Inflation and Supersymmetry after Planck

Inflation and Supersymmetry after ~~Planck~~ BICEP

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- Why Supersymmetry and Inflation
- Supergravity and the η problem
- No-Scale Supergravity
- Planck and $R+R^2$ inflation
- BICEP and quadratic chaotic inflation
- Supergravity models
- Stabilization
- One No-Scale model to them fit them all
- Phenomenology

Inflation- Cosmological Problems

Flatness Problem

Friedmann equation (with $\Lambda = 0$):

$$\frac{k}{R^2} = H^2(\Omega - 1)$$

Inflation- Cosmological Problems

Flatness Problem

Friedmann equation (with $\Lambda = 0$):

$$\frac{k}{R^2} = H^2(\Omega - 1)$$

Divide by T^2 and evaluate today:

$$\hat{k} = \frac{k}{R_0^2 T_0^2} = H_0^2(\Omega_0 - 1)/T_0^2 < 2 \times 10^{-58}$$

Represents an initial condition on the Universe

Inflation- Cosmological Problems

Horizon Problem

Causal volume V ~ t^3
but the Universe expands as $t^{2/3}$ (matter dominated)

Today's visible Universe contains (for t at recombination)

$$(\frac{t_0}{t})^3(\frac{R}{R_0})^3 = (\frac{t_0}{t}) \sim 10^5$$

different causal horizon volumes.

Inflation- Cosmological Problems

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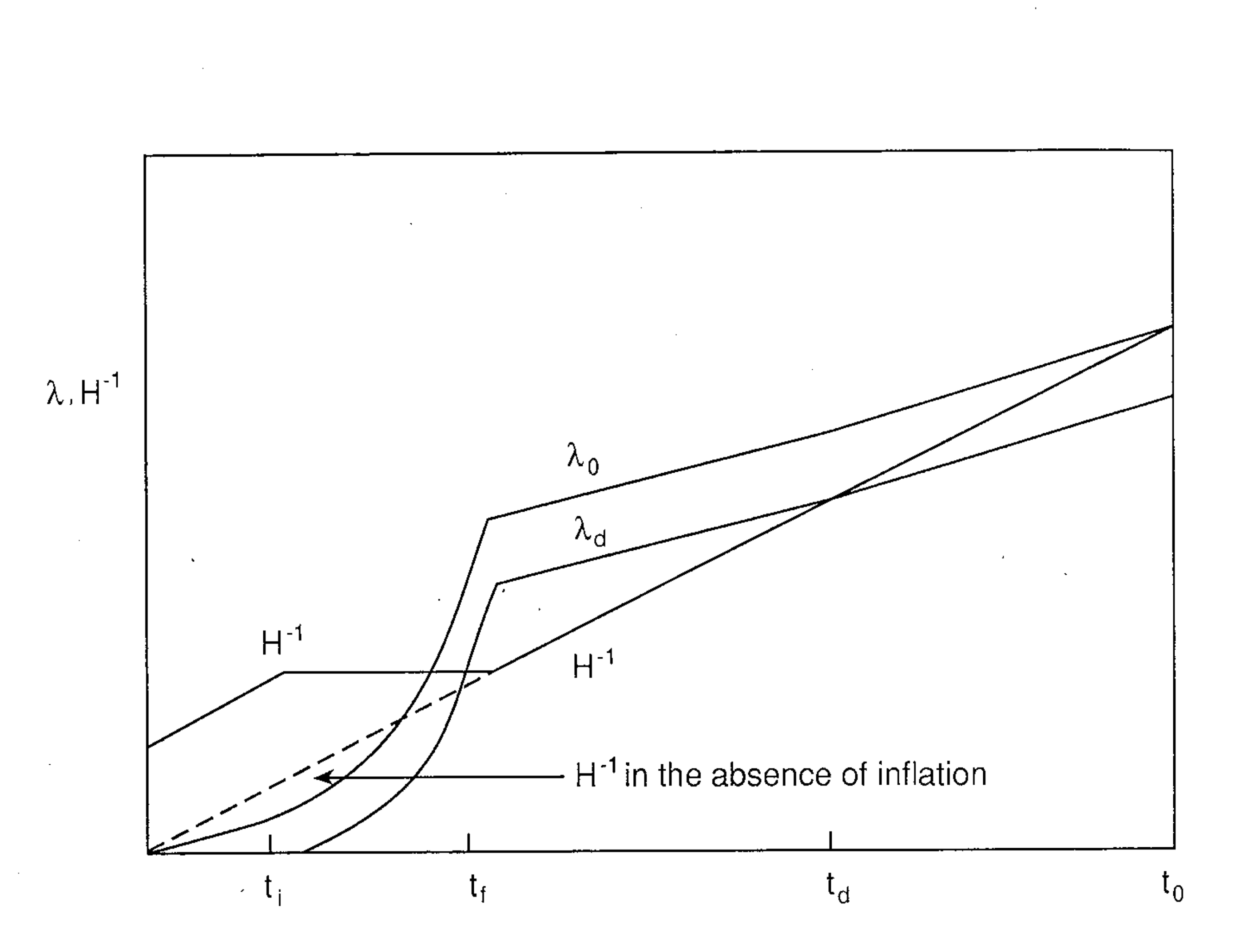
Why is

$$\frac{\Delta T}{T} \sim 10^{-5}$$

Inflation- Cosmological Problems

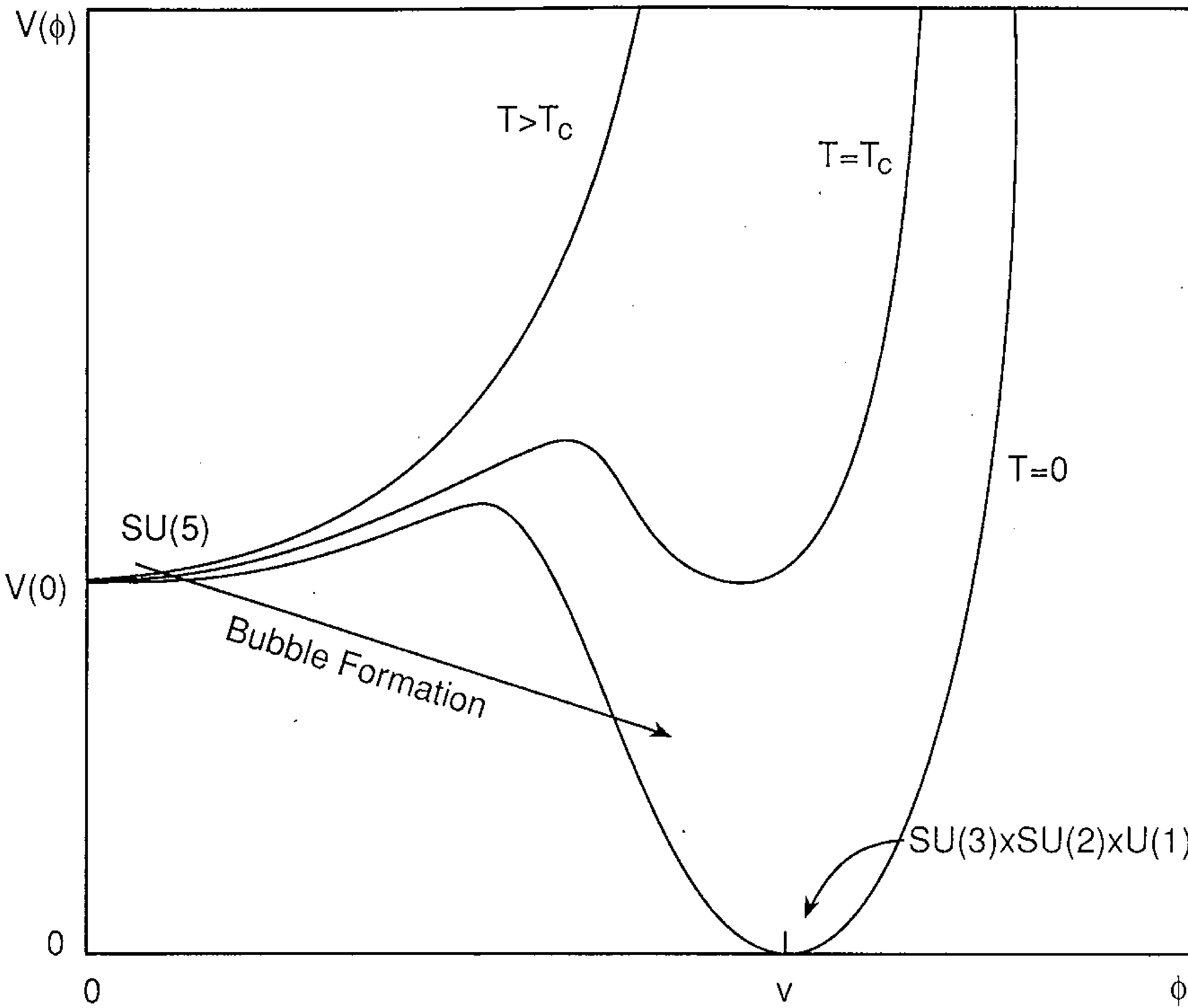
Perturbations Problem

Perturbations appear to have been produced outside our horizon.



