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The possible existence of a QUANTUM TIME OPERATOR and its possible refutation in meson experiments.

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Structure of the Talk.

1. Status of Time in the Quantum Theory

- 1 A. Some history: Schrödinger, von Neumann, Heisenberg, Dirac, Bohr and Pauli.
- 1 B. Standard versus non-standard (Time Operator, TimeSuperOperator) approaches.

2. Experimental proposals: decaying systems.

- 2A. exponential decay-measure of lifetime through energy distribution of decay products.
- 2B. non-exponential decay: single kaons and entangled kaons.

1. Status of Time in the Quantum Theory

1 A. Some history: Schrödinger, von Neumann, Heisenberg, Dirac, Bohr and Pauli.

The status of time is the source of much confusion![a]
MAIN SOURCE OF CONFUSION:

- Is time a classical variable (c-number)?

  (Is time an external parameter (universal time)?)

- Is it a quantum quantity (q-number) represented by an operator?

  (Is it an internal parameter (example: phase)?)

[a] Jan Hilgevoord: Time in Quantum Mechanics, a story of confusion, Studies in History and Philosophy of Science Part B 36 (1):29-60 (2005), see also the Book *"Time in Quantum Mechanics"*, volumes 1 and 2, Lecture Notes in Physics, Springer, Muga *et al.* Editors.

Is time is a c-number (external parameter)? VARIOUS ANSWERS[a]

- Bohr, yes, and there is no problem.
- von Neumann, yes, it is so in the non-relativistic quantum theory and it is a problem because x, y and z are described by operators (in the non-relativistic quantum theory), and Lorentz transformation treats space and time on the same footing so that there is a problem with the quantum theory.
- Consider e.g. the usual quantization rule that associates
  E to $i\hbar\frac{\partial}{\partial t}$,
  p_x to $\hbar\frac{\partial}{i\partial x}$,
  p_y to $\hbar\frac{\partial}{i\partial y}$,
  p_z to $\hbar\frac{\partial}{i\partial z}$...
  it has a strong relativistic flavour...
- Dirac for instance wrote his famous equation in order to formulate a Lorentz covariant quantum theory (of the electron), where space and time would be treated on the same footing.

[a] Jan Hilgevoord: Time in Quantum Mechanics, a story of confusion, Studies in History and Philosophy of Science Part B 36 (1):29-60 (2005)

1. Status of Time in the Quantum Theory

1 A. Some history: Schrödinger, von Neumann, Heisenberg, Dirac, Bohr and Pauli.

Is time is a q-number (internal parameter)? VARIABLE ANSWERS[a]

- Dirac considered so for a while, later he did not mention the question anymore.
- Heisenberg: sometimes yes, sometimes no.
- Schrödinger discussed the possibility of quantum clocks, and noticed that the ideal clocks of special relativity are idealizations (for instance they must have an infinite mass).
- Pauli remarked that one can live with idealized clocks FAPP, but also remarked that when an Hamiltonian possesses a continuous bounded spectrum, it is not possible to construct an operator T such that $[\hat{H}, \hat{T}] = i\hbar\hat{\mathbb{I}}$.

[a] Jan Hilgevoord: Time in Quantum Mechanics, a story of confusion, Studies in History and Philosophy of Science Part B 36 (1):29-60 (2005)

1. Status of Time in the Quantum Theory

1 B. **Standard** versus non-standard (Time Operator, TimeSuperOperator) approaches

What is the commonly accepted opinion TODAY?
Time IS a classical variable (c-number)!
Time IS an external parameter (universal time)!
This is the standard view...

1 B. **Standard** versus non-standard (Time Operator, TimeSuperOperator) approaches

- Example 1: Single electron Dirac equation: it is normalized over space, not over space time.

  The probability to find an electron "somewhere" at a given time is 1. The electron is sometimes here sometimes there but always somewhere...

- Example 2: Dirac equation in Quantum Field Theory.

  It is not possible to write a Lorentz covariant equation for, say, two electrons;

  one must jump from 1 to infinitely many electrons (QFT);

  then space AND time are external parameters (they are assigned to the space-time arena in which quantum fields evolve).

- Example 3: Newton-Wigner theorem and Hegerfeld theorem show that position itself is a ill-defined concept in QFT.

