Tests of Fundamental Principles

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DFG Research Training Group "Models of Gravity"

Questioning Fundamental Physical Principles
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Outline

1. The Equivalence Principle

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2. Implications of the UFF
3. Order of equations of motion

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2. Implications of the UFF
3. Order of equations of motion
4. Finsler geometry – Existence of inertial systems
5. Apparent violations of the Universality of Free Fall
6. Newton's third law

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The present situation

All predictions of General Relativity are experimentally well tested and confirmed

Foundations

The Einstein Equivalence Principle

- Universality of Free Fall
- Universality of Gravitational Redshift
- Local Lorentz Invariance

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Gravity is a metrical theory

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Implication

Gravity is a metrical theory

$\Rightarrow$

Predictions for metrical theories

- Solar system effects
  - Perihelion shift
  - Gravitational redshift
  - Deflection of light
  - Gravitational time delay
  - Lense–Thirring effect
  - Schiff effect
- Strong gravitational fields
  - Binary systems
  - Black holes
- Gravitational waves

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General Relativity

Description of tests of the universality principles

Purpose: parametrization of deviations, comparison of different experiments

Haugan formalism (Haugan, AP 1979)

Ansatz: effective atomic Hamiltonian (from modified Dirac and modified Maxwell)

$$H = mc^2 + \frac{1}{2m}\left(\delta^{ij} + \frac{\delta m_{\mathrm{i}}^{ij}}{m}\right) p_i p_j + m\left(\delta_{ij} + \frac{\delta m_{\mathrm{g}ij}}{m}\right) U^{ij}(\boldsymbol{x}) + \ldots$$

- additional anomalous spin terms (CL, CQG 1996, SME)
- additional anomalous charge terms (Dittus, C.L., Selig, GRG 2006)

can calculate (all quantities depend on all anomalous parameters)

- acceleration $\longrightarrow$ WEP tests
- frequency comparison $\longrightarrow$ redshift tests
- spin dynamics

Outline

Consequences of the UFF

Trajectories

- Trajectory of a particle $x = x(p; t)$
  $p =$ particle parameter (e.g. mass, charge, etc)
- UFF $\Rightarrow$ trajectory does not depend on particle parameters $x = x(t)$
  This is already the geometrization of the gravitational interaction
- The set of all trajectories is a path structure

Order of equations of notion / Cauchy problem

- Newton's setup: trajectory determined through
  - initial position $x_0 = x(t_0)$ and
  - initial velocity $v_0 = \dot{x}(t_0)$.

$\Rightarrow$ ordinary differential equations of second order: $\ddot{x}^\mu = H^\mu(p; x, \dot{x})$

Question: Why the fundamental equations of motion are of second order?
Equivalent to questioning Newton's second axiom

Consequences of the UFF

UFF + second order

equation of motion

$$\ddot{x}^\mu = H^\mu(x, \dot{x})$$

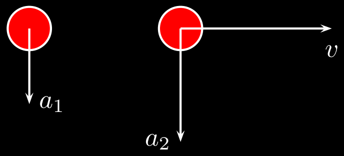
- equation of motion does not depend on particle parameter p
- equation of motion is of second order
- this defines a curve structure

Gravity cannot be transformed away:

Acceleration towards the center of Earth depends on horizontal velocity

exists no inertial system

Implies several effects: $G(T)$, violation of UGR



The free fall: The notions

Gravity can be transformed away

$\exists$ coordinate system $\forall$ particles : $\ddot{x} = 0$
Then in an arbitrary coordinate system

$$\ddot{x}^\mu = -\Gamma^\mu_{\rho\sigma}(x)\dot{x}^\rho\dot{x}^\sigma$$

autoparallel equation, projective structure (Ehlers, Pirani, Schild 1973, Coleman & Korte, many papers in the 80's)

- Need still relation between the connection $\Gamma^\mu_{\rho\sigma}(x)$ and the metric $g_{\mu\nu}$
  - properties of light and clocks as formulated in EPS axiomatics (Ehlers, Pirani, Schild 1993)
  - free turnability (Helmholtz, Lie)
- result: Riemannian geometry
- How to test whether gravity can be transformed away?
- equivalent to questioning Newton's first axiom

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1. The Equivalence Principle
2. Implications of the UFF
3. Order of equations of motion
4. Finsler geometry – Existence of inertial systems
5. Apparent violations of the Universality of Free Fall
6. Newton's third law
7. Summary and Outlook

Order of equation of motion?

Usual framework

$$L = L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) \qquad \Rightarrow \qquad \frac{d}{dt}\boldsymbol{p} = \boldsymbol{F}(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) \text{ with } \boldsymbol{p} = m\dot{\boldsymbol{x}}$$

more general equations?

- $\boldsymbol{p} = m\dot{\boldsymbol{x}}$ is a constitutive law. Can be more general (as is many cases)

  $$\boldsymbol{p} = \boldsymbol{f}(\dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}, \dddot{\boldsymbol{x}}, \dots)$$

  then higher order equations of motion
- Influence of external fluctuations (e.g. space–time fluctuations, gravitational wave background, Göklü, C.L., Camacho & Macias, CQG 2009): generalized Langevin equation with extra force term

  $$\int_0^t C(t - t')\dot{x}(t')dt'$$

Order of equation of motion?

