ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Higgs inflation at the critical point

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Questioning Fundamental Physical Principles

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based on Bezrukov, M.S., arXiv: 1403.6078

The starting point:

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Let us not question fundamental physical principles and see if the Higgs boson of the Standard Model can inflate the Universe

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Outline

- Higgs inflation, tree approximation
- Higgs inflation: radiative corrections
- Critical Higgs inflation
- Conclusions

Higgs inflation: tree approximation

Main idea of Higgs inflation: non-minimal coupling of the Higgs field to gravity:

$$S = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} \left(h^2 - v^2\right)^2 \right\} + \Delta S$$

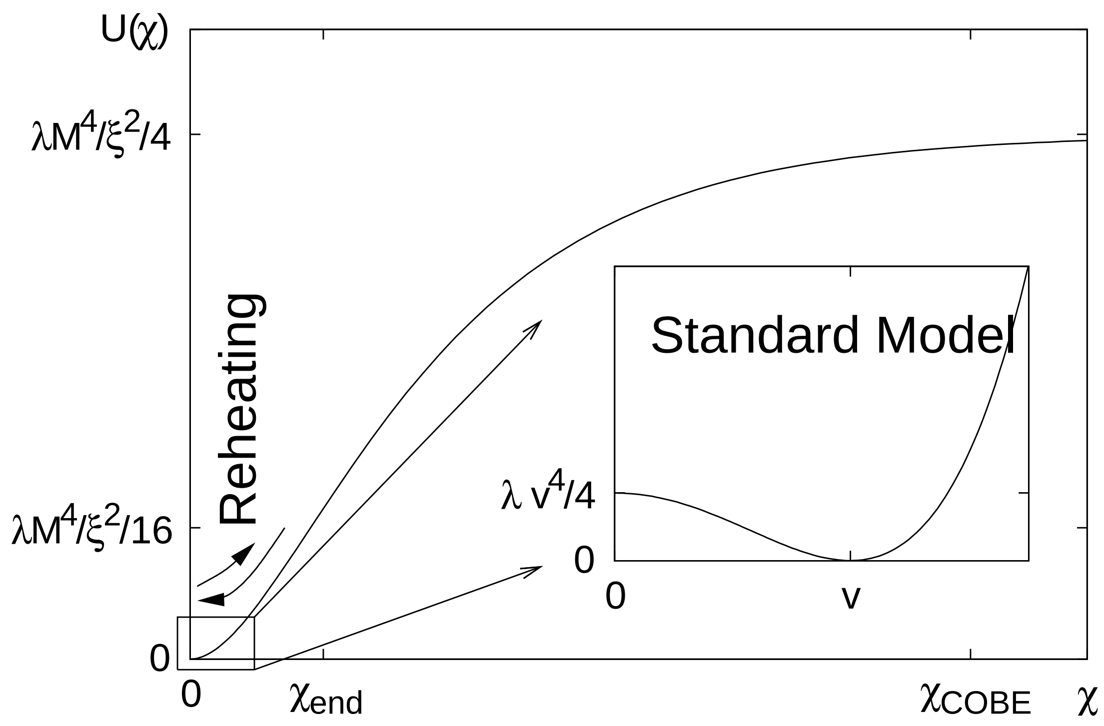
Extra term, necessary for renormalisability:

$$\Delta S = \int d^4x \sqrt{-g} \left\{ -\frac{\xi h^2}{2} R \right\}$$

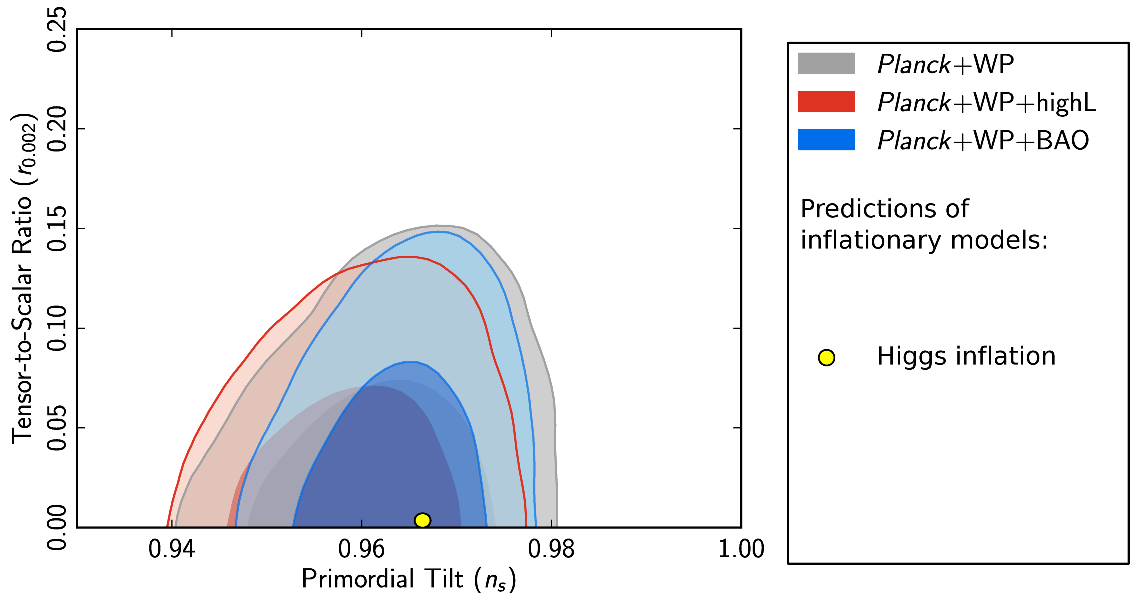
Feynman, Brans, Dicke,...

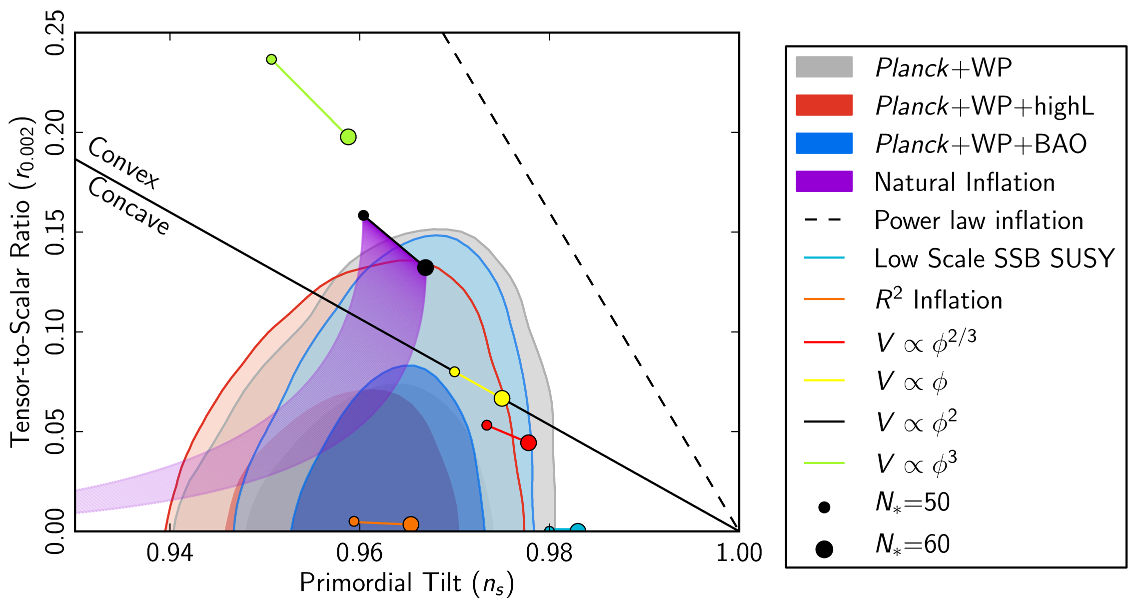
Potential in Einstein frame

$$U(\chi) = \begin{cases} \frac{\lambda}{4}\chi^4 & \text{for } h < M_P/\xi \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2 & \text{for } h > M_P/\xi \end{cases}$$



CMB parameters—spectrum and tensor modes





Higgs inflation: radiative corrections

Effect No 1

Radiative corrections: running of the coupling constants to inflationary scale. Main result: the form of the potential is universal for

$$M_H > M_{crit} - 0.1 \log \frac{\xi}{1000} \text{ GeV}$$

$$M_{crit} = [129.1 + \frac{y_t(173.2 \text{ GeV}) - 0.9361}{0.0058} \times 2.0 - \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \times 0.5] \text{ GeV}$$