IN SUMMARY: TODAY, TIME AND SPACE ARE MOST OFTEN CONSIDERED TO BE C-NUMBERS IN QFT; TIME IS AN EXTERNAL PARAMETER; IT IS NOT A QUANTUM OBSERVABLE.

1 B. **non-standard** (Time Operator-SuperOperator) approaches.

BUT...THE NON-STANDARD APPROACH STILL SURVIVES TODAY, AND IS AIMED AT DERIVING THE DISTRIBUTION OF DECAY TIMES OF AN UNSTABLE QUANTUM SYSTEM (photon in a cavity, particle tunneling from a trap, decaying radio-active particle and so on): In this non-standard approach, the decay time is a quantum quantity (q-number), an internal parameter, represented by an operator!

- Example 1: Time Super Operator approach (Misra Sudarshan Prigogine Courbage[a] *et al.*), makes it possible to associate a generalized Time Operator (Super Operator) to any Hamiltonian provided its spectrum is not bounded by above.
- Example 2: Time Operator approach[b]-described below.

IN ANALOGY WITH NON-RELATIVISTIC POSITION OPERATOR: ONE WOULD DERIVE THE STATISTICAL DISTRIBUTION OF DECAY TIMES OF AN UNSTABLE SYSTEM IN TERMS OF A TIME OPERATOR (SUPER OPERATOR).

[a] B. Misra, I. Prigogine and M. Courbage, in *Quantum theory and measurement*, eds. J.A. Wheeler and W.H. Zurek (Princeton, N-J, 1983).

[b] T. D., *Correlations of decay times of entangled composite unstable systems,* Int. Journ. of Mod. Phys. B 20072659, 2012

2. Experimental proposals: decaying systems.

MAIN QUESTION ADRESSED IN THIS TALK:
"IS IT POSSIBLE TO DISCRIMINATE BOTH APPROACHES BY CONSIDERING THE STATISTICAL DISTRIBUTION OF UNSTABLE QUANTUM SYSTEMS?"

Remark:

Decaying systems are good candidates for discriminating standard and non-standard approaches because

- 1. The temporal density of decay times is equal to $\frac{-dP_s(t)}{dt}$ where $P_s(t)$ is the survival probability at time t.

  It is properly normalized.

  $\int_0^{+\infty} dt \frac{-dP_s(t)}{dt} = -P_s(+\infty) + P_s(0) = -0 + 1 = 1.$

- 2. It is traditionally described in a standard manner:

  $\mathcal{H}$=$\mathcal{H}_{surviving} \oplus \mathcal{H}_{decayproducts}$, with $\Psi(t) = \Psi_S(t) \oplus \Psi_{decayproducts}(t)$.

  $i\hbar\frac{\partial}{\partial t}\Psi(t) = H\Psi(t)$, with H=$H_{surviving} + H_{decayproducts} + H_{interaction}$

  $P_s(t) = |\Psi_S(t)|^2$=1-$|\Psi_{decayproducts}(t)|^2$.

- 3. The STANDARD APPROACH WORKS VERY WELL AND HAS BEEN CONFIRMED IN NUMEROUS EXPERIMENTS.

2. Experimental proposals: decaying systems.

- MAIN CHALLENGE ADRESSED IN THIS TALK:
- "IS IT POSSIBLE TO SIMULATE/REPRODUCE THE STANDARD RESULTS IN A TIME-OPERATOR APPROACH?"
- **CONSTRAINT:**

  The temporal density of decay times is equal to $\frac{-dP_s(t)}{dt}$ where $P_s(t)$ is the STANDARD survival probability at time t.

  Is it possible to associate to the decay process a temporal wave function[a] $\tilde{\Psi}^{T.W.F.}(t)$ such that

  $|\tilde{\Psi}^{T.W.F.}(t)|^2 = \frac{-dP_s(t)}{dt}$???

[a]From now on the upperly tilded quantities will always refer to quantities derived in the framework of the T.W.F. approach.

2. Experimental proposals: decaying systems.

2A. Exponential decay process.

In the case of exponential decay: the answer is YES,
the Time Operator and Standard approaches cannot be discriminated.

