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Gravity, Lorentz Invariance and the Quantum RG

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Introduction

• Gravity stands apart from other known interactions in many respects.

• Progress in the theory of fundamental interactions in last 50 years made clear today that our understanding of gravity and its interplay with quantum field theory is at an all-time low.

• Some of the deepest/most puzzling problems of modern theoretical physics have gravity as the weak link:

♠ The cosmological constant problem and its avatars.

♠ The black hole information paradox.

• Physicists hope that with the advent of a "quantum theory of gravity" these problems will be solved.

• That did not happen with "string theory" that is the closest to what we can call "quantum theory of gravity" and which satisfies some non-trivial consistency conditions.

• There is another puzzle that consumes physicists from time to time.

♠ Why observed physics is Lorentz-invariant? Are there (small) violations of Lorentz invariance?

• This way of posing the problem is vacuous. Physics in any realistic situation is not Lorentz invariant as background fields of massless interactions break Lorentz invariance (trivially).

♠ I will argue later that the correct question is:

• "Are there almost massless interactions that we do not know?

• Do known massless interactions like gravity have different dynamics in some regimes from the one we think?

Diffeomorphism invariance and gravity

• Diffeomorphisms are general coordinate transformations.

• Standard QFTs are not diff-invariant.

• It was first observed by Einstein that the (appropriate) inclusion of a metric source in FT, can make diffeomorphisms to be (local) symmetries.

• This is a particular form of the Stuckelberg mechanism where a transformation is promoted to a symmetry by including additional fields (and in the simplest cases promoting the transformation functions to fields.

♠ Promoting diffeomorphisms to symmetries is a direct way to specify a rule for how a theory behaves under general coordinate charges.

♠ The same FT in two different coordinate systems is equivalent to the same theory coupled to two different (but related) types of metric.

• There are however many ways to promote diffeomorphisms to symmetries. Gravity is not the unique answer to this question.

A (contrived) example:

$$S(B_{\mu\nu},\phi^i)=\int d^dx\sqrt{\mathrm{det}(B_{\mu\nu})}\;B^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j\;G_{ij}(\phi^k)$$

• I will argue that:

(a) "String theory" is the general answer to this question.

(b) String theory is equivalent to the dynamics of the Schwinger Source functional of QFT.

(c) Gravity and other interactions via holography emerge from a large UV QFT coupled to the SM.

The generalized Schwinger source functional

• In QFT there is a duality between local operators $O_i(x)$ and their sources, $\phi_i(x)$.

• This is like a generalization of Poincaré duality. To any operator with n-down indices we associate a field (coupling constant/source) with n-up indices.

• A linearized coupling is the definition of this "duality":

$$S = \int d^d x \sum_i \left[\phi_i(x) O_i(x) + A^i_\mu(x) O^\mu_i(x) + h^i_{\mu\nu}(x) O^{\mu\nu}_i(x) + \cdots \right]$$

• A special operator of any translationally invariant QFT is the conserved stress tensor, $T_{\mu\nu}$.

• Standard intuition tells us that any QFT is expected to have a translation-invariant point.

Local symmetries from global symmetries

• The global translational invariance of the QFT always becomes a linearized local diff invariance of the Schwinger functional.

$$S(h'_{\mu\nu}, T'_{\mu\nu}, \cdots) \simeq S(h_{\mu\nu}, T_{\mu\nu}, \cdots)$$

• The Standard Noether procedure allows to construct order by order the non-linear diff-invariant theory.

• There are many subtleties in this, but it was proven that this procedure closes even after an infinite number of steps.

Barnich+Henneaux, 1993, Deser, 2004

• A similar statement holds for other continuous symmetries: they are all promoted to local symmetries, under which the sources transform, with the gauge fields being the sources dual to the conserved currents.

• An obstruction to this are anomalies. Even in that case, the symmetry can be implemented. The anomaly is reflected in "anomalous" transformation properties of sources.

Example: U(1)$_A$ in QCD $A_\mu \leftrightarrow J^A_\mu$, $a \leftrightarrow Tr[F \wedge F]$.

Under a local U(1)$_A$ transformation

$$A_\mu \to A_\mu + \partial_\mu \epsilon \quad , \quad a \to a - N_f \, \epsilon$$

• Even discrete global symmetries become local in a specific sense:

♠ In special points of QFT space they are embedded in continuous local symmetries,

or:

♠ They are the discrete remnants of continuous anomalous QFT symmetries.

The structure of the Schwinger functional

• The Schwinger functional was traditionally used as a generator of the QFT correlators.

• It is however much more physical. Consider QCD as an example:

$$S_{SM} \sim \int d^4x \;\; T^{\mu\nu,\rho\sigma} \; Tr[F_{\mu\nu}F_{\rho\sigma}] + e^{\mu}{}_{a}\bar{q}(\gamma^a(i\partial_\mu + A_m))q + H\bar{q}q + \theta F \wedge F$$

$$T^{\mu\nu,\rho\sigma} \sim \frac{\sqrt{g}\;\; g^{\mu\rho}g^{\nu\sigma}}{4g_{YM}^2} \to (\text{metric}) \quad , \quad H \to \quad \text{Higgs} \quad , \quad \theta \to \text{PQ} \quad \text{axion}$$

• We do believe that such sources correspond to existing, or yet to be discovered fields.