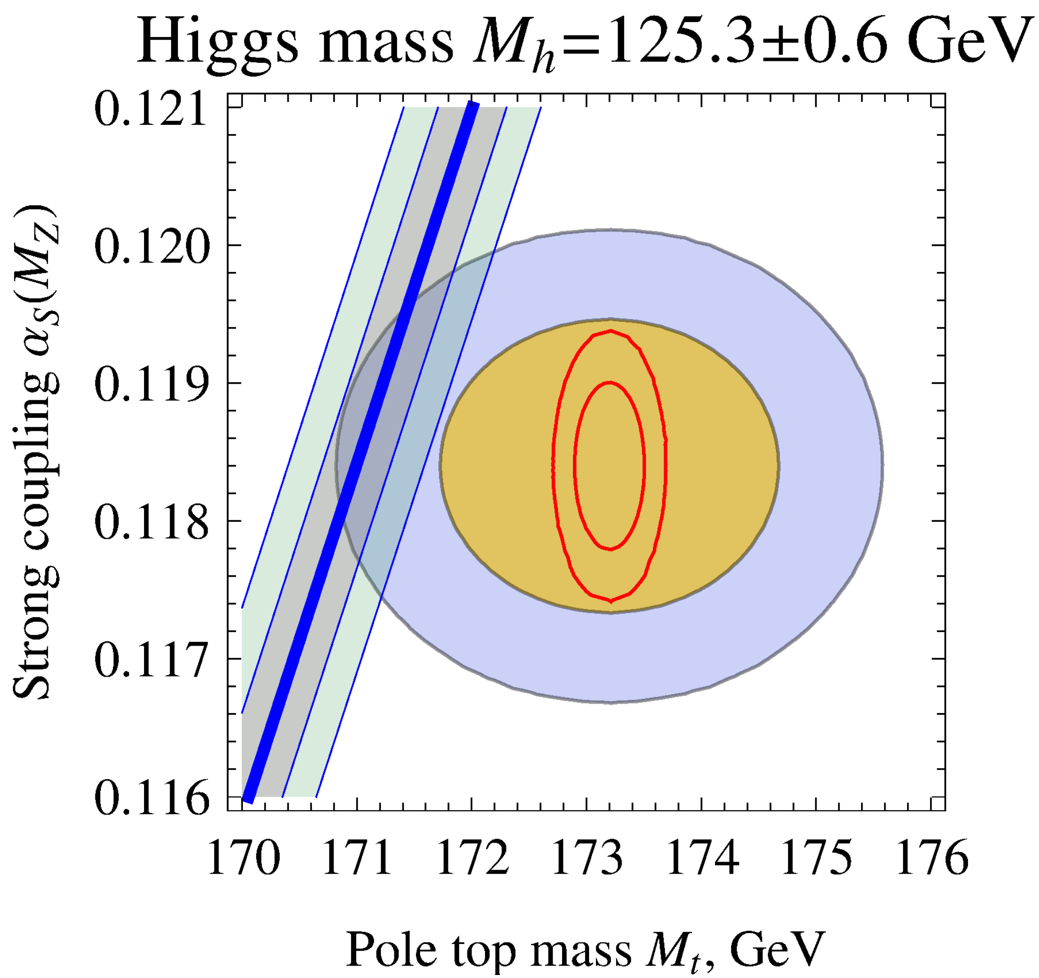
y_t - top Yukawa coupling

$\alpha_s(M_Z)$ - strong coupling

M_{crit}: Bezrukov et al, Degrassi et al, Buttazzo et al,

theoretical uncertainty ~ 70 MeV from Buttazzo et al

Comparison with experiments for $\xi \sim 1$



errors in y_t: theory + experiment

Tevatron - LHC combination : $M_t = 173.34 \pm 0.27 \pm 0.71$ GeV

$\alpha_s = 0.1184 \pm 0.0007$

Main uncertainty - top Yukawa coupling.

- 1 GeV experimental error in M_t leads to 2 GeV error in M_{crit}.
- Perturbation theory, $\mathcal{O}(\alpha_s^4)$. Estimate of Kataev and Kim: $\delta y_t/y_t \simeq -750(\alpha_s/\pi)^4 \simeq -0.0015$, $\delta M_{crit} \simeq -0.5$ GeV
- Non-perturbative QCD effects, $\delta M_t \simeq \pm\Lambda_{QCD} \simeq \pm 300$ MeV, $\delta M_{crit} \simeq \pm 0.6$ GeV
- Alekhin et al. Theoretically clean is the extraction of y_t from $t\bar{t}$ cross-section. However, the experimental errors in $p\bar{p} \to t\bar{t} + X$ are quite large, leading to $\delta M_t \simeq \pm 2.8$ GeV, $\delta M_{crit} \simeq \pm 5.6$ GeV.

