

Model Building with Heterotic String Theory

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1 August 2008

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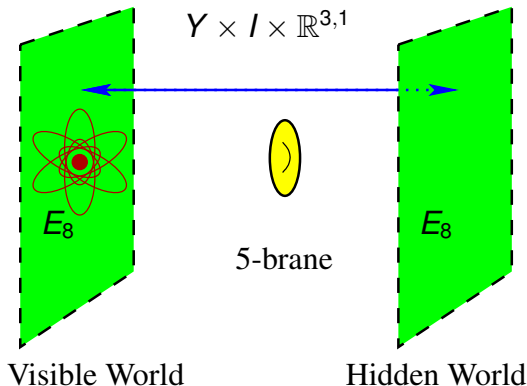
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- Straightforward to break E_8 gauge symmetry
- Gauge unification without Yukawa unification
- Natural hidden sector, ideal place for SUSY breaking and/or dark matter

Heterotic M-theory



Y Calabi-Yau manifold (6-dimensional)

I Interval (1-dimensional)

$\mathbb{R}^{3,1}$ 4-d spacetime

1 Introduction

2 Vector Bundles

3 A Heterotic Standard Model

4 Extra Higgs Pairs

Geometric Data

We need:

- A Calabi-Yau manifold Y .

Polynomial equations in projective spaces.

SUSY \Leftrightarrow Kähler and $c_1(TY) = 0$.

- Vector bundles.

Two basic constructions.

SUSY \Leftrightarrow slope-stable

Bundles on a Torus

Torus = Elliptic curve = Calabi-Yau 1-fold.

Atiyah classified the indecomposable bundles:

- Line bundles
- Extensions of line bundles

Line Bundles on a Torus

For simplicity: $c_1(\mathcal{L}) = 0 \Leftrightarrow \int F = 0$.

This fixes topology of the line bundle, but not its complex structure.

Theorem (Donaldson-Uhlenbeck-Yau)

$$\left\{ \begin{array}{c} \textit{Slope-stable} \\ \textit{holomorphic bundle} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} \textit{Solution to the} \\ \textit{Hermitian-Yang Mills} \\ \textit{equation} \end{array} \right\}$$

Line Bundles on a Torus

Fix a point $0 \in T^2$.

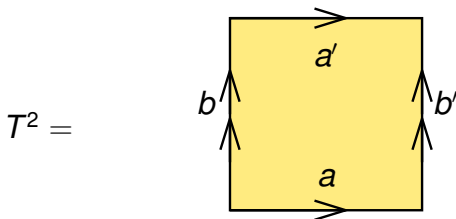
All line bundles (with $c_1 = 0$) are of the form

$$\mathcal{L} = \mathcal{O}_{T^2}(p - 0)$$

where p is a point in T^2 .

- Algebraic point of view: There is a meromorphic section with a zero at 0 and a pole at p .
- Geometric point of view: The HYM connection has two $U(1)$ holonomies, parametrize a point $p \in T^2$.

Extension of Bundles



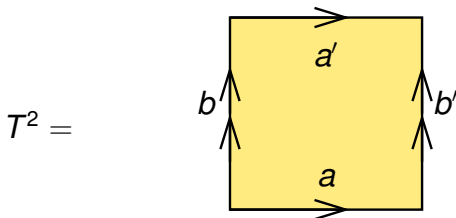
$O_{T^2} = O_{T^2}(0 - 0)$ has transition functions

$$\phi_{a,a'} = 1, \quad \phi_{b,b'} = 1$$

$O_{T^2} \oplus O_{T^2}$ has transition functions

$$\phi_{a,a'} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \phi_{b,b'} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Extension of Bundles



Define a new bundle with transition functions

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Extension of Bundles

Define a new bundle with transition functions

$$\hat{\phi}_{a,a'} = \begin{pmatrix} 1 & 0 \\ \mathbf{1} & 1 \end{pmatrix}, \quad \hat{\phi}_{b,b'} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- Embedding $f : (x) \mapsto (x, 0)$
- Projection $g : (y_1, y_2) \mapsto (y_2)$

