UV Physics and String Inflation

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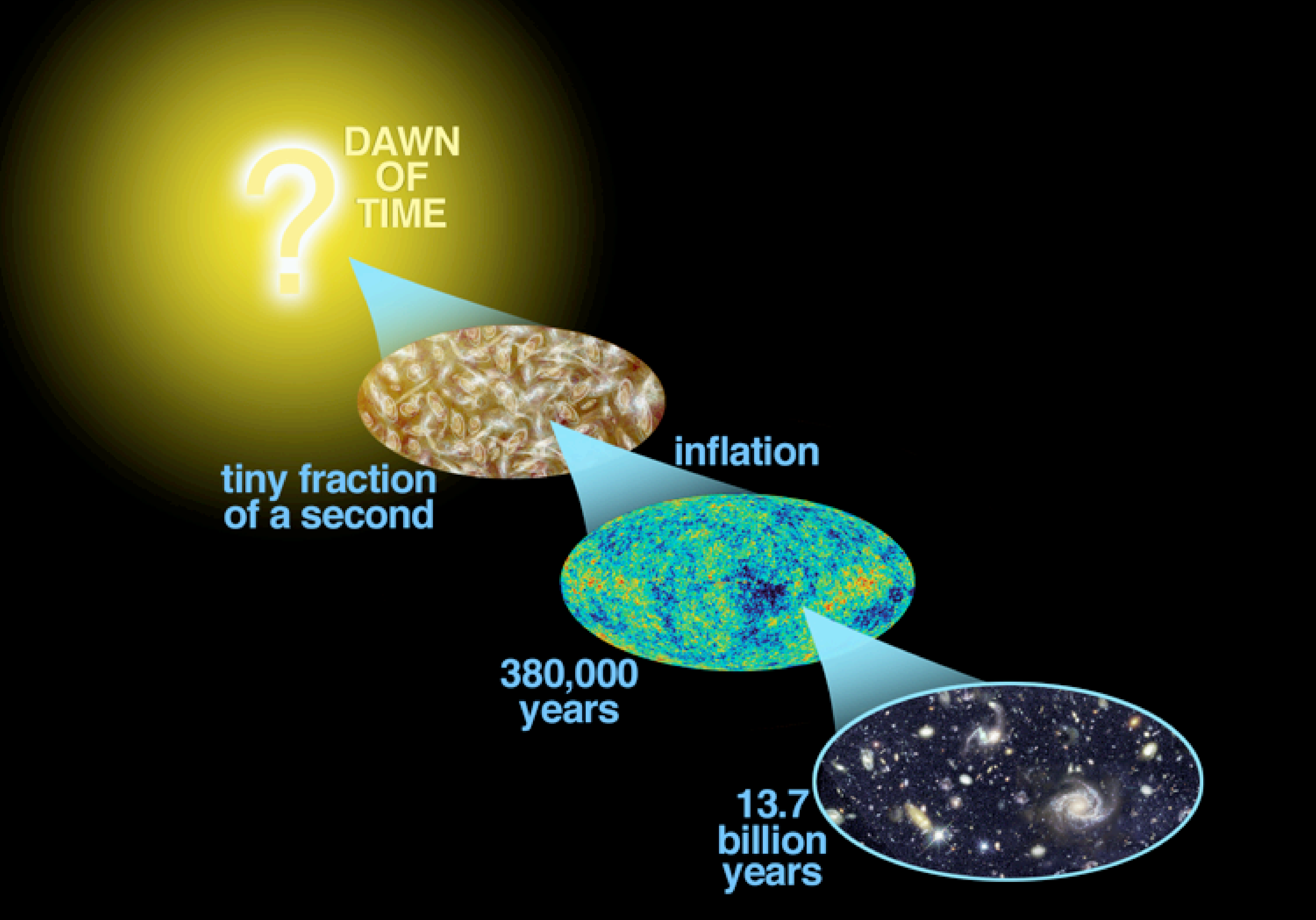
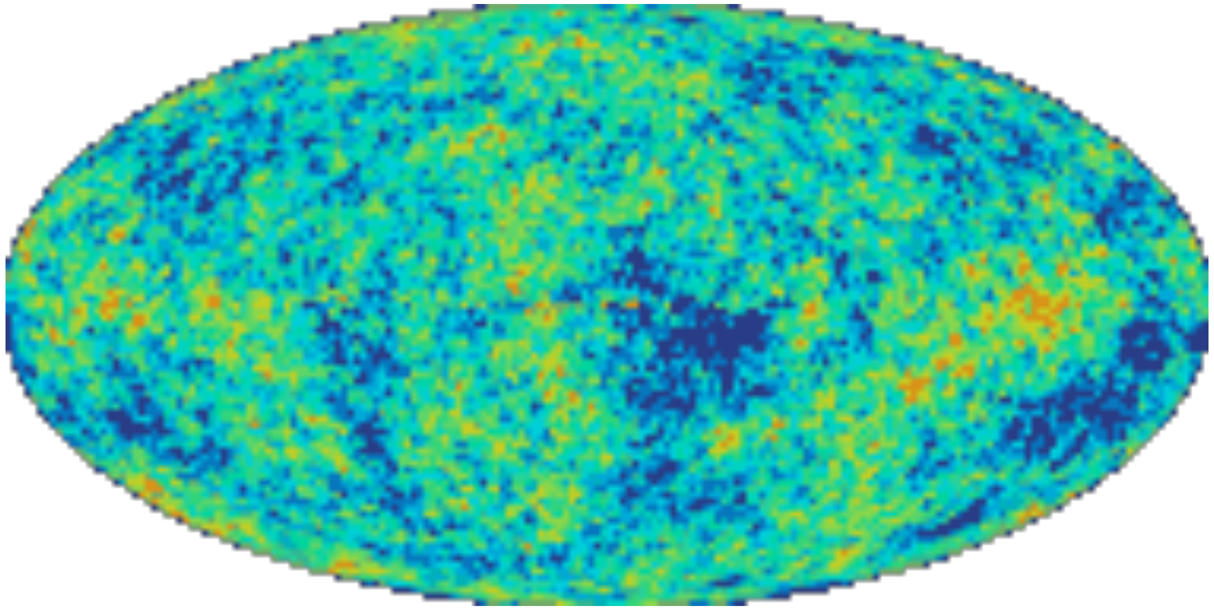
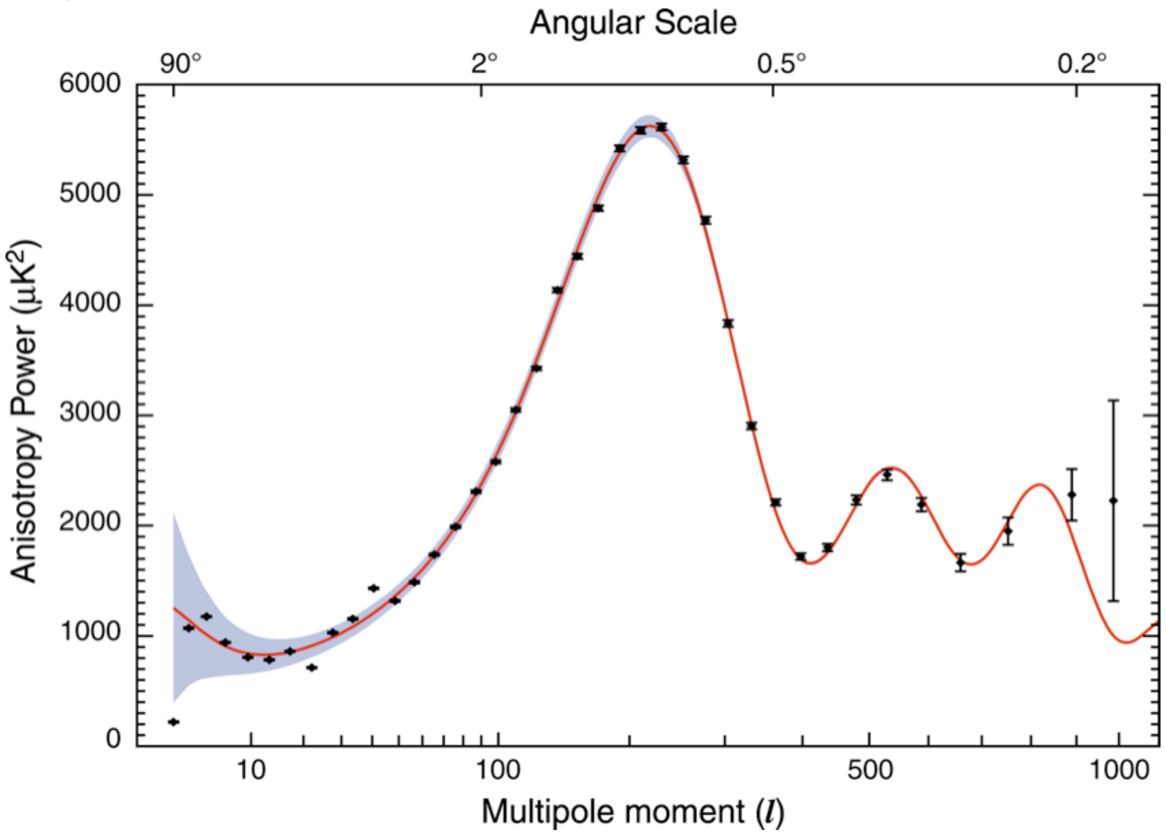