Generalized framework

$$L = L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) \qquad \Rightarrow \qquad \frac{d^2}{dt^2}(\epsilon \ddot{\boldsymbol{x}}) = \boldsymbol{F}(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}, \dddot{\boldsymbol{x}})$$

Our specific model

Gauge procedure in order to invent structure of interactions

$$L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) = L_0(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) \underbrace{-q_0 A_a \dot{x}^a}_{\text{1st order gauge fields}} + \underbrace{q_1 A_{ab} \dot{x}^a \dot{x}^b}_{\text{2nd order gauge fields}}$$

with (Pais–Uhlenbeck oscillator)

$$L_0(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) = -\frac{\epsilon}{2}\ddot{\boldsymbol{x}}^2 + \frac{m}{2}\dot{\boldsymbol{x}}^2$$

ϵ additional new particle parameter, dim$\epsilon = \mathrm{kg\,s^2}$

$\epsilon_{\mathrm{QG}} \sim m_{\mathrm{Planck}} t^2_{\mathrm{Planck}} \sim 10^{-95}\ \mathrm{kg\,s^2}$ $\qquad$ $\epsilon_{\mathrm{Ce}} \sim m_{\mathrm{Ce}} t^2_{\mathrm{Ce}} \sim 10^{-71}\ \mathrm{kg\,s^2}$

Equation of motion

simplest case: constant electric field

$$\epsilon \ddddot{\boldsymbol{x}} + m\ddot{\boldsymbol{x}} = q\boldsymbol{E}_0$$

solution in 1D with initial conditions $x(0) = 0$, $\dot{x}(0) = 0$, $\ddot{x}(0) = 0$, and $\dddot{x}(0) = 0$

$$x(t) = \frac{q}{m} E_0 \left(\frac{1}{2} t^2 + \frac{\epsilon}{m} \left(\cos(\omega t) - 1 \right) \right)$$ small deviation

$$\dot{x}(t) = \frac{q}{m} E_0 \left(t - \sqrt{\frac{\epsilon}{m}} \sin(\omega t) \right)$$ small deviation

$$\ddot{x}(t) = \frac{q}{m} E_0 \left(1 - \cos(\omega t) \right)$$ $\mathcal{O}(1)$ deviation

$$\dddot{x}(t) = \frac{q}{m} E_0 \sqrt{\frac{m}{\epsilon}} \sin(\omega t) \qquad \omega = \sqrt{\frac{m}{\epsilon}}$$ large deviation

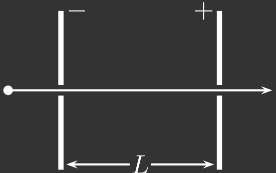
- zitterbewegung
- Limit $\epsilon \to 0$ exists for x and $\dot{x}$, not for $\dddot{x}$

Search for ϵ

Accelerated flight

Flight through accelerator

$$\frac{\langle \dot{x}(L)\rangle - \dot{x}_0}{\dot{x}_0} = \frac{\epsilon}{4m}\frac{\dot{x}_0^2}{L^2}$$



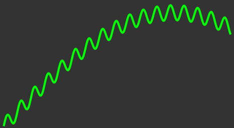
Ion interferometric measurement of acceleration

phase shift

$$\Delta\phi = A(\omega)\boldsymbol{k}\cdot\ddot{\boldsymbol{x}}(\omega)T^2$$

with transfer function

$$A(\omega) = C\frac{\sin^2(\omega t)}{\omega^2}$$

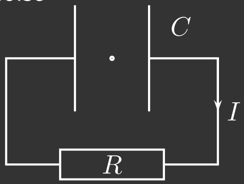


Search for ϵ

Electronic devices

Zitterbewegung of a charged particle induces voltage noise

$$\frac{1}{2} C \langle U^2 \rangle_t = m \langle \dot{x}^2 \rangle = \frac{1}{2} \epsilon \left(\frac{q}{m} E_0 \right)^2$$



- General estimate: $\epsilon \leq 10^{-50}\ \mathrm{kg\, s}^2$.
- Application to mirrors in gw interferometers?
- Adding a small higher derivative term is a mathematical method to analyze differential equations.

C.L. & Rademaker, PRD 2012

higher order time derivative in Schrödinger C.L, Bordé 2000

Outline

Reasons for Finsler geometry

Why Finsler?

