# Shower center of gravity and interaction characteristics

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ISVHECRI 2014

CERN 21th August

## Which hadron-air interaction characteristics define shower longitudinal development?

- ❑ Cross-sections?
- ❑ Inelasticities?
- ❑ Multilicities?
- ❑ Diffraction?

There is a long history of attempts to explicitely connect air shower longitudinal profile, in particular depth of shower maximum (Xmax), with hadronic interaction properties.
Very approximate approaches were tried, from toy models to extension of Heitler model for the electromagnetic shower to the hadronic shower. Not of big quantitative help.

## Why not to use directly cascade theory?

Because problem: shower maximum is inconvenient quantity for cascade equations.
Rather, convenient quantity is shower center of gravity (CG).

$$\overline{X(E)} = \int_0^\infty X\, N(X)\, dX \Big/ \int_0^\infty N(X)\, dX$$

Although shift between Xmax and CG is energy and generator dependent, having been successfully solved equations for CG would allow to look in detail into dynamics of shower longitudinal development, which governs both Xmax and CG, and to see explicitely how interaction characteristics enter that dynamics.

# Expression for center of gravity

**On assumptions of**

- Feinman scaling
- Cascading only two types of hadrons: barions (nucleons) and pions
- Neglect of decay of charged pions
- Logarithmically rising p-air and π-air cross-sections

it is possible to obtain **exact expression** for proton (i.e. nucleon) center of gravity (arXiv:1202.4989) :

$$\overline{X_N(E)} = X_0 \left( \ln \frac{E}{E_c} + \delta - \frac{1}{2} \right) + \frac{1}{1-g_{NN}} \left\{ \lambda_N(E_N^{eff}) + X_0 \cdot \mu_N + \frac{g_{N\pi}}{1-g_{\pi\pi}} \left[ \lambda_\pi(E_\pi^{eff}) + X_0 \cdot \mu_\pi \right] \right\}$$

Interaction lengths have proven to be taken not at the primary energy but at some lower effective energies:

$$E_N^{eff} = E \cdot exp\left( \frac{\gamma_{NN}}{1-g_{NN}} \right), \qquad E_\pi^{eff} = E_N^{eff} \cdot exp\left( \frac{\gamma_{N\pi}}{g_{N\pi}} + \frac{\gamma_{\pi\pi}}{1-g_{\pi\pi}} \right)$$

# Integrals of energy transition

**Characteristics of particle production enter two kinds of expressions.**

First kind reflects energy transition between different sorts of hadrons, i.e. from barion to charged or neutral pions or from charged pion to neutral pions (i,j below denote sort). The obtained expressions are simply mean relative energies contained in produced particles of some sort (like inelasticity):

$$g_{ij} = \frac{1}{\sigma^{i}_{inel}(E_i^{eff})} \int_0^1 x \, \frac{d\sigma_{i \to j}}{dx}(x, E_i^{eff}) \, dx = \int_0^1 x \, \frac{dn_{i \to j}}{dx}(x) \, dx$$

Energy transition governs pace of shower elongation through hadron cascading.

# Integrals of energy dissipation

Second kind reflects rate of energy dissipation:

$$\gamma_{ij} = \int_0^1 x \cdot log(x) \frac{dn_{i \to j}}{dx}(x)\, dx \ ,$$

splitting energy of particle of type **i** into energies of produced particles of type **j**

$$\mu_i = \gamma_{iX} = \int_0^1 x \cdot log(x) \frac{dn_{i \to X}}{dx}(x)\, dx$$

splitting energy of particle of type **i** into energies of all produced particles

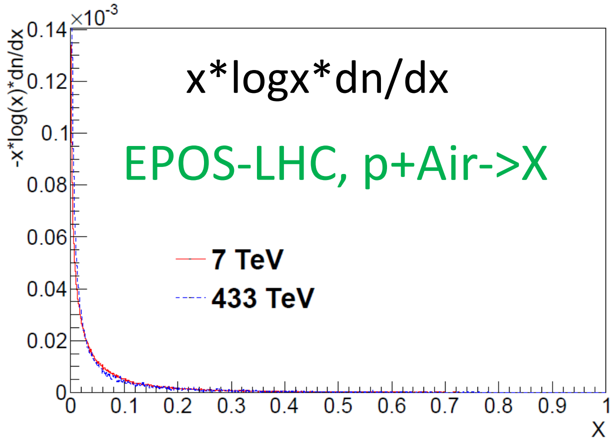
Meaning of weight **x · log(x)** is clear: each particle contributes to the total center of gravity proportionally to its energy and to longitudinal width of produced at this energy electromagnetic shower which is proportional to logarithm of energy.

