

String Junctions, Abelian Fibrations and Flux/Geometry Duality

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Goal

- The IIB T^6/Z_2 orientifold w. $\mathcal{N} = 2$ flux has purely geometric IIA CY duals with no flux. We wish to construct the dual manifolds explicitly.
- Many properties were deduced by classical sugra dualities (T-duality + M-theory circle swap).
- We now provide two explicit constructions:
 - Monodromy/junction based description:
analogous to F-theory description of K3,
but with T^4 rather than T^2 fibers.
 - Explicit algebro-geometric construction
(related to D(imensional) duality).
- Relation of CYs to one another? to connected web?

Motivations

- The T^6/Z_2 orientifold is the simplest IIB flux compactification. Any insight into this background is likely to shed light on flux compactifications in general (e.g., early analyses of moduli stabilization).
- The CY duals $X_{m,n}$ have $\pi_1 = Z_n \times Z_n$ for $n = 1, 2, 3, 4$.
 \Rightarrow useful for Heterotic phenomenology.
Few CYs with nontrivial π_1 are known (cf. recent work by M. Gross).
- D3 instantons in T^6/Z_2 (with $\mathcal{N} = 2$ flux) map to WS instantons wrapping P^1 sections of dual CY.
 \Rightarrow Nice check of our understanding of D3 instantons & zero mode counting, both at and away from O-planes.

Motivations (continued)

- \exists approx CY metric (valid for small fiber), precisely dual to classical sugra description of T^6/Z_2 .

Harmonic forms and low lying massive modes can be given explicitly in this approx CY metric. \Rightarrow Can in principle deduce warped KK reduction of the dual flux compactification using the duality (cf. recent work by Shiu, Torroba, Underwood, Douglas [STUD]).

- Flux breaks $\mathcal{N} = 4$ to $\mathcal{N} = 2$. \Rightarrow
Precise parametrization of extended SUSY breaking by CY topology.
IIA on CY as an $SU(2)$ structure compactification (cf. Spanjaard)
- Connection to D(imensional) duality: string theory on Riemann surfaces of genus $g \leftrightarrow$ string theory on their Jacobian tori T^{2g} (Green, Lawrence, McGreevy, Morrison, Silverstein).

Known properties of IIA CY duals $X_{m,n}$

- Abelian surface (T^4) fibration over P^1 , with $8 + N$ singular fibers.
($N =$ number of D3-branes in T^6/Z_2).
- Hodge #'s: $h^{11} = h^{21} = N + 2$, where $N + 4mn = 16$.
($F_3 \sim 2m$, $H_3 \sim 2n$, $N_{D3} + \int H \wedge F = \frac{1}{4}N_{O3}$ in dual.)
- Generic D_N lattice of sections (mod torsion)
(since N D-branes + O-plane can coalesce to $SO(2N)$ in T^6/Z_2 dual).
- $\pi_1 = Z_n \times Z_n$, isometry = $Z_m \times Z_m$.
(For nonminimal flux m, n , higgsing only partially breaks sugra $U(1)$ s in T^6/Z_2).
- The case $m = n = 0$ gives $K3 \times T^2$ instead of CY.

Known properties (continued)

- Polarization: $J_{\text{fiber}} \propto m dx^1 \wedge dx^2 + n dx^3 \wedge dx^4$.
- Intersections: $H^2 \cdot A = 2mn$, $H \cdot \mathcal{E}_I \cdot \mathcal{E}_J = -m\delta_{IJ}$
(from sugra EFT or explicit harmonic forms).
- $H \cdot c_2 = 8 + N$ (from F_1 topological amplitude and Green-Schwarz).
- Approximate metric, harmonic forms (small parameter = fiber/base).

Known properties (continued)

- Approximate metric is twisted product of Gibbons-Hawking and T^2 :

$$ds_{\text{CY}}^2 = Z \left(\frac{v_B}{\text{Im } \tau_1} |dx^1 + \tau dx^2|^2 + R_3^2 (dx^3)^2 \right) + Z^{-1} R_4^2 (dx^4 + A^4)^2 \\ + \frac{v_F}{\text{Im } \tau_3} |\eta^5 + \tau_1 \eta^6|^2, \quad R_3 R_4 = (n/m) v_F,$$

mod $\mathbb{Z}_2(x^{1,2,3,4})$.