- geometry of field equations
- EPS axiomatics (Ehlers, Pirani & Schild 1973)
- dynamical model for respecting UFF but violating Einstein's elevator
- from Quantum Gravity (Girelli, Liberati & Sindoni, PRD 2003)
- VSR (Gibbons, Gomis & Pope, PRD 2007)
- elegance of Lagrange and Hamilton formalism
- nontrivial generalization of Riemannian geometry
- example for violation of Schiff's conjecture
- and Finsler modifications not covered by PPN test theory

Two aspects

- Finsler geometry in the tangent space = Finsler relativity
- Finsler geometry of manifold = Finsler gravity

Finsler geometry

Finsler space

Finsler length function

$$dl^2 = F(x, dx)\,, \qquad F(x, \lambda dx) = \lambda^2 F(x, dx)$$

Finsler metric tensor $f_{\mu\nu}(x, dx)$ is defined as

$$dl^2 = g_{\mu\nu}(x, dx) dx^\mu dx^\nu\,, \quad \text{where} \quad g_{\mu\nu}(x, y) = \frac{1}{2}\frac{\partial^2 F(x^k, y^m)}{\partial y^\mu \partial y^\nu}$$

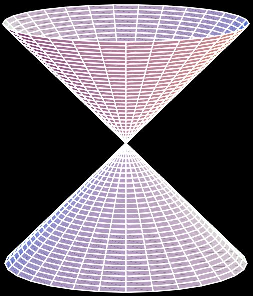
Light cones

Light cone defined by

$$ds^2 = dt^2 - dl^2$$

Finsler geometry

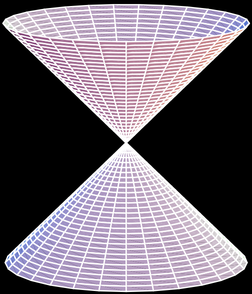
Euclidean light cone | Riemannian light cone | Finslerian light cone

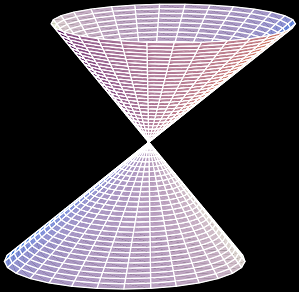


There is no coordinate transformation so that the Finslerian light cone can be locally written in Minkowskian form $0 = -dt^2 + \left(dx^2 + dy^2\right)$

Finsler geometry

Euclidean light cone | Riemannian light cone | Finslerian light cone





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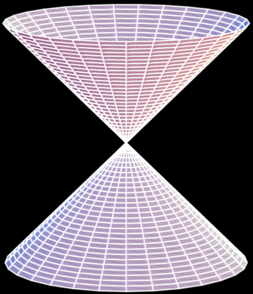
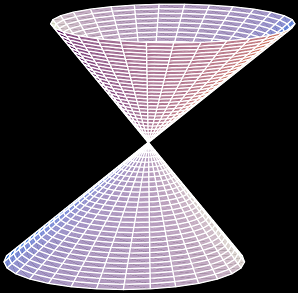
Finsler geometry

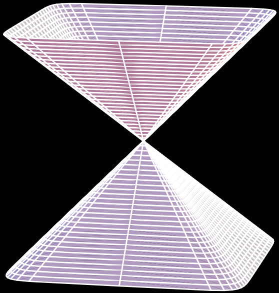
Euclidean light cone



Riemannian light cone



Finslerian light cone



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Finsler geometry

Geodesics

$$\delta \int ds = 0 \qquad \Rightarrow \qquad \boxed{0 = \frac{d^2 x^\mu}{ds^2} + \left\{ {}^{\mu}_{\rho\sigma} \right\}(x, \dot{x}) \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}}$$

with

$$\left\{ {}^{\mu}_{\rho\sigma} \right\}(x, \dot{x}) = g^{\mu\nu}(x, \dot{x}) \left(\partial_\rho g_{\sigma\nu}(x, \dot{x}) + \partial_\sigma g_{\rho\nu}(x, \dot{x}) - \partial_\nu g_{\rho\sigma}(x, \dot{x}) \right)$$

- UFF true, but gravity cannot be transformed away (no Einstein elevator)
- violates LLI: counterexample to Schiff's conjecture

Deviation from Riemann geometry

How to describe deviation from Riemannian geometry? (test theory)

Deviation from Riemann (C.L., Lorek & Dittus, GRG 2009)

- Special case: "power law" metrics (Riemann)

$$dl^2 = (g_{\mu_1\mu_2\ldots\mu_{2n}}(x)dx^{\mu_1}dx^{\mu_2}\cdots dx^{\mu_{2n}})^{\frac{1}{r}}$$

- From any given Riemannian metric g_{ij} and a tensor $\phi_{i_1\ldots i_{2r}}$ we can construct a Finslerian metric by

$$\begin{aligned} D^r(dx^i) &= \left(g_{ij}dx^i dx^j\right)^r + \phi_{i_1\ldots i_{2r}} dx^i \cdots dx^{i_{2r}} \\ &= \left(g_{i_1 i_2}\cdots g_{i_{2r-1}i_{2r}} + \phi_{i_1\ldots i_{2r}}\right) dx^i \cdots dx^{i_{2r}} \end{aligned}$$