- Let us consider Gamow's complex energy state[a]

  $\Psi_S(t) = \Psi_S(0) exp^{(\frac{mc^2}{i\hbar} - \frac{\Gamma}{2})\cdot t}$

  According to the standard interpretation, $\frac{|\Psi_S(t)|^2}{|\Psi_S(0)|^2}$ is interpreted to be equal to the SURVIVAL PROBABILITY $P_s(t)$ between time 0 and time t.

- Alternatively, let us define the Temporal Wave Function $\tilde{\Psi}^{T.W.F.}(t)$ through

  $\tilde{\Psi}^{T.W.F.}(t) = \tilde{\Psi}^{T.W.F.}(0) exp^{(\frac{mc^2}{i\hbar} - \frac{\Gamma}{2})\cdot t}$ with $\tilde{\Psi}^{T.W.F.}(0) = \Gamma$;

  It is straightforward to check that $\frac{-dP_s(t)}{dt} = |\tilde{\Psi}^{T.W.F.}(t)|^2$. SO BY A FORMAL RENORMALISATION WE OBTAIN SIMILAR PREDICTIONS IN BOTH APPROACHES.

[a] G. Gamow, Z. Phys. **51**, 537 (1928).

2. Experimental proposals: decaying systems.

2A. Exponential decay process.

Remark:
NOT ONLY A FORMAL TRICK...

- The Time Operator and Standard approaches cannot be discriminated EXPERIMENTALLY.
- Indeed, lifetimes of particles are often measured indirectly in particle physics, by fitting the energy distribution of decay products with a Breit-Wigner (Lorentzian) distribution,
- Also in this case the standard and time operator approaches cannot be distinguished[a]...

[a] C. Champenois and T. Durt: "Quest for the time-Operator with a Single Trapped Ion", IJQI, vol. 9, 189-202 (2011).

2. Experimental proposals: decaying systems.

2A. Exponential decay process.

STANDARD APPROACH.

To fit with a Breit-Wigner distribution is consistent with the "standard" recipe: $P_s(t) = | < \psi(0), \psi(t) > |^2$

- Suppose that at time 0 the amplitude of probability that the particle energy is E equals $\psi^E(E)$.
- Then, developing the wave function in the energy eigenbasis it is straightforward to show that $P_s(t) = | < \psi(0), \psi(t) > |^2 = | \int_{E_{min.}}^{E_{max.}} dE e^{-i\frac{Et}{\hbar}} |\psi^E(E)|^2|^2$, where the spectrum of the (supposedly time-independent) Hamiltonian runs from $E_{min.}$ to $E_{max.}$.
- When the spectrum of the Hamiltonian is large enough $| \int_{E_{min.}}^{E_{max.}} dE e^{-i\frac{Et}{\hbar}} |\psi^E(E)|^2|$ is close to $| \int_{-\infty}^{+\infty} dE e^{-i\frac{Et}{\hbar}} |\psi^E(E)|^2|$ and the survival probability is equal to the squared modulus of the Fourier transform of the energy distribution.
- Now, the characteristic function (Fourier transform) of the Breit-Wigner distribution is equal to $C.e^{-\mathrm{i}(mt-\frac{\mathrm{i}}{2}\Gamma|t|)}$ and we are done (disregarding negative times).

2. Experimental proposals: decaying systems.

2A. Exponential decay process.

Temporal Wave Function/Time Operator approach.

To fit with a Breit-Wigner distribution is ALSO consistent with the "non-standard" recipe: $\frac{-dP_s(t)}{dt} = \Gamma\theta(t)e^{-\Gamma t}$=$|\tilde{\Psi}^{T.W.F.}(t)|^2$

- Assume that the decay pdf (probability density function) is associated to a "temporal wave function": $\frac{-dP_s(t)}{dt} = \Gamma\theta(t)e^{-\Gamma t}$=$|\tilde{\Psi}^{T.W.F.}(t)|^2$ with $|\tilde{\Psi}^{T.W.F.}(t)|^2$=$|\sqrt{\Gamma}\theta(t)e^{-\mathrm{i}(m-\frac{\mathrm{i}}{2}\Gamma)t)}|^2$ (where $\theta(t)$ is the Heaviside function).