Inflation

- Standard cosmology assumes an adiabatically expanding Universe, $R \sim 1/T$
- Phase transitions can violate this condition
- Expect several phase transitions in the Early Universe
  - GUTS: SU5 $\rightarrow$ SU(3) x SU(2) x U(1)
  - SM: SU(2) x U(1) $\rightarrow$ U(1)
  - possibly other non-gauged symmetry breakings
- Entropy production common result
- Type of inflation will depend on the order of the phase transition



V(ϕ)
T>Tc
T=Tc
T=0
SU(5)
V(0)
Bubble Formation
SU(3)xSU(2)xU(1)
0
0
v
ϕ

Inflation

$$\Lambda = 8\,\pi\,G_N\,V_0$$

For $\varrho \ll V_0$,

$$H^2 = \frac{\dot{R}^2}{R^2} \approx \frac{8\pi G_N V_0}{3} = \frac{\Lambda}{3}$$

or

$$\frac{\dot{R}}{R} \approx \sqrt{\frac{\Lambda}{3}} \qquad R \sim e^{Ht}$$

For $H\tau > 65$, curvature problem solved

Inflation

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For $\varrho << V_0$,

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or

$$\frac{\dot{R}}{R} \approx \sqrt{\frac{\Lambda}{3}} \qquad R \sim e^{Ht}$$

For $H\tau > 65$, curvature problem solved

When the transition is over, the Universe reheats to $T < V_0^{1/4} \sim T_i$, but $R >> R_i$

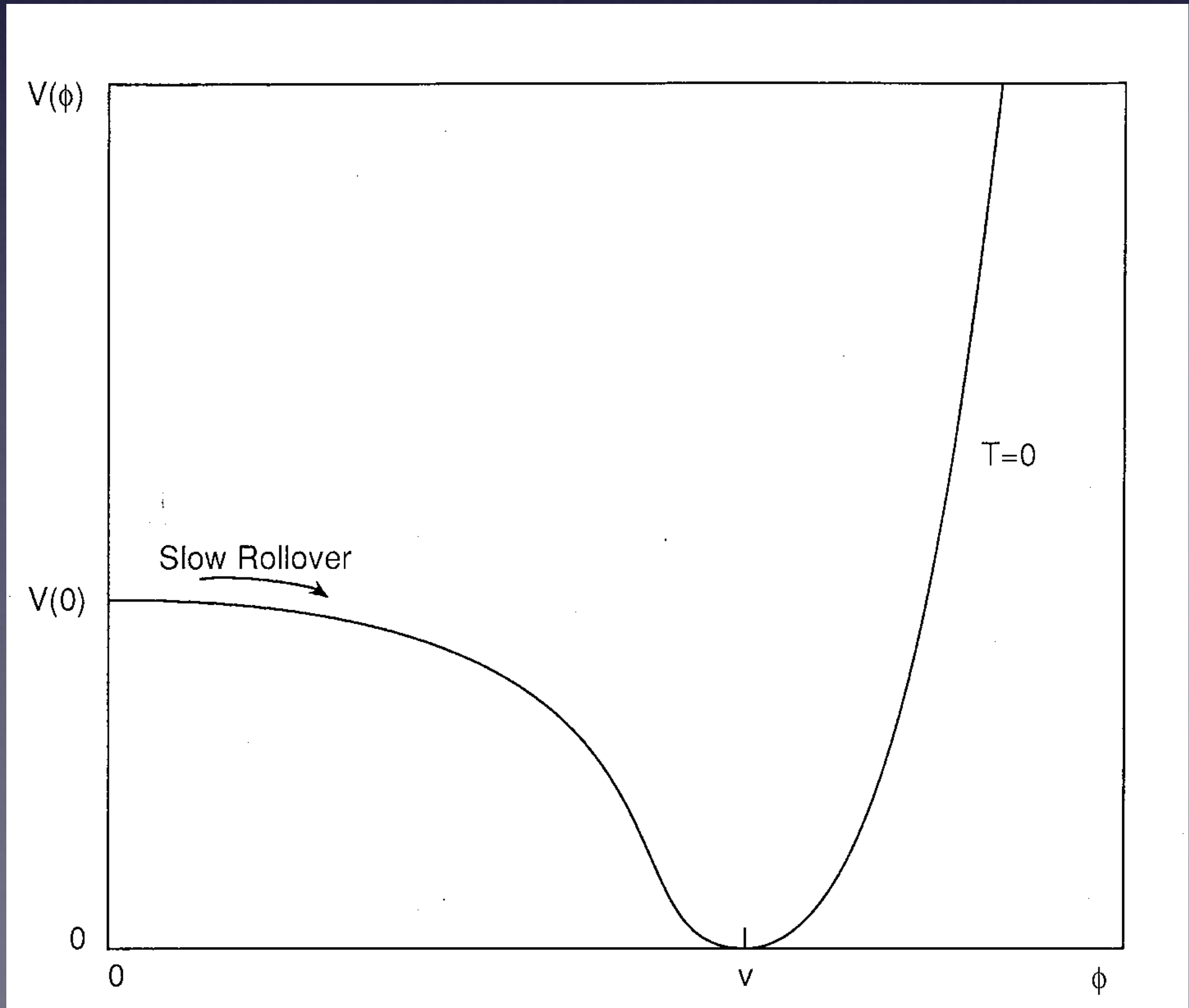
Old New Inflation

Great idea based on 1-loop corrected SU(5) potential for the adjoint:

$$V(\sigma) = A\sigma^4(\ln\frac{\sigma^2}{v^2} - \frac{1}{2})$$

$$A = \frac{5625}{1024\pi^2} g_5^4$$

Linde; Albrecht, Steinhardt



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Linde; Albrecht, Steinhardt

Problems:

- Vacuum structure
- destabilization through quantum fluctuations
- fine tuning (require curvature to be << M_X)
- density fluctuations - $\delta\rho/\rho \sim 100\ g_5^2$

How SUSY can help

Exact Susy - $V_{1\text{-loop}} = 0$

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$$A = \frac{1}{64\pi^2 v^4}(\sum_B g_B m_B^4 - \sum_F g_F m_F^4) = 0$$

How SUSY can help

Exact Susy - $V_{1\text{-loop}} = 0$

$$A = \frac{1}{64\pi^2 v^4}(\sum_B g_B m_B^4 - \sum_F g_F m_F^4) = 0$$

Broken Susy - $A = \frac{75}{32\pi^2 v^2} g_5^2 m_s^2$

fixes fine-tuning, $\delta\rho/\rho$, etc. -
but isn't really a model

Ellis,
Nanopoulos,
Olive,
Tamvakis

Supergravity

Start with a Kähler Potential

$$G = K + \ln|W|^2$$

Minimal N=1 defined by K = $\phi^i \phi_i^*$

and scalar potential

$$V = e^G \left[G_i (G^{-1})^i_j G^j - 3 \right] + \mathrm{D-terms}$$

or

$$V = e^{\phi^i \phi_i^*} \left[|\frac{\partial W}{\partial \phi^i} + \phi_i^* W|^2 - 3|W|^2 \right] + \mathrm{D-terms}$$

for minimal N=1

Typically, $m^2 \sim H^2$

η-problem!

Supergravity

Constructing Models $W = \mu^2 \sum_n \lambda_n \phi^n$

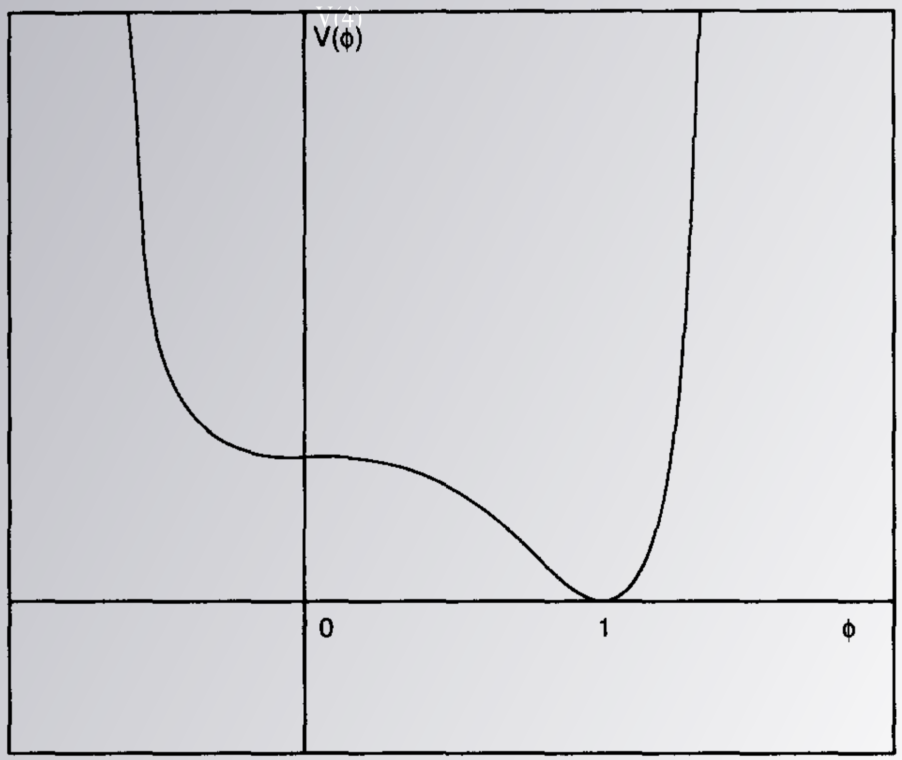
Nanopoulos, Olive, Srednicki, Tamvakis

μ^2 fixed by amplitude of density fluctuations, $\lambda_n \sim \mathcal{O}(1)$

Simplest example, $W = \mu^2 (1-\phi)^2$

Holman, Ramond, Ross

$$
\begin{aligned}
V &= \mu^4 e^{|\phi|^2} \Big[1 + |\phi|^2 - (\phi^2 + \phi^{*2}) - 2|\phi|^2(\phi + \phi^*) \\
&\quad + 5|\phi^2|^2 + |\phi|^2(\phi^2 + \phi^{*2}) - 2|\phi^2|^2(\phi + \phi^*) + |\phi^3|^2 \Big] \\
&\simeq \mu^4 \left(1 - 4\phi^3 + \frac{13}{2}\phi^4 + \cdots\right.
\end{aligned}
$$



Supergravity

Generic Models

Kawasaki, Yamaguchi, Yanagida; Kallosh, Linde, Rube; Kallosh, Linde, Olive, Rude

$$K = SS^* + (\phi - \phi^*)^2 + \cdots$$

with $\quad W = Sf(\Phi)$

resulting in $\quad V = |f(\phi)|^2$

Easily generates any potential
(which is a perfect square)

note $m^2\phi^2$ and Starobinsky models are perfect squares

No-Scale Supergravity

Natural vanishing of cosmological constant (tree level) with the supersymmetry scale not fixed at lowest order. (Also arises in generic 4d reductions of string theory.)