• However, without further specialization, there is little we can say about the Schwinger functional (that contains an infinite number of sources).

• It is a rather unwieldy functional that is only locally defined in the infinite dimensional space of sources.

Dynamical sources and the Quantum RG

• There are two cases where one can say more on the Schwinger functional.

♠ The case where we are expanding around free-field theory (and this is all we learned in perturbative QFT)

♠ The case where we consider a large-N theory.

• In both cases we can simplify the Source functional by integrating out polynomial/multitrace operators.

Simple example:

$$Z(\phi_1,\phi_2) \equiv \langle\ e^{-\int \phi_1 O+\frac{1}{2}\phi_2\ O^2}\ \rangle \sim \int \mathcal{D}s\ \langle\ e^{-\int \phi_1 O+sO+\frac{s^2}{2\phi_2}}\ \rangle$$

$$\sim \int \mathcal{D}s\ e^{-\frac{(s-\phi_1)^2}{2\phi_2}} \langle\ e^{-\int sO}\ \rangle \sim \int \mathcal{D}s\ e^{-\frac{(s-\phi_1)^2}{2\phi_2}} Z_1(s)$$

• Integrating out multi-trace makes single trace sources dynamical.

S. S. Lee, 2012

• There is an equivalent argument in open string theory that leads to a qualitatively similar conclusion.

Kiritsis, 2012

• This procedure can be done perturbatively for all higher multitrace couplings. At large-N this is a sensible procedure.

• It can be now put together with the standard Wilsonian RG group.

♠ Integrating a momentum slice generates an infinite new set of multitrace couplings.

♠ These can be again integrated out at the expense of introducing a new set of single trace couplings s'.

♠ Another momentum slice is integrated out introducing a new set s'', and so on.

• The end point of the RG flow is a a theory of single trace sources in one higher dimension.

• The RG equations now become second order $\to$ Quantum RG.

Holography and String Theory

• What we have found so far is not a surprise: It is precisely what holography advocates.

• There are many details that are missing and the appearance of $d+1$-dimensional diffeomorphism invariance is not fully understood.

• We can safely claim:

String theory (=generalized gravity) is the dynamics of sources of QFT.

• For example four dimensional CFTs define some known and some novel string theories.

Kiritsis, 2012

• Gravity and the whole gravitational sector is "induced" in QFT.

• There seems to be no generic reason why the equivalence principle should be valid.

Back to Lorentz invariance

• Conjecture: no conventional LI QFT has a Lorentz-violating ground state.

• It is arguable at weak coupling.

• It can in principle also be addressed at strong coupling via holography.

• It follows that Lorentz Violation is due to background fields typically in the generalized gravitational sector.

• This is environmental Lorentz violation. Once we identify the relevant background fields, this LV source is trivially understood.

♠ Lorentz-Violation $=$ a window into the generalized gravitational sector.

The UV Landscape of 4D gauge theories

♠ Our goal now will be to explore the emergence of (observable) gravity from the UV landscape of 4D gauge theories.

• We postulate that the UV theory is a 4D QFT (gauge theory) that is

1. Enormous and "Random". (I do not enter details on "statistics" here.)

H. Nielsen, 1982

2. UV complete (Conformal or AF).

• The gauge group structure is $\prod_i G_i$. The SM group is a tiny part of this.

• Generically the G_i are groups of large rank. Focus on $SU(N_i)$ but conclusions are general.

• UV completeness is a very strong constraint. It is more stringent for larger N_i. Matter can only be in the representations, (adjoint, $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ and $\square\!\square$, $\square$).

• Even at strong coupling, other representations are not allowed for large enough N_i.

• An important issue is communication between groups:

1. Matter ϕ_{ij} charged under both (G_i, G_j). Such fields must have non-zero (large) mass. They are the messengers.

• For $N_i \gg 1$ they must be generically bifundamentals to not spoil UV completeness (fundamental messengers).

• When integrated out, they generate double/multiple trace interactions between G_i and G_j.

The leading IR interactions

• A generic simple group factor G_i of the UV theory is characterized by a rank N_i, and a gauge coupling constant λ_i as well as other couplings (Yukawa, quartic etc).

• If the theory is AF, then the spin-two glueball (as well as others) will be massive. Its mass is given by the characteristic scale Λ_i generated by dimensional transmutation. Unless this mass is unnaturally low, such glueballs that will be eventually weakly coupled to the SM (via gravitational messengers) will not be easily visible.

• If the theory is conformal, then there is a continuum of spin-two modes and these will survive in IR physics. The conclusion is that (not surprisingly) only CFTs can give effects in the SM at the extreme IR.