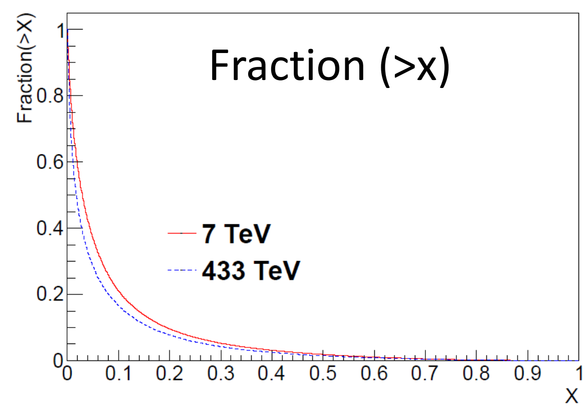
These factors are **negative**: dissipation of energy in hadronic cascade results in electromagnetic subcascades starting at smaller and smaller energies and thus in reducing total center of gravity relative to absense of dissipation.

**Total mutiplicity** is usually considered as interaction property responsible for energy dissipation. Instead, energy dissipation here is defined by above integrals which are **forward multiplicities** (next slide).

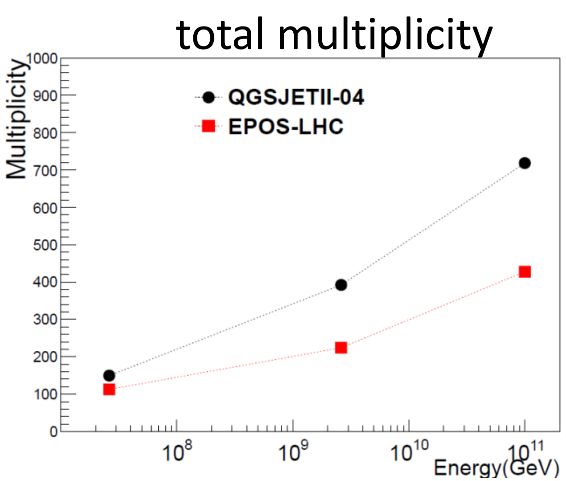
# Forward multiplicity

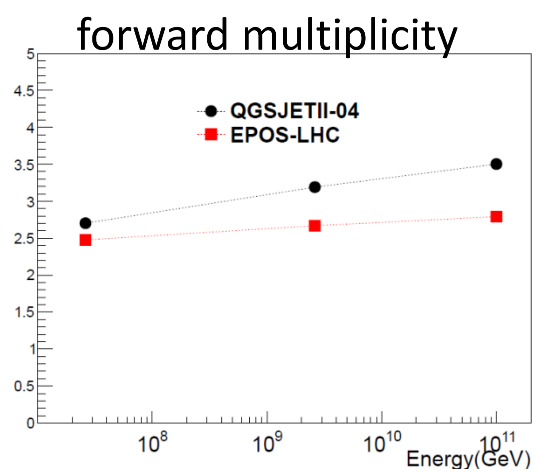
## Why forward?