Precision measurements of m_H, y_t and α_s are needed! ILC, TLEP stage of FCC.

Effect No 2

There are two essential scales for the Higgs field background h in the Higgs inflation

- $h \lesssim h^* \simeq M_P/2\sqrt{6}\xi$. For these fields
  - The running of the couplings coincides with the standard one
- $h \gtrsim h^* \simeq M_P/2\sqrt{6}\xi$. For these fields
  - The running of the couplings coincides with that for the Chiral SM
  - The couplings $\lambda,\ y_t,$ etc make a rapid change at $h \simeq h^*$ Bezrukov, Magnin, MS, Sibiryakov

Physics of the jump of the constants

Top quark Yukawa interaction in the Einstein frame:

$$L_t = \frac{y_t}{\sqrt{2}}\bar{t}tF(\chi)\ , F(\chi) = \frac{h}{\Omega}\ ,$$

Here $\Omega^2 = 1 + \xi h^2/M_P^2$ is the conformal factor, and the canonically normalised field χ is related to the Higgs field h via

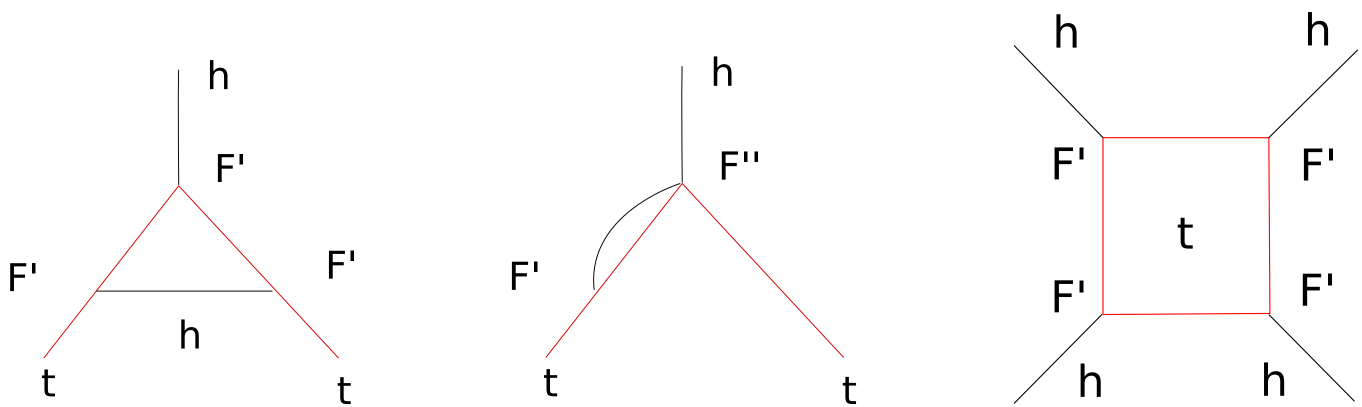
$$\frac{dh}{d\chi} = \frac{\Omega^2}{\sqrt{\Omega^2 + \xi(6\xi+1)h^2/M_P^2}}\ .$$

In the background field χ coupling of the top to χ is proportional to $dF/d\chi = F'$.

Counter-terms:

$$y_t \to y_t + \frac{y_t^3}{16\pi^2}\left(\frac{3}{\epsilon} + C_t\right) F'^2$$

$$\lambda \to \lambda - \frac{y_t^4}{16\pi^2}\left(\frac{6}{\epsilon} - C_\lambda\right) F'^4 \,,$$



$$F'(\chi) = \begin{cases} 1 & \text{for } h \lesssim M_P/2\sqrt{6}\xi \\ 0 & \text{for } h \gtrsim M_P/2\sqrt{6}\xi \end{cases}$$

For $h \lesssim h^*$ C is absorbed into definition of low energy coupling and is not observable: $y_t^{\text{phys}} = y_t + \frac{y_t^3}{16\pi^2}C_t$ and $\lambda^{\text{phys}} = \lambda - \frac{y_t^4}{16\pi^2}C_\lambda$.