• Two more factors are important: λ_i and N_i.

• Intuition from AdS/CFT suggests that at weak coupling, RG instabilities are generic and important.

• Relevant operators generically destroy the conformal invariance in the IR, and therefore the chance that the CFT is "visible" to other sectors at low energy.

• A stable CFT has no relevant operators. Weak coupling CFTs have ALWAYS, many relevant operators (fermion bilinears, scalar bilinears and trilinears etc.).

• Large N CFTs will also dominate smaller N CFTs. The reason is that they are IR stable against messenger perturbations.

• The conclusion is that the leading relevant IR couplings to the SM will come from a QFT that

1. Has messenger couplings to the SM
2. Is a CFT
3. Has the largest possible N and the largest possible λ.

It has therefore a dual realization in terms on AdS geometry in more than 4 dimensions. The (emergent) dimensionality depends on the details of that CFT, is at least 5 and can be more than 10.

Kiritsis, 2012

A messenger-friendly SM

• As mentioned, the messengers must be bi-fundamentals for UV completeness. They must have both bosons and fermions to couple to all SM particles.

• In order to have RENORMALIZABLE couplings of every SM field to two gravitational messenger fields, (for hidden color invariance) the SM must be written in a way that all representations are of the "bifundamental type".

• This can be done in several ways that have been classified when the embeddings of the SM spectrum in string-theory orientifolds was considered.

Anastasopoulos+Dijkstra+Kiritsis+Schellekens ('06)

• An orientable example is (including massive anomalous U(1)'s), with $Y = \frac{1}{6}Q_3 - \frac{1}{2}Q_1$.

particle	$U(3)_c$	$SU(2)_w$	$U(1)$
$Q(\mathbf{3},\mathbf{2},+\frac{1}{6})$	V	V	0
$U^c(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	$\bar{V}$	0	V
$D^c(\bar{\mathbf{3}},\mathbf{1},+\frac{1}{3})$	$\bar{V}$	0	$\bar{V}$
$L\ (\mathbf{1},\mathbf{2},-\frac{1}{2})$	0	$\bar{V}$	V
$e^c(\mathbf{1},\mathbf{1},+1)$	0	0	$\bar{S}$
$\nu^R(\mathbf{1},\mathbf{1},0)$	0	A	0
$H(\mathbf{1},\mathbf{2},-\frac{1}{2})$	0	$\bar{V}$	V

• If we denote the SM particles as B^{ij}_μ, q^{ij}, H^{ij} then the relevant couplings are

$$\bar{q}^{ij}\gamma^\mu\chi^a_i A^{a,j}_\mu \quad , \quad B^{ij}_\mu\bar{\chi}^a_i\gamma^\mu\chi^a_j \quad , \quad H^{ij}\bar{\chi}^a_i\chi^a_j$$

• There are several subtleties with anomalies and anomalous U(1)'s that may have observable consequences.

On the equivalence principle (I)

• In the absence of scalars in the SM, the issue of universality of the gravitational couplings is trivial.

• The metric couples to all spin two operators.

• Those that have dimension > 4 have a coupling suppressed by the gravitational messenger mass $\Lambda_{mes} \sim M_P$.

• If there are relevant couplings in the SM, then there could be "anomalous" gravitational couplings proportional to positive powers of Λ_{mes}.

• The SM (with a natural/composite Higgs) does NOT have relevant couplings.

• Marginal scalar operators of the large-N CFT can spoil the equivalence principle (more later).

Maldacena vs Randall+Sundrum

• After integrating out the messagers at $E \ll \Lambda_{mes}$, a caricature of the physics is given by a probe (stack of) branes (eg the SM) in a AdS_5-like background.

$$S_{CFT} = M_5^3 \int d^5x\sqrt{g}\left[R_5 + \frac{12}{\ell^2}\right] + S_{SM}(\hat{g}) \quad , \quad (M_5\ell)^3 \simeq N^2$$

This looks like Randall+Sundrum but:

• There is NO UV cutoff in the 5d-geometry

• There is no IR cutoff.

• There is no 4d-graviton zero mode as in RS.

• The probe brane stack is at a radial position associated with its energy scale. (as first considered by Lykken+Randall)

• This radial position is the mass of the messengers, Λ_{mes}. (This mass should be due to an expectation value).

• Λ_{mes} is a cutoff for 5d-gravity+SM.

Although there is an (AdS) geometry above Λ_{mes}, the SM does not "see it" as above Λ_{mes} it is not directly coupled to gravitons but only to the messengers.

DGP Revisited

• The main question now is: why gravity felt by the SM particles is 4d? The idea of an answer was given by

Dvali+Gabadadze+Porrati ('00)

• Loops of SM particles generate a four-dimensional Einstein term

$$S_{\text{grav}}^{\text{SM}-\text{loops}} = \int d^4x \sqrt{\hat{g}} \; \left[\Lambda_{mes}^4 + \Lambda_{mes}^2 R_4 + \log(\Lambda_{mes}^2) R_4^2 + \cdots \right]$$

The natural cutoff is the gravitational messenger scale.

