CPTV and neutrino asymmetry in strong gravitational field and its consequences

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Motivations

- Several astrophysical and cosmological problems are involved with neutrinos.
- Examples: leptogenesis and then baryogenesis, neutrino cooled accretion disks around black holes, r-process nucleosynthesis in supernova explosions etc.
- For successful description of many of these scenarios, one must investigate neutrinos in curved spacetime.
- One should start with Dirac equation in curved spacetime which gives rise to an interaction violating Lorentz symmetry, and further CPT symmetry, particularly in a local inertial frame.

Dirac Lagrangian in curved spacetime

$$\mathcal{L} = \sqrt{-g}\left(\frac{i}{2}\bar{\Psi}\gamma^a \overleftrightarrow{D}_a \Psi - \bar{\Psi} m \Psi\right),$$

where the covariant derivative is

$$D_a = \left(\partial_a - \frac{i}{4}\omega_{bca}\sigma^{bc}\right)$$

and the spin-connections are

$$\omega_{bca} = e_{b\lambda}\left(\partial_a e^{\lambda}{}_c + \Gamma^{\lambda}_{\gamma\mu} e^{\gamma}_c e^{\mu}_a\right).$$

Here

$$\sigma^{bc} = \frac{i}{2}\left[\gamma^b, \gamma^c\right].$$

In em field

$$D_a = (\partial_a - ieA_a)$$

Thus the Lagrangian can be rewritten as

$$\mathcal{L} = \det(e)\bar{\Psi}\left(\frac{i}{2}\gamma^a \overleftrightarrow{\partial}_a - m + \gamma^a\gamma^5 B_a\right)\Psi,$$

Lorentz and CPT violating

$$\overline{\Psi}\left(\frac{i}{2}\gamma^a \overleftrightarrow{\partial}_a - m + e\gamma^a A_a\right)\Psi$$

BM (CQG) 2005
Sinha, BM (PRD) 2008

Interaction Lagrangian

$$\bar{\Psi}\left(\gamma^a\gamma^5 B_a\right)\Psi$$

Axial vector: $$\bar{\Psi}\left(\gamma^a\gamma^5\right)\Psi$$

Under CPT:

$$\bar{\Psi}\left(\gamma^0\gamma^5\right)\Psi \to -\bar{\Psi}\left(\gamma^0\gamma^5\right)\Psi : pseudo-scalar$$

$$\bar{\Psi}\left(\gamma^i\gamma^5\right)\Psi \to +\bar{\Psi}\left(\gamma^i\gamma^5\right)\Psi : pseudo-vector$$

If background gravitational potential is constant: overall interaction is Lorentz symmetry violating, resulting CPT odd

↓↓↓

Like electrons moving in a medium: inside a crystal

↓

Dirac Lagrangian may break Lorentz and CPT symmetry in curved spacetime

Majorana neutrino

Standard model: neutrino is solely left-handed

antineutrino is solely right-handed

Majorana particle is self-conjugate: its own antiparticle

$$\psi = \psi_L + \psi_R = \psi_L + \psi_L^c = \psi^c$$

In Weyl representation for a left handed neutrino $\Psi = \begin{pmatrix} \psi_L^c \\ \psi_L \end{pmatrix}$

Mass term: $m\left(\overline{\Psi}_L \Psi_L{}^c + \overline{\Psi}_L{}^c \Psi_L\right)$

violates lepton number

Majorana neutrino

$$\mathcal{L} = \sqrt{-g}\left[\left(i\overline{\psi}_L\gamma^a\partial_a\psi_L + i\overline{\psi}^c_L\gamma^a\partial_a\psi^c_L\right) - m\left(\overline{\psi}^c_L\psi_L + \overline{\psi}_L\psi^c_L\right) + \left(\overline{\psi}^c_L\gamma^a\psi^c_L - \overline{\psi}_L\gamma^a\psi_L\right)B_a\right]$$