$$K = -3\ln(T + T^* - \phi^i {\phi_i}^*/3)$$

$$V = e^{\frac{2}{3}K} \left|\frac{\partial W}{\partial \phi^i}\right|^2$$

Globally supersymmetric potential once K (canonical) picks up a vev

No-Scale Supergravity

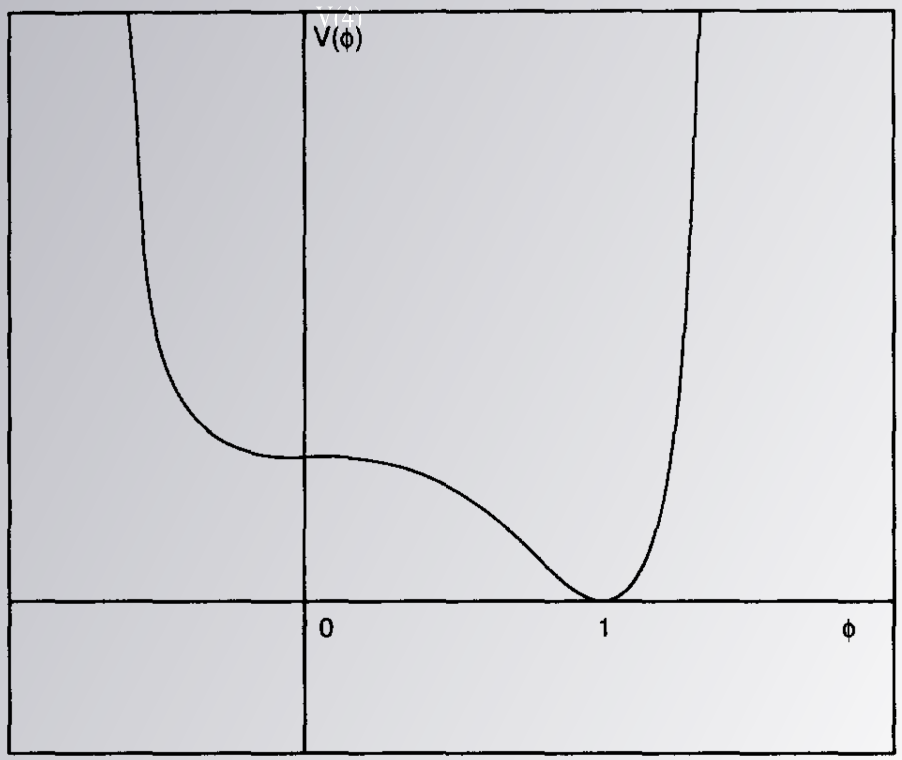
Constructing Models $$W = \mu^2 \sum_n \lambda_n \phi^n$$

Ellis, Enqvist, Nanopoulos, Olive, Srednicki

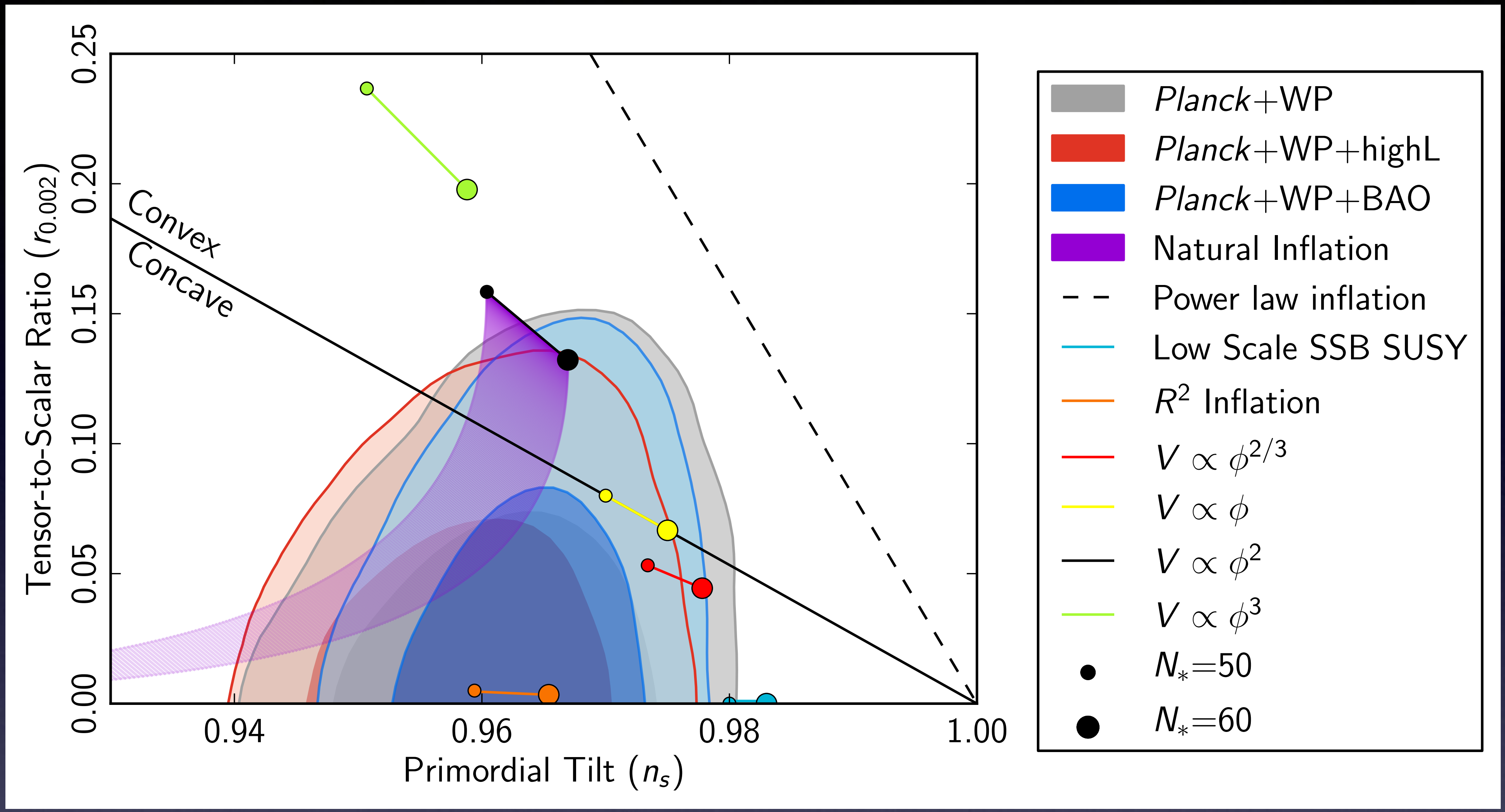
μ^2 fixed by amplitude of density fluctuations, $\lambda_n \sim \mathcal{O}(1)$

Simplest example, $$W = \mu^2(\phi - \phi^4/4)$$

$$V = \mu^4 |1 - \phi^3|^2$$



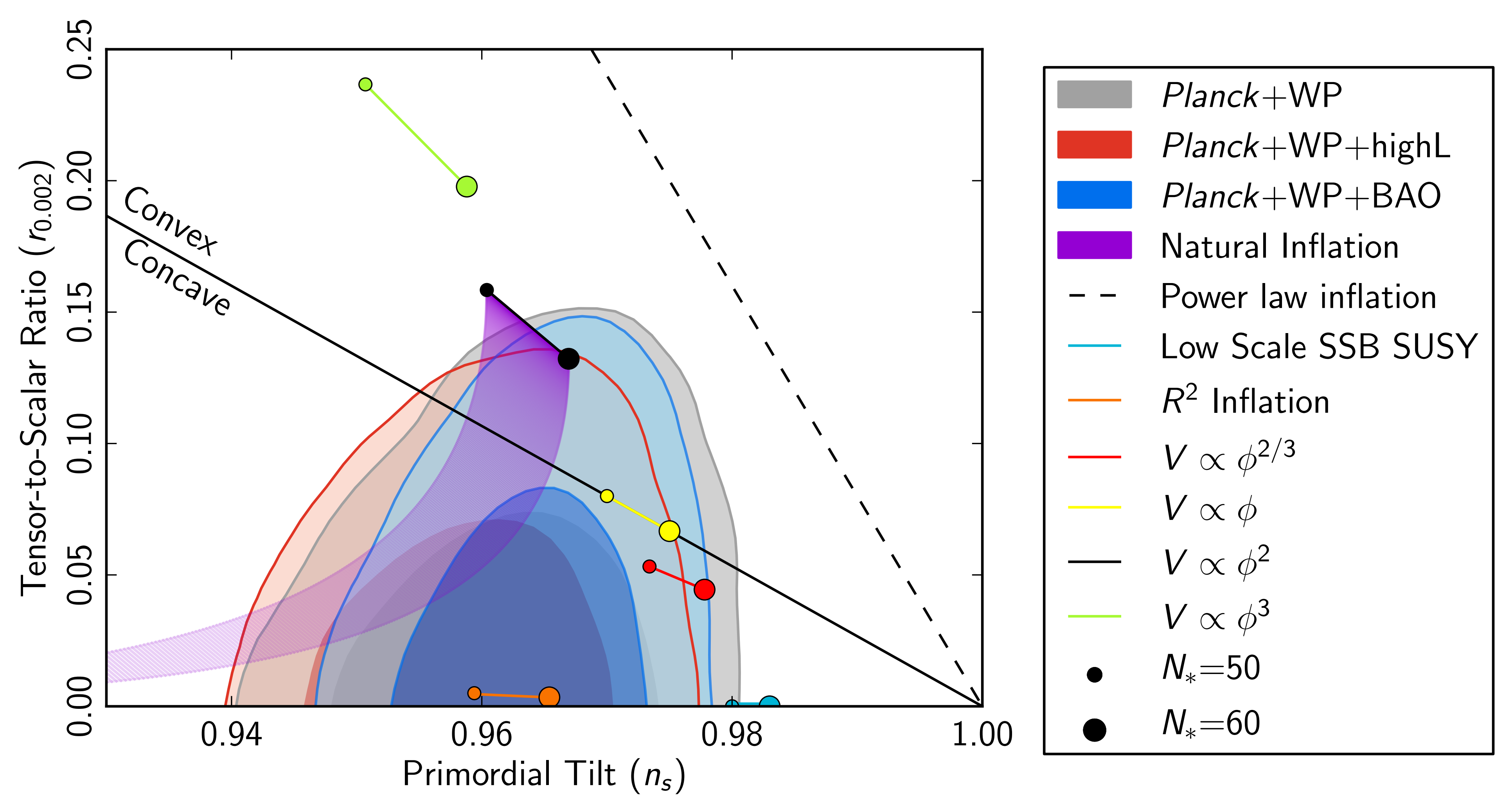
Planck Results



$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{V''}{V} \qquad n_s = 1 - 6\epsilon + 2\eta \qquad r = 16\epsilon$$