• The total gravity action is

$$S_{grav} = M_5^3 \int d^5x \sqrt{g} \left[R_5 + \frac{12}{\ell^2} \right] + S_{\text{grav}}^{\text{SM}-\text{loops}}$$

• The static graviton propagator (on the SM "brane") is (we ignore for the moment the Λ_{mes}^4)

$$G \sim \frac{1}{M_5^3} \frac{1}{|\vec{p}| + r_c \vec{p}^2} \quad , \quad r_c = \frac{\Lambda_{mes}^2}{M_5^3}$$

• At long distances $|\vec{p}|r_c \ll 1$ gravity is 5d: $V_{grav} \sim \frac{M_5^3}{r^2}$.

• At short distances $|\vec{p}|r_c \gg 1$ gravity is 4d: $V_{grav} \sim \frac{\Lambda_{mes}^2}{r}$.

Therefore : $M_{Planck} \sim \Lambda_{mes} \sim 10^{19}$ Gev

• In curved space (AdS_5) the story changes somewhat, but it turns out the physics remains 4d.

Kiritsis+Tetradis+Tomaras

The equivalence principle (II)

• We do not expect to have relevant operators, we may however have marginal scalar operators. (An example in N=4 is the dilaton $\to$ gauge coupling constant).

• Such operators will couple to the SM via the same gravitational messengers.

• They will correspond to scalar massless "gravitons". They might destroy the equivalence principle.

• The same SM quantum corrections will provide a localized effective action for them.

• (Unlike the graviton), nothing prohibits an induced mass for them.

$$S_{\text{induced}} = \Lambda^2_{mes} \int d^4x \sqrt{\hat{g}} \left[(\partial\phi)^2 + \Lambda^2_{mes}\phi^2 + \log(\Lambda^2_{mes})\phi^4 + \cdots \right]$$

Therefore they have Planck scale masses, and they are irrelevant for low scale physics. They do not violate the equivalence principle.

The axion

• There is always a single universal pseudoscalar marginal operator in the hidden group namely the instanton density $a \sim Tr[F \wedge F]$.

• Its dual bulk action is large-N suppressed (RR field, or θ angle)

$$S_a = \frac{M_5^3}{N^2} \int d^5x (\partial a)^2$$

• If the gravitational messengers generate a mixed anomaly $Tr[T_{\text{SM}_i} T_{\text{SM}_i} Q_{\text{m-chiral}}] = N \ I_i \neq 0,$ then the messengers induce a coupling of the axion to the pseudoscalar densities

$$S_{PQ} = \sum_i \int d^4x \ a \ \frac{I_i}{N} \ Tr[F_i \wedge F_i]$$

• Loop effects of the SM gauge bosons generate a 4d-kinetic term for the axion but no mass term or potential.

$$\delta S_{PQ} = \sum_i I_i^2 \ \frac{\Lambda_{SM}^2}{N^2} \ (\partial a)^2 \quad , \quad f_{PQ} \sim \frac{M_{\text{Planck}}}{N}$$

• QCD instantons generate a potential for the axion as usual $V_a \sim \Lambda_{QCD}^4 \cos a$.

• An analogue of the DGP mechanism is at work here for the axion with potentially interesting consequences.

Hyper-unification

• We have some evidence suggesting "unification" of SM couplings around "M_{GUT}"

• This may be affected by intermediate sectors, but is suggestive.

• The massive messenger vectors must belong to spontaneously broken gauge groups, for renormalizability.

• The natural expectation is that at $\Lambda_{mes} \sim M_P$ or above the messenger gauge fields and the Large-N hidden group "unify".

• This automatically entails the unification/inclusion of the SM group.

• This is a unification of "matter" and gravity.

Outlook

• Gravity and its avatars is induced by QFT

• Its structure is that of string theory.

• Its dynamics emerges from the quantum RG of QFT

• It can be realized from a large/random UV (almost conformal) CFT.

• It suggests that UV degrees of freedom of gravity are the "partons" of the large N, strongly coupled CFT.

• It suggests that the universality of the gravity couplings is an IR "accident".

• The PQ axion is as universal as gravity is.

• It also suggests novel sources of new gravitational physics from subleading interactions and subleading CFTs (gravitons)

• They suggest a mirage picture for cosmology.

• It remains to be seen whether these ideas will lead to a fruitful reconsideration of the mariage between QFT and gravity.

.

THANK YOU!

String theory and Gravity

• String theories have been traditionally defined via 2-d σ-models.

• The string coordinates (bosonic or fermionic) are 2d-quantum fields.

• Continuum σ-models are CFTs and are parametrized by "coupling constants" that correspond to the massless (or tachyonic) string modes.

• The relevant couplings involve the σ-model coupling constant $\frac{\ell}{\ell_s}$ and g_s that controls string interactions BOTH at tree level and loops.