CPT violating

$$(-g)^{-1/2}\mathcal{L} = \left(\psi^{c\dagger} \;\; \psi^\dagger\right)\frac{i}{2}\gamma^0\gamma^\mu\overleftrightarrow{\mathcal{D}}_\mu\begin{pmatrix}\psi^c\\ \psi\end{pmatrix} + \left(\psi^{c\dagger} \;\; \psi^\dagger\right)\gamma^5 B_0\begin{pmatrix}\psi^c\\ \psi\end{pmatrix} - \left(\psi^{c\dagger} \;\; \psi^\dagger\right)\gamma^0 m\begin{pmatrix}\psi^c\\ \psi\end{pmatrix}$$

where $\mathcal{D}_\mu \equiv (\partial_0, \partial_i + \gamma^5 B_i).$

Dispersion relation $(p^a \pm B^a)^2 = m^2$

$$E_\nu = \sqrt{(\vec{p} - \vec{B})^2 + m^2} + B_0,$$

$$E_{\nu^c} = \sqrt{(\vec{p} + \vec{B})^2 + m^2} - B_0$$

Debnath, BM, Dadhich (MPLA) 2006
BM (CQG) 2007
Sinha, BM (PRD) 2008

Neutrino Asymmetry

1) Possible under gravity if spacetime deviates from spherical symmetry
2) Lagrangian must be Lorentz and then CPT violating:
   B is a constant or an even function of space-time
3) Lepton number violating interactions must be present: If processes take place during GUT or inflation when primordial fluctuations are present:

$$\Delta n = \frac{g}{(2\pi)^3} \int d^3\mathbf{p} \left[\frac{1}{1+\exp(E_\nu/T)} - \frac{1}{1+\exp(E_{\bar{\nu}}/T)} \right]$$

$$\Delta n \sim g T^3 \left(\frac{\bar{B}_0}{T} \right)$$

Asymmetry per entropy is proportional to gravitational scalar potential

Singh, BM (MPLA) 2003
BM (MPLA) 2005

Gravity Wave perturbation in Early Universe

$$g_{\mu\nu} = a(\tau)^2 \begin{pmatrix} 1+2\phi & -\omega_1 & -\omega_2 & -\omega_3 \\ -\omega_1 & -(1+2\psi)+h_+ & h_\times & 0 \\ -\omega_2 & h_\times & -(1+2\psi)-h_+ & 0 \\ -\omega_3 & 0 & 0 & -(1+2\psi) \end{pmatrix}$$

- The space-time metric:

  $ds^2 = -(1 + 2\varphi)\, dt^2 - \omega_i\, dx^i\, dt + a(t)^2 [(1 + 2\Psi - h_+)\, dx^2 + (1 + 2\Psi + h_+)\, dy^2 - 2 h_x\, dx\, dy + (1 + 2\Psi)\, dz^2]$

- Components of gravitational coupling:

  $B^0 = \partial_z h_x$, $B^i = (\partial \times \omega)^i + \partial_t h_x \delta^{iz}$

$$\langle B_0 \rangle \equiv B_0 \simeq A_\times k \simeq A_\times \left(1.66 g_*^{1/2} \frac{T^2}{M_{Pl}}\right)$$

At GUT: $T \sim 10^{15}$ GeV
$M_{pl} \sim 10^{19}$ GeV

$B_0 \sim A_x\, 10^{12}$ GeV
For $A_x \sim 10^{-7}$, $\Delta n/s \sim 10^{-10}$

Mohanty, BM, Prasanna (PRD) 2002
Sinha, BM (PRD) 2008

Anisotropic phase of Early Universe

- Bianchi II model: Axially symmetric Universe

  $ds^2 = -dt^2 + S(t)^2\, dx^2 + R(t)^2\, [dy^2 + f(y)^2\, dz^2]$

  $\quad - S(t)^2\, h(y)\, [2dx - h(y)\, dz]\, dz$

  where $f(y) = y$ and $h(y) = -y^2/2$

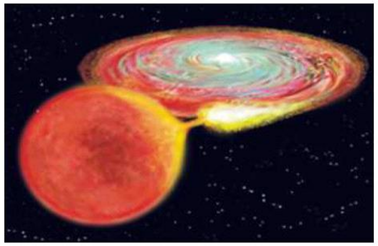
- At a particular situation: $R \sim (t/t_0)^{1/2}$, S is a constant

$$B_0 \sim \frac{S^2}{y}\left(\frac{t_0}{t}\right)$$

- "Mass varying neutrinos?"

