# Toric resolution of Heterotic orbifolds


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**Based on:**
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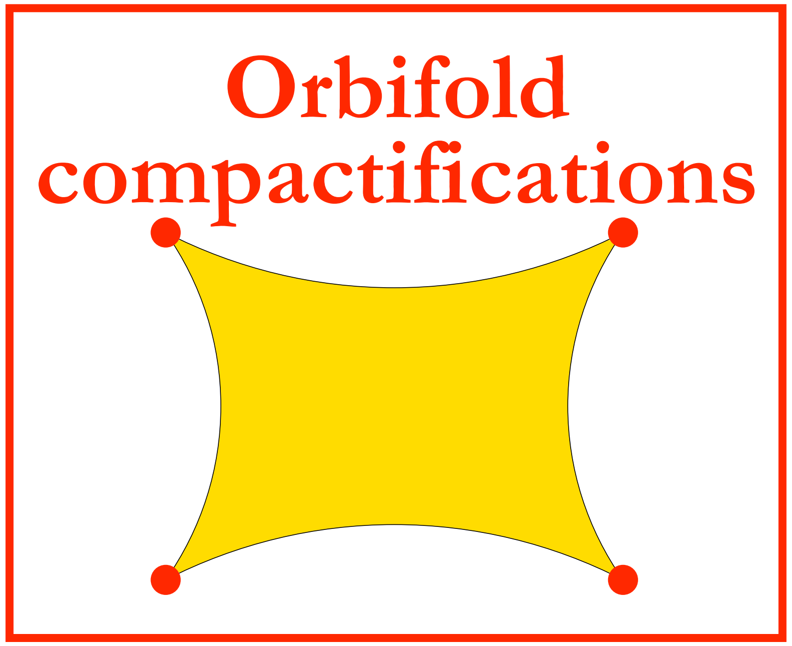
**+ work in progress**


**In collaboration with:**
**Stefan Groot Nibbelink, Tae Won Ha**
**Felipe Paccetti, Johannes Held, Fabian Ruehle**

# Introduction I: Motivations

Two main different paths to **heterotic string** phenomenology




Orbifold:

a space flat everywhere but in some singular points where (mostly) SUSY breaking, gauge symmetry breaking and chiral matter reside.

String theory on orbifolds:

Pure CFT approach (strong link with similar "non-geometric" approaches).

Some good properties:

- Exact quantization of the string;
- Allow for systematic (computer assisted) searches;
- Very successful!

Talk by A. Wingerter

Some disadvantages:

- Specific point in the moduli space (the orbifold point);
- Singular space! Difficult to make use of the net of dualities;
- Difficult to disentangle $M_{GUT}$ from $M_{Planck}$.

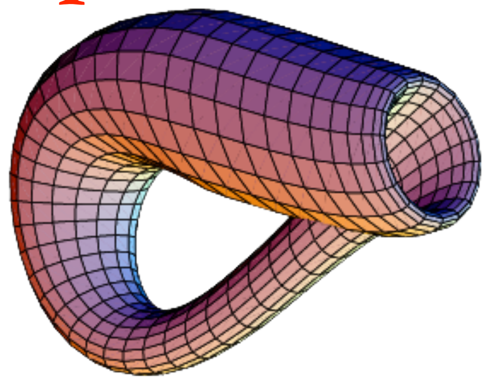
# Introduction I: Motivations

Two main different paths to **heterotic string** phenomenology

String theory on a smooth CY:

Pure SUGRA approach (KK reduction in the presence of gauge fluxes).

**Smooth manifold compactifications**



Some good properties:

- Properties of the model (gauge group, # of families etc) "easily" linked to topological properties of the model;
- Generic point in moduli space (introduction of fluxes, torsion, moduli stabilization mechanisms);
- Naturally embedded in the net of dualities with other strings;
- $M_{GUT}$ naturally linked to some internal volumes different from the string scale (but perturbativity requires volumes to be "not too large");
- $E_8$ x $E_8$ string: hidden sector "well hidden".

Talks by R. Tatar, V. Braun, B. Ovrut

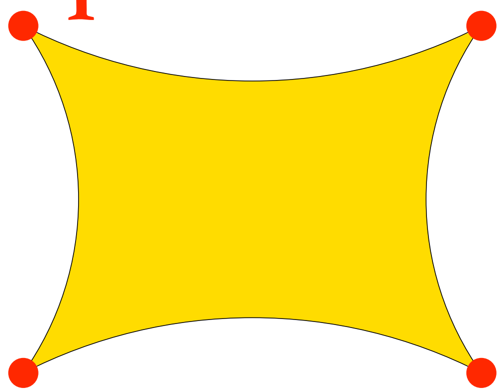
Some disadvantages:

- SUGRA approach;
- Difficult to get good CY's, good gauge fluxes etc.

# Introduction I: Motivations

Two main different paths to **heterotic string** phenomenology

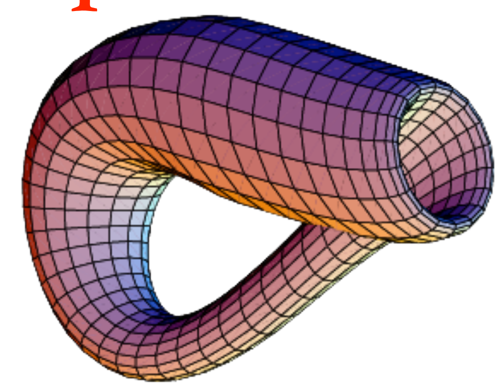
**Orbifold compactifications**



**My task:**

**Reconciliate them!**

**Smooth manifold compactifications**

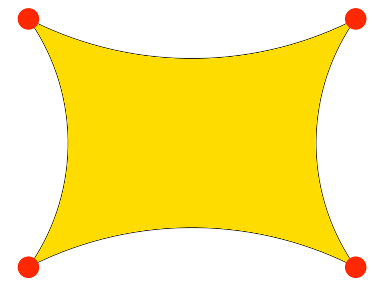


**Reproduce the orbifold models as**
**- compactifications of 10d SUGRA/SYM**
**- on smooth manifolds (blown-up orbifolds)**
**- in the presence of gauge fluxes.**