• In a sense, the "loop-expansion" is not inherent in the σ-model. It is an added ingredient. Also the space-time is "emergent": the coordinates are (2d) quantum fields and the metric+other fields are 2d coupling constants.

• Closed strings always include gravity. UV divergences are simply cutoff by the smart world-sheet cutoff of Riemann surfaces.

• The relevant conditions for conformal invariance have a simple expansion at weak σ-model coupling. For example, the dilaton β-function reads

$$\beta_\Phi = \left(D_b + \frac{1}{2}D_f\right) - D_{crit} + \frac{3}{2}\ell_s^2\left[4(\nabla\Phi)^2 - 4\Box\Phi - R + \frac{1}{12}H^2\right] + \mathcal{O}(\ell_s^4)$$

$D_{crit} = 26$ for the bosonic string and 15 for the fermionic strings.

• At weak coupling, conformal invariance imposes the critical dimension:

$$\left(D_b + \frac{1}{2}D_f\right) = D_{crit}$$

curvature corrections are small and the backgrounds are slowly varying.

• Subcritical strings, with $\left(D_b + \frac{1}{2}D_f\right) < D_{crit}$ quickly run to large curvatures and therefore to strong σ-model coupling. The relevant "flow" equations (summarized by the two derivative effective action) have AdS-like solutions.

• In the supercritical case with $\left(D_b + \frac{1}{2}D_f\right) > D_{crit}$ the equations have deSitter-like solutions.

Strings/Gravity from 4D gauge theories

• Strings emerge from higher-d QFTs in d=3,4 and maybe in d=6. I will focus in d=4 where the main QFT is a gauge theory coupled to fermions and scalars.

• Continuum string theories will emerge from conformal gauge theories.

• At weak coupling and large enough N, the main contributions to the β functions come from adjoints (orientable case)

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3} - \frac{2}{3}N_F - \frac{N_s}{6}\right\}N - \frac{g^5}{(4\pi)^4}\{34 - 16N_F - 7N_s\}\frac{N^2}{3} + \cdots$$

with N_f Majorana fermions and N_s scalars in the adjoint of SU(N). We may add $\square, \begin{smallmatrix}\square\\ \square\end{smallmatrix}, \square\square$ and they always contribute positively.

• Higher than "bi-fundamental" representations make the theory IR-free at sufficiently large N.

• The vanishing of the one-loop piece is analogous to being in the critical dimensions in the σ-model definition of string theory. There are two special cases:

$N_F = 4, N_s = 6$

that includes the case of $\mathcal{N} = 4$ sYM. The higher loop contributions to the β-functions are cancelled by Yukawa and quartic scalar contributions.

• The maximal global symmetry in this case is $SO(6)$, realized in a minimal geometrical fashion on an S^5.

• The "emergent" geometrical dual holographic picture (at large N) involves also AdS_5 that geometrically realizes the conformal invariance. The gauge theory develops "extra dimensions" to total of 10. This is type-II superstring theory.

Maldacena ('97)

• The theory contains fermionic gauge invariant operators, and therefore there are space-time fermions in the string theory.

• There are other fixed points with $N_F = 4, N_s = 6$ that should also be described by the same superstring theory.

$N_F = 0, N_s = 22$

• This is another special case. Although one-loop conformal, higher terms in β functions can only stabilized at strong coupling (presumably).

• The maximal global symmetry is SO(22), and in a holographic dual it should geometrically (and minimally) realized by an S^{21}.

• Together with the conformal factor, the backgrounds makes $AdS_5 \times S^{21}$ and is 26 dimensional.

• The associated gauge theory seems to correspond to **a bosonic string**. There are only bosonic gauge-invariant operators. This is however a false expectation. It is most probably a fermionic superstring with no space-time fermions.

• Therefore the theory is more like the Type-0 Theory. It is not obvious that this is the superstring behind the $N_s = 22$ case.

• There are also Bank-Zaks-like fixed points in 25 dimensions involving the condensation of flavor branes, and they may be related.

There are other cases that are "critical", for example:

- $N_s = 18, N_f = 1$. The maximal symmetry here is $O(18)$ as the fermionic U(1) is anomalous. The expectation therefore is that in the most symmetric case the background will be $AdS_5 \times S^{17}$ and may correspond to a novel fermionic non-supersymmetric string theory in 22 dimensions.

- $N_s = 14,\ N_f = 2$. The maximal symmetry is O(14) for the bosons and SU(2) for the fermions. As there are always Yukawas in this case, the SU(2) will be embedded in O(14), and the expected internal space will probably be a squashed S^{13} leading to a fermionic non-supersymmetric string theory in 18 dimensions.

- $N_s = 10,\ N_f = 3$. The maximal symmetry is O(10) for the bosons and SU(3) for the fermions. As there are always Yukawas in this case, the SU(3) will be embedded in O(10), and the expected internal space will probably be a squashed S^9 leading to a fermionic non-supersymmetric string theory in 14 dimensions.

Etc...

• The evidence for such more exotic fermionic string theories is so far slim, but can be made more solid by investigating the RG patterns of appropriate gauge theories.