- any deviation from Riemann encoded in coefficients $\phi_{i_1\ldots i_{2r}}$
- small deviation given by small $\phi_{i_1\ldots i_{2r}} \ll 1$, then

$$D(dx^i) = g_{ij}dx^i dx^j \left(1 + \frac{1}{r}\frac{\phi_{i_1\ldots i_{2r}} dx^i \cdots dx^{i_{2r}}}{(g_{kl}dx^k dx^l)^r}\right)$$

Testing Finsler

1. test of Finslerian Special Relativity:
   - Michelson–Morley type test (C.L., Lorek, Dittus, GRG 2009)
   - quantum tests are under consideration (Itin, C.L., Perlick, in preparation)
2. test of Finslerian gravity: Finslerian deviation from given solutions of Einstein equation

First model: Finsler modification of Schwarzschild

for $h_{\mu\nu}$ Schwarzschild metric: simplest Finsler modification

$$2L = \left(h_{tt} + c^2\psi_0\right)\dot{t}^2 + \left(\left(h_{ij}h_{kl} + \phi_{ijkl}\right)\dot{x}^i\dot{x}^j\dot{x}^k\dot{x}^l\right)^{\frac{1}{2}}$$

by spherical symmetry

$$\phi_{ijkl} = \psi_1\dot{r}^4 + \psi_2 r^2\dot{r}^2(\sin^2\vartheta\dot{\varphi}^2 + \dot{\vartheta}^2) + \psi_3 r^4(\sin^2\vartheta\dot{\varphi}^2 + \dot{\vartheta}^2)$$

Solar system: Approximation, Specifications

- linearization with respect to Finslerian perturbations
- restriction to equatorial plane

then

$$L = \frac{1}{2}\left((1+\phi_0)h_{tt}\dot{t}^2 + (1+\phi_1)h_{rr}\dot{r}^2 + r^2\dot{\varphi}^2 + \phi_2 \frac{h_{rr}r^2\dot{r}^2\dot{\varphi}^2}{h_{rr}\dot{r}^2 + r^2\dot{\varphi}^2}\right)$$

with

- $\phi_0 := \frac{c^2}{h_{tt}}\psi_0$ modifies temporal metric
- $\phi_1 := \frac{\psi_1}{2h_{rr}^2}$ modifies radial metric
- $\phi_2 := \frac{h_{rr}\psi_2 - \psi_1}{2h_{rr}^2}$ is “Finslerity” – not covered by standard PPN ansatz

Kepler's third law

for circular orbits

$$\frac{r^3}{T^2}\left(1 - \frac{c^2 r^2}{2GM}\left(\phi_0\left(1 - \frac{2GM}{c^2 r}\right)\right)'\right) = \frac{GM}{4\pi^2}$$

from observations

$$r_1 \left|\frac{\phi_0(r_2) - \phi_0(r_1)}{r_2 - r_1}\right| \leq 10^{-16}$$

for all r_1 and r_2 between Mercury and Neptune

Radial acceleration

acceleration from rest

$$\frac{d^2r}{d\tau^2} = -\frac{GM}{r^2}\left(1 - \phi_1 - \phi_0' r \left(1 - \frac{c^2 r}{2GM}\right)\right)$$

from observations

$$|\phi_1(r)| \leq 10^{-6}$$

so far no effect related to Finslerity

Effects for Finslerity

- for access to the Finslerity one needs $\dot\varphi \neq 0$ and $\dot r \neq 0$
- this is for light deflection, gravitational time delay, perihelion shift
- calculations are a bit involved ....
  - light deflection
    $$|10^4\,\phi_1 + \phi_2| \leq 50$$
    will be improved by Gaia
  - gravitational time delay
    $$|20\,\phi_1 + \phi_2| \leq 10^{-3}$$
  - perihelion shift
    $$|\phi_2| \leq 10^{-3}$$
- effect most pronounced for perihelion shift (periodic motion)

C.L., Perlick, Hasse: PRD 2012

Quantum mechanics in Finsler space

Finslerian Hamilton operator

$$H = H(p) \qquad \text{with} \qquad H(\lambda p) = \lambda^2 H(p)$$

"Power–law" ansatz (non–local operator)

$$H = \frac{1}{2m} \left(g^{i_1 \ldots i_{2r}} \partial_{i_1} \cdots \partial_{i_{2r}} \right)^{\frac{1}{r}}$$

Simplest case: quartic metric

$$H = \frac{1}{2m} \left(g^{ijkl} \partial_i \partial_j \partial_k \partial_l \right)^{\frac{1}{2}}$$

Deviation from standard case

$$\begin{aligned} H &= -\frac{1}{2m} \left(\Delta^2 + \phi^{ijkl} \partial_i \partial_j \partial_k \partial_l \right)^{\frac{1}{2}} \\ &= -\frac{1}{2m} \Delta \sqrt{1 + \frac{\phi^{ijkl} \partial_i \partial_j \partial_k \partial_l}{\Delta^2}} \end{aligned}$$

Quantum mechanics in Finsler space

$$H = -\frac{1}{2m}\Delta\left(1 + \frac{1}{2}\frac{\phi^{ijkl}\partial_i\partial_j\partial_k\partial_l}{\Delta^2}\right)$$