# Introduction II: the Spirit

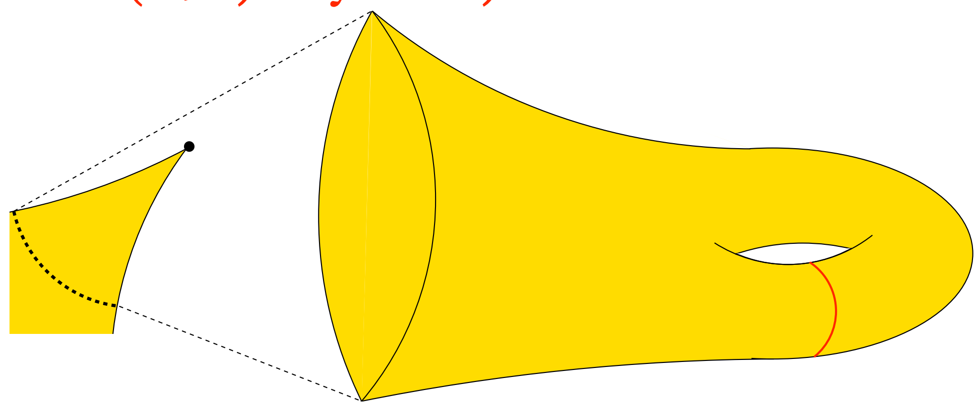
## I - Resolve the orbifold geometry

Ia - Given the orbifold



Ib - Cut apart each singularity and resolve it:

characterize the local geometric structure “hidden” in the singularity (localized (1,1)-cycles)



Ic - Glue together the resolved singularities:

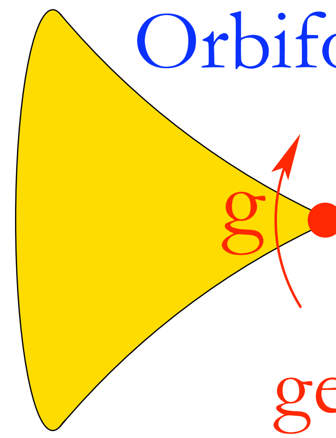
characterize the topology of the whole CY space (non-localized cycles)

**Get a smooth compact CY space**
**(having the original orbifold as singular limit)**

# II - Compactify 10d SUGRA/SYM on the smooth CY

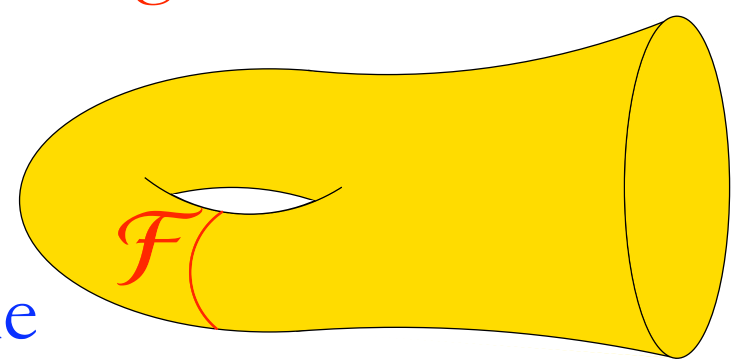
**- A crucial detail:**

**Orbifold models:**




Orbifold action g embedded in the gauge degrees of freedom.

The freedom in doing this generates a vast set of models!




**SUGRA models:**

Gauge flux wrapped on the new localizes cycles, to be embedded in SO(32) or $E_8 \times E_8$.
The freedom in the embedding generates a vast set of models

**Reproduce each string orbifold model as a compactification of 10d SUGRA + SYM on a smooth CY embedding the "right" gauge flux**

# Introduction III - Outline

1) **Getting the smooth CY space (toric geometry)**
   - Local resolution of orbifold singularities
   - Gluing the resolved singularities
2) **10d SUGRA on the smooth CY space**
   - Consistency conditions (flux quantization, SYM e.o.m, ... )
   - Matching the orbifold models: local & global informations
3) **An example: $T^4/Z_3$**
4) **Conclusions, outlook and working plan**

# 1 - Orbifold resolution

## Some definitions

Lust, Reffert, Scheidegger, Stieberger '07

### Divisors

- Given a complex n-dim space (parameters $z^i$), a divisor X is locally an analytic hypersurface (e.g. $z^1 = 0$).
- To each divisor X we can associate a complex line bundle.

### Linear equivalence

- Given two divisors X and Y we say that they are equivalent $X \sim Y$ if the associated line bundles differ by a trivial one.
- The set of divisors corresponds, modulo linear equivalence, to the (1,1)-forms on the space.

### Intersection of divisors

- An intersection of divisors defines curves in the space.
- Intersecting n divisors we get points, the intersecting number $X_1 X_2 \ldots X_n = p$ means that the hypersurface $X_1$ intersects the curve $X_2 \ldots X_n$ in p points (or that $X_2$ intersects ... ).
- Equivalently, we can read $X_1 X_2 \ldots X_n = p$ as the integral of the (1,1)-form $X_1$ on $X_2 \ldots X_n$ (or the integral of $X_2$ on ... ).

## Resolution of local singularities

- Each singularity (we treat) has form $\mathbf{C}^n/\mathbf{Z}_m$, with parameters $z^i$.
- Before resolution, the space has n divisors $D_i$, the surfaces $z^i = 0$.
- The singularity is resolved
  - adding new exceptional divisors, E's to the set of D's
  - specifying the n linear relations between E's and D's: $D_i \sim a_{ij} E_j$.
  - fixing the intersection numbers between D's and E's

## Gluing together the singularities into $T^{2n}/Z_m$

- Each resolved singularity is equipped with
  - a set of divisors $\{D_i, E_j\}$;
  - a set of linear equivalences $D_i \sim a_{ij} E_j$;
  - the local intersection numbers.
- Gluing:
  - "put together" the divisors in a single set (add the $\mathbf{T}^{2n}$ divisors $R_i$)
  - extend the linear equivalences to include all the objects
  - compute the intersections among the various divisors.