• The $a-c$ argument from holography, together with perturbative β-functions suggests that the only weakly-coupled theories are the ten-dimensional ones.

• We can go further by allowing the 4d gauge couplings to be space-time dependent. The β-functions are not known except is the simplest possible case of constant but non-Lorentz invariant context. (H. Nielsen, ('78))

Generalized Bank-Zaks fixed points

Consider the general β function coefficients and set

$$\frac{11}{3}-\frac{2}{3}N_F-\frac{N_s}{6}=a \quad , \quad 0\leq a\leq \frac{11}{3}$$

and choose the number of flavors so that

$$b_1=aN-\frac{2}{3}n_F-\frac{n_s}{6}=\epsilon>0 \quad , \quad \epsilon \ll N$$

$$b_2=-\left[\frac{50+4N_F+5N_s}{4}N^2+\frac{n_s}{4N}(N^2-3)\right]+\mathcal{O}(\epsilon)<0$$

For $\epsilon\to 0$ there is a Bank-Zaks fixed point at

$$\frac{\lambda_*}{(4\pi)^2}=\frac{g_*^2 N}{(4\pi)^2}\simeq\frac{4N\epsilon}{(50+4N_F+5N_s)N^2+\frac{n_s}{N}(N^2-3)}$$

The maximum number of emerging dimensions is obtained by $N_F=0, N_s=21$, where $a=\frac{1}{6}$ and $\epsilon=\frac{N}{6}-\frac{2}{3}n_F-\frac{n_s}{6}$. Take $n_F=0$ and $n_s=N-1$, so that $\epsilon=1$ and

$$\frac{\lambda_*}{(4\pi)^2}\simeq\frac{4}{155N+(N-1)\frac{N^2-3}{N^2}}\simeq\frac{1}{39N}+\mathcal{O}(N^{-2}) \quad , \quad \mathit{RETURN}$$

Bank-Zaks-YM theories

$$\beta_{BZ}(\hat{\lambda}) = -\epsilon\hat{\lambda}^2 + b_*\hat{\lambda}^3 + \cdots \quad , \quad \left(\frac{\epsilon}{b_*\hat{\lambda}(\mu)} - 1\right) e^{\frac{\epsilon}{b_*\hat{\lambda}(\mu)}} = \left(\frac{\mu}{\Lambda}\right)^{\frac{\epsilon^2}{b_*}}$$

In the BZ region, $\epsilon = \mathcal{O}\left(\frac{1}{N}\right) \ll 1$, $b_* = \mathcal{O}(1)$. We have

$$\hat{\lambda}(\mu \to \infty) = \frac{1}{\epsilon \log \frac{\mu}{\Lambda}} + \cdots \quad , \quad \hat{\lambda}(\mu \to 0) = \frac{\epsilon}{b_*} - \frac{e^{-1}\epsilon}{b_*}\left(\frac{\mu}{\Lambda}\right)^{\frac{\epsilon^2}{b_*}} + \cdots$$

- We now consider the fundamentals having a common mass m.

- For $\mu \gg m$ the flavors are effectively massless, and the flow is as above.

- Below m however the flavors decouple and the theory is asymptotically free with a one-loop β function $b_0 = \mathcal{O}(1)$.

• For $\mu \ll m$ we obtain

$$\beta_{YM}(\hat{\lambda}) = -b_0 \hat{\lambda}^2 + \cdots \quad , \quad \frac{1}{\hat{\lambda}(\mu)} = \frac{1}{\hat{\lambda}(m)} + b_0 \log \frac{\mu}{m}$$

$$\left(\frac{\epsilon}{b_* \hat{\lambda}(m)} - 1\right) e^{\frac{\epsilon}{b_* \hat{\lambda}(m)}} = \left(\frac{m}{\Lambda}\right)^{\frac{\epsilon^2}{b_*}}$$

In this case the theory in the ultimate IR is AF, and the coupling is driven to infinity. We can calculate the effective IR scale associated with the AF running of the coupling as

$$\Lambda_{IR} = m \ e^{-\frac{1}{b_0 \hat{\lambda}(m)}}$$

• For $\frac{m}{\Lambda} \ll 1$, we obtain

$$\Lambda_{IR} \sim m \ e^{-\frac{b_*}{b_0} \frac{1}{\epsilon}} \ll m \quad , \quad \frac{m}{\Lambda} \ll 1$$

Bosonic string or superstring?

• Consider the axion a dual to $Tr[F\wedge F]$. We can show that it must come from a RR sector.