- Then, $|\tilde{\Psi}^{T.W.F.}(t)|^2$=$|\int_{-\infty}^{+\infty} dE e^{+i\frac{Et}{\hbar}}\hat{\tilde{\Psi}}^{T.W.F.}(E)|^2$, with $\hat{\tilde{\psi}}^{T.W.F.}(E)$=$\frac{1}{2\pi}\int_{-\infty}^{+\infty} dt e^{-i\frac{Et}{\hbar}}\tilde{\Psi}^{T.W.F.}(t)$=$-i\sqrt{\frac{\Gamma}{2\pi}}\frac{1}{(E-(m-\frac{\mathrm{i}}{2}\Gamma))}$.

- The modulus squared of $\hat{\tilde{\psi}}^{T.W.F.}(E)$ is precisely equal to the Breit-Wigner distribution: $|\hat{\tilde{\psi}}^{T.W.F.}(E)|^2$=$N.\frac{1}{(E-m)^2+(\Gamma/2)^2}$ with N a normalisation factor. To fit with a Breit-Wigner distribution is thus also consistent with the "non-standard" recipe: $\frac{-dP_s(t)}{dt} = \Gamma\theta(t)e^{-\Gamma t}$=$|\tilde{\Psi}^{T.W.F.}(t)|^2$.

2. Experimental proposals: decaying systems.

2A. Exponential decay process.

CONCLUSION of section 2A.

- EXPONENTIAL DECAY PROCESSES DO NOT MAKE IT POSSIBLE TO DISCRIMINATE BETWEEN STANDARD AND TIME OPERATOR APPROACHES.
- LET US THEREFORE CONSIDER NON-EXPONENTIAL DECAY PROCESSES.
- KAONS ARE GOOD CANDIDATES: THEIR DECAY DISTRIBUTIONS EXHIBIT OSCILLATIONS DUE TO CP VIOLATION AND/OR REGENERATION: RICH AND COMPLEX PHYSICS!

2B. non-exponential decay: single kaons and entangled kaons.

IN ABSENCE OF CP VIOLATIONS: IMPOSSIBLE TO DISCRIMINATE STANDARD AND TIME OPERATOR (T.W.F.) APPROACHES.
WHY?

- The probability that a kaon prepared in the $|K_0>$ state at time $t=0$ decays in the CP=+1 sector between time t and time $t+dt$ obeys

$$P_+(t) = \frac{\Gamma_S}{2}|exp^{-iE_S t/\hbar} + \epsilon exp^{-iE_L t/\hbar}|^2 \tag{1}$$

- The probability that a kaon prepared in the $|K_0>$ state at time $t=0$ decays in the CP=-1 sector between time t and time $t+dt$ obeys

$$P_+(t) = \frac{\Gamma_S}{2}|exp^{-iE_S t/\hbar} + \epsilon exp^{-iE_L t/\hbar}|^2 \tag{2}$$

- When ϵ=0 (no CP violation), both processes are purely exponential processes. They can be simulated thanks to a pseudo-spinorial T.W.F. defined as follows:

$$(\tilde{\Psi}_+^{T.W.F.\epsilon=0}(t), \tilde{\Psi}_-^{T.W.F.\epsilon=0}(t)) = (\sqrt{\Gamma_S/2}exp^{-iE_S t/\hbar}, \sqrt{\Gamma_L/2}exp^{-iE_L t/\hbar}). \tag{3}$$

2B. non-exponential decay: single kaons and entangled kaons.

IN PRESENCE OF CP VIOLATIONS: POSSIBLE TO DISCRIMINATE STANDARD AND TIME OPERATOR (T.W.F.) APPROACHES.

HOW?

SOME PHENOMENOLOGY:

The main decay channels in the CP=+1 and -1 channels are respectively pairs ("2π") and triplets of kaons ("3π").

In absence of CP-violation the short-lived state is a CP=+1 eigenstate K_1 and the long-lived one a CP=-1 eigenstate K_2.