## A heuristic picture

- The R's are the $\mathbf{T}^{2n}$ inherited (1,1)-forms/cycles.
- The D's are auxiliary objects, defined in order to deal with the local case (where no R is there): they have fixed point index.
- Before of the resolution, the linear equivalence looks like

$$R_i \sim n\, D^{a}{}_{i}$$

where n is the order of the orbifold, and there is an equivalence per each different D.
- The resolution is the introduction of the localized (hidden) topological objetcs, the E's. They do not come with extra equivalence relations, rather they modify the old equivalence relations.

# 2 - Gauge bundles on the resolved space

**Consistency conditions**

**1)** Flux quantization: $\int_\gamma F \in \mathbf{Z}$

**2)** Equations of motion/SUSY:

- $F$ must be a (1,1)-form, fulfilling the DUY condition

**3)** The Bianchi Identity for H must be fulfilled

$$\int_{C_2} (\mathcal{R} \wedge \mathcal{R} - F \wedge F) = 0$$

**In the language of divisors:**

- $F$ can be written as $F = E_i V^{iI} H^I$
  - $E_i$ the localized (1,1)-forms (flux invisible in blow-down)
  - $H^I$ elements in the Cartan algebra of SO(32) or $E_8$ x $E_8$

- Quantization: $V_i^I$ must be integers (half-integers)
- E.o.m.: conditions on the Kaehler moduli
- Bianchi Identity: use the splitting principle and the intersections

model dependent conditions

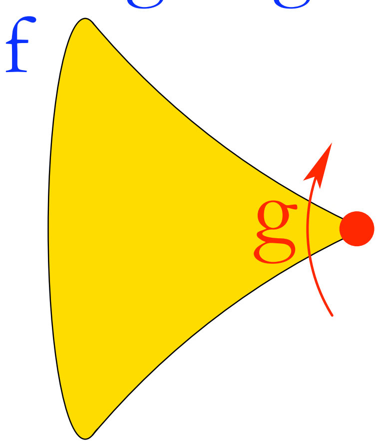
**Spectrum:** from the Dirac index.

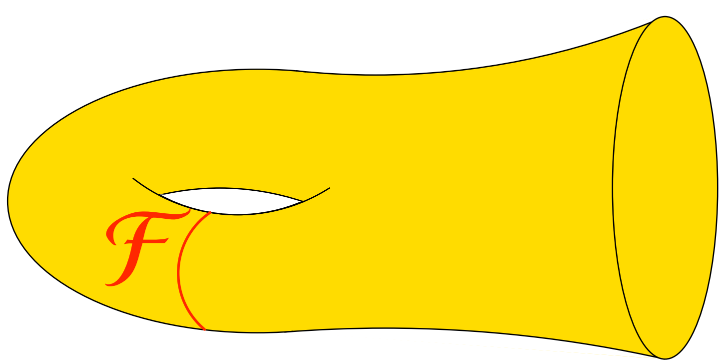
## Matching the orbifolds: local informations

### Basic idea:

- on the orbifold side there are non-trivial identifications "going "round" the singularity, dictated by the embedding of the orbifold action in the gauge degrees of freedom

$$\mathrm{g}: T^a \;\to\; e^{2\pi i H^I V_I/n}\; T^a\; e^{-2\pi i H^I V_I/n}$$




- on the bundle side the same identifications are generated by the presence of the flux (depending on how it is embedded in SO(32) or $E_8$ x $E_8$)

### "Trivial" example: $C^3/Z_3$

- the resolution is obtained adding a single exceptional divisor E.
- take then $\mathcal{F} = V_I^g H^I E/3$, quantization fixes the vector to integer or half integer values, the boundary effect (and identification) is

$$\int_{D_2 D_3} \mathcal{F} = \frac{V_I^g}{3} H^I\; E D_2 D_3 = \frac{V_I^g}{3} H^I \sim \frac{V_I}{3} H^I$$

N.B. The Bianchi identity is $V^{g^2} = 12$ , to be compared with the modular invariance condition $V^2 = 0 \bmod 6$ !

**Less trivial example: $C^2/Z_3$**

- the resolution needs two exceptional divisors $E_1$ and $E_2$.
- we have then two possible shift vectors, since we can have

$$\mathcal{F} = V^g_{I1} H^I E_1/3 + V^g_{I2} H^I E_2/3$$

- but we also have two different identifications (in the previous case we had three, but all equivalent), so we still have a single choice (up to SO(32) or $E_8$ x $E_8$ lattice elements)

$$V \sim V^g_2 \sim -V^g_1$$

- again we can check the Bianchi identity and see

$$V^{g\,2}_1 + V^{g\,2}_2 + V^g_1 V^g_2 \sim V^{g\,2}_1 = 8$$

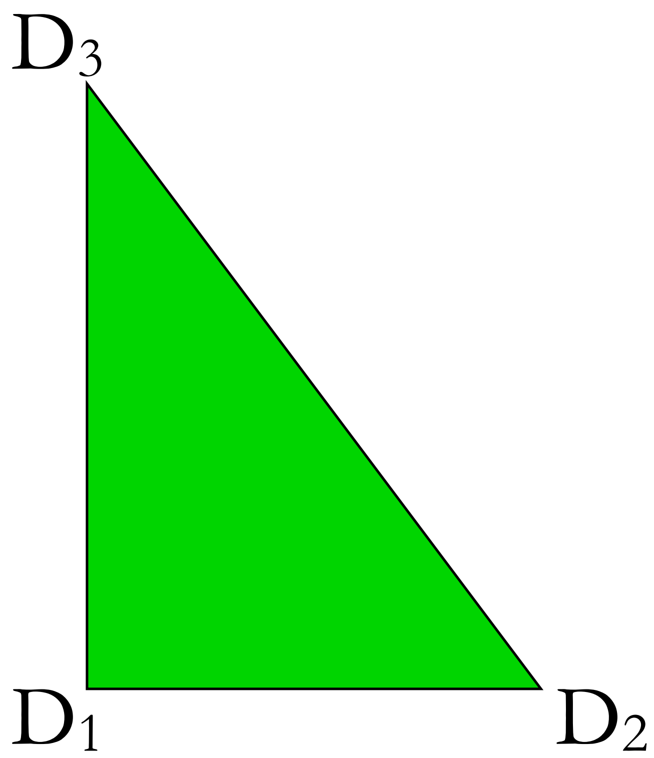
that should be compared with the modular invariance condition