Defines “extension” bundle

$$0 \longrightarrow \mathcal{O}_{T^2} \xrightarrow{f} \mathcal{A}^{(2)} \xrightarrow{g} \mathcal{O}_{T^2} \longrightarrow 0$$

For the experts: $\text{Ext}^1(\mathcal{O}_{T^2}, \mathcal{O}_{T^2}) = \mathbb{C}$

Classification of Bundles on T^2

A rank-2 bundle¹ on T^2 is one of the following two possibilities:

- $\mathcal{O}_{T^2}(p_1 - 0) \oplus \mathcal{O}_{T^2}(p_2 - 0)$

(Spectral cover)

- $\mathcal{O}_{T^2}(p - 0) \otimes \mathcal{A}^{(2)}$

(Extension)

¹with $c_1 = 0$ and a HYM connection

Spectral Covers on Calabi-Yau Threefolds

Use elliptically fibered Calabi-Yau threefold over a base surface. Patch together sums of line bundles on each fiber.

Local coordinates (x, y) on the base

$$\mathcal{V}|_{f(x,y)} = \mathcal{O}_{T^2}(p_1(x, y) - 0) \oplus \mathcal{O}_{T^2}(p_2(x, y) - 0) \oplus \dots$$

Bundle defined by the surface

$$\mathcal{C}_V = \{p_1(x, y) = 0\} \cup \{p_2(x, y) = 0\} \cup \dots$$

Extensions on Calabi-Yau Threefolds

Rank-1 ingredients:

- Line bundles, defined by divisors (codimension 1)
- Sheaves, like bundles but with codimension ≥ 2 degenerations.

Extensions give higher rank bundles:

$$0 \longrightarrow \mathcal{L}_1 \longrightarrow \mathcal{V}^{(2)} \longrightarrow \mathcal{L}_2 \longrightarrow 0$$

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Constructions

- Our "Heterotic Standard Model", MSSM via $SO(10)$
[Braun-HuiHe-Pantev-Ovrut]
- MSSM via $SU(5)$, using extensions of spectral covers
[Bouchard-Donagi]
- Standard embedding + intermediate scale breaking
[Greene-Kirklin-Miron-Ross]
- Orbifolds [Buchmuller-Hamaguchi-Lebedev-Ratz,
Wingerter]
- $U(n)$ bundles instead of $SU(n)$
[Blumenhagen-Honecker-Weigand]
- Free fermions [Faraggi]

Particle Spectrum

The massless fields can be counted without knowing the metric and gauge connection explicitly.

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We found a Calabi-Yau manifold and slope-stable bundles with “nice” 4d low energy effective action:

- $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
- 3 families of quarks and leptons
- No anti-families
- Anti-fivebrane breaks SUSY in hidden sector

Calabi-Yau Threefold

\tilde{X} is a complete intersection of 2 equations of degree $(3, 0, 1)$ and $(0, 3, 1)$ in $\mathbb{P}^2 \times \mathbb{P}^1 \times \mathbb{P}^2$.

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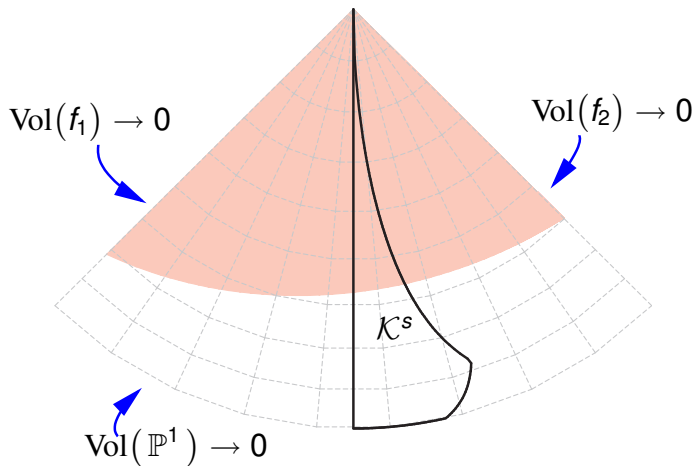
\downarrow quotient

$X = \tilde{X} / (\mathbb{Z}_3 \times \mathbb{Z}_3) = \text{Calabi-Yau with } \pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$

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Three Kähler moduli $t_1, t_2, t_3 \geq 0$ on X .