CP-violation means that the long-lived kaon can also decay to "2π". Then, the CP symmetry is slightly violated by weak interactions so that the CP eigenstates K_1 and K_2 are not exact eigenstates of the decay interaction. The exact states are denoted (K_S) and long-lived state (K_L).

$$
\begin{aligned}
|\mathrm{K}_L\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}}\big[\epsilon\,|\mathrm{K}_1\rangle + |\mathrm{K}_2\rangle\big], \\
|\mathrm{K}_S\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}}\big[|\mathrm{K}_1\rangle + \epsilon\,|\mathrm{K}_2\rangle\big],
\end{aligned}
\tag{4}
$$

where ϵ is a complex CP-violation parameter, $|\epsilon| \ll 1$: $|\epsilon| = (2.27 \pm 0.02) \times 10^{-3}, \quad \arg(\epsilon) = 43.37°$.

2. Experimental proposals: decaying systems.

2B. non-exponential decay: single kaons and entangled kaons.

- We know already that in the limit where $\epsilon = 0$ (no violation), $\tilde{\Psi}_+^{T.W.F.\epsilon=0}(t) = \sqrt{\Gamma_S/2} exp^{-iE_S t/\hbar}$, corresponds to the K_1 (CP=+1) state, and $\tilde{\Psi}_-^{T.W.F.\epsilon=0}(t) = \sqrt{\Gamma_L/2} exp^{-iE_L t/\hbar}$, corresponds to the K_2 (CP=-1) state,
- and that Gamow states and T.W.F. states differe by a mere normalisation factor,
- Therefore, in presence of CP violation, we must try $\tilde{\Psi}_+^{T.W.F.}(t) = \sqrt{\Gamma_S/2} exp^{-iE_S t/\hbar} + \tilde{\epsilon}\sqrt{\Gamma_L/2} exp^{-iE_L t/\hbar}$, $\tilde{\Psi}_-^{T.W.F.}(t)$=$\sqrt{\Gamma_L/2} exp^{-iE_L t/\hbar} + \tilde{\epsilon}\sqrt{\Gamma_S/2} exp^{-iE_S t/\hbar}$, where $\tilde{\epsilon}$ must be fixed on the basis of experimental data.

2. Experimental proposals: decaying systems.

2B. non-exponential decay: single kaons and entangled kaons.

- T.W.F. predictions are

$$\tilde{P}_+(t) \approx \frac{\Gamma_S}{2} |exp^{-iE_S t/\hbar} + \tilde{\epsilon}\sqrt{\frac{\Gamma_L}{\Gamma_S}} exp^{-iE_L t/\hbar}|^2 \tag{5}$$

$$\tilde{P}_-(t) \approx \frac{\Gamma_L}{2} |exp^{-iE_L t/\hbar} + \tilde{\epsilon}\sqrt{\frac{\Gamma_S}{\Gamma_L}} exp^{-iE_S t/\hbar}|^2, \tag{6}$$

- To compare with

$$P_+(t) = \frac{\Gamma_S}{2} |exp^{-iE_S t/\hbar} + \epsilon exp^{-iE_L t/\hbar}|^2 \tag{7}$$

$$P_-(t) = \frac{\Gamma_L}{2} |exp^{-iE_L t/\hbar} + \epsilon exp^{-iE_S t/\hbar}|^2 \tag{8}$$

NOW, FITCH-CRONIN EXPERIMENT IMPOSES THAT $\tilde{\epsilon}\sqrt{\frac{\Gamma_L}{\Gamma_S}}=\epsilon$,
BUT THEN EQUATIONS (6) and (8) ARE NO LONGER COMPATIBLE.

2. Experimental proposals: decaying systems.

2B. non-exponential decay: single kaons and entangled kaons.

- THE T.W.F. APPROACH PREDICTS MORE OR LESS 10^3 MORE TRIPLET PRODUCTION AT SHORT TIMES COMPARED TO THE STANDARD APPROACH.
- DIFFICULT TO OBSERVE (CLOSE TO THE SOURCE, AND CONTAMINATED BY PAIRS OF PIONS),
- BUT SIMILAR INCOMPATIBILITIES/INCONSISTENCIES BETWEEN THE T.W.F. AND STANDARD APPROACHES CAN BE SHOWN IN THE CASE OF ENTANGLED KAON STATES (APPENDIX)
- THE SINGLET EPR-BOHM KAON STATES PREDICTS STRONG ANTI-CORRELATIONS[a], SIMILAR TO A DARK FRINGE EXPERIMENT IN INTERFEROMETRY
- EASIER TO OBSERVE, BETTER SIGNAL/NOISE RATIO.[b] (APPENDIX).