$$V^2 = 2 \bmod 6$$

again, the introduction of SO(32) / $E_8$ x $E_8$ lattice vectors plays an important role in the matching (these are irrelevant from the orbifold perspective).

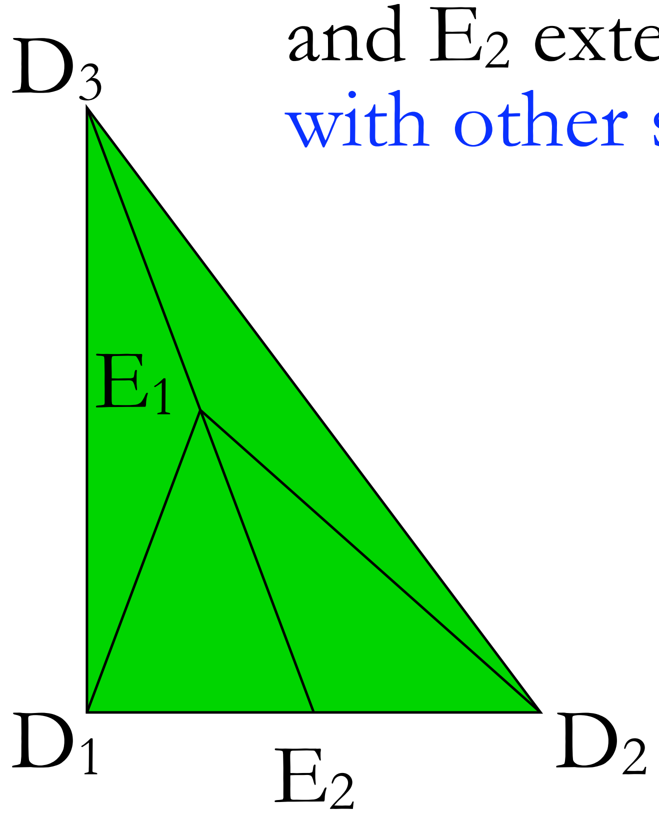
## Another example: $C^3/Z_4$

- complex coordinates $z_1$, $z_2$, $z_3$
- $\mathbf{Z_4}$ fixed points: singular case, only 3 $D_i$ divisors, planes $z_i=0$.



## Another example: $C^3/Z_4$

- complex coordinates $z_1$, $z_2$, $z_3$
- $\mathbf{Z_4}$ fixed points: resolved case, add $E_1$ and $E_2$, with $E_1$ compact and $E_2$ extending in the $z_3$ direction -- shared with other singularities in the third torus.



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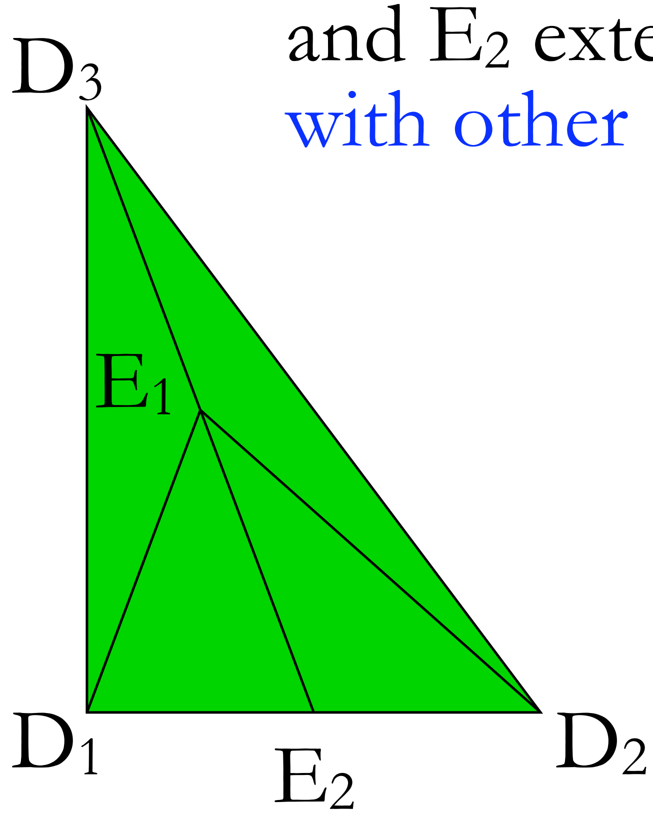


- Point: in $T^6/Z_4$ there are $\mathbf{Z_2}$ fixed points: singular case, two divisors $D_1$, $D_2$