DAWN
OF
TIME
tiny fraction
of a second
inflation
380,000
years
13.7
billion
years

Inflation and UV Physics

- Almost scale invariant, Gaussian primordial spectrum predicted by inflation: good agreement with data.



WMAP

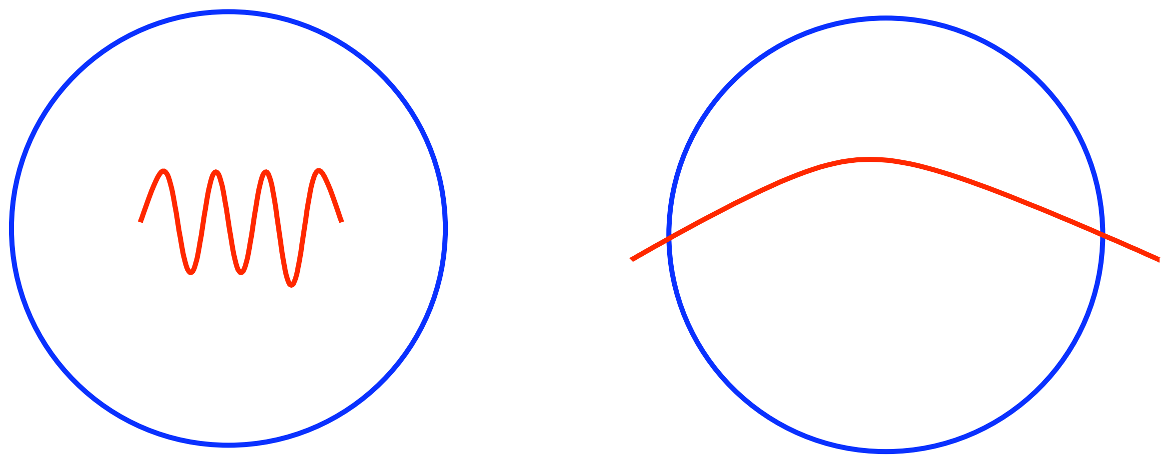


- Tantalizing upper bound on the inflaton energy density:

$$V \leq M_{GUT}^4 \sim (10^{16} GeV)^4, \ i.e., H \leq 10^{14} GeV$$

Inflation as a Short Distance Probe

Quantum Fluctuations | "Freeze In" | Structure



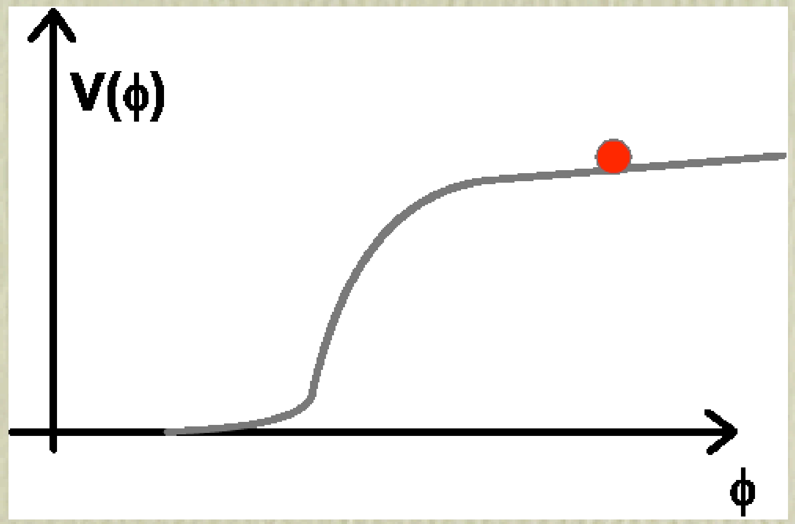
Quantum Fluctuations	"Freeze In"	Structure
$H^{-1} \sim$ constant	$H^{-1} \sim$ constant	H^{-1} increases
$\lambda < H^{-1}$	$\lambda \sim H^{-1}$	$\lambda < H^{-1}$

Imprints of short distance physics

[Brandenberger];[Chu,Greene,GS];[Easther, Greene, Kinney, GS];
[Kaloper, Kleban, Lawrence, Shenker, Susskind];[Einhorn, Larsen][Danielsson];
[Goldstein, Lowe];[Burgess, Cline, Holman];[Schalm, GS, van der Schaar], ...