**Because ~75% of integral is contained at x>0.01**

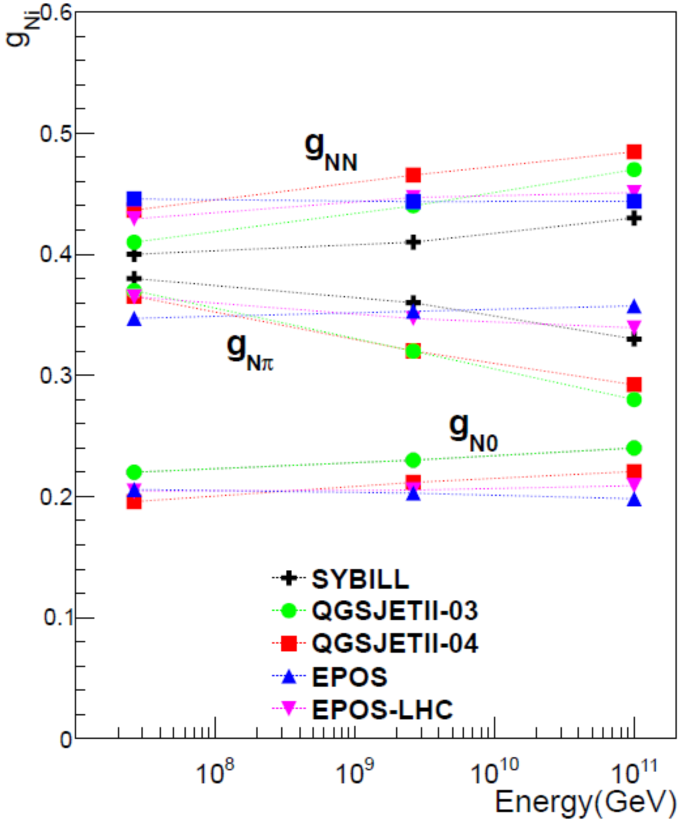


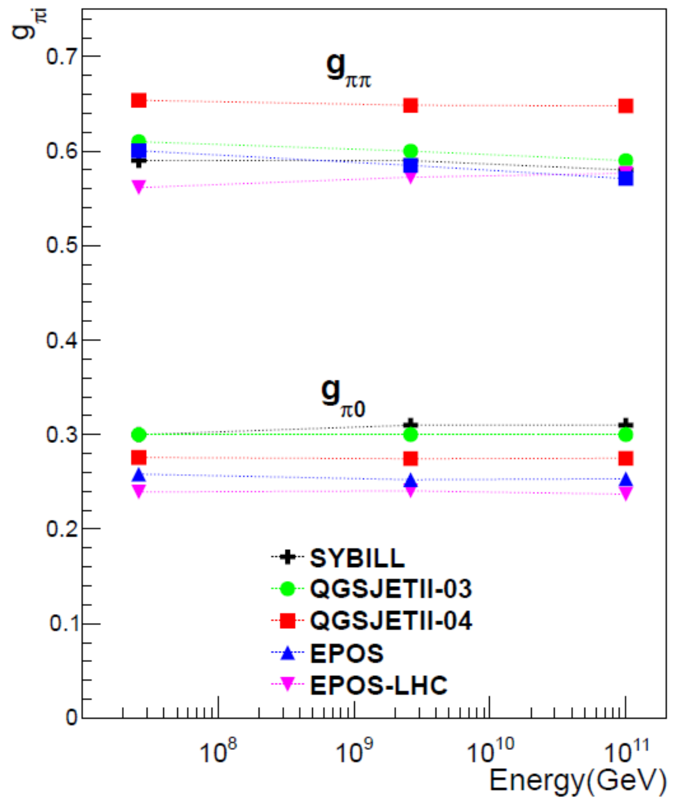

- Total multiplicity rises much faster
- Difference in total multiplicity between MC is bigger