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- complex coordinates $z_1$, $z_2$, $z_3$
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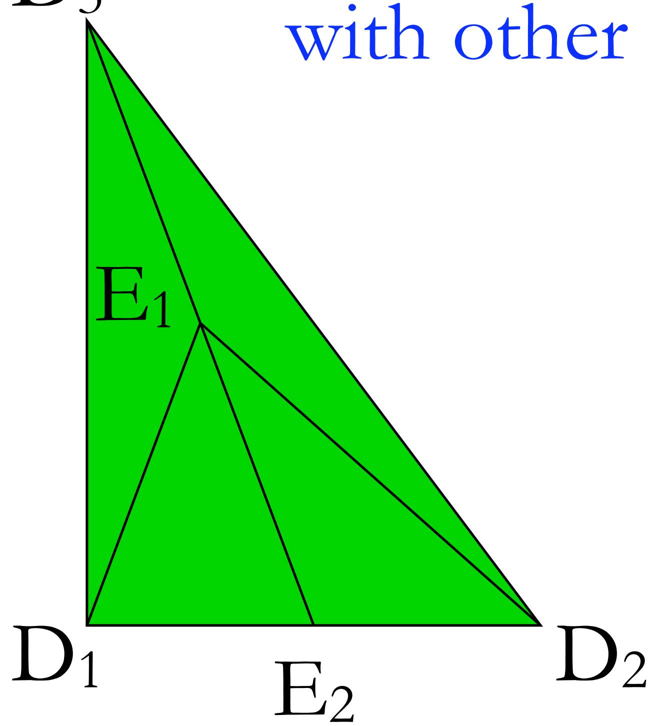
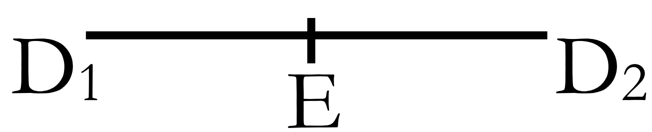



- Point: in $T^6/Z_4$ there are $\mathbf{Z_2}$ fixed points: resolved case, add E, compact from the $\mathbf{Z_2}$ perspective, but extending in the third torus




- The $\mathbf{Z_4}$ singularity contains informations on the gauge embedding of the $\mathbf{Z_4}$ and of the $\mathbf{Z_2}$ orbifold rotation!

## Another example: $\mathbf{C}^3/\mathbf{Z}_4$

- in detail, take $\mathcal{F} = \frac{1}{4}E_1 V_1^g \cdot H + \frac{1}{2}E_2 V_2^g \cdot H$

- we have the $\mathbf{Z}_4$ identification

$$\frac{1}{4}V_{Z_4} \cdot H \sim \int_{D_1 D_3} \mathcal{F} = \int_{D_2 D_3} \mathcal{F} = \frac{1}{4}V_1^g \cdot H$$

- and the $\mathbf{Z}_2$ identification

$$\frac{1}{2}V_{Z_2} \cdot H \sim \int_{D_1 E_2} \mathcal{F} = \int_{D_2 E_2} \mathcal{F} = \frac{1}{2}(V_1^g - V_2^g) \cdot H$$

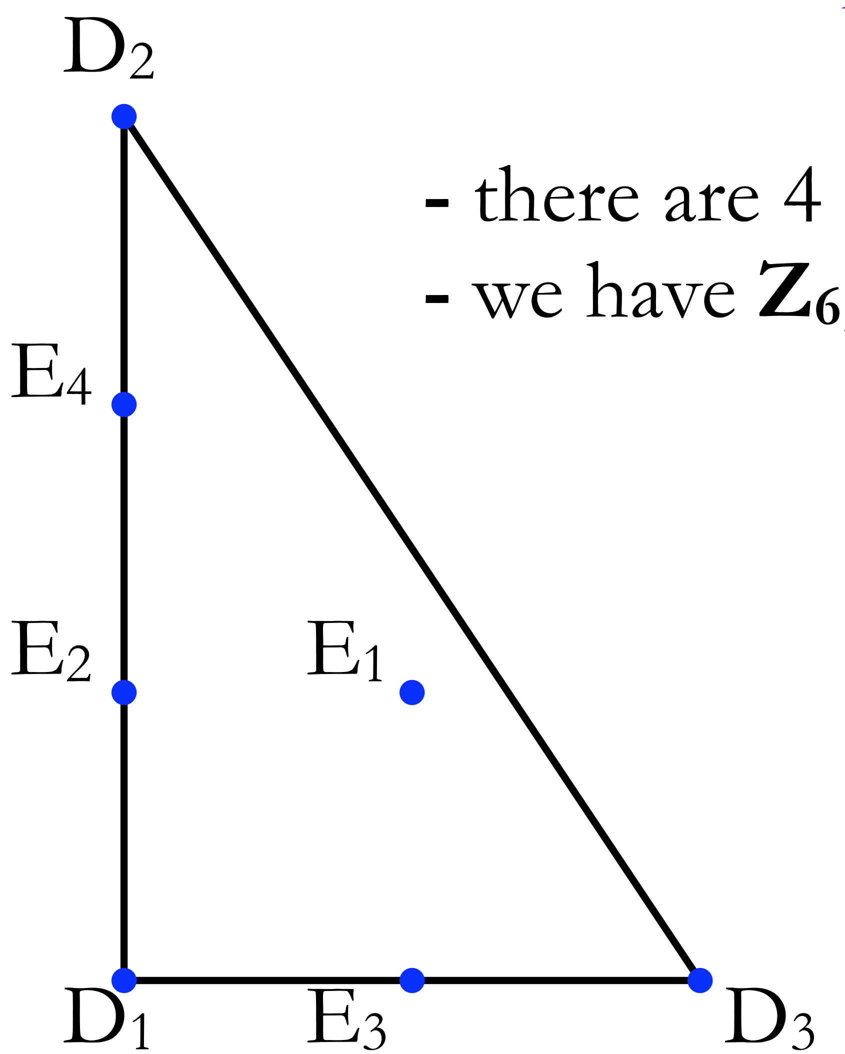
- but the orbifold vectors are not independent!

$$V_{Z_4} \sim V_1^g \sim -V_2^g$$

- The orbifold identification highly constrains the possible models!