- For $m, n = 0$:
 - 1st line (Gibbons-Hawking) approximates K3 metric,
 - 2nd line is a T^2 metric.
- For $m, n \neq 0$, the two pieces are twisted:

$$dA^4 = R_4 *_3 dZ - 2m(dx^1 \wedge \eta^6 - dx^2 \wedge \eta^5), \\ d\eta^5 = 2ndx^1 \wedge dx^3, \quad d\eta^6 = 2ndx^2 \wedge dx^3.$$

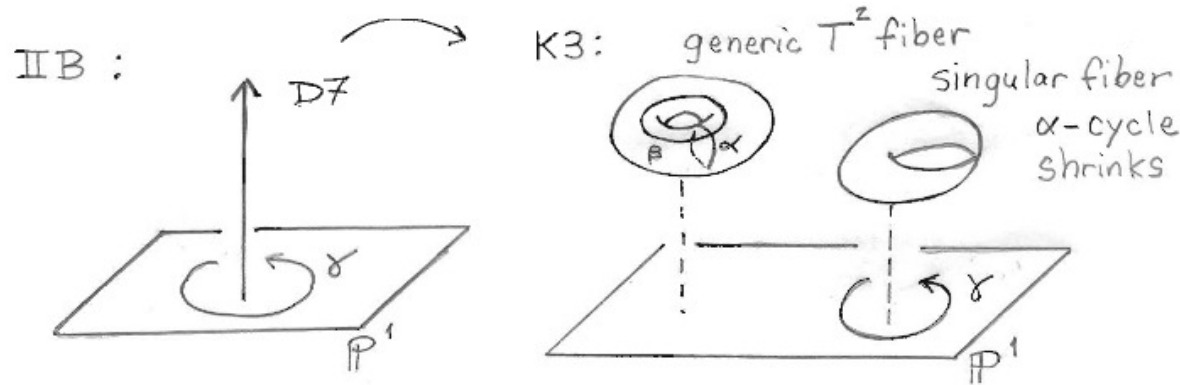
- Can interpret metric as that of a $T_{\{3,4,5,6\}}^4$ fibration over $T_{\{1,2\}}^2 / \mathbb{Z}_2 \cong P^1$.

Construction 1:

Monodromy/junction based description

Warm-up: F-theory on K3

Recall IIB encoding of the geometry of a T^2 fibration over P^1 (e.g., K3):



- $\oint_{\gamma} F_1 = 1$ unit RR charge
 $\tau \rightarrow \tau + 1$ about γ
 $(\tau = C_0 + i/g_s \text{ dilaton-axion})$
 Here D7 = object on which fundamental strings can end.
- Similarly (p, q) 7-brane = object on which (p, q) string can end.

- $\tau = \text{cpx modulus of } T^2$
 $\tau \rightarrow \tau + 1$ about γ .
- $a\alpha + b\beta$ cycle in T^2 : $\begin{pmatrix} a \\ b \end{pmatrix} \rightarrow K \begin{pmatrix} a \\ b \end{pmatrix}$,
 $K = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$ monodromy matrix.
- $p\alpha + q\beta$ (instead of α) cycle shrinks,
 $K_{[p,q]} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix}$.

F-theory on K3 = IIB on T^2/Z_2

- Let (p, q) charges $A = (1, 0)$, $B = (1, -1)$ and $C = (1, 1)$.
- Perturbative description of T^2/Z_2 orientifold: 16 D7s + 4 O7s.
- Nonperturbative description: each O7 resolves to BC pair.
(B,C 7-branes are determined [up to equivalences] by factorization of K_{O7} into $K_{[p,q]}$'s.)
- So, F-theory manifold: base $P^1 \cong T^2/Z_2$ and 24 singular fibers $A^{16} BC BC BC BC$, with monodromies
$$K_A = \begin{pmatrix} 1 & -1 \\ & 1 \end{pmatrix}, \quad K_B = \begin{pmatrix} & -1 \\ 1 & 2 \end{pmatrix}, \quad K_C = \begin{pmatrix} 2 & -1 \\ 1 & \end{pmatrix}.$$
- These nonperturbative IIB data *define* the topology of K3.

CY duals of T^6/Z_2 are abelian fibrations

CY duals $X_{m,n}$ are T^4 fibration over P^1 . Why?

● No flux:

$$T^6/Z_2 \text{ orientifold} \leftrightarrow \text{IIA on K3} \times T^2$$

(K3 = T^2 fibration over P^1)

(both dual to type I or het-SO on T^6).