For $h \gtrsim h^*$ contribution from F' disappears: determines a jump of the coupling around h^*

"Inflationary" top and Higgs masses m_t^* and M_h^* : lead to y_t and λ at high energies through the renormalisation group evolution *without any jumps* at $h = h^*$

$$m_t^* = m_t \left(1 - \frac{y_t^2 C_t}{16\pi^2}\right), \;\; M_h^* = M_h \left(1 - \frac{y_t^4 C_\lambda}{16\pi^2}\frac{h_0^2}{M_h^2}\right) \,,$$

Numerically (in the units of GeV, and for $M_h \simeq 126$ GeV),

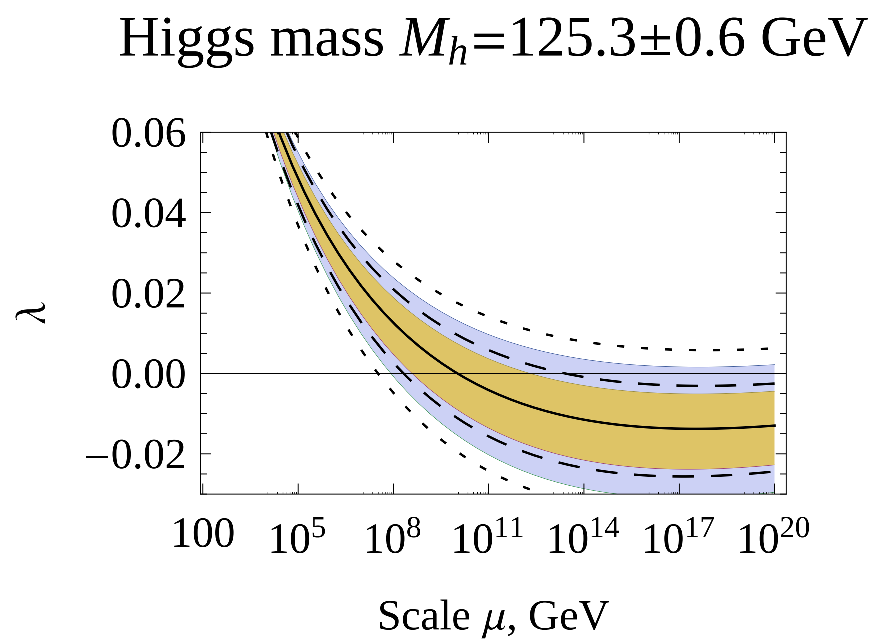
$$m_t^* \simeq m_t - C_t, \;\; M_h^* \simeq M_h - 3C_\lambda \,.$$

Effect No 3:

Critical Higgs inflation

Critical point

Behaviour of λ:



$$\lambda(z) = \lambda_0 + b\,(\log z)^2\,, \quad z = \frac{\mu}{qM_P}, \quad M_P = 2.44 \times 10^{18}\mathrm{GeV}$$

Numerically $\lambda_0 \ll 1, \;\; q \sim 1, \;\; b \simeq 2.3 \times 10^{-5}$.

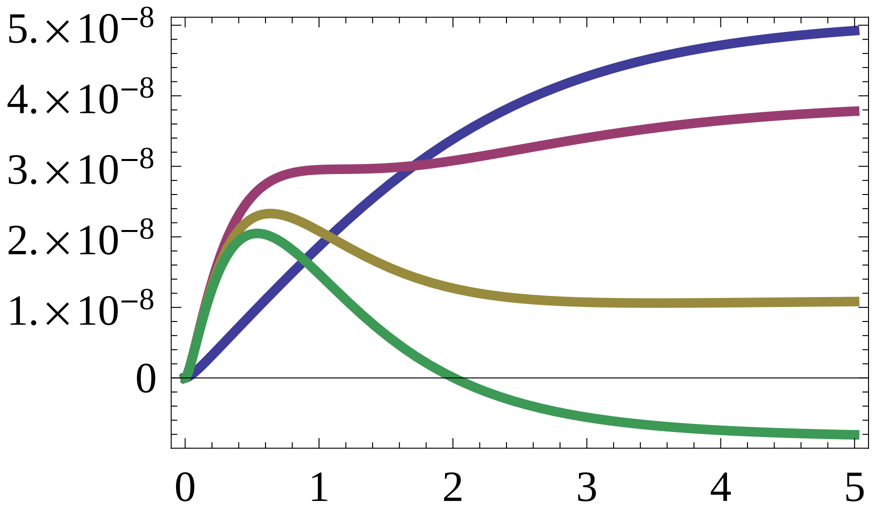
Effective potential

$$U(\chi) \simeq \frac{\lambda(z')}{4\xi^2}\bar{\mu}^4\,, \quad z' = \frac{\bar{\mu}}{\kappa M_P}, \quad \bar{\mu}^2 = M_P^2\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$