Sinha, BM (PRD) 2008

Accretion disk around a rotating compact object



- Kerr geometry:
  $ds^2 = \eta_{ij}\, dx^i\, dx^j - [2\alpha/\rho\, s_i\, v_j + \alpha^2\, v_i\, v_j]\, dx^i\, dx^j$

$$\alpha = \frac{\sqrt{2Mr}}{\rho},\ \rho^2 = r^2 + \frac{a^2 z^2}{r^2},$$

$$v_i = \left(1,\ \frac{ay}{a^2+r^2},\ \frac{-ax}{a^2+r^2},\ 0\right),\ s_i = \left(0,\ \frac{rx}{\sqrt{r^2+a^2}},\ \frac{ry}{\sqrt{r^2+a^2}},\ \frac{z\sqrt{r^2+a^2}}{r}\right)$$

- Gravitational scalar potential coupling:

$$B^0 = e_{1\lambda}\left(\partial_3 e_2^\lambda - \partial_2 e_3^\lambda\right) + e_{2\lambda}\left(\partial_1 e_3^\lambda - \partial_3 e_1^\lambda\right) + e_{3\lambda}\left(\partial_2 e_1^\lambda - \partial_1 e_2^\lambda\right) = -\frac{4a\sqrt{M}z}{\bar{\rho}^2\sqrt{2r^3}}$$

where $\bar{\rho}^2 = 2r^2 + a^2 - x^2 - y^2 - z^2$

BM (MPLA) 2005
BM (CQG) 2007

Oscillations in neutrino and antineutrino

This is very similar to neutral kaon anti-kaon oscillation

$$|\psi^c\rangle = \cos\theta|\nu_1\rangle - \sin\theta|\nu_2\rangle,$$
$$|\psi\rangle = \sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle.$$

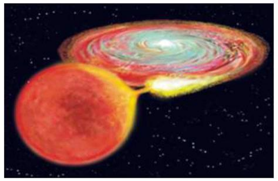
$$\mathcal{M}_n = \begin{pmatrix} m_n - B_0 & -m \\ -m & m_n + B_0 \end{pmatrix},$$

$$m_{n(1,2)} = m_n \mp \sqrt{B_0^2 + m^2}.$$

where m_n is the so called lepton number nonviolating mass.

Oscillation probability $$\mathcal{P}_n(t) = \frac{m^2}{B_0^2 + m^2}\sin^2\left(\frac{m_n\sqrt{B_0^2 + m^2}}{E}t\right)$$

Oscillation length $$\lambda_n = \frac{\pi E}{m_n\sqrt{B_0^2 + m^2}}.$$



- **Considering neutrinos coming out off inner accretion disks around a black hole of mass $10M_{Sun}$: Length ~ 10km ~ Schwarzschild radius**
- **For a supermassive black hole of mass 10^8M_{Sun}: Length ~ 10^9km**

BM (CQG) 2007
Sinha, BM (PRD) 2008

Effect in white dwarfs

- At high density electrons are captured by ions:

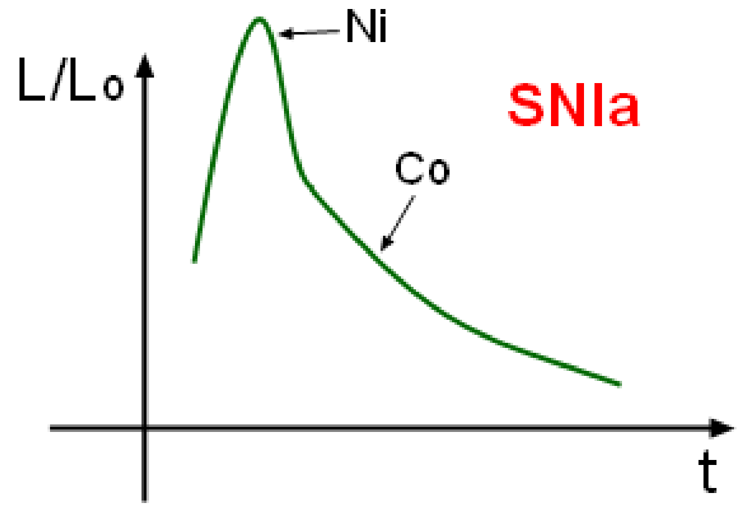
For ^{12}C: $\rho = 3.89 \times 10^{10}$ gm/cc