- Hughes–Drever: $H_{\rm tot} = H + \boldsymbol{\sigma}\cdot\boldsymbol{B}$
- Atomic interferometry, atom–photon interaction

$$\delta\phi \sim H(p+k) - H(p) = \frac{k^2}{2m} + \frac{1}{m}\left(\delta^{il} + \frac{\phi^{ijkl}p_j p_k}{p^2}\right)p_i k_l$$

  modified Doppler term: gives different Doppler term while rotating the whole apparatus (even in Finsler light still propagates on straight lines, anisotropy – deformed mass shell)

- incorporation of gravity needs relativistic framework

Maxwell in Finsler space

Maxwell in Minkowski

$$\partial_{[a} F_{bc]} = 0 \qquad \partial^b F_{ab} = J_a$$

Maxwell in Riemann

$$\partial_{[\mu} F_{\nu\rho]} = 0 \qquad \partial^\nu F_{\mu\nu} = J_\mu$$

Maxwell in Finsler

$$\partial_{[\mu} F_{\nu\rho]} = 0 \qquad H^\nu(\partial) F_{\mu\nu} = J_\mu$$

with

$$H^\mu(x, k) = \frac{1}{2} \frac{\partial H(x, k)}{\partial k_\mu} \qquad \text{with} \qquad H(x, k) = k_\mu \dot{x}^\mu - L(x, \dot{x})$$

Then

- characteristics are Finslerian null geodesics
- Finsler modified Coulomb law in flat Finsler space

$$\Delta V + 2 \frac{\phi^{abcd} \partial_a \partial_b \partial_c \partial_d}{\Delta} V = q \delta(r) \quad \Rightarrow \quad V = \frac{q}{r} \left(1 - \frac{3}{4 r^4} \phi^{abcd} x_a x_b x_c x_d \right)$$

Hydrogen atom

$$-\frac{\hbar^2}{2m}\left(\Delta + 2\frac{\phi^{abcd}\partial_a\partial_b\partial_c\partial_d}{\Delta}\right)\Psi(\boldsymbol{r}) - \frac{e^2}{r}\left(1 + \frac{3}{4r^4}\phi^{abcd}x_a x_b x_c x_d\right)\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r})$$

can calculate shifts of energy levels

Itin, Perlick, C.L. in preparation

Outline

The basic equations

The model

- Klein–Gordon equation

$$g^{\mu\nu} D_\mu D_\nu \varphi + m^2 \varphi = 0 \,, \qquad D = \partial + \{ \therefore \}$$

- Fluctuating metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \,, \qquad |h_{\mu\nu}| \ll 1$$

- noise

$$\langle h_{\mu\nu}(x) \rangle_{\rm st} = \gamma_{\mu\nu} \,, \qquad \delta^{\rho\sigma} \langle h_{\mu\rho}(x) h_{\nu\sigma}(x) \rangle_{\rm st} = \sigma^2_{\mu\nu}$$

- small amplitude of fluctuations
- frequency might be large, wavelength might be small
- $\langle \cdot \rangle_{\rm st}$ = averaging over a space–time volume
- we do not require the $h_{\mu\nu}$ to obey a wave equation

The basic equations

Approximations

- Weak field up to second order $\tilde{h}^{\mu\nu} = h^{\mu\rho} h_\rho{}^\nu$
- Relativistic approximation of metric and quantum field (á la Kiefer & Singh, PRD 1994)

$$H\psi = -({}^{(3)}g)^{\frac{1}{4}} \frac{\hbar^2}{2m} \Delta_{\text{cov}} \left(({}^{(3)}g)^{-\frac{1}{4}} \psi' \right) + \frac{m}{2} \left(\tilde{h}^{00}_{(0)} - h^{00}_{(0)} \right) \psi$$
$$-\frac{1}{2} \left\{ i\hbar\partial_i, h^{i0}_{(1)} - \tilde{h}^{i0}_{(1)} \right\} \psi$$

manifest hermitean w.r.t. flat scalar product

- only second order terms do not vanish by averaging
- Dirac equation

Short wavelength

Spatial average

- spatial average

$$\langle A\psi\rangle_{\rm s}(x) := \frac{1}{V_x}\int_{V_x} A(y)\psi(y)d^3{\rm y}$$

- short wavelength of fluctuations: V small
- spatial average of Schrödinger equation

$$H = \frac{1}{2m}\left(\delta^{ij} + \alpha^{ij}(x)\right) p_i p_j + \alpha_0$$

with $\alpha(x) = \langle \tilde{h}^{ij} - h^{ij}\rangle_{\rm s}(x)$

- $\alpha^{ij}(x)$: small variation w.r.t. ${\rm x}$, fluctuations w.r.t. t.
- decompose $\alpha^{ij}(x) = \tilde{\alpha}^{ij}(x) + \gamma^{ij}(x)$ with $\langle\gamma^{ij}\rangle_{\rm t} = 0$