Example 1: Eta Problem

- In a wide class of models, the inflaton potential takes a peculiar shape:



$$\epsilon = \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2 << 1$$

$$\eta = M_P^2 \frac{V''}{V} << 1$$

- Dimension 6, Planck suppressed operators can stop inflation:

$$\delta V \sim \frac{V}{M_P^2} \phi^2$$

- A sufficient degree of UV completeness is needed to estimate such corrections.

Example 2: Tensor Modes

- Lyth bound: $$\frac{\Delta\phi}{M_P} \sim \sqrt{\frac{r}{0.05}}$$
- A detection of primordial gravitational wave will imply the inflaton rolled over super-Planckian distances in field space.
- Naturalness suggests order one corrections to inflaton potential, unless UV completion shows otherwise.

$$V(\phi) = V_{\text{renomalizable}}(\phi) + \phi^4 \sum_{n\geq 1} c_n \left(\frac{\phi}{M_P}\right)^n$$

Example 3: Non-Gaussianities

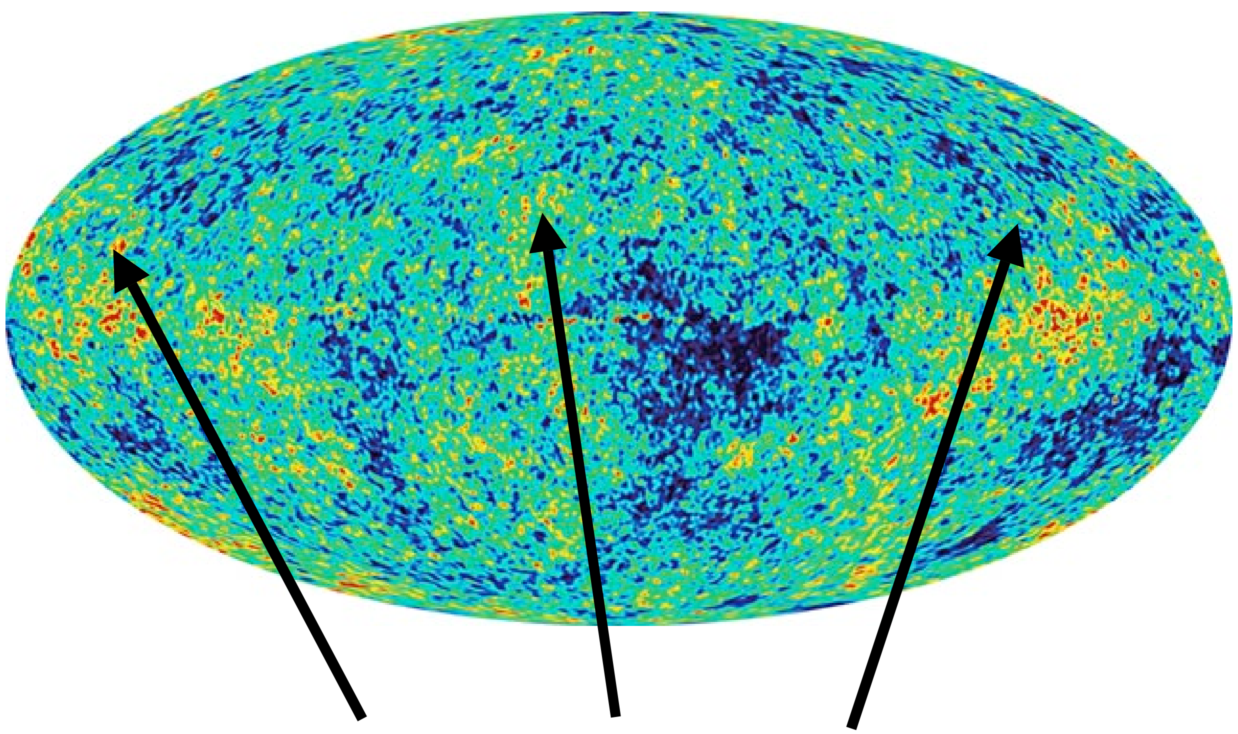
- Models of large non-Gaussianities tend to involve crucially higher derivative terms. [Chen, Huang, Kachru, GS]

- Models of this sort have been proposed:

  - K-inflation — Mukhanov
  - DBI inflation — Silverstein, Tong
  - Ghost inflation — Arkani-Hamed, Creminelli, Mukohyama, Zaldarriaga

- UV completion is needed to argue why some terms suppressed by a high mass scale are present while others are absent.

More about Non-Gaussianities



$$< \frac{\Delta T(x_1)}{T}\frac{\Delta T(x_2)}{T}\frac{\Delta T(x_3)}{T} >$$

Size of 3-point function:

$$f_{NL} \sim \frac{\text{Bispectrum}}{(\text{Power Spectrum})^2}$$

For slow-roll:

$$f_{NL} \sim \mathcal{O}(\epsilon)$$

Maldacena 02
Acquaviva et al 02

Bispectrum: $$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

Power spectrum: $$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle \sim \delta^3(\mathbf{k}_1+\mathbf{k}_2)\frac{P_k^\zeta}{k_1^3}$$

Bispectrum gives much richer info because of its shape.

General Results

Chen, Huang, Kachru, GS

- General kinetic term:

$$\mathcal{L}(\phi, X) \qquad \text{where} \qquad X = \frac{1}{2} g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$$

- Bi-spectrum depends on 5 parameters:

$$c_s^2 = \frac{\mathcal{L}_{,X}}{\mathcal{L}_{,X} + 2X\mathcal{L}_{,XX}}$$

$$\lambda/\Sigma = \frac{X^2\mathcal{L}_{,XX} + \frac{2}{3}X^3\mathcal{L}_{,XXX}}{X\mathcal{L}_{,X} + 2X^2\mathcal{L}_{,XX}}$$

$$s = \frac{\dot{c}_s}{c_s H}.$$

$$\epsilon = -\frac{\dot{H}}{H^2}$$

$$\eta = \frac{\dot{\epsilon}}{\epsilon H},$$

Large non-Gaussianities

$\Longrightarrow$

small c_s or large λ/Σ

Final Results (Chen, Huang, Kachru, GS, 06)

- The 3-pt function for a general single field inflation to $\mathcal{O}(\epsilon)$:

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(\tilde{P}_K^\zeta)^2\frac{1}{\prod_i k_i^3} \times \left(\mathcal{A}_\lambda+\mathcal{A}_c+\mathcal{A}_o+\mathcal{A}_\epsilon+\mathcal{A}_\eta+\mathcal{A}_s\right) ,$$

where we have decomposed the shape into six parts ($K \equiv k_1+k_2+k_3$)

$$\mathcal{A}_\lambda = \left(\frac{1}{c_s^2}-1-\frac{\lambda}{\Sigma}[2-(3-2\mathbf{c}_1)l]\right)_K \frac{3k_1^2k_2^2k_3^2}{2K^3} ,$$

$$\mathcal{A}_c = \left(\frac{1}{c_s^2}-1\right)_K \left(-\frac{1}{K}\sum_{i>j}k_i^2k_j^2+\frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3+\frac{1}{8}\sum_i k_i^3\right) ,$$

$$\mathcal{A}_o = \left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)_K (\epsilon F_{\lambda\epsilon}+\eta F_{\lambda\eta}+sF_{\lambda s}) + \left(\frac{1}{c_s^2}-1\right)_K (\epsilon F_{c\epsilon}+\eta F_{c\eta}+sF_{cs}) ,$$

$$\mathcal{A}_\epsilon = \epsilon\left(-\frac{1}{8}\sum_i k_i^3+\frac{1}{8}\sum_{i\neq j}k_ik_j^2+\frac{1}{K}\sum_{i>j}k_i^2k_j^2\right) ,$$

$$\mathcal{A}_\eta = \eta\left(\frac{1}{8}\sum_i k_i^3\right) ,$$

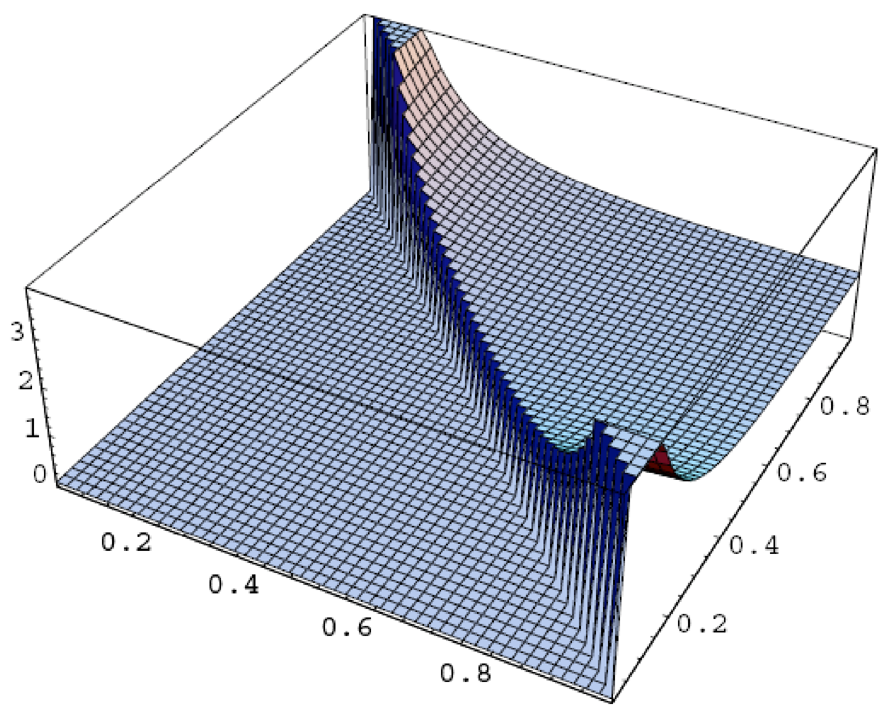
$$\mathcal{A}_s = sF_s .$$

- Completely specified by 5 parameters: c_s , $\frac{\lambda}{\Sigma}$, ϵ , η , s.

Shape of Non-Gaussianities

(Babich, Creminelli, Zaldarriaga, 04; Chen, Huang, Kachru, GS, 06)

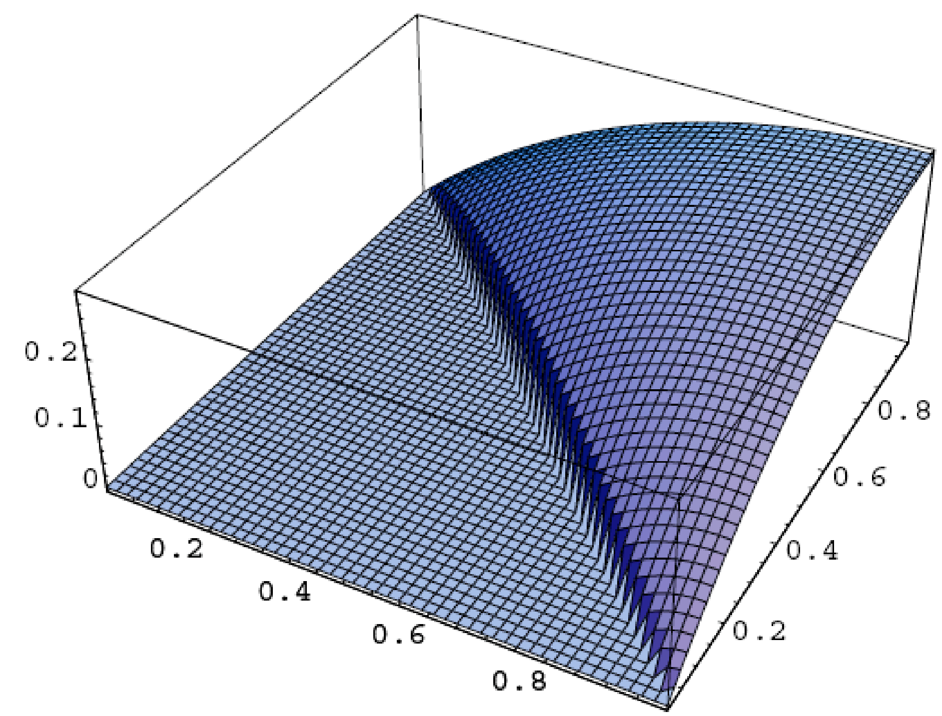
Plot $\mathcal{A}(1, x_2, x_3)/x_2 x_3$



Local shape (Slow-roll)