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 Cross-section of the Kählercone:



$$\text{Vol}(X) = t_1 t_2 (t_1 + t_2 + 6t_3)$$

Visible Sector Gauge Group

- Rank 4 vector bundle \mathcal{V} breaks visible E_8 to $Spin(10)$.

$$\underline{248} = (\underline{1}, \underline{45}) \oplus (\underline{15}, \underline{1}) \oplus (\underline{4}, \underline{16}) \oplus (\overline{\underline{4}}, \overline{\underline{16}}) \oplus (\underline{6}, \underline{10})$$

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- $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson line breaks

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Each quark and lepton corresponds to a different **16**.

Doublet-Triplet Splitting

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$$\underline{10} \longrightarrow \underbrace{(\underline{1}, \underline{2}, 3, 0) \oplus (\underline{1}, \underline{2}, -3, 0)}_{\text{Higgs}} \oplus \underbrace{(\underline{\mathbf{3}}, \underline{1}, -2, -2) \oplus (\underline{\mathbf{3}}, \underline{1}, 2, 2)}_{\text{Triplets}}$$

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On \tilde{X} , there are

$$h^1(\tilde{X}, \wedge^2 \mathcal{V}) = 4$$

10's. Depending on the $\mathbb{Z}_3 \times \mathbb{Z}_3$ group action on each **10**, get either Higgs, triplet, or nothing.

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In our “heterotic standard model”, all triplets are projected out while one Higgs (with its conjugate) is kept.

Hidden Sector

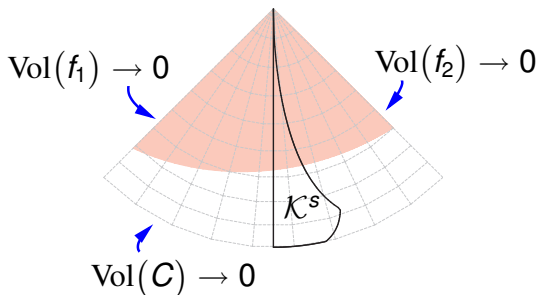
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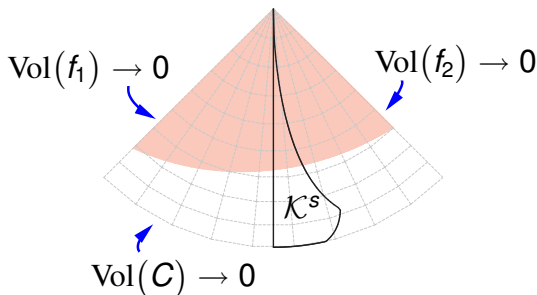
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- Flip of $C \Leftrightarrow$ Decay into SUSY vacuum.

Yukawa Textures

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In our “heterotic standard model”:

- Only two of the three families have Yukawa couplings
 \Rightarrow One light family.
- No μ -term, avoids hierarchy problem.

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Another Model

- Same Calabi-Yau manifold as before
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Leads to:

- Again 3 generations, no anti-generations
- Triplets still projected out
- But: 2 Higgs and 2 conjugate Higgs.

Massless Fields

Massless fields \Leftrightarrow Bundle-valued one-forms.

Yukawa coupling

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In order for $Y_{ijk} \neq 0$, the three forms $\alpha_i, \alpha_j, \alpha_k$ must have legs in

- fiber 1 direction,
- fiber 2 direction, and
- base \mathbb{P}^1 direction.