# Integrals defining energy transitions

$$g_{ij} = \int_0^1 x \, \frac{dn_{i\to j}}{dx}(x)\, dx$$




$\lambda_{\pi\pi}$

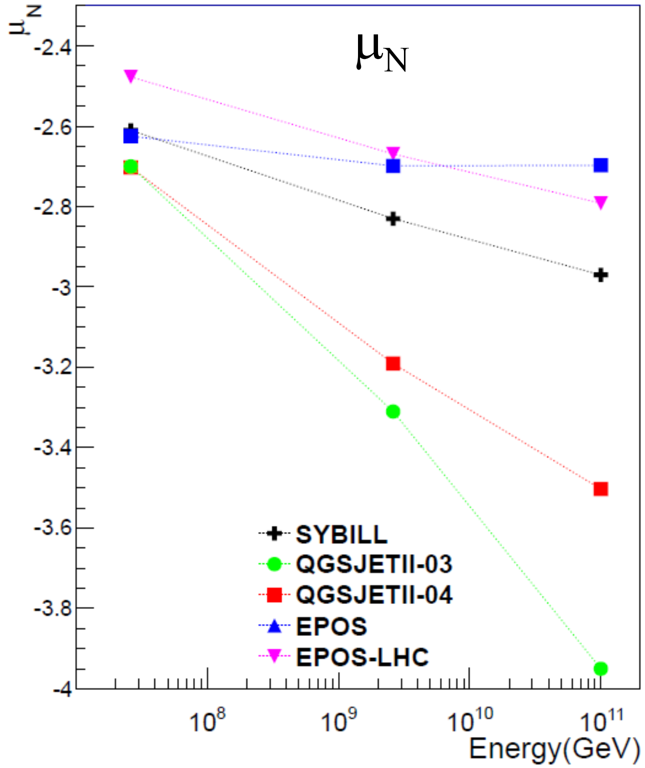


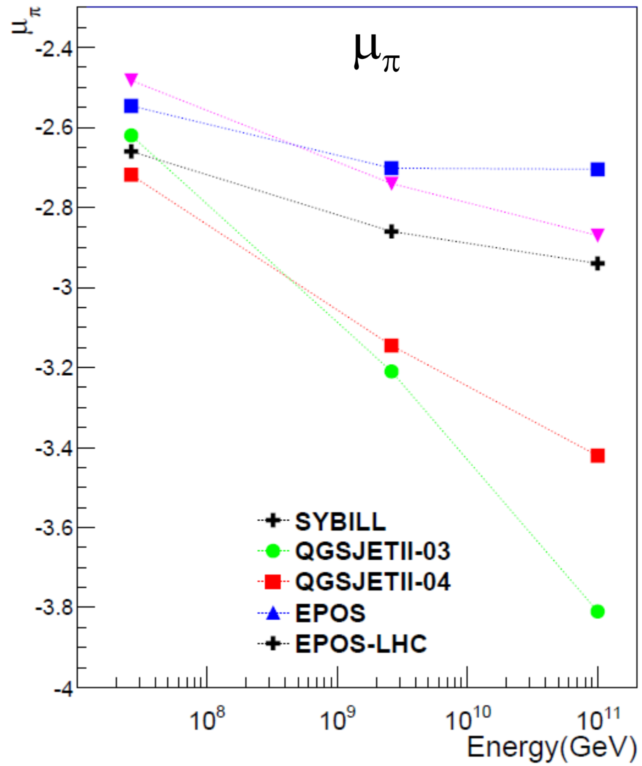

Big spread, starting from LHC energy (first point)!

Pion elasticity ($g_{\pi\pi}$) in QGSJETII-04 much above other MC

# Integrals defining energy dissipation

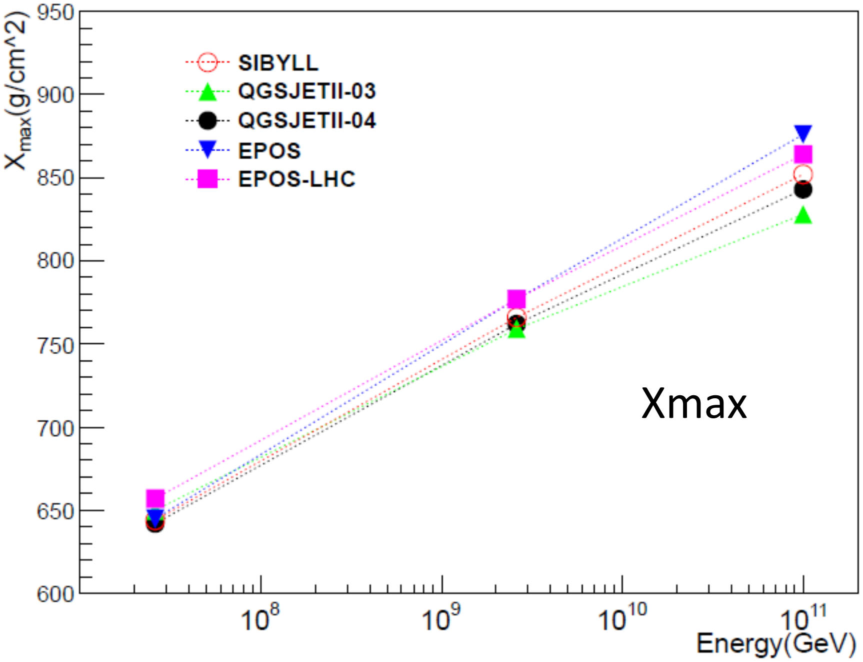
$$\mu_i = \int_0^1 x \cdot log(x) \frac{dn_{i \to X}}{dx}(x)\, dx$$