Trouble for simple supergravity
and no-scale models:

Planck Results



$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{V''}{V} \qquad n_s = 1 - 6\epsilon + 2\eta \qquad r = 16\epsilon$$

Trouble for simple supergravity and no-scale models:

$$\epsilon \simeq \frac{1}{72N^4} \ll 1 \quad \cancel{\text{OK}}$$

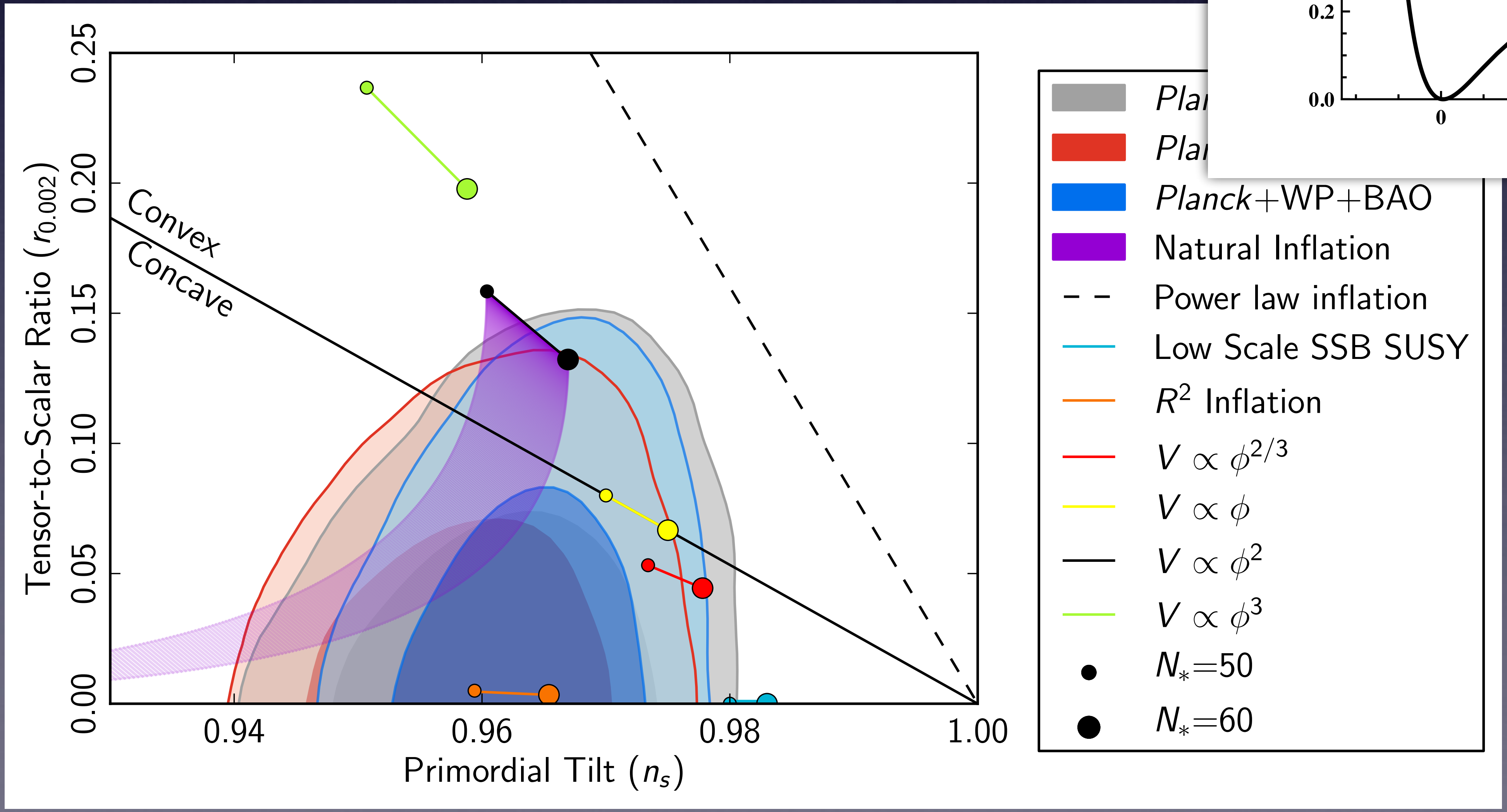
$$\eta \simeq -\frac{1}{2N} \quad \text{not OK}$$