In large-N$_c$ YM, the proper scaling of couplings is obtained from

$$\mathcal{L}_{YM} = N_c\ Tr\left[\frac{1}{\lambda}F^2 + \frac{\theta}{N_c}F\wedge F\right] \quad , \quad \zeta \equiv \frac{\theta}{N_c} \sim \mathcal{O}(1)$$

It can be shown (Witten, '79)

$$E_{YM}(\theta) = N_c^2\ E_{YM}(\zeta) = N_c^2\ E_{YM}(-\zeta) \simeq C_0\ N_c^2 + C_1\theta^2 + C_2\frac{\theta^4}{N_c^2} + \cdots$$

In the string theory action

$$S \sim \int e^{-2\phi}\,[R + \cdots] + (\partial a)^2 + e^{2\phi}(\partial a)^4 + \cdots \quad , \quad e^{\phi} \sim g_{YM}^2 \quad , \quad \lambda \sim N_c e^{\phi}$$

$$\sim \int \frac{N_c^2}{\lambda^2}\,[R + \cdots] + (\partial a)^2 + \frac{\lambda^2}{N_c^2}(\partial a)^4 + \cdots \quad , \quad a = \theta[1 + \cdots]$$

RETURN

Many string universes and their mixing

• What is the dual (geometrical) description of two strongly coupled, large-N CFTs, $CFT_{1,2}$?

• The product of two AdS spaces with their own string theory on them

$$AdS_1 \times X_1 \cup AdS_2 \times X_1$$

(with in general different, M_5, ℓ_{AdS}, N).

• They share a common boundary.

• They contain two distinct massless NON-interacting gravitons.

• We now couple such CFTs by (multiple trace) operators, $h\ O_1 O_2$?

• This is the $m \to \infty$ limit of a coupling with bifundamental “messenger” fields.

• The two AdS spaces ("Universes") are coupled via their common boundary.

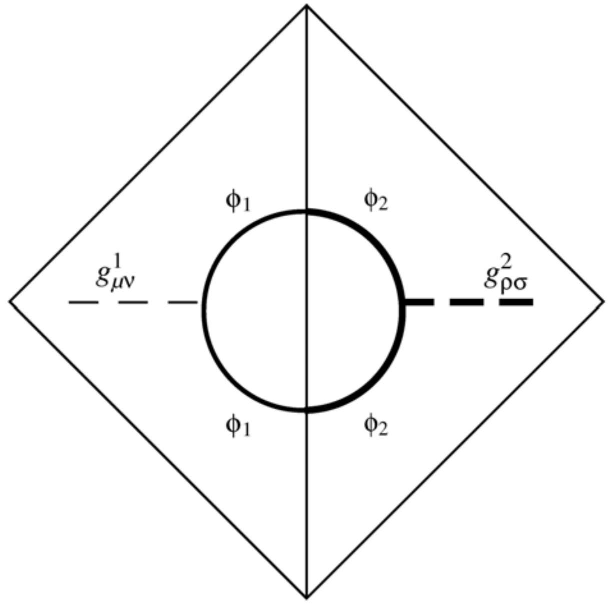
• One of the two gravitons remains massless while the other acquires a mass at one-loop.

Kiritsis ('05), Aharony+Adam+Karch ('05), Kiritsis+Niarchos ('08)

The reason is that now only one of the stress tensors is conserved and the graviton mass is proportional to the anomalous dimension of the spin-two operator.

$$M_g^2\ell^2 = h^2\left(\frac{1}{c_1}+\frac{1}{c_2}\right)\frac{\Delta_1\Delta_2 d}{(d+2)(d-1)} \sim h^2\left(\frac{1}{N_1^2}+\frac{1}{N_2^2}\right)\frac{\Delta_1\Delta_2 d}{(d+2)(d-1)}$$

with $C_i \sim N_i^2$.



- In the bulk theory, $\mathcal{O}_1 \sim \Phi_1$ and $\mathcal{O}_2 \sim \Phi_2$, with the same mass.

- The double trace deformation induces mixed boundary conditions for Φ_1, Φ_2

Witten ('01), Berkooz+Sever+Shomer ('01), Muck ('02)

- This allows the one-loop diagram that provides a term $g_1^{\mu\nu} g_{2,\mu\nu}$ mixing the two gravitons.

- This generalizes to multiple QFTs.

• According to intuition from string theory and holography, If a large-N CFT is coupled at UV by multiplet trace interactions to a finite-N QFT, the bulk picture is of a "probe brane" carrying the finite-N QFT at the boundary of the bulk space generated by the large-N CFT. The bulk graviton couples to the stress tensor of the probe-brane.

Therefore:

• large-N/large-N $\to g_1^{\mu\nu} g_{2,\mu\nu}$

• large-N/finite-N $\to g_1^{\mu\nu} T_{2,\mu\nu}$

• finite-N/finite-N $\to T_1^{\mu\nu} T_{2,\mu\nu}$

• If several large-N CFTs are coupled to a single finite-N QFT, then the geometrical picture is that of a brane embedded and interacting simultaneously with several distinct geometries.

• Out of all gravitons, only one is massless. Thinking about groups of CFTs and their interconnections: per connected component of CFTs there is a single unbroken diffeomorphism invariance associated to a single energy conservation law.

• Of all spin-two glueballs that are coupled via messengers to SM stress tensors,

1. Some correspond to AF theories or weakly coupled CFTs that have been destabilized: they are massive with masses of O(1).
UV Stability prefers strong coupling: all weakly coupled CFTs have relevant operators.