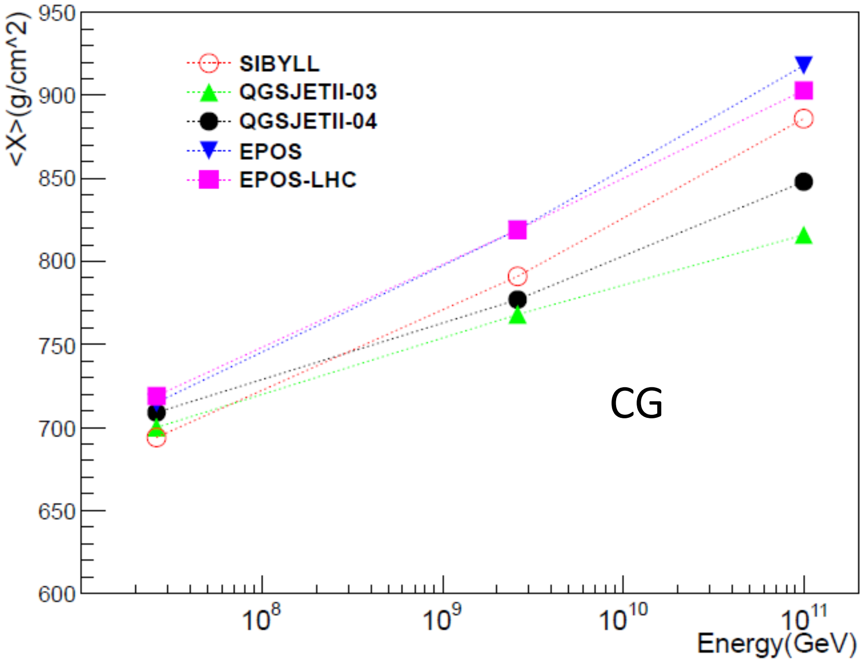



- Very similar behavior for pion and nucleon.
- Spread significantly increases with energy
- Forward multiplicity in two QGSJETII much higher than in other MCs => larger energy dissipation => smaller CG (smaller Xmax)

# Center of gravity vs shower maximum




Disposition of MCs is similar for CG and Xmax but spread of CG is larger.

The reason is that CG represents the whole shower while Xmax represents developing stage of shower. Accordingly number of contributing generations of interactions is larger for CG. Since effect of variation of interaction characteristics multiplicates with generations spread in CG exceeds spread in Xmax.

# Expression for center of gravity

**Expression for the center of gravity splits into two terms:**

- center of gravity of the purely electromagnetic cascade at the proton primary energy

$$X_0 \left( \ln \frac{E}{E_c} + \delta - \frac{1}{2} \right)$$

- modification of this by hadronic cascading.

$$\frac{1}{1 - g_{NN}} \left\{ \lambda_N(E_N^{eff}) + X_0 \cdot \mu_N + \frac{g_{N\pi}}{1 - g_{\pi\pi}} \left[ \lambda_\pi(E_\pi^{eff}) + X_0 \cdot \mu_\pi \right] \right\}$$