^{56}Fe: $\rho = 1.06 \times 10^{9}$ gm/cc

$X(A,Z)+e^{-} \rightarrow X(A,Z-1)+\nu_e$

By oscillation: $\nu_e \rightarrow \vec{\nu}_e$

$\vec{\nu}_e + X(A,Z-1) \rightarrow X(A,Z-2)+e^{+}$



Supernova at limiting mass White dwarf

Das, BM (PRL) 2013

Chandrasekhar limit : $5.76\ M_{\odot}/\mu^2$

μ = mean molecular weight = $A/[x(A-Z)+Z]$

Effect in white dwarfs

- When H is completely destroyed : $x=0$

For $^{12}C/^{16}O$ white dwarf: $\mu = 2$ when $\mu = A/[x(A-Z)+Z]$

Chandrasekhar limit: $1.44\ M_\odot$ ($5.76\ M_\odot/\mu^2$)

When $X(A,Z)$ is ^{12}C and $X(A,Z-2)$ is ^{12}Be

Hence, $\mu=3$ and limiting mass $0.64M_\odot$ → lower limit

Indeed, there are several under-luminous type Ia supernovae observed: presumably exploding at a significantly sub-Chandrasekhar mass

Effect in white dwarfs

Introducing a Dirac mass for neutrinos

$$\mathcal{M}_n = \begin{pmatrix} m_n - B_0 & -m \\ -m & m_n + B_0 \end{pmatrix},$$

Masses for mass eigenstates

$$m_{n(1,2)} = m_n \mp \sqrt{B_0^2 + m^2}.$$

Neutrinos and anti-neutrinos are expressed as linear combinations of mass eigenstates : oscillation between neutrinos and anti-neutrinos →

$$\mathcal{P}_n(t) = \frac{m^2}{B_0^2 + m^2} \sin^2\left(\frac{m_n\sqrt{B_0^2 + m^2}}{E} t\right)$$

$$\lambda_n = \frac{\pi E}{m_n\sqrt{B_0^2 + m^2}}.$$

For $m \sim m_n \sim 0.0001$ eV >> B_0 → $\lambda \sim 100$km for T~ keV

Bounds on B_a

- In nonrelativistic limit, gravitational interaction becomes: s.B
  ≡ interaction between fermion spin and external field
- Eot-Wash II experiment (Adelberger et al. 1999): macroscopic number of fermions can be polarized in same direction → ΔE=|B|: difference in energy between fermion spins polarized parallel and antiparallel to B

- ❑ Measuring net magnetization in a paramagnetic materials using a squid (Ni et al. 1999): External field B appears as an effective magnetic field $B_{eff} = B/\mu_B$
- ❑ B_{eff} can be probed in this experiment ~ 10^{-12} G → B ~ 10^{-29} GeV

- ➢ In an inner accretion disk around a $10M_{Sun}$ black hole: B ~ 10^{-23} GeV
- ➢ In a satellite orbiting Earth with v_Φ ~ 1 km/sec: B ~ 10^{-37} GeV

Summary

- Neutrino couples to spacetime curvature, violating Lorentz and CPT symmetry with a suitable gravitational background.

- Conditions for CPT symmetry violation: 1) Space-time must NOT be spherical symmetric. 2) Background curvature coupling is either a constant or an even function of space-time.

- This results in possible neutrino-antineutrino asymmetry and then leptogenesis/baryogenesis. This also leads to neutrino-antineutrino oscillation.

- Early curved Universe is a very feasible situation to have this occurred.

- This oscillation is useful to explain under-luminous type Ia supernovae, which presumably have sub-Chandrasekhar progenitor white dwarfs

Neutrino equation in curved spacetime

The Euler-Lagrange equation in two-component form for a Majorana neutrino

$$(-g)^{-1/2}\mathcal{L} = \begin{pmatrix} \psi^{c\dagger} & \psi^\dagger \end{pmatrix} \frac{i}{2}\gamma^0\gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} + \begin{pmatrix} \psi^{c\dagger} & \psi^\dagger \end{pmatrix} \gamma^5 B_0 \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} - \begin{pmatrix} \psi^{c\dagger} & \psi^\dagger \end{pmatrix} \gamma^0 m \begin{pmatrix} \psi^c \\ \psi \end{pmatrix}$$

where $\mathcal{D}_\mu \equiv (\partial_0, \partial_i + \gamma^5 B_i).$

Mass matrix: $\mathcal{M} = \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix}$

$$m_{(e,\mu)1} = -\sqrt{B_0^2 + m_{e,\mu}^2},$$
$$m_{(e,\mu)2} = \sqrt{B_0^2 + m_{e,\mu}^2}$$

In presence of a Majorana mass, they appear as mixed states: their mass eigenstates are linear combination of neutrino and antineutrino states.