Massless Fields

Origin	Direction	Field
<u>16</u>	Fiber 1	$Q_1, u_1, d_1, L_1, e_1, \nu_1$
<u>16</u>	Base	—
<u>16</u>	Fiber 2	$Q_{2,3}, u_{2,3}, d_{2,3}, L_{2,3}, e_{2,3}, \nu_{2,3}$
<u>10</u>	Fiber 1	—
<u>10</u>	Base	H_1, \bar{H}_1
<u>10</u>	Fiber 2	H_2, \bar{H}_2

Yukawa Couplings

For example, up-quark mass matrix

$$\begin{pmatrix} 0 & \lambda_{u,12}\langle H_1 \rangle & \lambda_{u,13}\langle H_1 \rangle \\ \lambda_{u,21}\langle H_1 \rangle & 0 & 0 \\ \lambda_{u,31}\langle H_1 \rangle & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_1\langle H_1 \rangle & 0 \\ 0 & 0 & \lambda_2\langle H_1 \rangle \end{pmatrix}$$

\Rightarrow One light generation of quarks and leptons.

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Second Higgs Pair

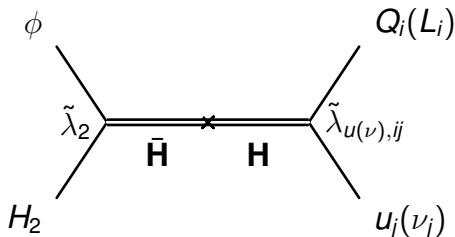
H_2, \bar{H}_2 has no Yukawa couplings except μ -term

$$W_\mu = \lambda_1 \phi H_1 \bar{H}_2 + \lambda_2 \phi H_1 \bar{H}_2$$

Beyond Tree-Level

The superpotential can have higher order terms:

- Integrating out massive Kaluza-Klein modes, for example

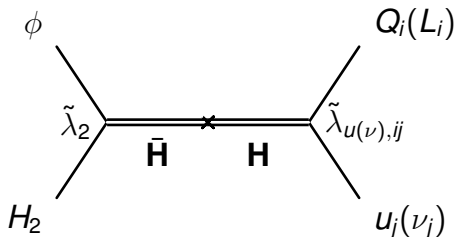


- Non-perturbative corrections.

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But: Suppressed by $\frac{1}{M_c}$.

μ -Term and Moduli Vevs

Classical μ -term:

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Quartic term

$$W_\mu = \frac{\lambda_{mnij}}{M_c} \phi_m \phi_n H_i \bar{H}_j$$

must be electroweak scale, hence

$$\frac{(\tilde{\lambda}^2)^2}{M_c} \langle \phi_m \rangle \langle \phi_n \rangle \lesssim M_{\text{EW}} \quad \Leftrightarrow \quad \tilde{\lambda}^2 \frac{\langle \phi \rangle}{M_c} \lesssim \sqrt{\frac{M_{\text{EW}}}{M_c}} \approx 10^{-7}$$

Simplified Model

Take normal (non-SUSY) Standard Model and add

- Single scalar ϕ
- second Higgs H_2
- \mathbb{Z}_2 symmetry $\phi \mapsto -\phi$, $H_2 \mapsto -H_2$.
- Dimension 5 operators

$$\begin{aligned} \mathcal{L}_5 = & \tilde{\lambda}_{u,ij} \frac{\phi}{M_c} \bar{Q}_i H_2^* u_j + \tilde{\lambda}_{d,ij} \frac{\phi}{M_c} \bar{Q}_i H_2 d_j + \\ & + \tilde{\lambda}_{\nu,ij} \frac{\phi}{M_c} \bar{L}_i H_2^* \nu_j + \tilde{\lambda}_{e,ij} \frac{\phi}{M_c} \bar{L}_i H_2 e_j + \text{h.c.} \end{aligned}$$

with couplings $\lesssim 10^{-7}$.

Result

We obtain FCNC-induced mass splittings of mesons

F^0	$\Delta M_F^{SM}/GeV$	$\Delta M_F^{Exp}/GeV$	$\Delta M_F^{2-Higgs}/GeV$
K^0	$1.4 - 4.6 \times 10^{-15}$	3.51×10^{-15}	4.72×10^{-19}
B_d^0	$10^{-13} - 10^{-12}$	3.26×10^{-13}	$.88 \times 10^{-20}$
D^0	$10^{-17} - 10^{-16}$	$< 1.32 \times 10^{-13}$	4.56×10^{-21}

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Flavor-changing neutral currents are far below the Standard Model contributions.

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