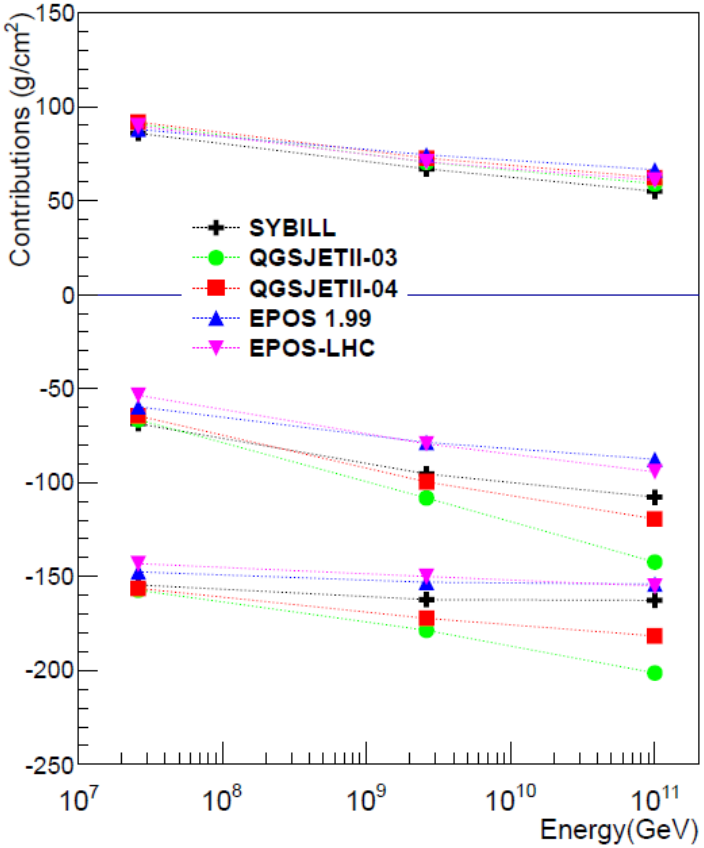
There is a competition between λ-terms and μ-terms in the hadronic part.

- The former are positive and reflect elongation of shower by carrying through energy by hadrons in cascading.
- The latter are negative and reflect dissipation of energy in interactions.

According to which terms prevail, proton center of gravity will be either larger or smaller than that of purely electromagnetic shower of the same energy.

It is worth to consider nucleon and pion contributions separately=>

# Nucleon contribution



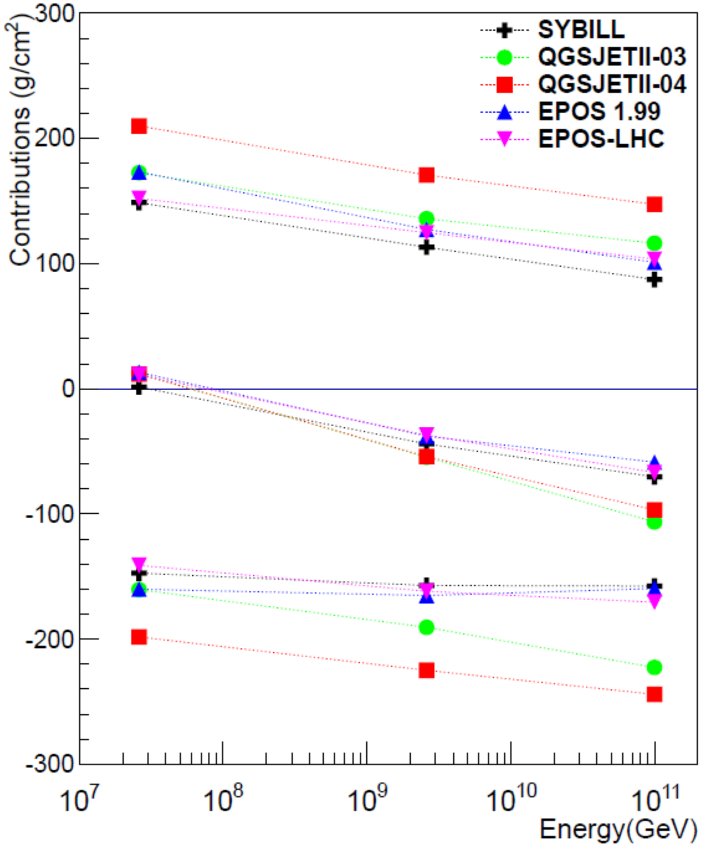

$\lambda_N$ contribution = $\dfrac{\lambda_N(E_N^{eff})}{1 - g_{NN}}$

$\lambda_N$ contribution + $\mu_N$ contribution

$\mu_N$ contribution = $\dfrac{X_0 \cdot \mu_N}{1 - g_{NN}}$

- Small spread of $\lambda$ term.
- Main spread in nucleon contribution comes from $\mu$ term.
- For Xmax, spread due to difference in nucleon interaction characterisics should be smaller due to smaller number of contributing generations.