Similar to neutral kaon

$$|\nu_1\rangle = \frac{1}{N}\{(B_0 + \sqrt{B_0^2 + m^2})|\psi^c\rangle + m|\psi\rangle\},$$
$$|\nu_2\rangle = \frac{1}{N}\{-m|\psi^c\rangle + (B_0 + \sqrt{B_0^2 + m^2})|\psi\rangle\}$$

Neutrino-antineutrino states couple together with modified energy

$$E_\nu = \sqrt{(\vec{p} - \vec{B})^2 + m^2} + B_0,$$
$$E_{\nu^c} = \sqrt{(\vec{p} + \vec{B})^2 + m^2} - B_0$$

Due to difference in energy (effective mass), their time evolution is different: Results in possible asymmetry and oscillation.

BM 2007; Sinha, BM 2008

Flavor Mixing

- Neutrino Lagrangian under gravity:

$$(-g)^{-1/2}\mathcal{L} = (\psi^{c\dagger}\psi^{\dagger})\frac{i}{2}\gamma^0\gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \begin{pmatrix}\psi^c\\ \psi\end{pmatrix} + (\psi^{c\dagger}\psi^{\dagger})\gamma^5 B_0 \begin{pmatrix}\psi^c\\ \psi\end{pmatrix} - (\psi^{c\dagger}\psi^{\dagger})\gamma^0 m \begin{pmatrix}\psi^c\\ \psi\end{pmatrix}$$

- The modified masses:

$$\mathcal{M} = \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix}$$

$$m_{(e,\mu)1} = -\sqrt{B_0^2 + m_{e,\mu}^2},$$
$$m_{(e,\mu)2} = \sqrt{B_0^2 + m_{e,\mu}^2}$$

$$|\nu_1\rangle = \frac{1}{N}\{(B_0 + \sqrt{B_0^2 + m^2})|\psi^c\rangle + m|\psi\rangle\},$$
$$|\nu_2\rangle = \frac{1}{N}\{-m|\psi^c\rangle + (B_0 + \sqrt{B_0^2 + m^2})|\psi\rangle\}$$

In presence of a Majorana mass, neutrinos appear as mixed states: their mass eigenstates are linear combination of neutrino and antineutrino.

- Mass Lagrangian of ν_μ, ν_e mixed by a Majorana mass $m_{e\mu}$

$$(-g)^{-1/2}\mathcal{L}_m = -\frac{1}{2}\Big(\nu_{e1}^\dagger m_{e1}\nu_{e1} + \nu_{e2}^\dagger m_{e2}\nu_{e2} + \nu_{\mu 1}^\dagger m_{\mu 1}\nu_{\mu 1} + \nu_{\mu 2}^\dagger m_{\mu 2}\nu_{\mu 2}$$
$$+ \nu_{\mu 1}^\dagger m_{\mu e}\nu_{e1} + \nu_{\mu 2}^\dagger m_{\mu e}\nu_{e2} + \nu_{e1}^\dagger m_{\mu e}\nu_{\mu 1} + \nu_{e2}^\dagger m_{\mu e}\nu_{\mu 2}\Big).$$

- This leads to modified flavor oscillation due to gravity.