# More complicated example: $\mathbb{C}^3/\mathbb{Z}_{6\text{-}II}$

in progress with S. Groot Nibbelink, J. Held, F. Ruehle




- there are 4 exceptional divisors
- we have $\mathbf{Z_6}$, $\mathbf{Z_3}$ and $\mathbf{Z_2}$ identifications

# More complicated example: $\mathbb{C}^3/\mathbb{Z}_{6\text{-}II}$

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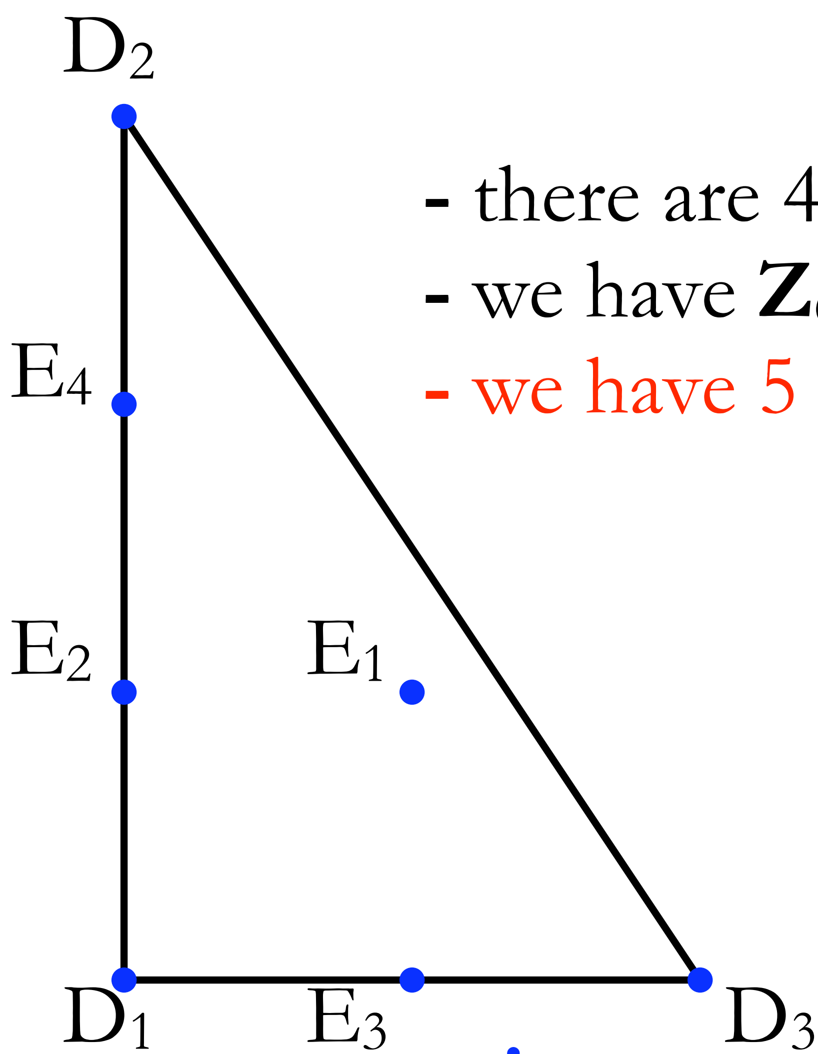
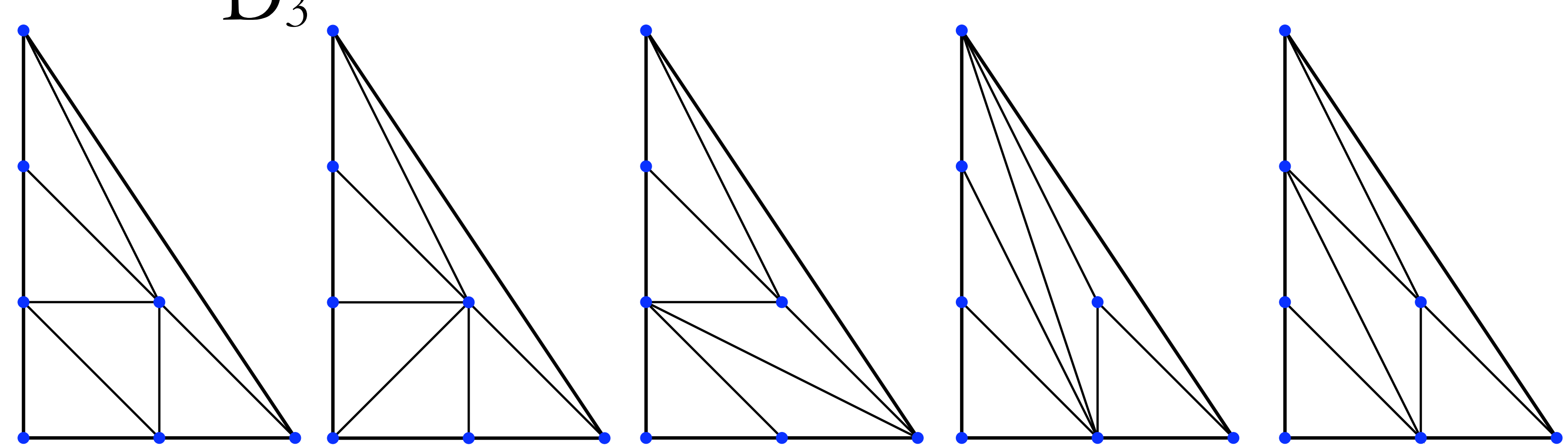



- there are 4 exceptional divisors
- we have $\mathbf{Z_6}$, $\mathbf{Z_3}$ and $\mathbf{Z_2}$ identifications
- we have 5 different possible resolutions!



## Matching the orbifolds: global informations

- When we glue together the various singularity in a compact manifold we have

1) More choices for the flux

**Ex. $T^4/Z_3$**

local case: $\mathcal{F} = V_{I1}^{g} H^{I} E_1/3 + V_{I2}^{g} H^{I} E_2/3$

global case: $\mathcal{F} = \frac{1}{3}\sum_{a,b=1}^{3}\left(V_1^{g_{ab}} \cdot H\, E_1^{ab} + V_2^{g_{ab}} \cdot H\, E_2^{ab}\right)$

all shift the same: no discrete Wilson lines

different shifts: discrete Wilson lines there!