- $\tilde{\alpha}^{ij}(x)$ acts like an anomalous inertial mass tensor

Space–time fluctuations

Fluctuation model

- $\alpha^{ij} \leftrightarrow$ spectral noise density of fluctuations
- particular model:

$$\tilde{\alpha}^{ij}(x) = \frac{1}{V_x} \int_{V_x} \tilde{h}^{ij}(\mathrm{x}, t) d^3\mathrm{x} = \frac{1}{V_x} \int_{1/V_x} (S^2(\mathrm{k}, t))^{ij} d^3\mathrm{k}$$

- model: power law spectral noise density

$$(S^2(\mathrm{k}, t))^{ij} = (S^2_{0n})^{ij} |\mathrm{k}|^n \quad \overset{\text{integration}}{\Longrightarrow} \quad \alpha^{ij}(x) = (S^2_{0n})^{ij} \lambda_p^{-(6+n)}$$

with $\dim(S^2_{0n})^{ij} = \mathrm{length}^{3+\frac{n}{2}}$

- $V_x \sim \lambda_p^3$
- $\lambda_p =$ invariant length scale of quantum object $= \lambda_{\mathrm{Compton}}$
- $\lambda_p =$ de Broglie wave length
- $\lambda_p =$ geometric extension l_p of quantum object (Bohr radius of atom)

Space–time fluctuations

Fluctuation model

- assumption: $S_{0n} \sim l_{\mathrm{Planck}}^{3+\frac{n}{2}}$, then

$$\alpha^{ij}(x) \sim \left(\frac{l_{\mathrm{Planck}}}{l_p}\right)^{\beta} a^{ij}(x)\,, \qquad \beta = 6 + n\,,\ \ a^{ij}(x) = \mathcal{O}(1)$$

- effective Hamiltonian

$$H = \frac{1}{2m}\left(\delta^{ij} + \left(\frac{l_{\mathrm{Planck}}}{l_p}\right)^{\beta} a^{ij}(x)\right) p_i p_j = \frac{1}{2m}\left(\delta^{ij} + \frac{\delta m^{ij}(x)}{m}\right) p_i p_j$$

$\delta m^{ij} =$ anomalous inertial mass tensor, depends on particle

- δm^{ij} leads to violation of Universality of Free Fall

- $\beta = \frac{1}{2}$ $\leftrightarrow$ random walk
- $\beta = \frac{2}{3}$ $\leftrightarrow$ holographic noise

Result

Result

metric fluctuations $\Rightarrow$ anomalous inertial mass $\rightarrow$ apparent violation of UFF

- alternative route for violation of UFF and LLI
- need of quantum tests

Example

for Cesium and Hydrogen and geometric extension of atoms

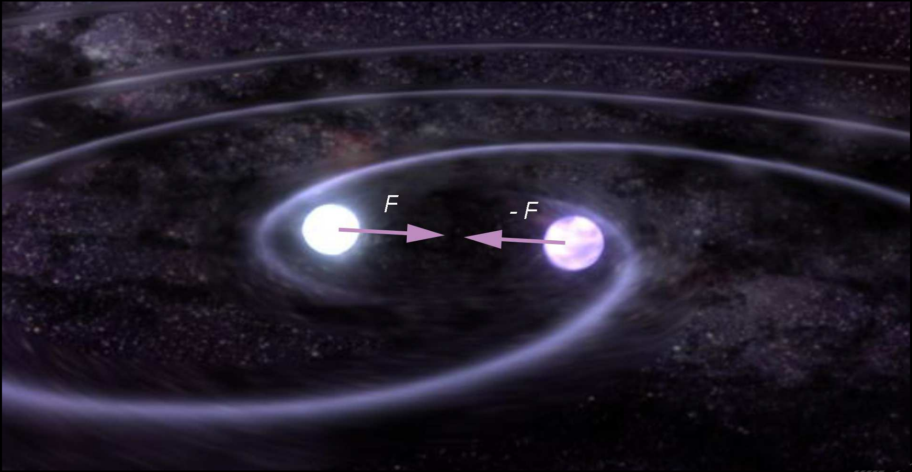
$$\eta_{\beta=1} = 10^{-20}\,, \quad \eta_{\beta=2/3} = 10^{-15}\,, \quad \eta_{\beta=1/2} = 10^{-12}$$

accuracy 10^{-15} is planned for the next years

(Göklü & C.L. CQG 2008)

Outline

actio = reactio ?