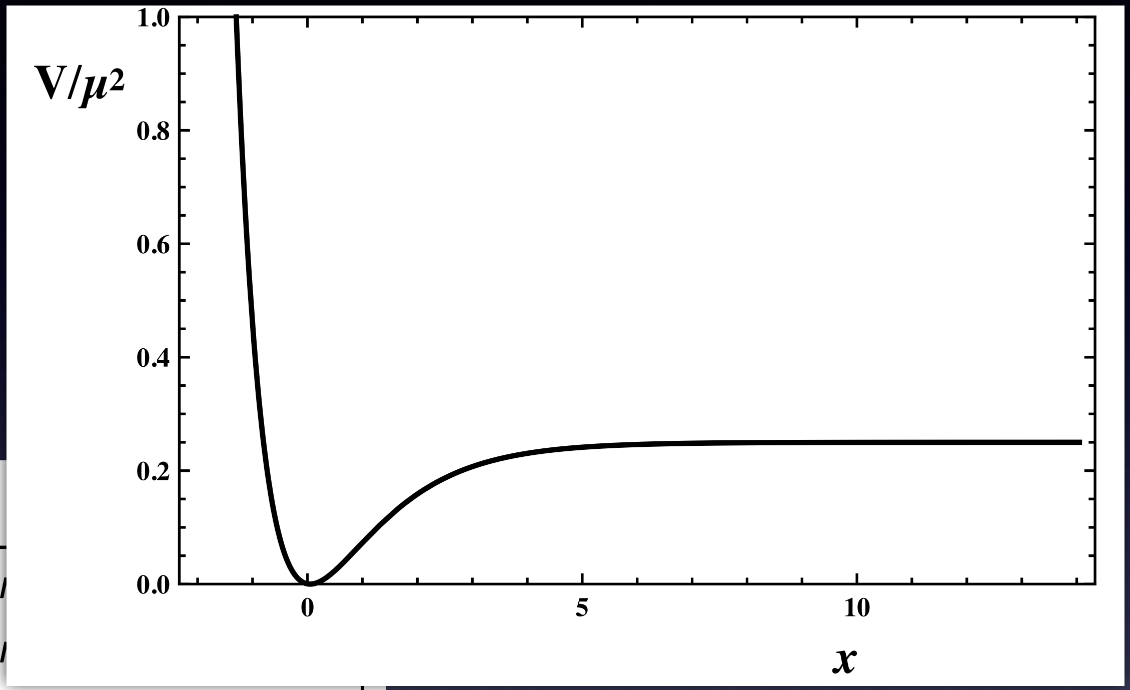
$$n_s \sim .933$$

Planck-friendly Models

R+R^2 Inflation

For N=55, n_s = 0.965; r = .0035

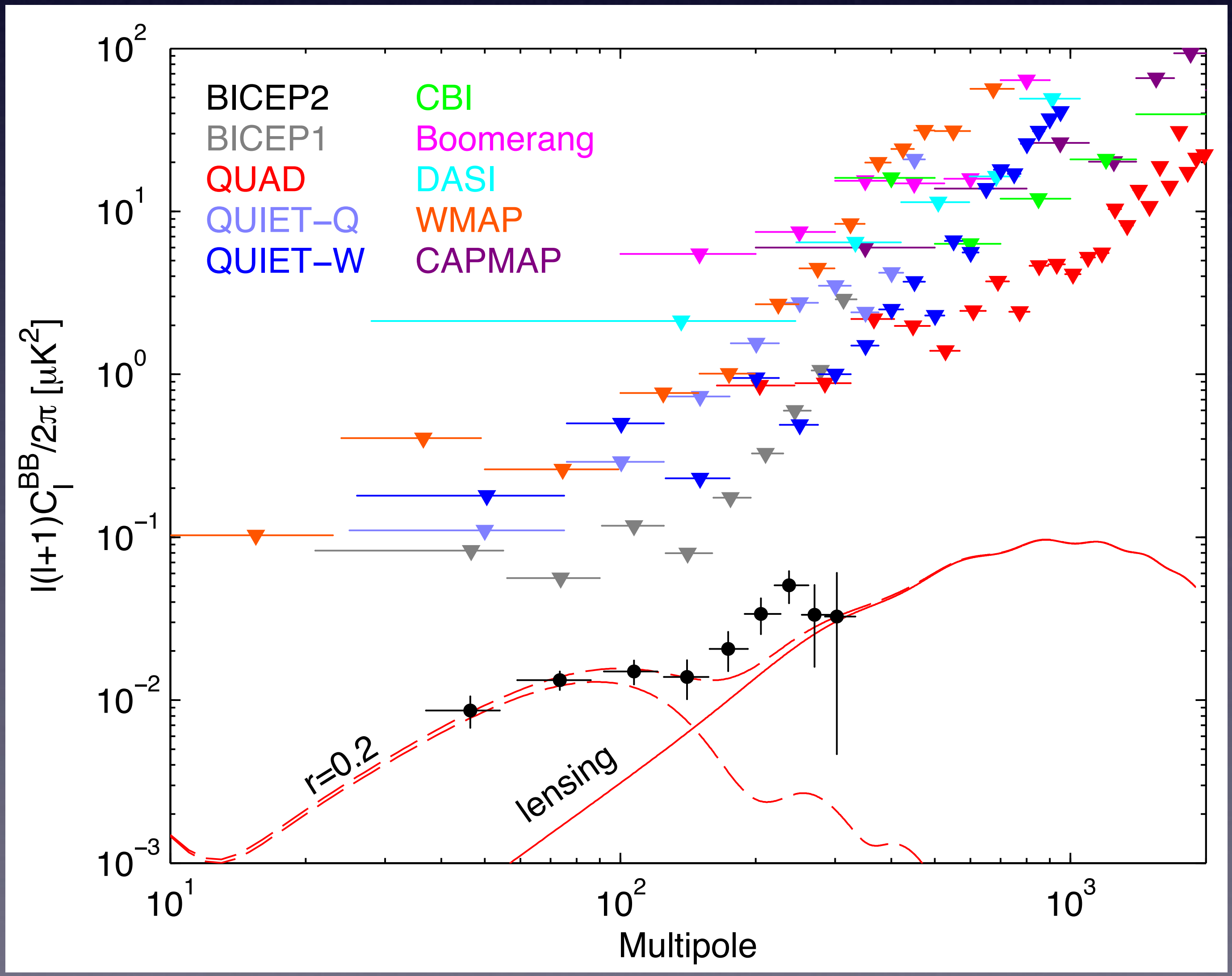




Bicep-friendly Models

Quadratic Chaotic Inflation

For N=55, n_s = 0.964; r = .145



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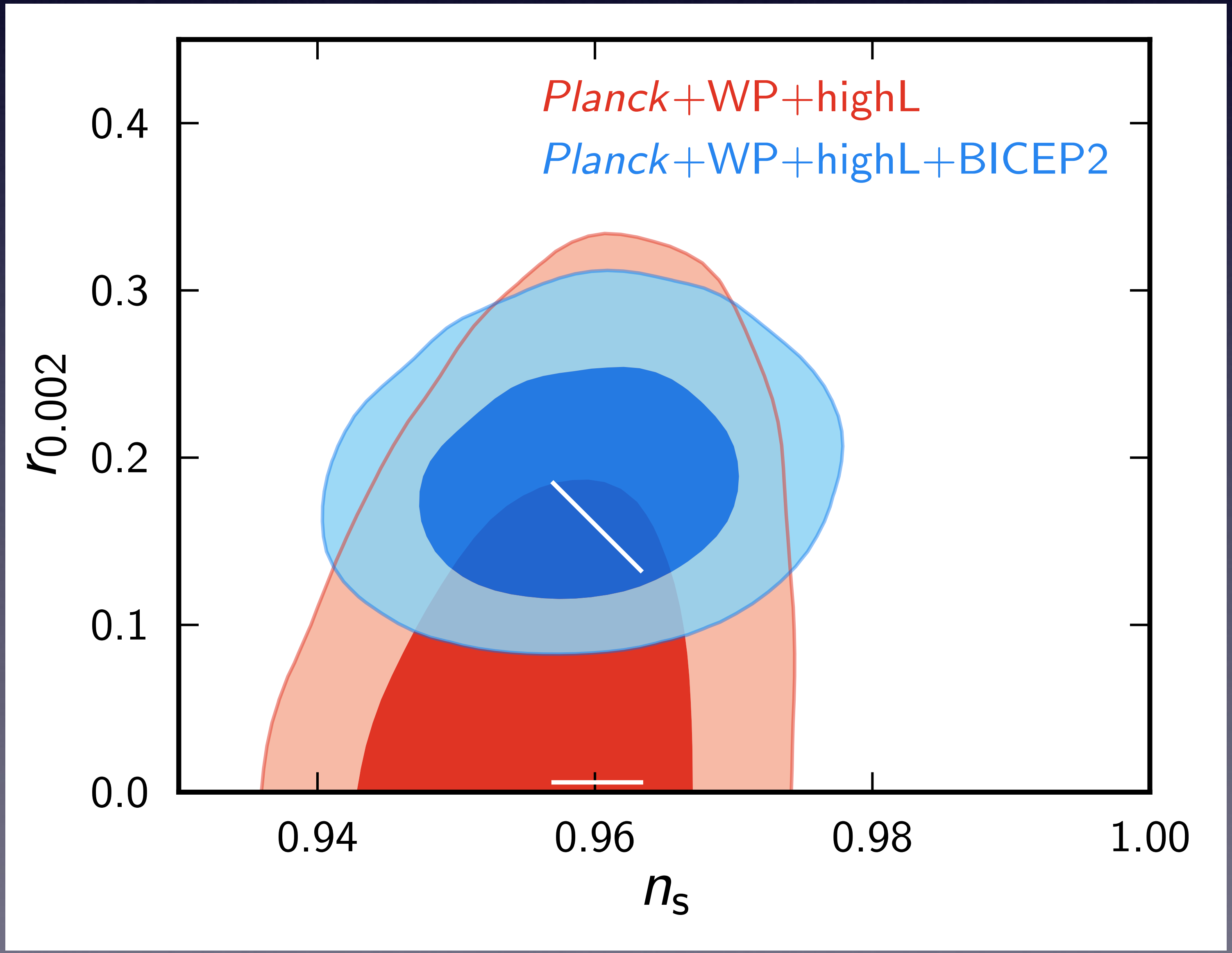

← Includes running of the spectral index

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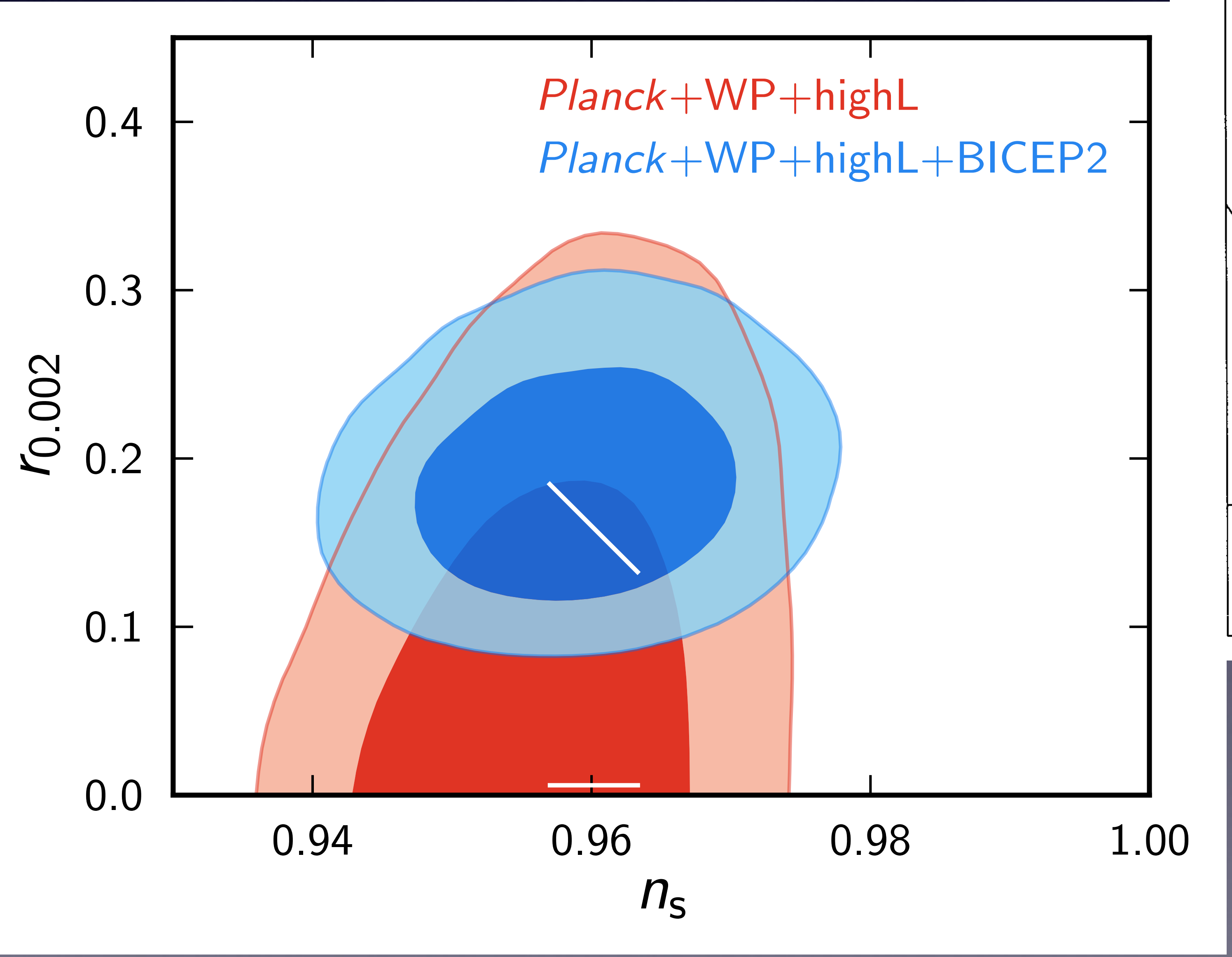