Sinha, BM 2008

Effect of modified mass

- Modified masses and mass eigenstates

Thus we obtain all together four mass eigenstates χ_1, χ_2, χ_3 and χ_4 described as

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \mathcal{F}_1^\dagger \begin{pmatrix} \nu_{e1} \\ \nu_{\mu 1} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \chi_3 \\ \chi_4 \end{pmatrix} = \mathcal{F}_2^\dagger \begin{pmatrix} \nu_{e2} \\ \nu_{\mu 2} \end{pmatrix}$$

$$\mathcal{F}_{1,2} = \begin{pmatrix} \cos\phi_{1,2} & -\sin\phi_{1,2} \\ \sin\phi_{1,2} & \cos\phi_{1,2} \end{pmatrix}.$$

$$\tan\phi_{1,2} = \frac{m_{\mu e}}{m_{i(1,2)} + \sqrt{m_{i(1,2)}^2 + m_{\mu e}^2}}$$

$$2m_{i(1,2)} = m_{\mu(1,2)} - m_{e(1,2)}$$

Sinha, BM 2008

Flavor Oscillation

- Flavor states are described by

$$
\begin{aligned}
|\nu_{e1}\rangle &= \cos\phi_1|\chi_1\rangle - \sin\phi_1|\chi_2\rangle \\
|\nu_{\mu 1}\rangle &= \sin\phi_1|\chi_1\rangle + \cos\phi_1|\chi_2\rangle
\end{aligned}
$$

and

$$
\begin{aligned}
|\nu_{e2}\rangle &= \cos\phi_2|\chi_3\rangle - \sin\phi_2|\chi_4\rangle \\
|\nu_{\mu 2}\rangle &= \sin\phi_2|\chi_3\rangle + \cos\phi_2|\chi_4\rangle .
\end{aligned}
$$

- Oscillation probability $\mathcal{P}_{fg} = \sin^2 2\phi \sin^2 \delta_{fg}(t)$
- In original Ψ $\mathcal{P}_{ig} = \sin^2 2\phi \left\{ (\sin\theta_e \sin\theta_\mu + \cos\theta_e \cos\theta_\mu)^2 \sin^2\left(\frac{\Delta M^2}{4E} t \right) \right\}$

$$
\delta_{fg} = \frac{\Delta M^2}{4E} = \frac{|M_1{}^2 - M_2{}^2|t}{4E} = \frac{|M_3{}^2 - M_4{}^2|t}{4E}, L_{osc} = \frac{4\pi E}{\Delta M^2}
$$

$$
\Delta M^2 = \left(\sqrt{B_0^2 + m_\mu^2} + \sqrt{B_0^2 + m_e^2} \right) \sqrt{\left\{ \left(\sqrt{B_0^2 + m_\mu^2} - \sqrt{B_0^2 + m_e^2} \right)^2 + 4m_{e\mu}^2 \right\}}
$$

- B_0 very large: $\Delta M^2 \rightarrow 4B_0 m_{e\mu}$: situation of GUT scale

Sinha, BM 2008

Early Universe: GUT

- At GUT: $t \sim 10^{-35}$ sec, $B_0 \sim 10^{45}$ eV >> m_e, m_μ, $m_{e\mu}$ ($\sim 10^{-2}$ eV)

$$P = 0.999 \sin^2\left(\frac{1.4\times10^8 eV^2 \sec}{4E\hbar}\right)$$

- Oscillation is completely controlled by gravity
- Oscillation takes place vigorously
- Enormous muon neutrinos produce: not understood if we don't consider gravity effect

Sinha, BM 2008

Early Universe: Nucleosynthesis

- At BBN: t ~ 1 sec, $B^0 > 10^{-8}$ eV

  choose $B_0 \sim 5 \times 10^{-2}$ eV ~ m_e, m_μ, $m_{e\mu}$

$$P = 0.999 \sin^2\left(\frac{7\times 10^{-4} eV^2 \sec}{4E\hbar}\right)$$

- For TeV neutrinos gravity effect increases Probability two orders of magnitude, while for thermal neutrinos it is 1.5 times.

Sinha, BM 2008

Around Primordial Black Holes

- Gravitational coupling: $$B^0 = -\frac{4a\sqrt{M}\,z}{\sigma^2\sqrt{2r}}$$

  when $\sigma^2 = 2r^2 + a^2 - x^2 - y^2 - z^2$

- Consider neutrinos at around 20 Schwarzschild radius around a black hole of $M \sim 10^{22}$ gm; $B_0 \sim 5\text{x}10^{-2}$ eV

$$P = 0.999\sin^2\left(\frac{7\times10^{-4}eV^2 t}{4E\hbar}\right)$$

$$L = \frac{4\pi E\hbar c}{7\times10^{-4}eV^2}$$

L for thermal neutrinos decreases to 0.54cm from 4.6cm obtained without gravity

BM 2007
Sinha, BM 2008