Active and passive mass

Gravitationally bound two–body system (Bondi, RMP 1957)

$$m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 = m_{1\mathrm{p}} m_{2\mathrm{a}} \frac{\boldsymbol{x}_2 - \boldsymbol{x}_1}{|\boldsymbol{x}_2 - \boldsymbol{x}_1|^3}$$

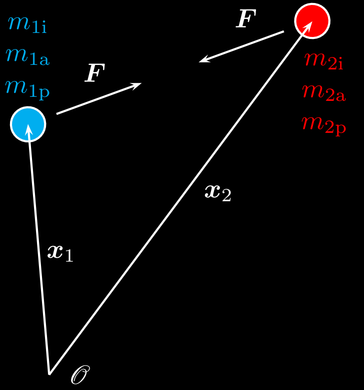
$$m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 = m_{2\mathrm{p}} m_{1\mathrm{a}} \frac{\boldsymbol{x}_1 - \boldsymbol{x}_2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|^3}$$

center–of–mass and relative coordinate

$$\boldsymbol{X} := \frac{m_{1\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_1 + \frac{m_{2\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_2$$

$$\boldsymbol{x} := \boldsymbol{x}_2 - \boldsymbol{x}_1$$

$M_\mathrm{i} = m_{1\mathrm{i}} + m_{2\mathrm{i}} =$ total inertial mass. Then



Active and passive mass

Decoupled dynamics of relative coordinate

$$\ddot{\boldsymbol{X}} = \frac{m_{1\mathrm{p}} m_{2\mathrm{p}}}{M_{\mathrm{i}}} C_{21} \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} \qquad \text{with} \qquad C_{21} = \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} - \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}}$$

$$\ddot{\boldsymbol{x}} = -\frac{m_{1\mathrm{p}} m_{2\mathrm{p}}}{m_{1\mathrm{i}} m_{2\mathrm{i}}} \left(m_{1\mathrm{i}} \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}} + m_{2\mathrm{i}} \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} \right) \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}$$

- $C_{21} = 0$: ratio of the active and passive masses are equal for both particles
- $C_{21} \neq 0$: $\Rightarrow$ self–acceleration of center of mass

Interpretation

$\ddot{\boldsymbol{X}} \neq 0 \quad \Leftrightarrow \quad C_{12} \neq 0 \quad \Leftrightarrow$

- Violation of law of reciprocal action or of *actio* $=$ *reactio* for gravity
- The gravitational field created by masses of same weight depends on its composition. Has the same status as the Weak Equivalence Principle.

Requires experimental tests ...

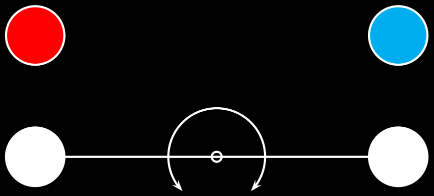
Experiment testing $m_{\rm ga} = m_{\rm gp}$

Measurement of relative acceleration

Step 1: Take two masses with $m_{\rm pg1} = m_{\rm pg2}$ (equal weight)

Step 2: Test active equality of these two masses with torsion balance

Experimental setup: Torsion balance with equal passive masses reacting on $m_{\rm ag1}$ and $m_{\rm ag2}$



No effect has been seen: $C_{12} \leq 5 \cdot 10^{-5}$ (Kreuzer, PR 1868)

Experiment testing $m_{\mathrm{ga}} = m_{\mathrm{gp}}$

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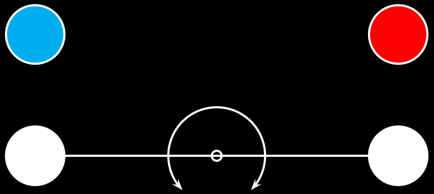

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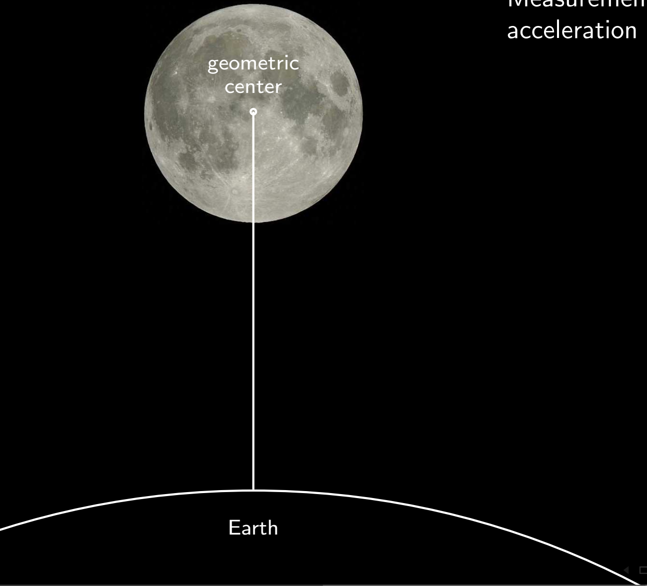
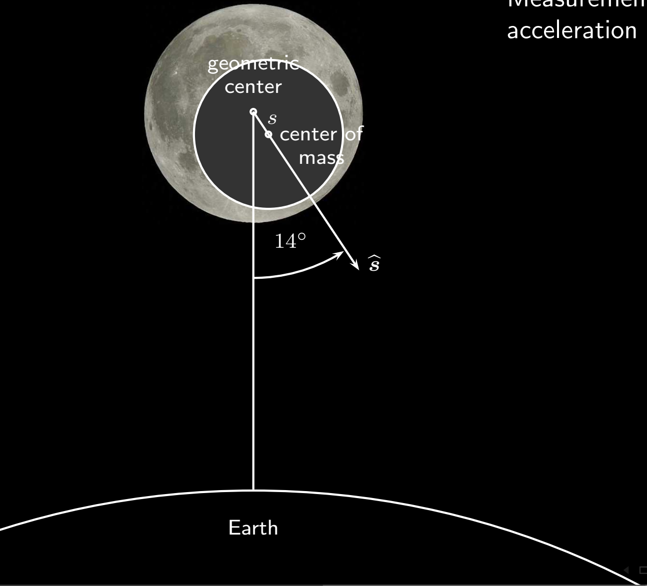