2. Others correspond to strongly coupled CFTs that are connected to the SM and may be connected to other large N CFTs. They contain one massless and several massive components with a mass $min\left(\frac{1}{N_i\ell_i}\right)$. Therefore the lightest graviton beyond the massless one is determined by the smallest $N_i\ell_i$ that mixes with the largest N_i.

Anomalies and extra U(1)'s

• There are several subtleties with anomalies:

♠ It can be shown that in every attempt to write the SM in terms of bi-fundamentals there is at least one and typically more than one EXTRA U(1)'s

Antoniadis+Kiritsis+Tomaras ('00), Anastasopoulos+Dijkstra+Kiritsis+Schellekens ('06)

• Unless an extra U(1) is ∼ B-L it is anomalous.

• If $B-L$, then it must be broken by strong dynamics beyond the SM.

• If anomalous, other degrees of freedom must cancel the anomaly. There are two possibilities:

Anastasopoulos+Bianchi+Dudas+Kiritsis, ('06)

A. The associated U(1) is broken and there are additional chiral fermions that are massive because of the Higgs effect that cancel the anomaly.

B. There is an axion-like field that breaks the "anomalous" U(1) and cancels the anomaly by $aF\wedge F$ type couplings. Consistency requires that the residual global U(1) symmetry to be broken by instanton effects. (more later)

• In all cases: at least one extra U(1) massive gauge boson is expected.RETUR

RS meets DGP

• The standard DGP analysis is valid in 5 flat dimensions.

• In the standard fine-tuned RS model, we can superpose an extra four-dimensional Einstein term $M_P^2\ R_4$ coming from SM loops.

• We have two characteristic length scales, ℓ the AdS scale and $r_c = \frac{M_P^2}{M_5^3}$, the DGP scale.

♠ When $r_c \gg \ell$, gravity is 4d at all scales with 4d Plank scale equal to M_P.

♠ When $\ell \gg r_c$ gravity is 4d at length scales shorter than r_c with Planck scale M_P, 5D when the length scale is between r_c and ℓ and 4d with Planck scale $M_5^3 \ell$, when the length scale is longer than ℓ.

Kiritsis+Tetradis+Tomaras ('02)

Here effectively, as there is no RS cutoff, $\ell \to \infty$, and physics is five dimensional (and AdS-like) at scales longer than r_c.

Therefore, $M_P = 10^{19}$ GeV, and

- Asking for the 5d gravity scale to be perturbative $10^{-3} eV \lesssim M_5$

- Asking for the transition scale to be at the size of the universe, $M_5 \lesssim 100 MeV$.

In total we have a range spanning 11 orders of magnitude

$$10^{-3} \ eV \quad \lesssim \quad M_5 \quad \lesssim \quad 100 \ \ MeV$$

- The dark energy observed today could be due to the DGP acceleration mechanism or mixing with other light gravitons.

Cosmology

• Cosmological evolution as felt by the SM "starts when it is coupled to gravity".

• The underlying paradigm is "mirage cosmology" understood best in its bulk formulation.

Kraus (99), Kehagias+Kiritsis (99)

• The SM branes start at $r = \Lambda_{mes}$ in the bulk and they "fall" gravitationally, inducing a cosmological evolution for the SM fields.

• As $N_c \gg 1$ they can be treated as probe branes in the background geometry.

• We can "start" cosmological evolution by an initial SM energy density $> \Lambda_{mes}^4$. This triggers the gravitational couplings and affects the evolution of the SM energy density

• The SM brane "falls" in the bulk.

• The detailed analysis of various effects of the cosmological evolution remains to be done

Plan of the presentation

- Title page 1 minutes
- Introduction 3 minutes
- Diffeomorphism invariance and Gravity 5 minutes
- The Schwinger Source Functional 6 minutes
- Local vs Global symmetries 8 minutes
- The structure of the Schwinger functional 9 minutes
- Dynamical sources and the quantum RG 11 minutes
- Holography and String Theory 12 minutes
- Back to Lorentz Invariance 13 minutes
- The UV landscape 15 minutes
- The leading IR interactions. 18 minutes
- A messenger friendly SM 22 minutes
- The equivalence principle: I 23 minutes
- Maldacena vs Randall+Sundrum 26 minutes

- DGP Revisited 29 minutes
- The equivalence principle(II). 30 minutes
- The Axion 32 minutes
- Hyper-unification 33 minutes
- Outlook 34 minutes

- String Theory and gravity 37 minutes
- Strings/gravity and 4D gauge theories 52 minutes
- Generalized Bank-Zaks fixed points 55 minutes
- Bank-Zaks-YM theories 58 minutes
- Bosonic String or Superstring 61 minutes
- Many string universes and their mixing 71 minutes
- Anomalies and extra U(1)'s 73 minutes
- RS meets DGP 76 minutes
- Cosmology 79 minutes