[a] B. Hiesmayr, A. Di Domenico, C. Curceanu, A. Gabriel, M. Huber, J-A Larsson and P. Moskal, *Eur. Journ. Phys. C* **72**, 1856 (2012).

[b] Ambrosino *et al.*, *Phys. Lett. B* **642**, 315 (2006), A.Di Domenico (Editor), *Handbook of neutral kaon interferometry at a ϕ-factory*, (Frascati Physics Series, Vol. XLIII, Frascati Roma, 2007).

CONCLUSIONS

- CP violation is responsible of the occurence of a fine structure in kaon correlations that makes it possible to discriminate the standard approach in which Time is treated as an external, classical, parameter at one side, from the Time Operator (in particular T.W.F.) approach in which, in analogy with the spin 1/2 theory, the time of occurence of decay processes in the CP=± 1 sectors would obey a generalized Born rule.
- If there was no CP-violation ($\epsilon = 0$), it would be impossible to discriminate the T.W.F. model from the standard model, in the single particle case and as well in the entangled pair case.
- Ultimately, our analysis allows us to bring to the realm of experiments an old debate that can be traced back to the original developments of the quantum theory concerning the role and status of Time, and CP violation is an essential ingredient of our derivation.

APPENDIX.

non-exponential decay: entangled kaons.

The so-called ϕ resonance is the source of kaon pairs in the EPR-Bohm (so-called *singlet*) state

$$
\begin{aligned}
|\phi\rangle &= \frac{1}{\sqrt{2}}(|\mathrm{K}_1\rangle_l|\mathrm{K}_2\rangle_r - |\mathrm{K}_1\rangle_l|\mathrm{K}_2\rangle_r) \\
&= \frac{1+|\epsilon|^2}{\sqrt{2}}\Big(|\mathrm{K}_S\rangle_l|\mathrm{K}_L\rangle_r - |\mathrm{K}_L\rangle_l|\mathrm{K}_S\rangle_r\Big),
\end{aligned}
\tag{9}
$$

where the indices l and r refer to the fact that those kaons are sent along opposite directions, left and right (with equal velocities v close to the speed of light). At the lowest order in $|\epsilon|$,

$$
\begin{aligned}
|\phi(t_l, t_r)\rangle \;=\; & \frac{1}{\sqrt{2}}\Bigg(\begin{pmatrix} 1 \\ \epsilon \end{pmatrix}_l \otimes \begin{pmatrix} \epsilon \\ 1 \end{pmatrix}_r e^{-\mathrm{i}(m_S - \frac{\mathrm{i}}{2}\Gamma_S)t_l} \cdot e^{-\mathrm{i}(m_L - \frac{\mathrm{i}}{2}\Gamma_L)t_r} - \\
& \begin{pmatrix} \epsilon \\ 1 \end{pmatrix}_l \otimes \begin{pmatrix} 1 \\ \epsilon \end{pmatrix}_r e^{-\mathrm{i}(m_L - \frac{\mathrm{i}}{2}\Gamma_L)t_l} \cdot e^{-\mathrm{i}(m_S - \frac{\mathrm{i}}{2}\Gamma_S)t_r}\Bigg)
\end{aligned}
\tag{10}
$$

APPENDIX.

non-exponential decay: entangled kaons.

In the standard approach, the probability $p_d^{ij}(t_l, t_r)$ of a decay in the CP=i sector at time t_l and a decay in the CP=j sector at time t_l obeys

$$p_d^{ij}(t_l, t_r) = \Gamma_i \Gamma_j P_S^{ij}(t_l, t_r) \tag{11}$$

where we define the survival probabilities, projected along the CP sectors as follows:

$$P_S^{ij}(t_l, t_r) = \psi_{ij}^*(t_l, t_r)\psi_{ij}(t_l, t_r) \tag{12}$$

In the Temporal Wavefunction approach we find (neglecting global normalisation factors of the order of $|\epsilon|^2$)