ϵ, η, s

$f_{NL} \sim \mathcal{O}(\epsilon)$

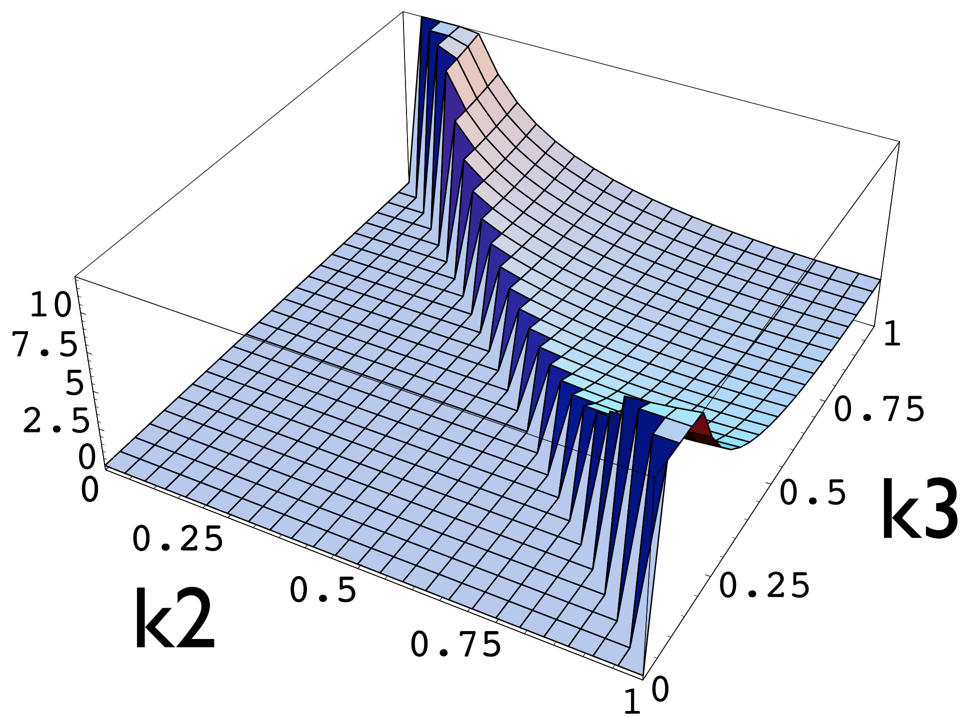


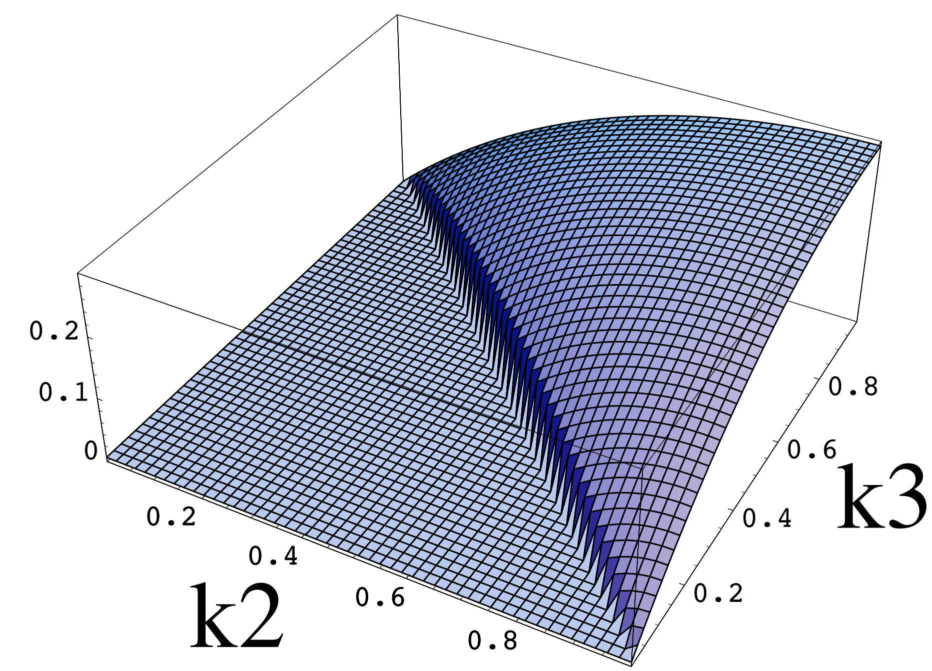
Equilateral shape (e.g., DBI)

c_s, λ

$f_{NL} \sim \mathcal{O}\left(c_s^{-2}\right)$

Experimental Bound





Current bound (WMAP5):

$-9 < f_{NL} < 111$ at 95% C.L. $-151 < f_{NL} < 253$ at 95% C.L.

Future expectation:

$|f_{NL}(\text{local})| \leq 20$ (WMAP) $|f_{NL}(\text{local})| \leq 5$ (PLANCK)

UV Physics & String Inflation

- All these UV questions about inflation boil down to a controllable effective theory.
- Answers to these questions have important observational consequences as well.
- In addition to the usual α' & g_s corrections, there is yet another expansion parameter in warped compactifications: $g_s N \alpha'$
- Warping ubiquitous in string inflation models: important to understand such corrections.


A Gentle Landscape


A Warped Landscape

Dynamics of Warped Flux Compactifications

GS, Torroba, Underwood, Douglas

Dynamics of Warped Flux Compactifications

GS, Torroba, Underwood, Douglas

Dynamics of Warped Flux Compactifications

GS, Torroba, Underwood, Douglas
(STUD)
See also: Douglas & Torroba

Warped Kahler Potential

- The warping corrected Kahler potential for the complex moduli sector was conjectured to be:

$$\mathrm{K} = -\log\left(\int e^{-4A}\Omega\wedge\overline{\Omega}\right) \Rightarrow G_{\alpha\overline{\beta}} = -\frac{1}{V_W}\int e^{-4A}\chi_\alpha\wedge\chi_{\overline{\beta}}$$

DeWolfe-Giddings

suggested by the fact that

$$\mathrm{V}_{CY} = \int d^6y\sqrt{g_6} \rightarrow V_W = \int d^6y\sqrt{\tilde{g}_6}e^{-4A(y)}$$

- For the warped deformed conifold:

$$\mathrm{G}_{S\overline{S}} = -\partial_S\partial_{\overline{S}}K = \frac{1}{V_W}\left[c\log\frac{\Lambda_0^3}{|S|} + c'\frac{(g_sN\alpha')^2}{|S|^{4/3}}\right]$$