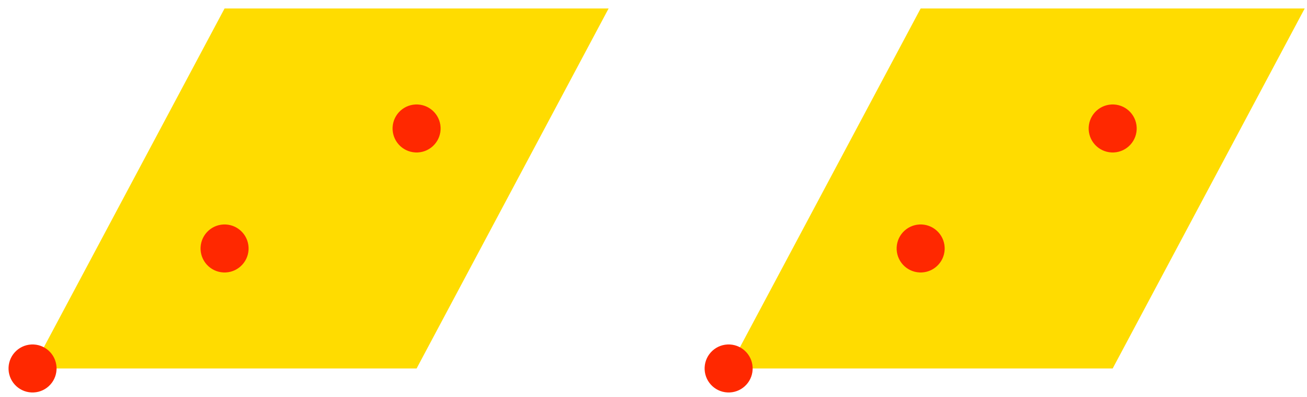
2) More compact 4-cycles: more conditions from the Bianchi identity

- Simple resolutions: easy to introduce the new Bianchi's keeping a local study
- **$T^6/Z_{6\text{-}II}$**: need a genuine global study (in progress)

# 3 - $T^4/Z_3$ orbifold

in progress with S. Groot Nibbelink & Felipe Paccetti

- $\mathbf{T}^4 = \mathbf{T}^2 \times \mathbf{T}^2$, complex coordinates $z_1$, $z_2$.
- $\mathbf{Z}_3$ has 3 x 3 equivalent fixed points (singularities).



**Local information:**

- Each singularity has form $\mathbf{C}^2/\mathbf{Z}_3$, with 2 divisors (pre-resolution):

$D_1$ corresponding to $z^1=0$ (fills the second **C**-plane)

$D_2$ corresponding to $z^2=0$ (fills the first **C**-plane)

- Resolution: add two exceptional divisors $E_1$ and $E_2$.

Linear equivalences: $0 \sim 3\,D_1 + E_1 + 2\,E_2$

$0 \sim 3\,D_2 + E_2 + 2\,E_1$

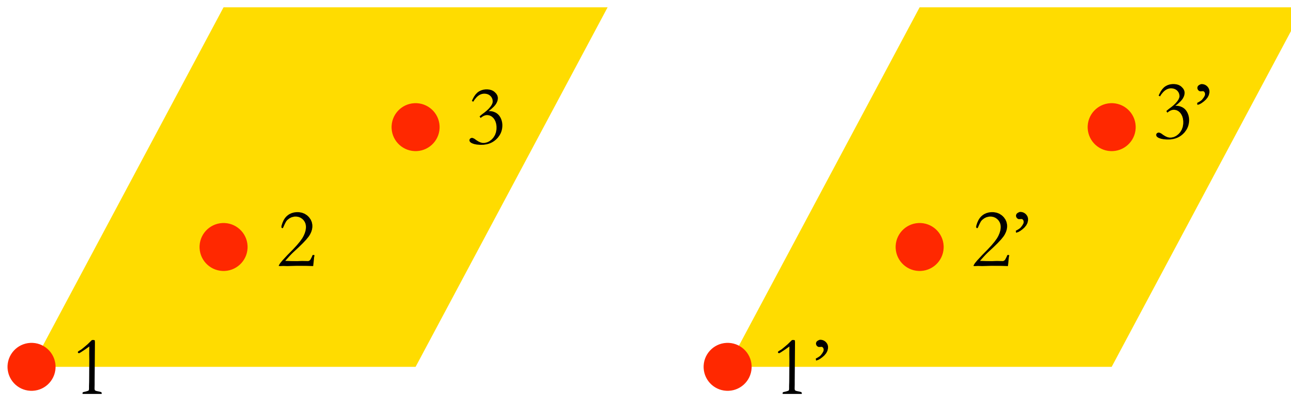
Intersections: $D_1E_2 = E_2E_1 = E_1D_2 = 1$

$D_1E_1 = D_2E_2 = 0 \quad E_1E_1 = E_2E_2 = -2$

## Gluing:

**1) "Assign fixed point indices"**

- The $E_i$'s are "localized" in the singularities, named 11', 12', 32', ...




for each $E_i$ we assign two extra indices: $E_i^{jk'}$.

- $D_1$ extends in the second torus and is localized in the first:

  we assign an extra index: $D_1^i$, similarly for $D_2$: $D_2^{j'}$.

- The D's are shared among various fixed points!