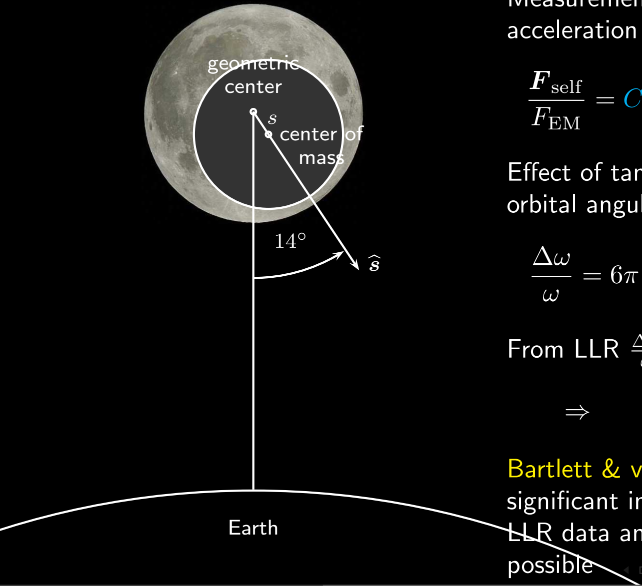
Measurement of center–of–mass acceleration

Experiment testing $m_{\rm ga} = m_{\rm gp}$



Measurement of center–of–mass acceleration

Experiment testing $m_{\rm ga} = m_{\rm gp}$



Measurement of center–of–mass acceleration

$$\frac{\boldsymbol{F}_{\rm self}}{F_{\rm EM}} = C_{\rm Al-Fe} \frac{M_{\rm M}}{M_\oplus} \frac{r_{\rm EM}^2}{r_{\rm M}^2} \frac{s}{r_{\rm M}} \frac{\rho}{\Delta\rho} \widehat{\boldsymbol{s}}$$

Effect of tangential part: increase of orbital angular velocity

$$\frac{\Delta\omega}{\omega} = 6\pi \frac{F_{\rm self}}{F_{\rm EM}} \sin 14° \text{ per month}$$

From LLR $\frac{\Delta\omega}{\omega} \leq 10^{-12}$ per month

$$\Rightarrow \qquad C_{\rm Al-Fe} \leq 7 \cdot 10^{-13}$$

Bartlett & van Buren, PRL 1986
significant improvement with new LLR data and moon orbiter data possible

Active and passive charges: Dynamics

C.L., Macias, Müller, PRA 2007

Dynamics of two electrically bound particles ($\boldsymbol{E}$ = external electric field)

$$\begin{aligned} m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 &= q_{1\mathrm{p}}q_{2\mathrm{a}}\frac{\boldsymbol{x}_2-\boldsymbol{x}_1}{|\boldsymbol{x}_2-\boldsymbol{x}_1|^3}+q_{1\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_1) \\ m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 &= q_{2\mathrm{p}}q_{1\mathrm{a}}\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|^3}+q_{2\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_2) \end{aligned}$$

- Similar phenomena
- New feature: Active and passive neutrality
- Very good neutrality measurements $\Rightarrow C_{12} \leq 10^{-21}$
- Other approach through fine structure constant for H and He^+
- Also: active and passive magnetic moment
- Theory: no Hamiltonian for total system, only for relative motion

One should do dedicated experiments!

Outline

Summary and outlook

- discussion of underlying assumptions influencing the meaning of UFF and EEP
- order of equation of motion
- Finsler geometry as example for no inertial system / violation of local Minkowski
- no test theory so far for Finslerian modification of gravity, needs considerations beyond PPN
- Finslerian modification of Schwarzschild
- Solar system effects
- Finsler is further example for violation of Schiff's conjecture

- Earth–Moon system in field of Sun, should lead to extra polarization, comparison with LLR data
- Finslerian extension of Kerr

Main theme

Gravity and its structure can only be explored through the motion of test particles

Test particles

- Orbits and clocks
- Massive particles and light
- quantum fields

What is gravity depends on the structure of the equation of motion

- Existence of inertial systems
- Order of differential equation
- Dependence on particle parameters

Summary

What determines gravity?

GR = UFF + CP + LLI + UGR + Newton potential + UGF + ...

- scheme not complete as far as Einstein's equations are concerned
- part of it can be interpreted as test of Newton's axioms: IS + CP + UGF
- fundamental violation of principle vs. apparent violation of principle

What are the fundamental principles?

Summary

Thank you!

Thanks to

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- DFG
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- Center of Excellence QUEST