$$\tilde{p}_d^{ij}(t_l, t_r) = \tilde{P}_S^{ij}(t_l, t_r) \tag{13}$$

$$\tilde{P}_S^{ij}(t_l, t_r) = \tilde{\psi}_{ij}^{T.W.F.*}(t_l, t_r)\tilde{\psi}_{ij}^{T.W.F.}(t_l, t_r) \tag{14}$$

$$\begin{aligned} |\tilde{\phi}^{T.W.F.}\rangle(t_l, t_r) \; = \; & \frac{1}{\sqrt{2}}\Bigg(\begin{pmatrix} 1 \\ \tilde{\epsilon} \end{pmatrix}_l \otimes \begin{pmatrix} \tilde{\epsilon} \\ 1 \end{pmatrix}_r \sqrt{\Gamma_S \Gamma_L} e^{-\mathrm{i}(m_S - \frac{\mathrm{i}}{2}\Gamma_S)t_l} \cdot e^{-\mathrm{i}(m_L - \frac{\mathrm{i}}{2}\Gamma_L)t_r} - \\ & \begin{pmatrix} \tilde{\epsilon} \\ 1 \end{pmatrix}_l \otimes \begin{pmatrix} 1 \\ \tilde{\epsilon} \end{pmatrix}_r \sqrt{\Gamma_L \Gamma_S} e^{-\mathrm{i}(m_L - \frac{\mathrm{i}}{2}\Gamma_L)t_l} \cdot e^{-\mathrm{i}(m_S - \frac{\mathrm{i}}{2}\Gamma_S)t_r} \Bigg) \end{aligned} \tag{15}$$

APPENDIX.

non-exponential decay: entangled kaons.

We get thus for instance[a]

$$\tilde{p}_d^{11}(t_l,t_r)=\frac{1}{2}\Gamma_S\Gamma_L|\tilde{\epsilon}|^2|e^{-\mathrm{i}(m_L-\frac{\mathrm{i}}{2}\Gamma_L)t_l}e^{-\mathrm{i}(m_S-\frac{\mathrm{i}}{2}\Gamma_S)t_r}-e^{-\mathrm{i}(m_S-\frac{\mathrm{i}}{2}\Gamma_S)t_l}e^{-\mathrm{i}(m_L-\frac{\mathrm{i}}{2}\Gamma_L)t_r}|^2$$

$$=\frac{1}{2}\Gamma_S\Gamma_S|\epsilon|^2\left(e^{-\Gamma_L t_l-\Gamma_S t_r}+e^{-\Gamma_S t_l-\Gamma_L t_r}-2e^{-\frac{(\Gamma_S+\Gamma_L)(t_l+t_r)}{2}}cos(\Delta m(t_l-t_r))\right) \quad (16)$$

and

$$\tilde{p}_d^{22}(t_l,t_r)=\frac{1}{2}\Gamma_S\Gamma_L|\tilde{\epsilon}|^2|e^{-\mathrm{i}(m_L-\frac{\mathrm{i}}{2}\Gamma_L)t_l}e^{-\mathrm{i}(m_S-\frac{\mathrm{i}}{2}\Gamma_S)t_r}-e^{-\mathrm{i}(m_S-\frac{\mathrm{i}}{2}\Gamma_S)t_l}e^{-\mathrm{i}(m_L-\frac{\mathrm{i}}{2}\Gamma_L)t_r}|^2$$

$$=\frac{1}{2}\frac{\Gamma_S}{\Gamma_L}\Gamma_L\Gamma_L|\epsilon|^2\left(e^{-\Gamma_L t_l-\Gamma_S t_r}+e^{-\Gamma_S t_l-\Gamma_L t_r}-2e^{-\frac{(\Gamma_S+\Gamma_L)(t_l+t_r)}{2}}cos(\Delta m(t_l-t_r))\right) \quad (17)$$

where we made use of the fact that $\tilde{\epsilon}\sqrt{\frac{\Gamma_L}{\Gamma_S}}=\epsilon$.

In particular $\tilde{p}_d^{22}(t_l,t_r)/\tilde{p}_d^{11}(t_l,t_r)$ is of the order of 10^3 larger than its standard counterpart.

CAN BE CHECKED EXPERIMENTALLY.