● With $\mathcal{N} = 2$ flux $F_3 \sim 2m, H_3 \sim 2n$:

$$T^6/Z_2 \text{ orientifold} \leftrightarrow \text{IIA on CY } X_{m,n}$$

($X_{m,n} = T^4$ fibration over P^1)

Roughly, twists mix previous T^2 factor with T^2 fiber of K3.

Monodromy matrices for CY duals

$$N \text{ D3s} + \text{O3s of } T^6/Z_2 \quad \leftrightarrow \quad A^N B_1 C_1 B_2 C_2 B_3 C_3 B_4 C_4$$

singular T^4 fibers of $X_{m,n}$.

Can again determine the monodromy matrices explicitly. We find

$$K_A = \left(\begin{array}{cc|cc} 1 & -1 & & \\ & 1 & & \\ \hline & & 1 & \\ & & & 1 \end{array} \right) = (\text{old } K_A) \otimes (\text{identity}) \text{ on } T^2 \times T^2,$$

but B_i, C_i differ for $i = 1, 2, 3, 4$. For example,

$$K_{B_1} = \left(\begin{array}{cc|cc} & -1 & & -m \\ & 2 & & m \\ \hline & & 1 & \\ -n & -n & & 1 \end{array} \right) = (\text{old } K_B) \otimes (\text{identity}) \text{ on } T^2 \times T^2 + m, n \text{ twists.}$$

The monodromies uniquely determine the topology of $X_{m,n}$.

CY dual interpretation of RR tadpole

- Since the base of $X_{m,n}$ is P^1 , a loop that encloses all singular fibers is contractible (to the point at infinity).

⇒ Total monodromy must be unity:

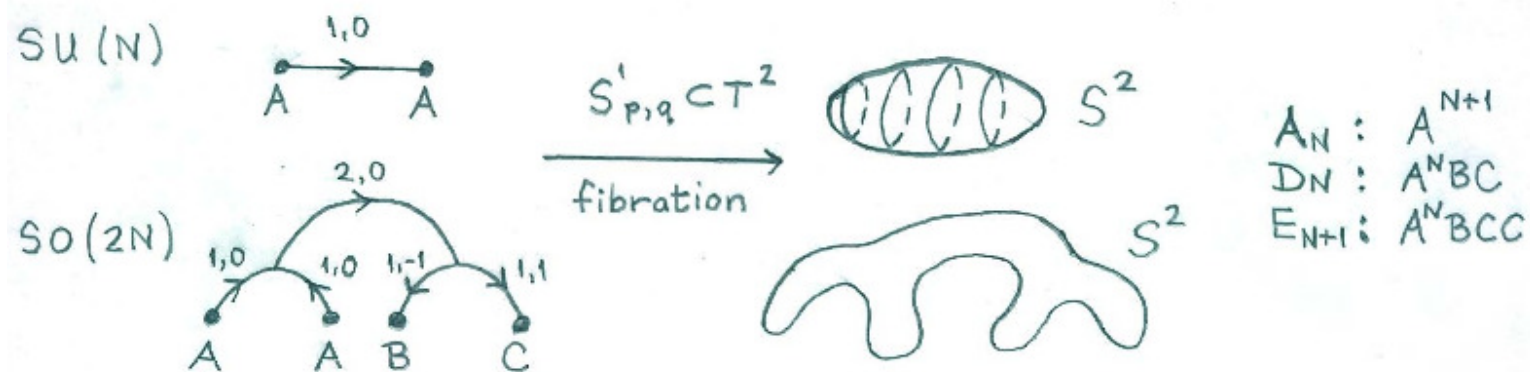
$$\begin{aligned} 1 &= K_{\text{total}} \\ &= K_{C_4} K_{B_4} \dots K_{C_1} K_{B_1} K_A^N \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ & 1 & -Q & 0 \\ & & 1 & 0 \\ & & & 1 \end{pmatrix}, \end{aligned}$$

where $Q = N - 16 + 4mn$.

- Topological constraint reproduces T^6/Z_2 D3 charge constraint $Q = 0$.

String junctions & Mordell-Weil lattice

- String junctions:
- are W-bosons of 7-brane gauge theory,
 - encode homology of F-theory elliptic fibration,
 - equivalence classes (charges) form lattice.
- (DeWolfe et al.)