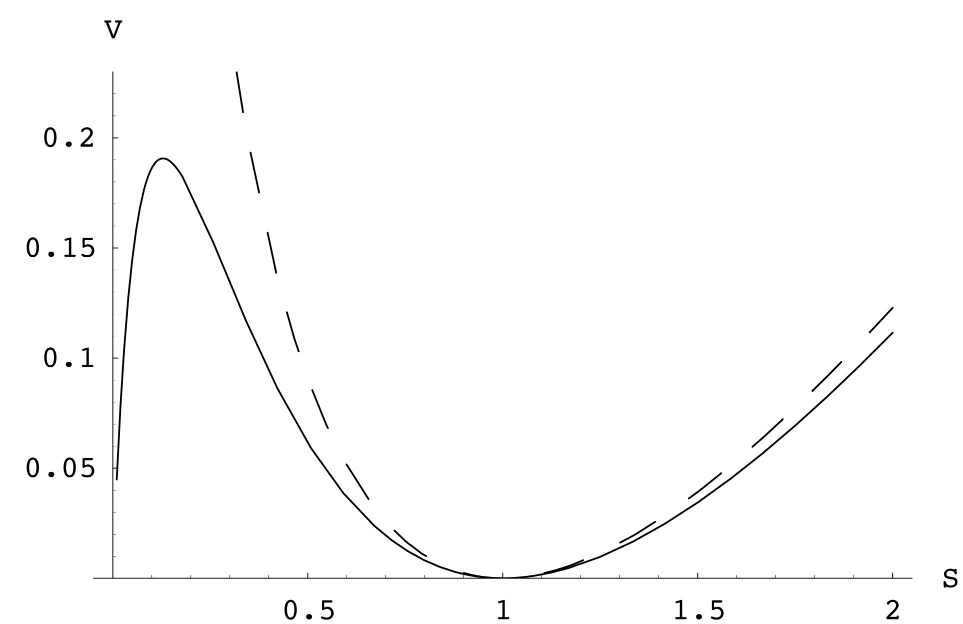
Douglas, Shelton, Torroba

Applications of Warped EFT

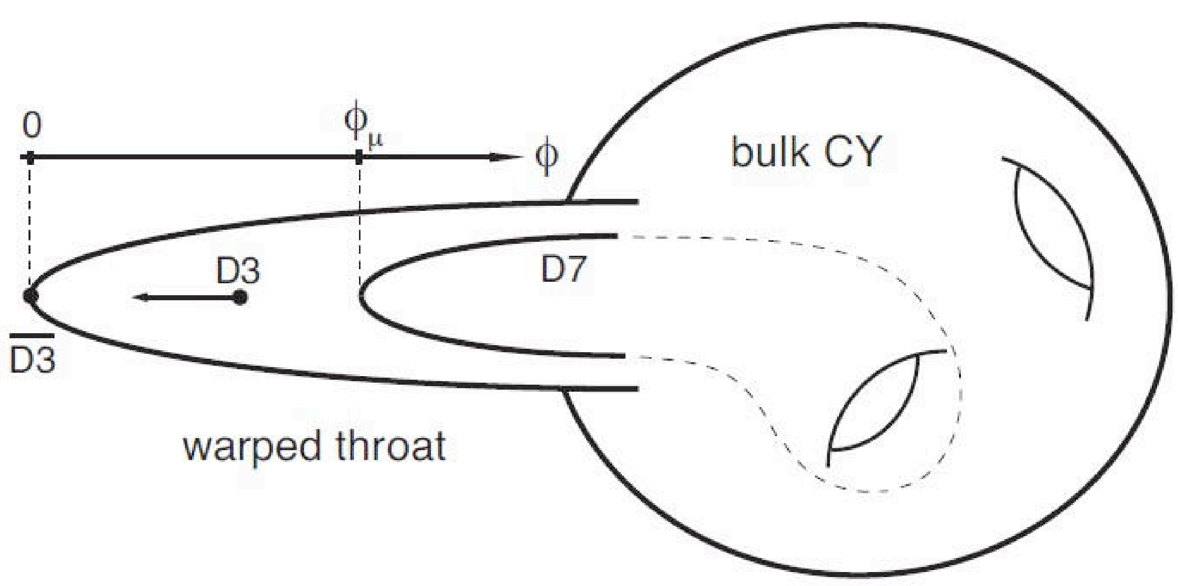
- Moduli (and hierarchy) stabilization potential:

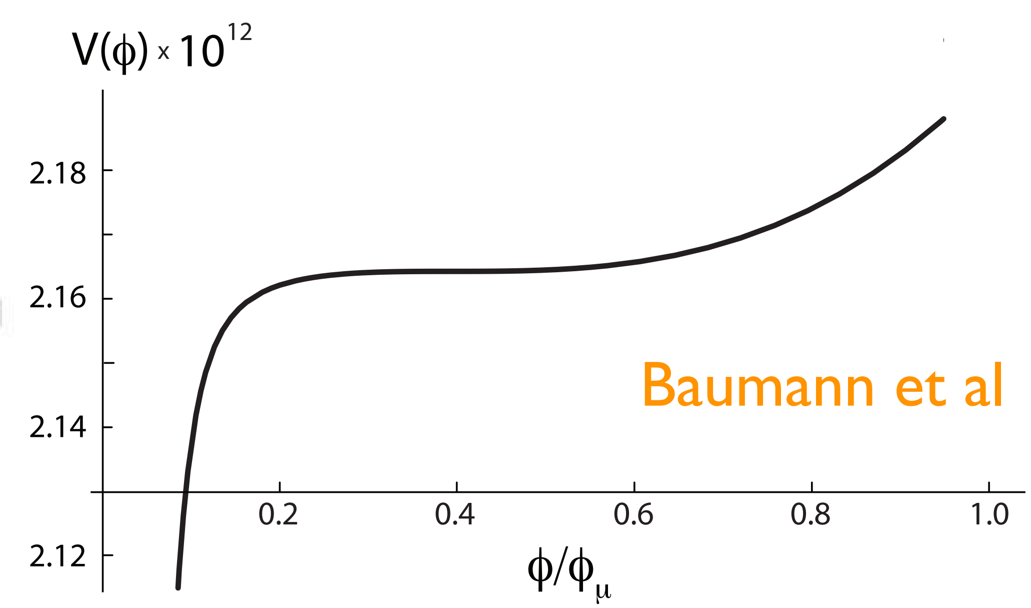
Near the conifold point:

$$V \simeq |S|^{4/3}|D_S W|^2$$



- Inflation potential, soft SUSY breaking terms, etc





Issues with Strong Warping

D=10 String Theory

↓ Low Energy

D=10 SUGRA with fluxes

↓ KK Dimensional Reduction

D=4 N=1 SUGRA EFT

↓

String vacua, inflation, de-Sitter, MSSM…

Ex: GKP and KKLT

Type IIB String Theory in D=10

↓ Low Energy

IIB Supergravity in D=10

$$S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{|g|}\left\{R_{10} - \frac{|G_3|^2}{2\mathrm{Im}\tau} - \frac{1}{4}|\tilde{F}_5|^2\right\} + \mathrm{CS} + \mathrm{local}$$

↓ KK Dimensional Reduction

N=1 SUGRA in D=4

$$K = -3\log(\rho+\bar{\rho}) - \log(\tau+\bar{\tau}) - \log\left(\int J^3\right) - \log\left(\int \Omega\wedge\bar{\Omega}\right)$$

$$W = \int G_3\wedge\Omega + W_{np}$$

Issues with Strong Warping

D=10 String Theory

↓ Low Energy

D=10 SUGRA with fluxes

↓ KK Dimensional Reduction

D=4 N=1 SUGRA EFT

↓

String vacua, inflation, de-Sitter, MSSM…

Many subtleties with warped KK reduction:

- General KK ansatz (compensators)
- Mixing/sourcing of KK modes with moduli
- Backreaction of moduli on warp factor
- 10D Gauge redundancies
- 10D Constraint equations

In *warped backgrounds* these issues are all highly coupled to each other!