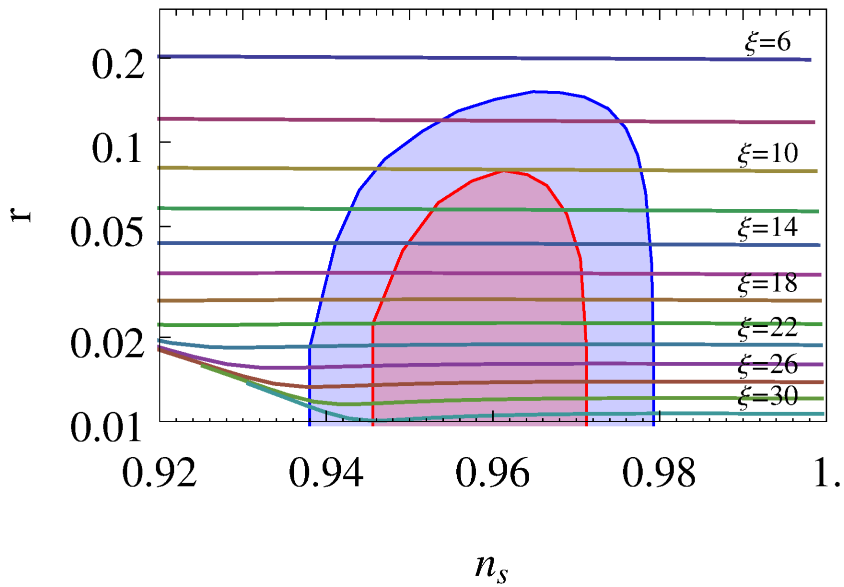
The parameter μ that optimises the convergence of the perturbation theory is related to $\bar{\mu}$ as

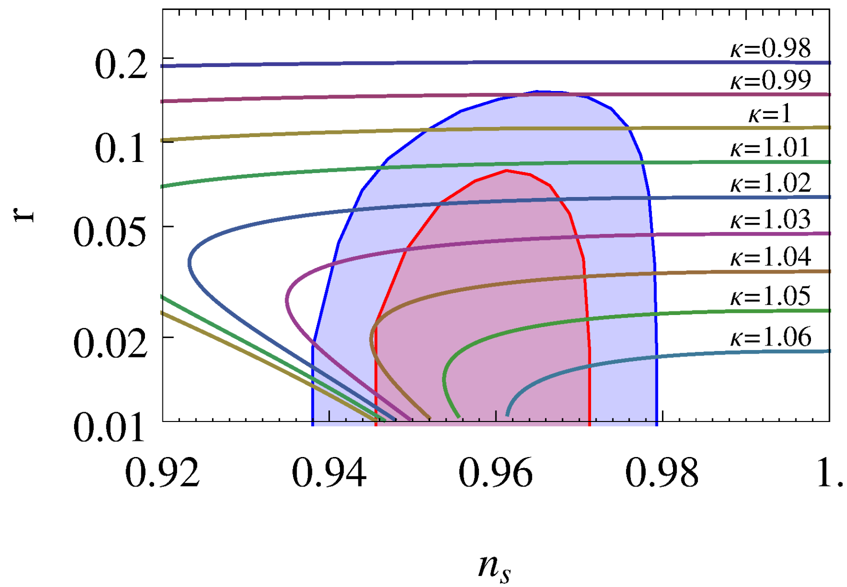
$$\mu^2 = \alpha^2 \frac{y_t(\mu)^2}{2}\frac{\bar{\mu}^2}{\xi(\mu)}\,, \quad \alpha \simeq 0.6$$

Behaviour of effective potential for $\lambda_0 \simeq b/16$:



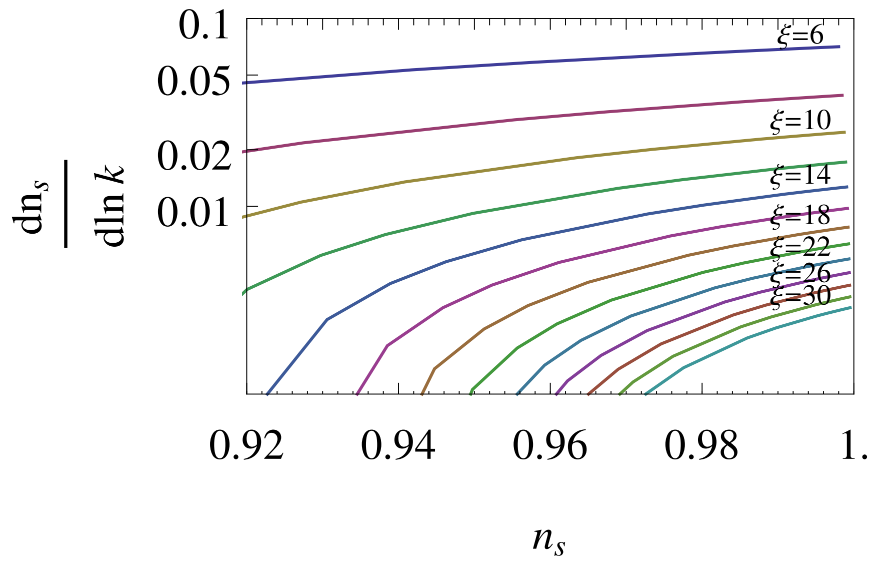
The inflationary indexes



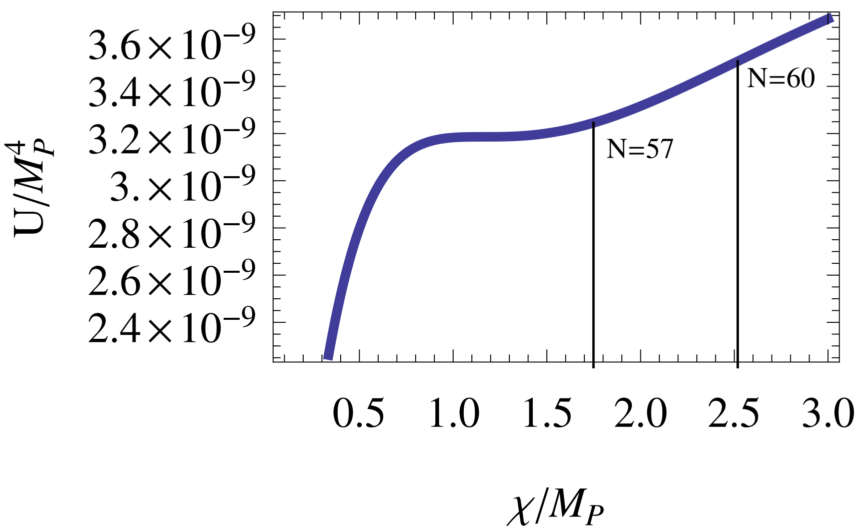


r can be large!

see also Hamada, Kawai, Oda and Park



0.1
0.05
0.02
0.01
dn_s / dln k
ξ=6
ξ=10
ξ=14
ξ=18
ξ=22
ξ=26
ξ=30
0.92
0.94
0.96
0.98
1.
n_s



3.6×10^-9
3.4×10^-9
3.2×10^-9
3.×10^-9
2.8×10^-9
2.6×10^-9
2.4×10^-9
U/M_P^4
N=57
N=60
0.5
1.0
1.5
2.0
2.5
3.0
χ/M_P

Features of critical Higgs inflation

- Relatively small non-minimal coupling $\xi \sim 10$
- No new scale in addition to the Planck scale, as $M_P/\xi \sim M_P/\sqrt{\xi} \sim M_P$
- Radiation dominated universe right after inflation