- H_2 is generated by:
- generic fiber,
 - components of singular fibers,
 - sections. \leftarrow string junctions, $H^1(\mathbb{P}^1, R^1\pi_*\mathbb{Z})$

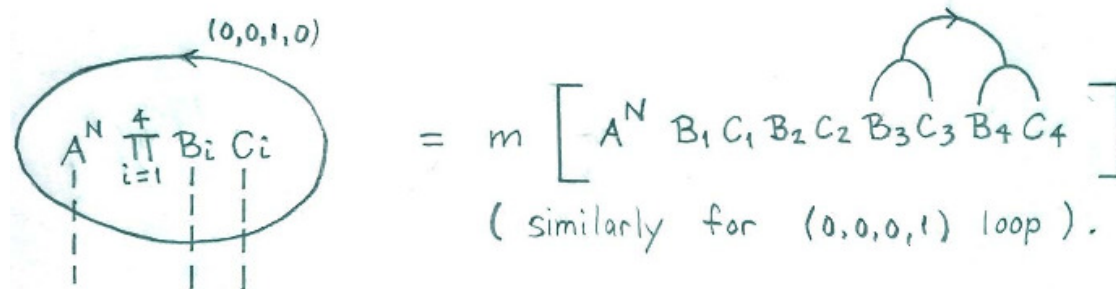
Mordell-Weil lattice of sections = junction lattice/null loops (Fukae et al.).

MW and junction lattice for $X_{m,n}$

- In CY $X_{m,n}$: a (p, q, r, s) 1-cycle in T^4 fiber shrinks at each A, B_i, C_i .
- Obtain 2-cycles in $X_{m,n}$ from $S^1_{[p,q,r,s]}$ fibration over (p, q, r, s) junction graphs in base P^1 .
- Again, MW lattice of sections = junction lattice/null loops.

$A^N \prod_{i=1}^4 B_i C_i \Rightarrow$ Obtain D_N from $A^N B_i C_i$ ($A + A = B_i + C_i$)
but not E_{N+1} from $A^N B_i C_i C_j$ ($C_i \neq C_j$).

D_N = free part of MW lattice.



$$= m \left[A^N B_1 C_1 B_2 C_2 B_3 C_3 B_4 C_4 \right]$$

(similarly for $(0, 0, 0, 1)$ loop).

- $Z_m \times Z_m$ = torsion part of MW lattice = isometry group.

Relations between CYs

- $N + 4mn = 16$. Complete set of 8 $X_{m,n}$ is $\{X_{1,1}, X_{m,1}, X_{1,n}, X_{2,2}\}$.
- Relations:
 - $X_{m,1} \xrightarrow[Z_m \times Z_m \text{ isometry}]{\text{quotient by}} X_{1,m}, \quad \text{for } m = 2, 3,$
 - $X_{4,1} \xrightarrow[Z_2 \times Z_2 \subset Z_4 \times Z_4 \text{ isometry}]{\text{quotient by}} X_{2,2} \xrightarrow[Z_2 \times Z_2 \text{ isometry}]{\text{quotient by}} X_{1,4}.$
- When singular fibers coalesce, additional isometries can develop.
Have new MW torsion from “weakly integer” junctions. For example, a $(1, 0)$ string can end on a coalesced A^2 pair: “ $(1/2, 0)$ on each.”
- Quotienting by these isometries gives *new* CY manifolds
(with nontrivial π_1 if action is free).
- Do not appear to have extremal transitions to other interesting CYs.
(Cpx def away from singular loci \rightarrow new section; Kähler resolution \rightarrow new Kodaira component; same Hodge numbers either way.)

Construction 2:

Explicit algebro-geometric construction

Relative Jacobian of a complex surface

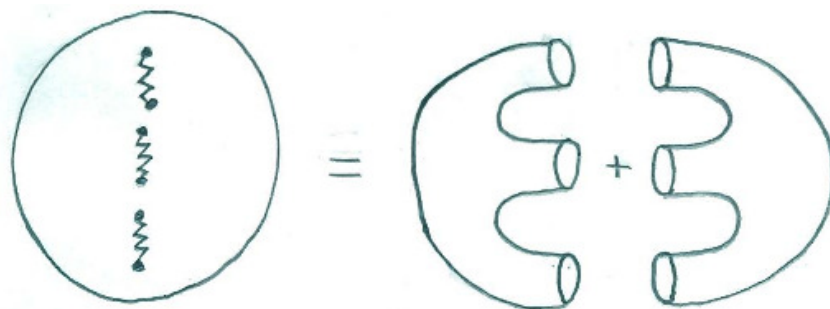
- Restrict to $m, n = 1, 1$ (principle polarization).
- Idea: complex surfaces much easier than 3-folds.
- To every genus- g curve, can associate a principally polarized Jacobian torus T^{2g} with the same H_1 (same space of 1-cycles (p, q, r, s)):

$$g = 2 : \quad \begin{array}{c} \text{Diagram of a genus-2 surface } C_2 \text{ with cycles } \alpha_1, \beta_1, \alpha_2, \beta_2 \end{array} \xrightarrow[\text{"Wilson lines"}]{\text{Abel-Jacobi map}} T^4.$$