**2) Include the inherited divisors:**

- The R's and D's are linked, on the singular space: $R_i \sim 3D_i$.
- This link is the same for each of the D's : $R_1 \sim 3D_1^i,\ R_2 \sim 3D_2^{j'}$
- After resolution this linear equivalence is modified as

$$R_2 \sim 3D_2^{j'} + \sum_{i=1}^{3}\left(E_2^{ij'} + 2E_1^{ij'}\right),\ R_1 \sim 3D_1^{i} + \sum_{j'=1'}^{3'}\left(E_1^{ij'} + 2E_2^{ij'}\right)$$

**3) Compute the global set of intersections:**

- Use of the local information
- Input on the intersection of the R's

$$\mathrm{E}_1^{\mathrm{ij}'}\mathrm{E}_2^{\mathrm{pq}'} = \delta^{\mathrm{ip}}\delta^{\mathrm{j}'\mathrm{q}'}, \quad \mathrm{E}_1^{\mathrm{ij}'}\mathrm{E}_1^{\mathrm{pq}'} = \mathrm{E}_2^{\mathrm{ij}'}\mathrm{E}_2^{\mathrm{pq}'} = -2\delta^{\mathrm{ip}}\delta^{\mathrm{j}'\mathrm{q}'},$$

$$\mathrm{R}_1\mathrm{R}_2 = 3, \quad \mathrm{R}_1\mathrm{R}_1 = \mathrm{R}_2\mathrm{R}_2 = 0, \quad \mathrm{R}_\mathrm{i}\mathrm{E}_\mathrm{j}^{\mathrm{pq}'} = 0.$$

**Outcome:**

**- Number of (1,1)-forms:**

9 x 2 exceptional divisors
\+ 2 x 3 "normal divisors"
– 2 x 3 equivalences
\+ 2 inherited divisors
= **20**

**- Characteristic classes (splitting principle)**

$$c(\mathcal{R}) = (1+\mathrm{R}_1)(1+\mathrm{R}_2)\prod_{i=1}^{3}(1+\mathrm{D}_2^i)\prod_{j'=1'}^{3'}(1+\mathrm{D}_1^{j'})\prod_{i=1}^{3}\prod_{j=1'}^{3'}(1+\mathrm{E}_1^{ij'})(1+\mathrm{E}_2^{ij'})$$

from linear equivalence and intersections:

$$c_1(\mathcal{R}) = 0, \quad c_2(\mathcal{R}) = 24.$$

## Gauge bundles

- in general we have $\mathcal{F} = \frac{1}{3}\sum_{a,b=1}^{3}\left(V^{g\,ab}_{1}\cdot H\,E^{ab}_{1} + V^{g\,ab}_{2}\cdot H\,E^{ab}_{2}\right)$
- given the orbifold identification we can choose $V^{g\,ab}_{1} = -V^{g\,ab}_{2}$
- assuming no Wilson lines we can take the same flux in all the fixed points

$$\mathcal{F} = \frac{1}{3}V^{g}\cdot H\sum_{a,b=1}^{3}\left(E^{ab}_{1} - E^{ab}_{2}\right)$$

- and consider the Bianchi Identity, using the intersections given before

$$\int \mathcal{F}^{2} = \frac{V^{g\,2}}{9}\left[\sum_{ab}\left(E^{ab}_{1} - E^{ab}_{2}\right)\right]^{2} = \frac{V^{g\,2}}{9}9\left(E^{2}_{1} + E^{2}_{2} - 2E_{1}E_{2}\right) = -6V^{g\,2}$$

that means $V^{g\,2} = 8$

**Matching the orbifold models**

1) Orbifold shifts vs. line bundle embeddings;

| $V$ | $V_1^g = V + \Lambda_1$ | $V_2^g = -V + \Lambda_2$ |
|---|---|---|
| $(1^2, 0^{14})$ | $(2^2, 0^{14})$<br>$(2, 1, 0^{14})$ | $-(2^2, 0^{14})$<br>$(1, -1, 0^{14})$ |
| $(2, 1^4, 0^{11})$ | $(2, 1^4, 0^{11})$ | $-(2, 1^4, 0^{11})$ |
| $(1^8, 0^8)$ | $(1^8, 0^8)$ | $-(1^8, 0^8)$ |
| $(1^{14}, 0^2)$ | $\frac{1}{2}(1^{14}, 3^2)$ | $-\frac{1}{2}(1^{14}, 3^2)$ |
| $(2, 1^{10}, 0^5)$ | // | // |

## 2) Gauge group and matter: an example

| orbifold | resolution |
|---|---|
| $V = (1^{14}, 0^2)$ | $V_1^g = \frac{1}{2}(1^{14}, 3^2) \sim V,\ V_2^g = -V_1^g$ |
| $\mathrm{U}(14) \times \mathrm{SO}(4)$ | $\mathrm{U}(14) \times \mathrm{U}(2)$ |
| $(14,4) + (91,1) + 2(1,1)$<br>$9(1,1) + 9(14,2_+) + \underline{18(1,2_-)}$<br>higgsing<br>$(91,1) + 11(14,2) + 45(1,1)$ | $(91,1) + 11(14,2) + 45(1,1)$ |

in the blow-down regime we can have gauge enhancement or, in the blow-up there is a gauge symmetry breaking).

# 4 - Conclusions & working plan

**1)** We show how to resolve the $\mathbf{C^n/Z_m}$ and $\mathbf{C^n/Z_m \times Z_p}$ singularities, how to wrap U(1) flux on them and match heterotic orbifold models, at the gauge group/chiral spectrum level

S. Groot Nibbelink, MT, M. Walter;
T.-W. Ha, S. Groot Nibbelink, MT.

**2)** Using toric geometry we can glue the singularities and recover compact $\mathbf{T^n/Z_m}$ and $\mathbf{T^n/Z_m \times Z_p}$ orbifolds.

**3)** Study of compact heterotic models

- done the $\mathbf{T^6/Z_3}$ model.

S. Groot Nibbelink, D. Klevers, F. Ploger, MT, P. Vaudrevenge

- <u>in progress:</u> the K3 models S. Groot Nibbelink, F. Paccetti, MT
  - reobtain the results of G. Honecker, MT with explicit control on the line bundles
  - tool for a study of Heterotic/IIA duality
- <u>in progress:</u> the appealing $\mathbf{T^6/Z_{6\text{-}II}}$ model

S. Groot Nibbelink, MT, J.Held, F. Ruehle

**4)** Non-abelian bundle case

- <u>in progress:</u> the K3 models S. Groot Nibbelink, F. Paccetti, MT