KK Scale in Warped Background

	Moduli	KK modes
Unwarped	$m_z^2 \sim \frac{1}{\alpha'}$	$m_{KK}^2 \sim \frac{1}{L^2}$

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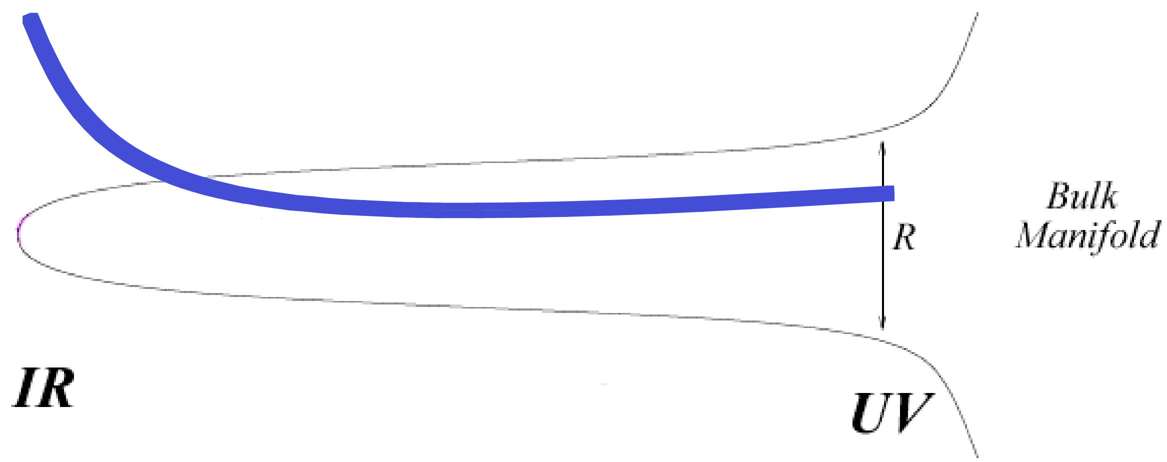
Strong warping



DeWolfe, Giddings; Giddings, Maharana; Frey, Maharana; Burgess, Camara, de Alwis, Giddings, Maharana, Quevedo, Suruliz; ...

KK Scale in Warped Background

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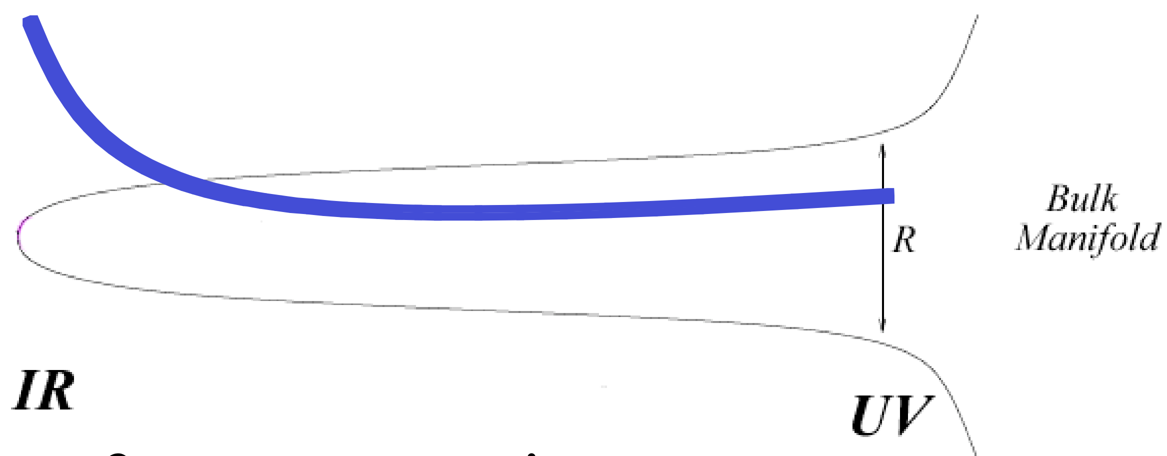
Strong warping



DeWolfe, Giddings; Giddings, Maharana; Frey, Maharana; Burgess, Camara, de Alwis, Giddings, Maharana, Quevedo, Suruliz; ...

Fields localize to region of strong warping.

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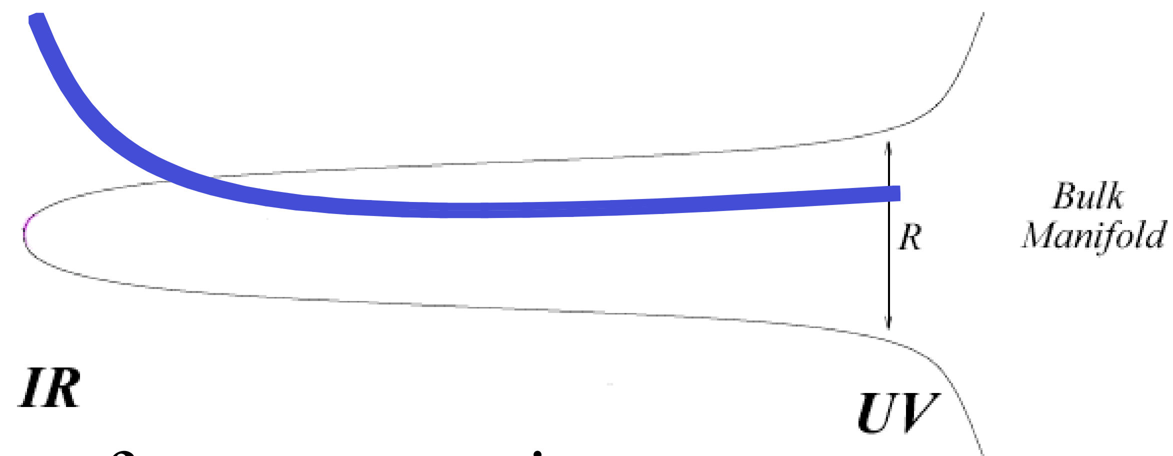
Unwarped	$m_z^2 \sim \frac{1}{\alpha'}$	$m_{KK}^2 \sim \frac{1}{L^2}$

Strong warping



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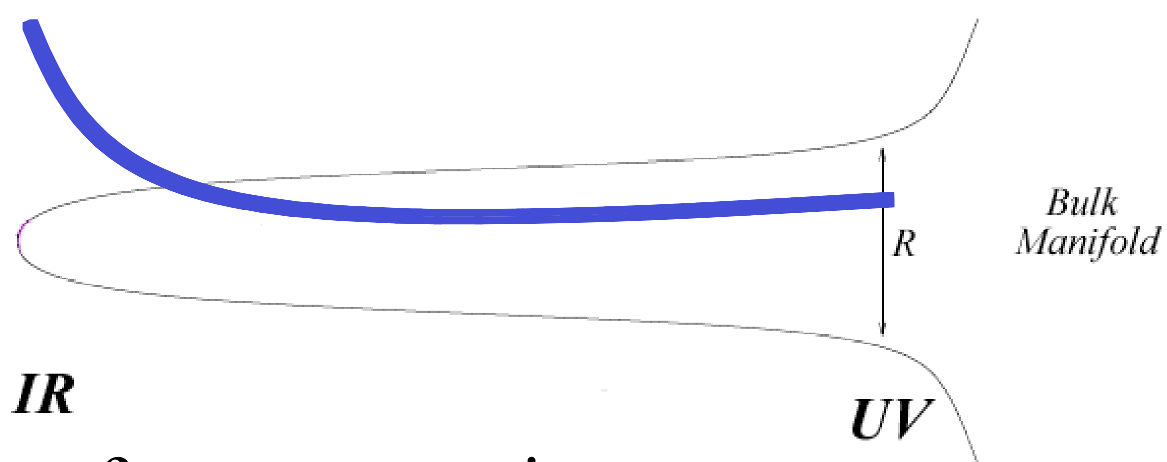
Fields localize to region of strong warping.