- So, can try to realize CY $X_{1,1}$ as the fiberwise Jacobian of a surface S , where S is a genus-2 fibration over P^1 .
- S could be made physical if desired via a D(imensional)-duality type solution that interpolates between S and $X_{1,1}$.
(Green, Lawrence, McGreevy, Morrison, Silverstein)

Construction of the surface S

- A genus-2 curve = double cover of \mathbb{P}^1 with 6 branch points.



$$\begin{aligned} \Rightarrow S &\equiv \text{genus-2 fibration over } \mathbb{P}_{(1)}^1 \\ &= \text{branched double cover of } \mathbb{P}_{(1)}^1 \times \mathbb{P}_{(2)}^1. \end{aligned}$$

- Degree of branch curve $B \subset S$ is $(a, 6)$
(for 6 branch pts in generic fiber of $S \rightarrow \mathbb{P}_{(2)}^1$, i.e., for genus-2).
If you like, can view as S as 2-fold section of $\mathcal{O}(a/2, 3)$.
- For $a = 2$, obtain a candidate for $X_{1,1}$ from $\text{Jacobian}(S/\mathbb{P}^1)$
(construction first studied by Saito).

Checks

- $c_1(X_{1,1}) = 0, \quad h^{1,1} = h^{2,1} = 14.$
- 20 degenerations of genus-2 fiber $f_2 \Rightarrow$ also of T^4 fibration $X_{1,1}$.
- $c_2 = 20$ elliptic curves (singular loci of special fibers).

- Sections of S

Other projection $S \rightarrow \mathbb{P}_{(2)}^1$ has genus-0 fibers C_0 ($2\mathbb{P}^1 - 2$ br pts) with 12 degenerations: $C_0 \rightarrow 2 \mathbb{P}^1$ s ℓ_I, ℓ'_I meeting at a point ($I = 1, \dots, 12$).

$\Rightarrow 2 \times 12$ sections of genus-2 fibration (w. relations $\ell_I + \ell'_I = C_0$).

- Sections of $X_{1,1}$

Given a choice of zero section $\sigma_0 \in \{\ell_I, \ell'_I\},$

$\text{MW}(X_{1,1}) \cong \langle \sigma_0, f_2 \rangle^\perp$ (with S intersection pairing).

\Rightarrow 12 dimensional lattice, D_{12} .

Checks (continued)

● Intersections:

● $\ell_I \subset S \mapsto$ “theta surface” $\Theta_I \subset X_{1,1}$.

● Writing

$$A = \text{abelian fiber}, \quad \mathcal{E}_I = \frac{1}{2}(\Theta_I - \Theta'_I), \quad H = \frac{1}{2}(\Theta_I + \Theta'_I) - \frac{1}{6}A,$$

gives the quoted intersections.

(N.B. Basis A, \mathcal{E}_I, H from sugra duality is not an integer basis:

- $\frac{1}{2}$'s are expected (roots versus half roots of D_N).
- $\frac{1}{6}$ follows from correct definition of warped volume of T^6/Z_2 .)

● So, can apply Wall's classification theorem:

c_1, c_2 , intersections \Rightarrow unique CY up to homotopy type.

Conclusions

- We have seen two explicit constructions of the IIA CY duals of T^6/Z_2 :
 1. Monodromy/junction description (analog of F-theory for T^4 fibers),
 2. Relative Jacobian of a genus-2 fibered surface S (for $m, n = 1, 1$).
- In each case, we have computed the Mordell-Weil lattice of sections, to obtain the required D_N lattice (using junctions, and sections S , resp.).
 - In Case 1, D3 tadpole condition \Leftrightarrow total monodromy = 1.
 - All criteria for Wall's theorem (c_1, c_2, C_{IJK}) satisfied in Case 2.
- Related projects in progress:
 - Map between D3 instantons in T^6/Z_2 and WS instantons in CY dual.
 - Warped KK reduction of T^6/Z_2 using approximate CY metric, which is an exact dual to the classical sugra description of T^6/Z_2 .