# Pion contribution



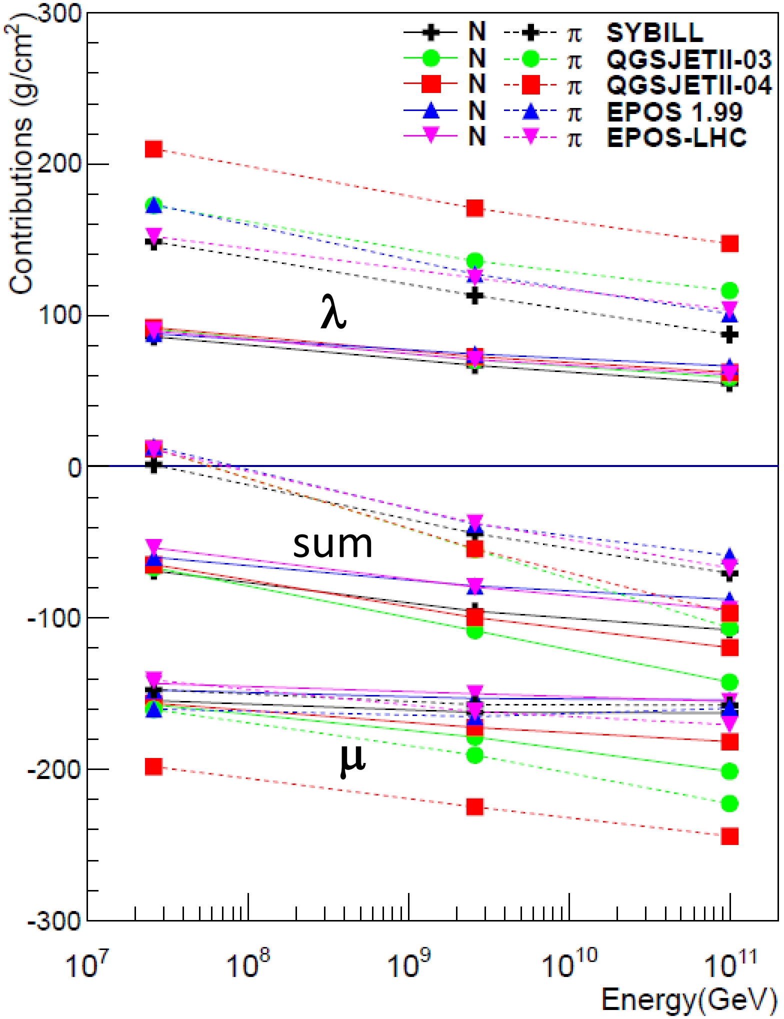

$$\lambda_\pi \text{ contribution} = \frac{1}{1-g_{NN}} \cdot \frac{g_{N\pi}}{1-g_{\pi\pi}} \cdot \lambda_\pi(E_\pi^{eff})$$

$\lambda_\pi$ contribution + $\mu_\pi$ contribution

$$\mu_\pi \text{ contribution} = \frac{1}{1-g_{NN}} \cdot \frac{g_{N\pi}}{1-g_{\pi\pi}} \cdot X_0 \cdot \mu_\pi$$

$\lambda_\pi$ contribution for QGSJETII-04 stands out of others due to distinct value of $\mathbf{g}_{\pi\pi}$ (slide 8). It proves to be compenstated in the sumed contribution by much smaller than in other MC value of $\mu_\pi$.

# All contributions




Spread of pion contributions bigger

Total elongation rate more influenced by pions
Spread increases with energy

Spread of pion contributions bigger

# About diffraction

**How nesessary is accurate descripion of hadron projectile diffraction to low masses? How nesessary is accurate descripion of target diffraction?**

First, let's answer question: how important is correct description of elastic scattering?

Answer: **not important at all**.