Relation to inflationary Higgs and top masses

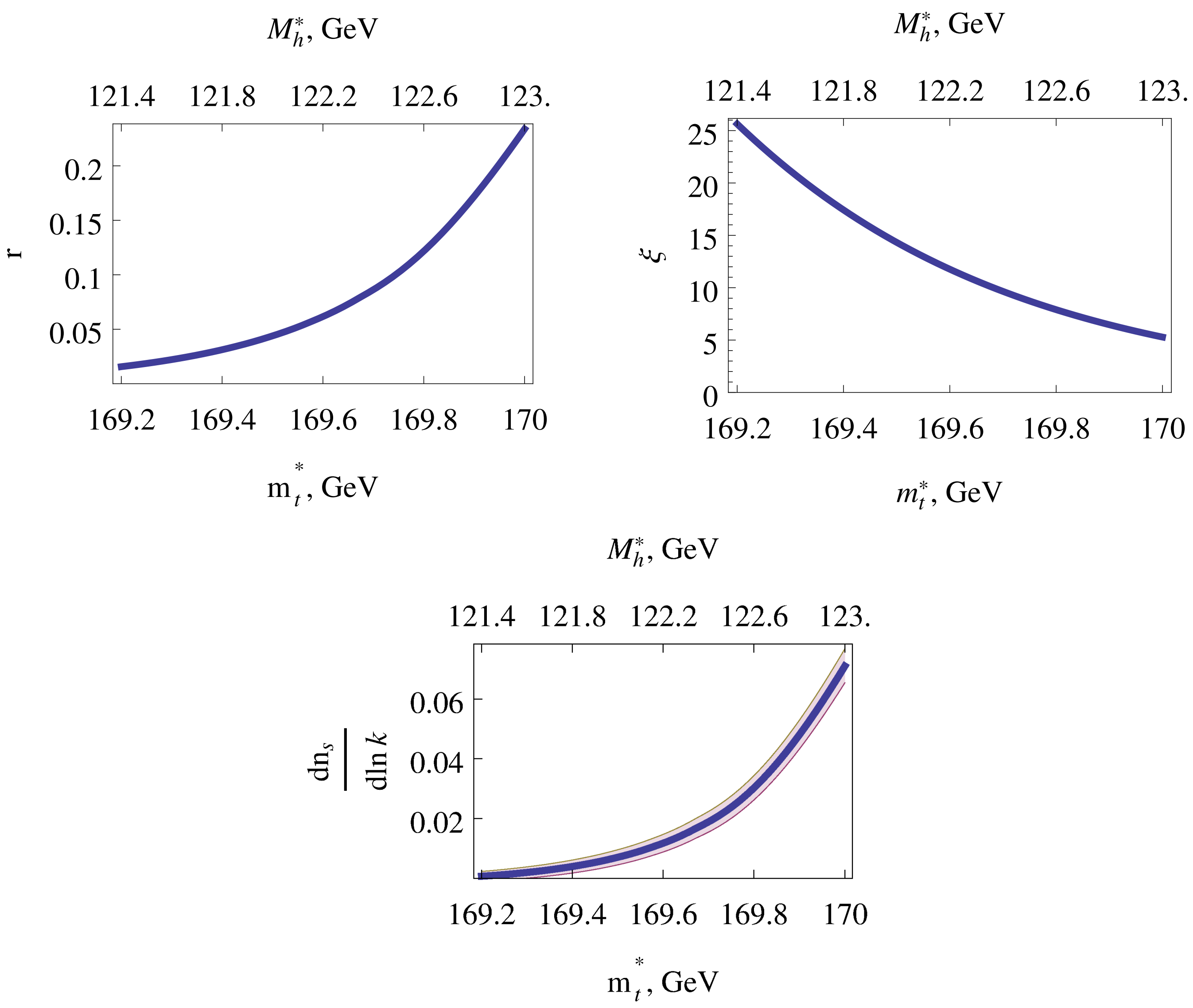
Parameters of the inflationary potential and inflationary masses m_t^* and M_h^*:

$$\lambda_0 = 0.003297\big((M_h^* - 126.13) - 2(m_t^* - 171.5)\big),$$

$$q = 0.3\exp\big(0.5(M_h^* - 126.13) - 0.03(m_t^* - 171.5)\big),$$

$$b = 0.00002292 - 1.12524\times 10^{-6}\big((M_h^* - 126.13) - 1.75912(m_t^* - 171.5)\big),$$

Indexes for $n_s \in \{0.94, 0.98\}$



Relation to physical top and Higgs masses

Example: for $r = 0.12$ we have $M_h^* \simeq 122.6$ GeV and $m_t^* \simeq 169.8$ GeV, $\xi \simeq 8$.

To get the physics low energy value of the Higgs mass $M_h = 125.6$ GeV we need $C_\lambda \simeq 1$.

The value of $C_t \simeq 1.5$ would bring the physical top mass to $m_t \simeq 171.5$ GeV , consistent with the measured top quark mass within 2 σ uncertainties.

The relative jump of the top Yukawa coupling at $h = h^*$ is $(y_t^{\text{phys}} - y_t)/y_t \simeq 0.024$, while the jump in λ is very sensitive to the physical Higgs and top masses and can be made equal to zero by tuning m_t and M_h within few MeV.

Conclusions

- For $M_h > M_{\text{crit}}$ is a predictive theory for *cosmology*: inflationary indexes are practically independent of the SM parameters
- For $M_h = M_{\text{crit}}$: strong dependence of n_s and r on inflationary masses of the top quark and the Higgs boson m_t^* and M_h^*
- If r is large, the required m_t^* and M_h^* are remarkably close to the physical values m_t and M_h; uncertainties related to the transition from low and high energies corresponding to the Higgs field $h^* \propto M_P/\xi$ are quite small.
- Higgs inflation near the critical point is quite far from a power-low inflation - a detailed re-analysis of the cosmological data is needed