1 - \left\langle \prod_{j=1}^{A} \left(1 + i\widehat{T}_{\rm pN}(\vec{b})\right) \right\rangle$$

p-air distributions:

$$\frac{1}{2}\frac{d^2\sigma_{pA}^{\rm tot}}{d^2\vec{b}}(s,\vec{b}) = \left\langle 1 - \prod_{i=1}^{A}\left(1 - \frac{1}{2}\frac{d^2\sigma_{pp}^{\rm tot}}{d^2\vec{b_i}}(s,\vec{b}-\vec{b}_i)\right)\right\rangle$$

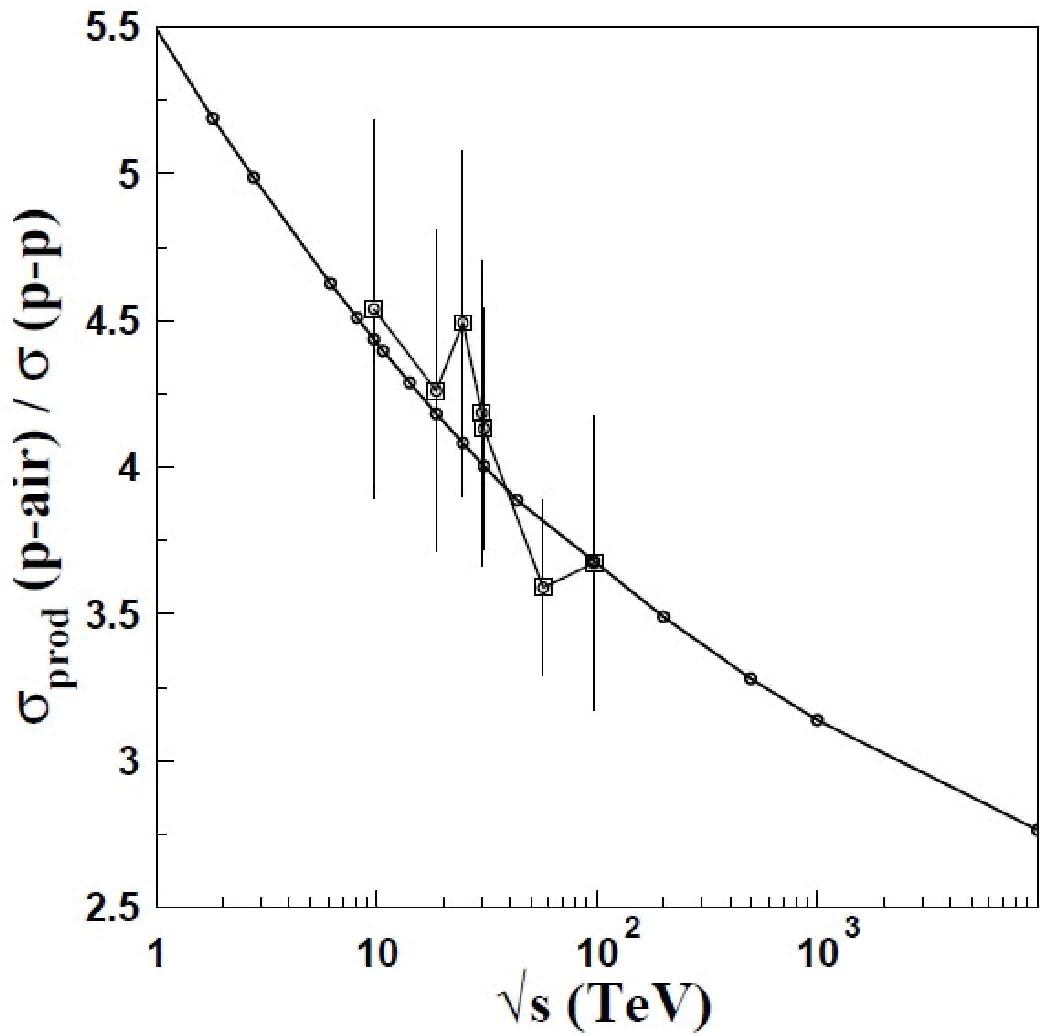
$$\frac{d^2\sigma_{pA}^{\rm el}}{d^2\vec{b}}(s,\vec{b}) = \left\langle \left[1 - \prod_{i=1}^{A}(1 - \frac{d^2\sigma_{pp}^{\rm tot}}{d^2\vec{b_i}}(s,\vec{b}-\vec{b}_i))\right]^2 \right\rangle$$