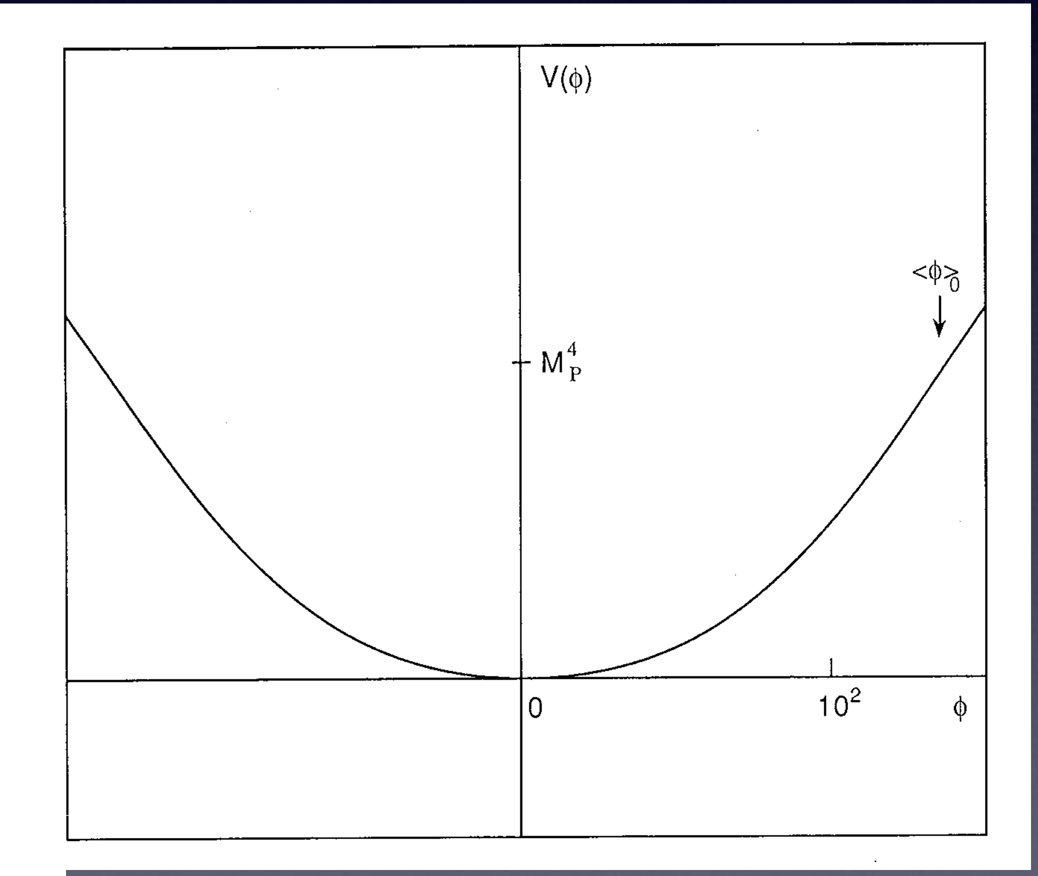

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No-Scale models revisited

Can we find a model consistent with Planck?

Ellis, Nanopoulos, Olive

Start with WZ model: $$W = \frac{\hat{\mu}}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$$

Assume now that T picks up a vev: 2<Re T> = c

$$\mathcal{L}_{eff} = \frac{c}{(c - |\phi|^2/3)^2}|\partial_\mu \phi|^2 - \frac{\hat{V}}{(c - |\phi|^2/3)^2}$$

Redefine inflaton to a canonical field χ

$$\hat{V} = |W_\Phi|^2$$

$$\phi = \sqrt{3c}\tanh\left(\frac{\chi}{\sqrt{3}}\right)$$

No-Scale models revisited

The potential becomes:

$$V = \mu^2 \left| \sinh(\chi/\sqrt{3}) \left(\cosh(\chi/\sqrt{3}) - \frac{3\lambda}{\mu} \sinh(\chi/\sqrt{3}) \right) \right|^2$$

For $\lambda=\mu/3$, this is exactly the $R + R^2$ potential and Starobinsky model of inflation

$$V = \mu^2 e^{-\sqrt{2/3}x} \sinh^2(x/\sqrt{6})$$

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and Starobinsky model of inflation

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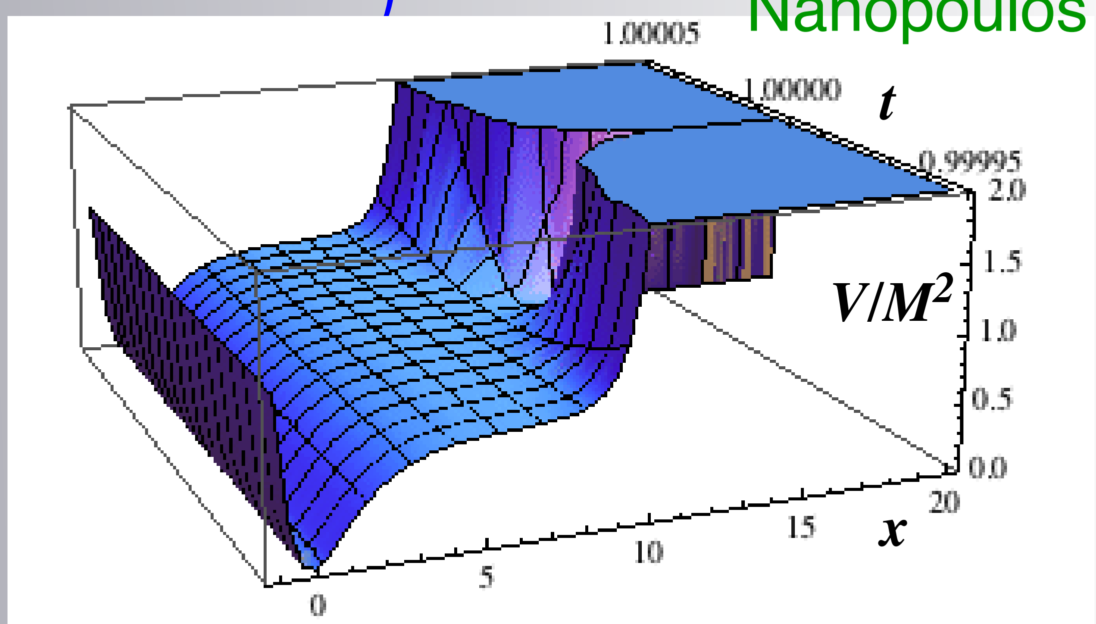
Have so far *assumed* a vev for one of the two fields

$$K = -3 \ln \left(T + T^* - \frac{|\phi|^2}{3} + \frac{(T + T^* - 1)^4 + d(T - T^*)^4}{\Lambda^2} \right)$$

Ellis, Kounnas, Nanopoulos



Classes of R+R^2 in No-Scale Supergravity

So is the inflaton T or ϕ?

1) T-fixed (ϕ-inflaton)

Starobinsky potential found when

Ellis, Nanopoulos, Olive

$$\hat{V} = M^2|\phi|^2|1 - \phi/\sqrt{3}|^2$$

$$W = M\left[\frac{\phi^2}{2} - \frac{\phi^3}{3\sqrt{3}}\right]$$

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Ellis, Nanopoulos, Olive

$$W \;=\; M\left[\frac{\phi^2}{2} - \frac{\phi^3}{3\sqrt{3}}\right]$$

2) ϕ-fixed (T-inflaton)

Starobinsky potential found when

$$\hat{V} = 3M^2|T-1/2|^2$$

Cecotti; Kallosh, Linde

$$W \;= \sqrt{3}M\phi(T-1/2)$$

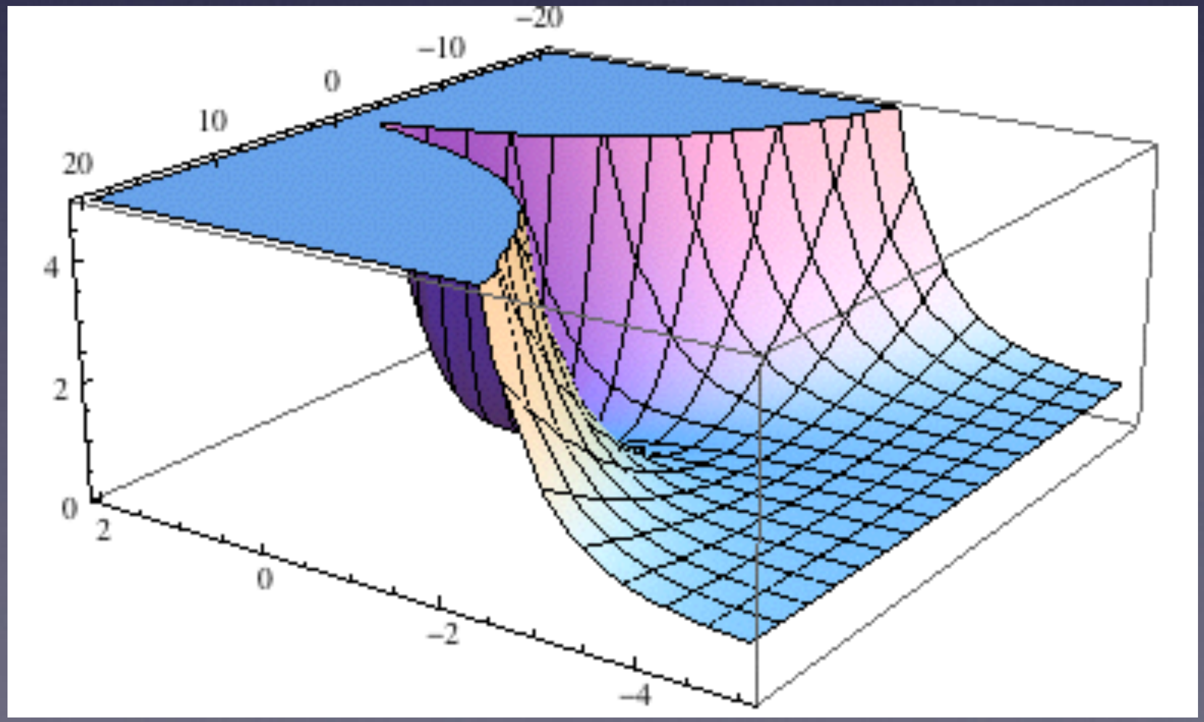
Qadratic Chaotic Inflation:

with $$W = \sqrt{3} M \phi (T - 1/2)$$

Cecotti

write $$T = \frac{1}{2}\left(e^{-\sqrt{\frac{2}{3}}\rho} + i\sqrt{\frac{2}{3}}\,\sigma\right)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \rho \partial^\mu \rho + \frac{1}{2} e^{2\sqrt{\frac{2}{3}}\rho} \partial_\mu \sigma \partial^\mu \sigma - \frac{3}{4} m^2 \left(1 - e^{-\sqrt{\frac{2}{3}}\rho}\right)^2 - e^{2\sqrt{2/3}\rho} \frac{1}{2} m^2 \sigma^2 \,,$$