The only requirement is increase of cross-section due to elastic scattering being equal to fraction of elastic scatterings in simulated events.

Target diffraction corresponds to elastic scattering.

The lower mass of projectile diffractive system the closer the interaction to elastic scattering and the smaller contribution of such events to shower development and the less sensitive shower to description of such events (with above requirement fulfilled).

For CG that can be illustrated explicitely=>

# About diffraction

Expression for center of gravity could be rewritten as follows:

$$L_N = \frac{C + t_0 \cdot \phi_N}{f_{N\pi}} + \frac{f_{N\pi}}{f_{N\pi}} \frac{(C + t_0 \cdot \phi_\pi)}{f_{\pi\pi 0}} + t_0 \, ( ln \frac{E}{\beta} + 1.2)$$

$$f_{ij} = \int_0^1 x \, \frac{d\sigma_{i \to j}}{dx} (x, E_i^{eff}) \, dx$$

$$\phi_i = \int_0^1 x \cdot log(x) \, \frac{d\sigma_{i \to X}}{dx} (x, E_i^{eff}) \, dx$$

$$C = 24100 \, g/cm^2 \cdot mbarn$$

Diffraction to low masses is characterized by small number of secondary particles with small total energy. These particles would provide small contributions to both above integrals.

# Notes, lessons

- Although equations are written for the case of Feynman scaling, integrals entering final formula were obtained for each primary proton energy from MC simulations for that particular energy with accounting for whatever physics these MC managed. Thus scaling violation proves to be partially accounted for (may be considered as to first order).
- Spread of integrals characterizing pion interactions is larger than spread for nucleon interactions.
- Influence on the CG elongation rate of pions is larger than that of nucleons.
- These two above items should be less relevant for Xmax since pions populate tail of shower to which Xmax insensitive.
- Nevertheless tuning of pion interactions seems to be not less important than tuning of proton interactions which usually attract main attention.
- Since same dynamics of longitudinal development of shower define Xmax and CG energy dissipation for both should be represented by same integrals. That means that forward multiplicities should work rather than total mutiplicities for Xmax as well.

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- These two above items should be less relevant for Xmax since pions populate tail of shower to which Xmax insensitive.
- Nevertheless tuning of pion interactions seems to be not less important than tuning of proton interactions which usually attract main attention.
- Although relations between interaction characteristics (integrals) and shower properties for Xmax should be different from those for CG, energy dissipation for both should be represented by same integrals which reflect dynamics of longitudinal development of shower. That means that forward multiplicities should matter rather than total mutiplicities for Xmax as well.

# Notes, lessons

- Leading hadron is not in any way distinct  from other produced hadrons here. Usually it is separated because it defines energy going to multiple production and thus multiplicity of produced particles, i.e. energy dissipation. But the latter here is explicitely defined by another integral,  forward multiplicity, and there is no need to engage leading hadron for that.
- Importance of low-mass diffraction for shower development is minor and accordingly loose is requirement to accuracy of its description, the only condition is consistency in cross-section setting and event simulation.

# Main question

**Which hadron-air interaction characteristics define shower longitudinal development?**

- Cross-section? Yes
- Inelasticity? Not quite, leading hadron is not distinct, integral over all hadrons of given type works
- Multilicity? Not quite, forward multiplicity instead
- Diffraction? Diffraction to low masses matters little

# Conclusions

- Equations for the proton shower center of gravity can be solved within certain assumptions
- The obtained expression for the proton center of gravity has a transparent form, it explicitely splits into center of gravity of the purely electromagnetic cascade at the primary proton energy and modification of this by hadronic cascading
- In the latter, two competeng processes are directly expressed:
  energy transition between different sorts of hadrons governing shower elongation and energy dissipation governing shower shortening
- The expression includes interaction lengths (cross-sections), integrals over inclusive distribution defining energy transition and integrals defining energy dissipation
- The latter are forward multiplicities rather than total multiplicities
- Lack of attention to tuning pion interactions results in bigger spread
  in integrals for pion than those for proton and accordingly bigger spread in pion contributions to the total hadron center of gravity.  Tuning of pion interactions, presumably, deserves not less attention than that of protons