[a] T. D., *Correlations of decay times of entangled composite unstable systems,* Int. Journ. of Mod. Phys. B 20072659, 2012

EXTRA MATERIAL: Time Super Operator Formalism.

As Bell said: What is proved by impossibility theorems is a lack of imagination....

- Consider an Hamiltonian of which the energy spectrum is bounded by below, but not upperly bounded.
- Instead of considering pure-states evolution (Schrödinger equation $i\hbar\partial_t|\Psi\rangle = H|\Psi\rangle$),
- consider mixed states evolution (Liouville-von Neumann equation $i\hbar\partial_t\rho = [H, \rho]$),
- the superoperator $[H, ...]$ is no longer bounded by below ($\langle E|[H, \rho]|E'\rangle = (E - E')\langle E|\rho|E'\rangle$ and $E - E'$ belongs to $[-\infty, +\infty]$ when E, E' belong to $[0, +\infty]$.

THEREFORE IT IS POSSIBLE TO CIRCUMVENT PAULI'S OBJECTIONS AND TO DERIVE A TIME SUPEROPERATOR THAT SATISFIES CANONICAL COMMUTATION RULES WITH THE LIOUVILLE-VON NEUMANN SUPEROPERATOR (SUDARSHAN, PRIGOGINE, MISRA AND COURBAGE[a]).

[a] B. Misra, I. Prigogine and M. Courbage, in *Quantum theory and measurement*, eds. J.A. Wheeler and W.H. Zurek (Princeton, N-J, 1983).

SEE ARXIVES AND J. PHYS. G[a] FOR DETAILS

...

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[a]Time Super Operator: M. Courbage, T. Durt and M. Saberi, "A new formalism for the estimation of the CP-violation parameters" *J. Phys. G: Nucl. Part. Phys.* **39**, 045008 (2012).

After long computations we derive the survival probability in the CP=+1 sector

$$P_\rho^S(s=-t)$$

$$\simeq \frac{\pi}{2}\left[|\epsilon_1|^2 e^{2b_1 s} + \frac{3}{2}|\epsilon_2|^2\, e^{2b_2 s} + \left(\frac{\mathrm{i}\epsilon_1^*\epsilon_2\lambda_1^*\lambda_2\, e^{(b_1+b_2)s}\, e^{\mathrm{i}(\widetilde{\omega}_1-\widetilde{\omega}_2)s}}{(\widetilde{\omega}_2-\widetilde{\omega}_1)+\mathrm{i}(b_1+b_2)} + \mathrm{C.C.}\right)\right]$$

$$\simeq \frac{\pi}{2}|\epsilon_1|^2\left[e^{2b_1 s} + \frac{3}{2}|\epsilon|^2\, e^{2b_2 s} + \left(\frac{\mathrm{i}\epsilon\lambda_1^*\lambda_2\, e^{(b_1+b_2)s}\, e^{\mathrm{i}(\widetilde{\omega}_1-\widetilde{\omega}_2)s}}{(\widetilde{\omega}_2-\widetilde{\omega}_1)+\mathrm{i}(b_1+b_2)} + \mathrm{C.C.}\right)\right].$$

where

$$\epsilon = |\epsilon|\, e^{\mathrm{i}\phi} := \frac{\epsilon_2}{\epsilon_1}.$$

The derivative of this equation yields the density of the probability or intensity $p_d(s=-t) := -\frac{dP_\rho^S(s)}{ds}$

$$= p_d(0)\left[e^{2b_1 s}+|\epsilon|^2\frac{3}{2}\frac{b_2}{b_1}\, e^{2b_2 s}+2|\epsilon|\sqrt{\frac{b_2}{b_1}}\, e^{(b_1+b_2)s}\, \cos((\widetilde{\omega}_1-\widetilde{\omega}_2)s+\phi+\theta_2-\theta_1)\right].$$

where $\lambda_i = \sqrt{b_i}\, e^{\mathrm{i}\theta_i},\ (i=1,2)$.
What is important is the appearance of a factor $\frac{3}{2}$ in front of the purely exponential contribution with the "Long" ($b_2 = \Gamma_L$) lifetime...the Time Super-Operator approach is experimentally FALSIFIABLE!