Ferrara, Kehagias, Riotto

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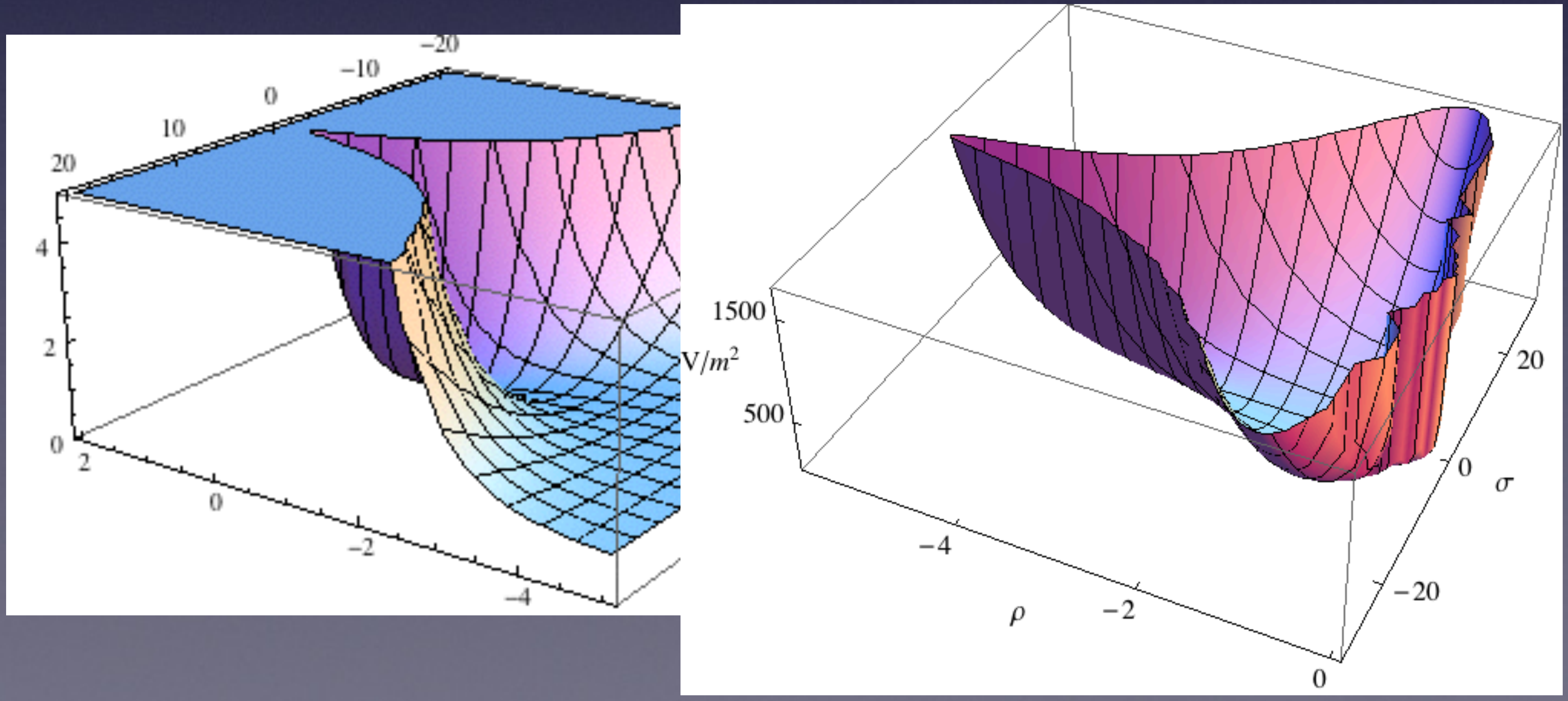

Ferrara, Kehagias, Riotto

Ellis, Garcia, Nanopoulos, Olive

SU(1,1)/U(1) models

Ellis, Garcia,
Nanopoulos, Olive

$$K = -3\ln(T+\bar{T}) + \frac{|\phi|^2}{(T+\bar{T})^w}$$

with $W = \sqrt{3}M\phi(T-1/2)$

$$V \propto e^{|\phi|^2/(T+\bar{T})^w} \simeq e^{|\phi|^2}$$

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with $W = \sqrt{3}M\phi(T-1/2)$