p-air cross section predictions



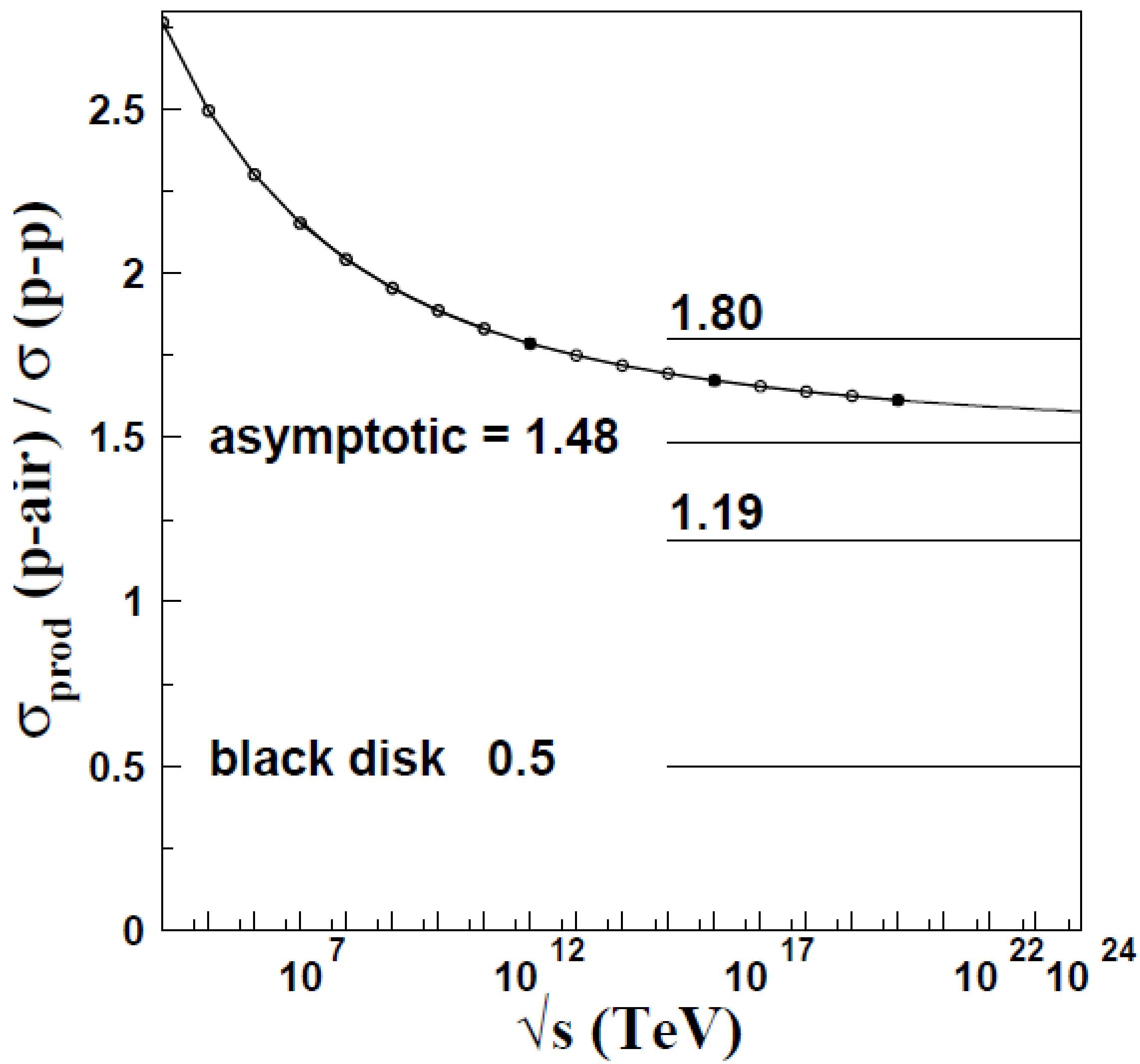
ratio of p-air/pp cross sections

$$\frac{\sigma^{\mathrm{prod}}_{\mathrm{p-air}}(s)}{\sigma_{\mathrm{pp}}(s)} = \frac{383.474 + 33.158\log\sqrt{s} + 1.3363\log^2\sqrt{s}}{69.3286 + 12.6800\log\sqrt{s} + 1.2273\log^2\sqrt{s}}$$



The ratio tends to a finite value in the asymptotic limit

asymptotic limit of ratio p-air / pp



A. K. Kohara , E. Ferreira and T. Kodama
arXiv:1406.5773v1 [hep-ph] 22 Jun 2014

Conclusions

- We believe that we have realistic pp inputs, with energy dependence.
- The simplest Glauber calculation accounts for the C.R measurements of p-air production cross section at all energies 1 TeV – 100 TeV.
- We give predictions for energies beyond present experiments and for an asymptotic regime.

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Thanks