$$V \propto e^{|\phi|^2/(T+\bar{T})^w} \simeq e^{|\phi|^2}$$

$$V = 3m^2|T-1/2|^2(T+T^*)^{(w-3)}$$

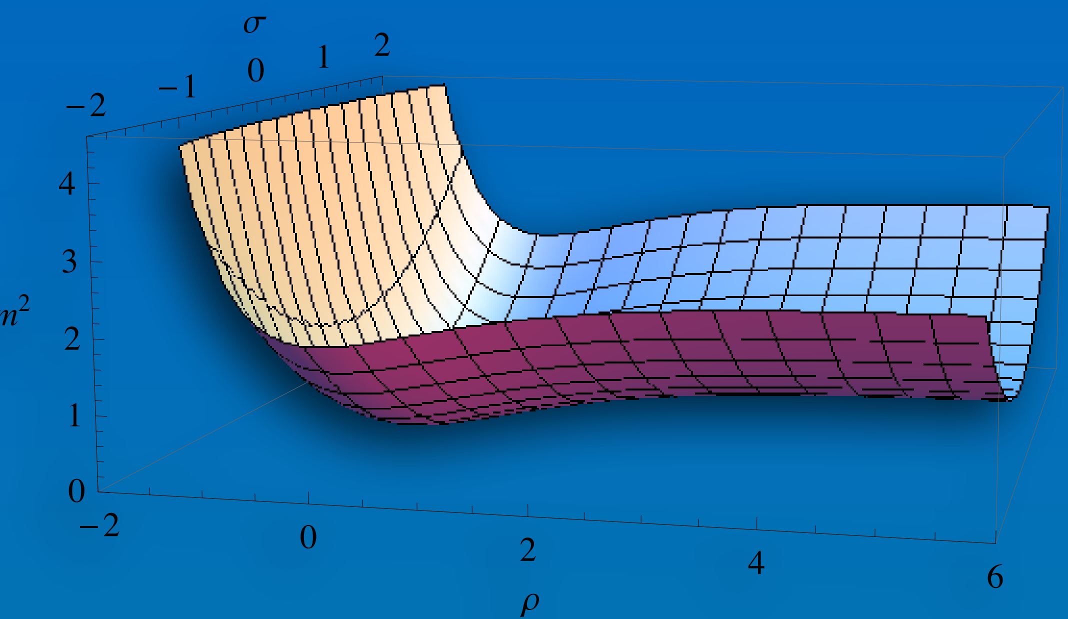
choose $w = 3$

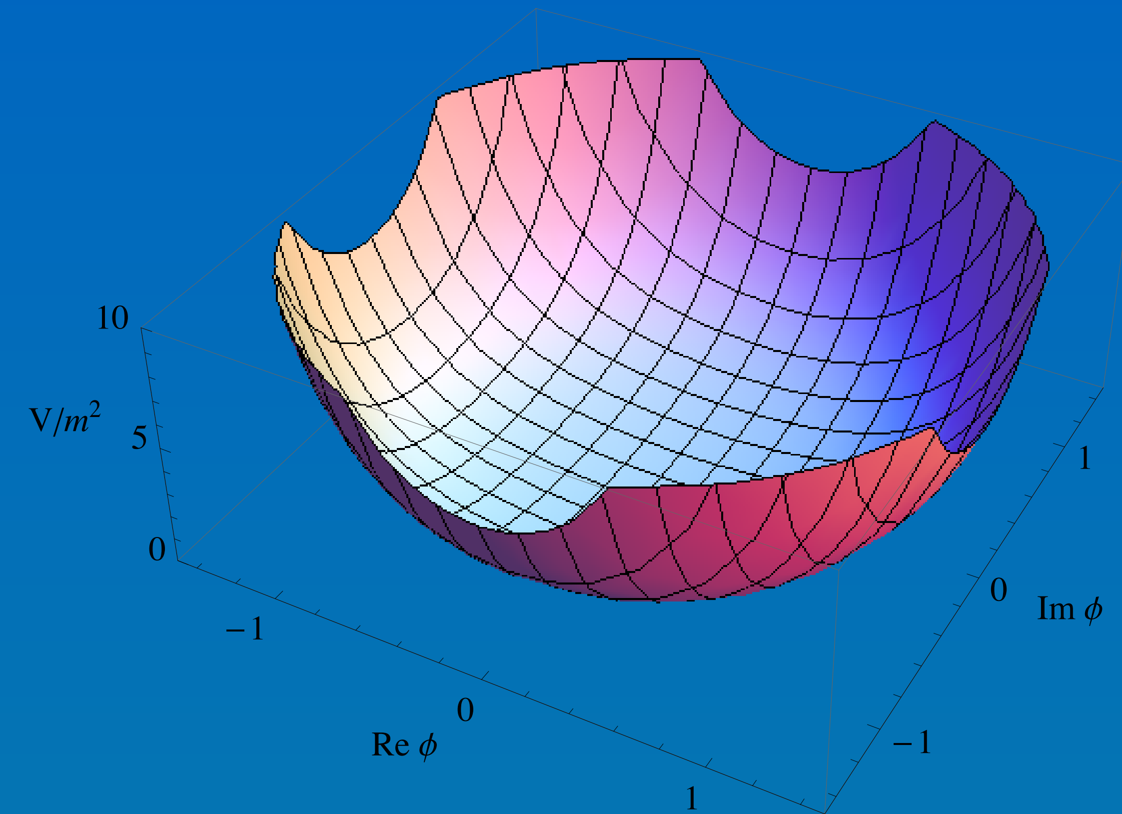
One No-Scale Model to Fit Them All

Ellis, Garcia, Nanopoulos, Olive

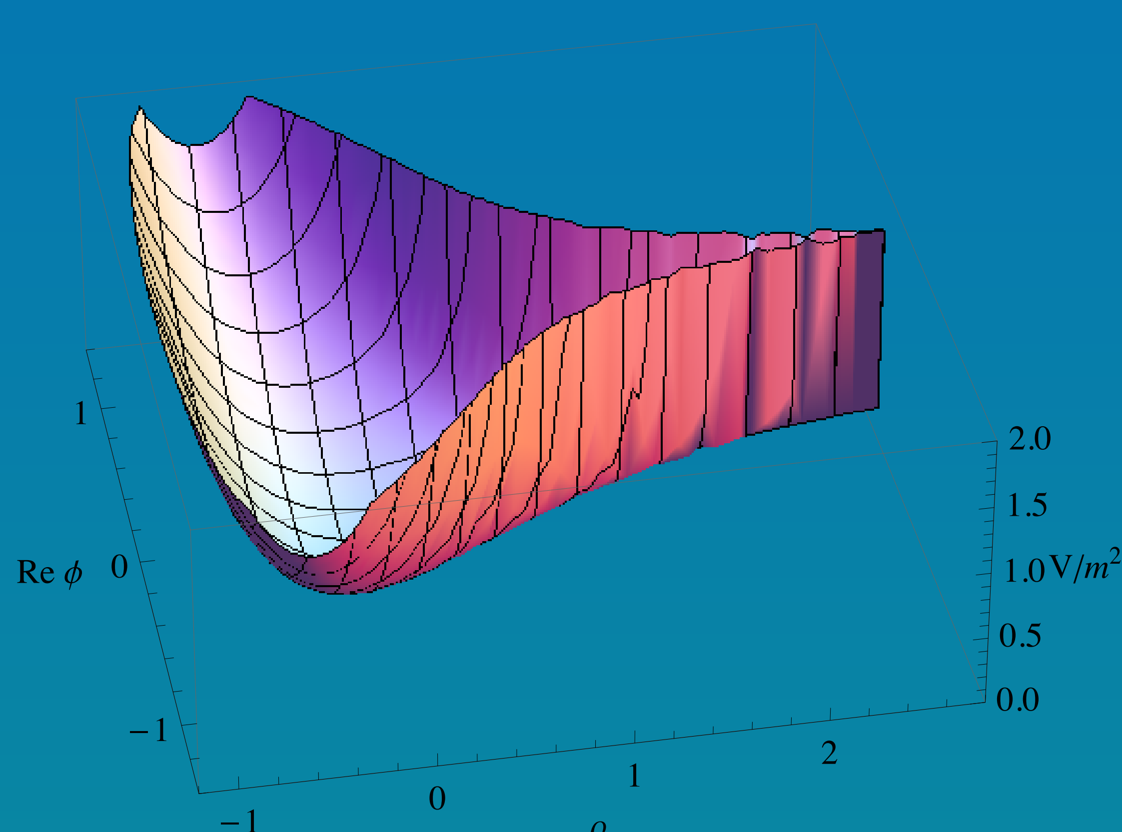
again write $T = \frac{1}{2}\left(e^{-\sqrt{\frac{2}{3}}\rho} + i\sqrt{\frac{2}{3}}\,\sigma\right)$

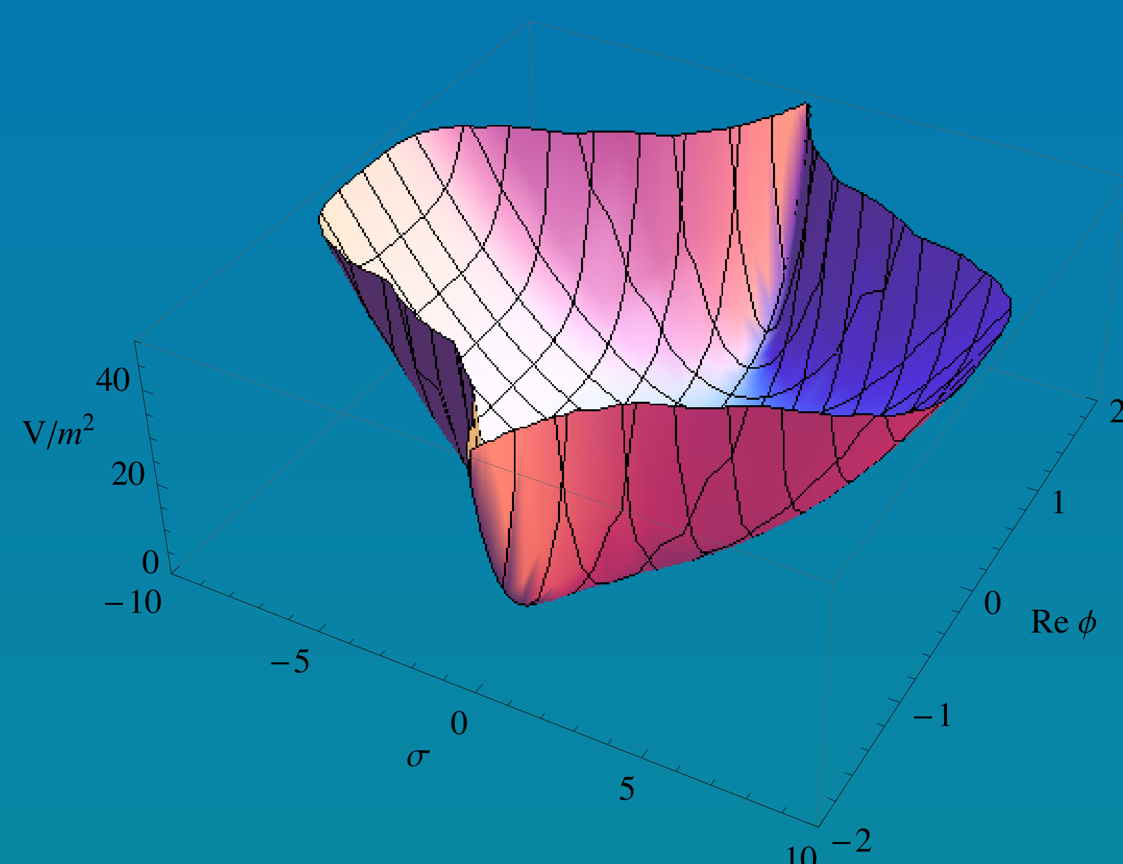
$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{1}{2}e^{2\sqrt{\frac{2}{3}}\rho}\partial_\mu\sigma\partial^\mu\sigma - \frac{3}{4}m^2\left(1 - e^{-\sqrt{\frac{2}{3}}\rho}\right)^2 - \frac{1}{2}m^2\sigma^2$$



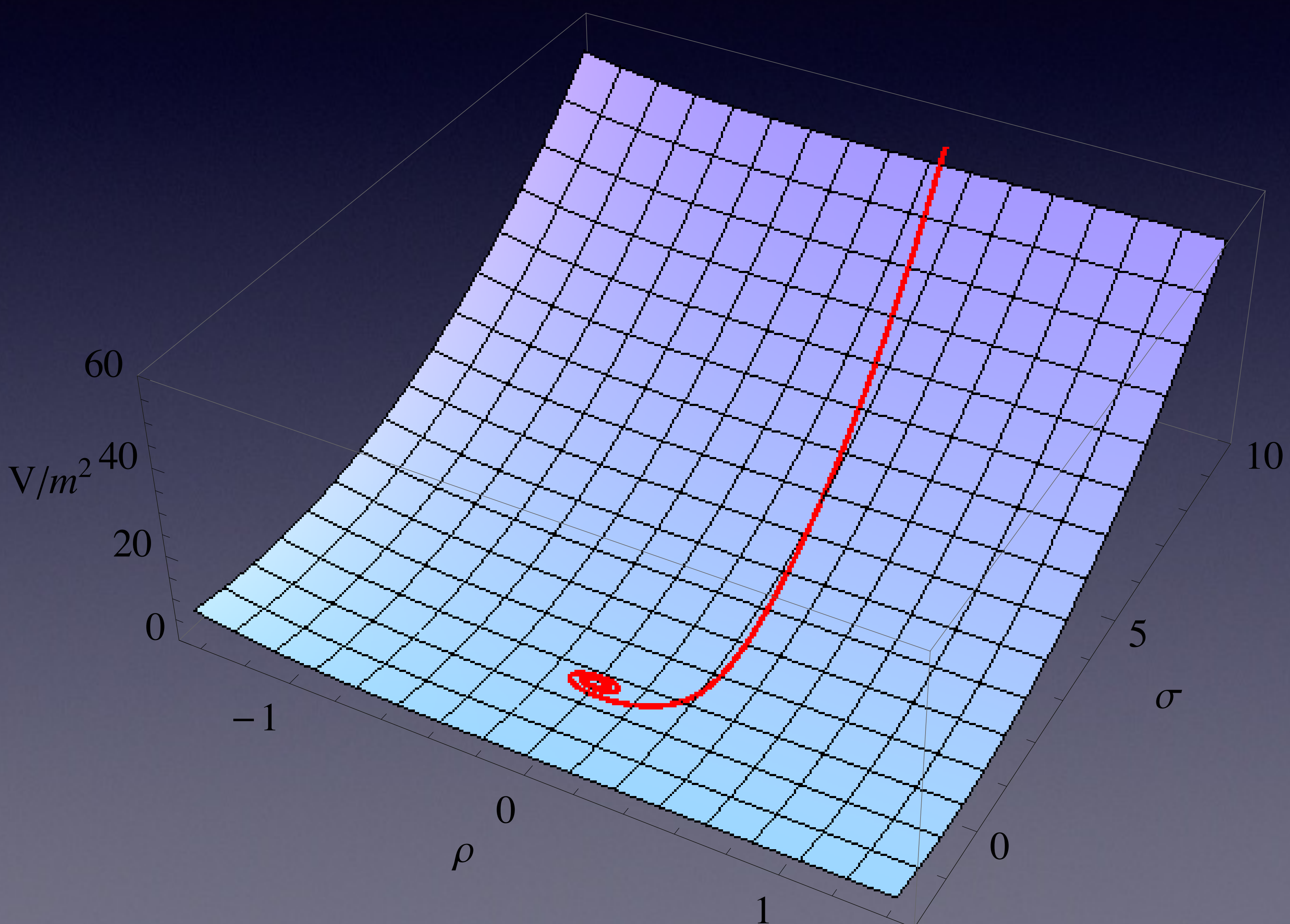


The Potential





Evolution



SUSY breaking and LE phenomenology

- Add SUSY breaking sector, e.g. Polonyi fields
- Add matter fields
  - either in or out of the log
- resulting in mSUGRA type or Split SUSY type models

Summary

- Inflation ingrained in Standard Cosmology.
- The Starobinsky model of inflation can be realized in no-scale supergravity with either modulus T or 'matter' field ϕ with a simple WZ superpotential.
- The latter lends itself nicely to equating the inflaton with a right-handed sneutrino
- Can construct a simple potential which interpolates between the Planck-friendly (small r) solution, or the BICEP-friendly (large r) solution
- Can easily integrate low energy susy phenomenology