Masses redshifted	$m_z^2 \sim e^{2A_0}\frac{1}{\alpha'}$	$m_{KK}^2 \sim e^{2A_0}\frac{1}{\alpha'}$

KK Scale in Warped Background

	Moduli	KK modes
Unwarped	$m_z^2 \sim \frac{1}{\alpha'}$	$m_{KK}^2 \sim \frac{1}{L^2}$

Strong warping



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Fields localize to region of strong warping.

	Moduli	KK modes
Masses redshifted	$m_z^2 \sim e^{2A_0}\frac{1}{\alpha'}$	$m_{KK}^2 \sim e^{2A_0}\frac{1}{\alpha'}$

No mass hierarchy between moduli and KK modes for integrating out heavy fields.

Warped Kahler Potential

Previous proposal: (DeWolfe, Giddings)

$$\mathrm{K} = -\log\left(\int e^{-4A}\Omega \wedge \overline{\Omega}\right) \Rightarrow G_{\alpha\overline{\beta}} = -\frac{1}{V_W}\int e^{-4A}\chi_\alpha \wedge \chi_{\overline{\beta}}$$

did not account for all these subtle issues with warping.

Ansatz for fluctuations: (DeWolfe, Giddings)

$$ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}(\tilde{g}_{mn} + \delta\tilde{g}_{mn})dy^m dy^n$$

... does not solve 10D EOM! Giddings, Maharana; STUD

More general ansatz does, but extremely messy ...

$$ds^2_{10} \rightarrow ds^2_{10} + 2\partial_\mu\partial_\nu S^\alpha e^{2A} K_\alpha(y) dx^\mu dx^\nu + 2e^{2A} B_{\alpha m}(y)\partial_\mu S^\alpha dx^\mu dy^m \,.$$

Linearized Einstein Equations

$$\delta G^\mu_\nu = \delta^\mu_\nu u^I \delta_I \left\{ e^{2A} \left[-2\tilde\nabla^2 A + 4(\widetilde{\nabla A})^2 - \frac{1}{2}\tilde R \right] \right\} + e^{-2A}\left(\partial^\mu \partial_\nu u^I - \delta^\mu_\nu \Box u^I\right)\left(4\delta_I A - \frac{1}{2}\delta_I \tilde g\right)$$
$$+ \left(\partial^\mu \partial_\nu u^I - \delta^\mu_\nu \Box u^I\right) e^{2A} \tilde\nabla^p (B_{Ip} - \partial_p K_I)$$
$$+ e^{-2A} f^K \delta_K G^{(4)\mu}_\nu - \frac{1}{2}\left(\delta_K g^\mu_\nu - \delta^\mu_\nu \delta_K g^\lambda_\lambda\right) e^{2A} \tilde\nabla^2 f^K \ , \tag{A.14}$$

$$\delta G^\mu_m = \delta R^\mu_m = e^{-2A} \partial^\mu u^I \Bigg\{ 2\partial_m \delta_I A - 8\partial_m A \delta_I A - \frac{1}{2}\partial_m \delta_I \tilde g + \partial_m A \delta_I \tilde g$$
$$- 2\partial^{\tilde p} A \delta_I \tilde g_{mp} + \frac{1}{2}\tilde\nabla^p \delta_I \tilde g_{mp}$$
$$- \frac{1}{2}\tilde\nabla^p \left[e^{4A}\left(\tilde\nabla_p B_{Im} - \tilde\nabla_m B_{Ip} \right) \right] + 2(\partial_m A B_{Ip} - \partial_p A B_{Im}) \tilde\nabla^p e^{4A}$$
$$+ \frac{1}{2} e^{8A} B_{Im} \tilde\nabla^2 e^{-4A} - e^{4A} \tilde R^n_m B_{In} \Bigg\} \ , \tag{A.15}$$

$$\delta G^m_n = u^I \delta_I \left\{ e^{2A} \left[\tilde G^m_n + 4(\widetilde{\nabla A})^2 \delta^m_n - 8\nabla_n A \tilde\nabla^m A \right] \right\} - \frac{1}{2} e^{-2A} \Box u^I \tilde g^{mk} \delta_I \tilde g_{kn}$$
$$+ \delta^m_n e^{-2A} \Box u^I \left(-2\delta_I A + \frac{1}{2}\delta_I \tilde g\right)$$
$$\Box u^I \Bigg(\frac{1}{2} e^{-2A} \left\{ \tilde\nabla^m \left[e^{4A} \left(B_{In} - \partial_n K_I\right)\right] + \tilde\nabla_n \left[e^{4A}\left(B_I^{\tilde m} - \partial^{\tilde m} K_I\right)\right] \right\}$$
$$- \delta^m_n \tilde\nabla^p \left[e^{2A} \left(B_{Ip} - \partial_p K_I\right)\right] \Bigg)$$
$$+ \frac{1}{2}\delta_K g^\mu_\mu \left\{ -\frac{1}{2} e^{-2A} \left[\tilde\nabla^m \left(e^{4A}\partial_n f^K\right) + \tilde\nabla_n \left(e^{4A}\partial^{\tilde m} f^K\right)\right] + \delta^m_n \tilde\nabla^p \left[e^{2A} \partial_p f^K \right] \right\}$$
$$- \frac{1}{2}\delta^m_n f^K e^{-2A} \delta_K R^{(4)} \ . \tag{A.16}$$

$$\delta T^\mu_\nu = -\delta^\mu_\nu \frac{1}{4\kappa_{10}^2} \left\{ u^I \delta_I \left[e^{-6A} (\widetilde{\nabla\alpha})^2 \right] - 2e^{-6A} \Box u^I S_{Im} \partial^{\tilde m}\alpha - 2\Box u^I K_I e^{-6A} (\widetilde{\nabla\alpha})^2 \right\} \ , \tag{A.37}$$

$$\delta T^\mu_m = \frac{1}{2\kappa_{10}^2} \partial^\mu u^I e^{-6A} \left[\partial_m S_{Ip} - \partial_p S_{Im} + \partial_m \alpha B_{Ip} - \partial_p \alpha B_{Im} \right] \partial^{\tilde p} \alpha \ , \tag{A.38}$$

$$\delta G^M_N = \kappa_{10}^2 \delta T^M_N$$

$$\delta T^m_n = -\frac{1}{2\kappa_{10}^2} u^I \delta_I \left\{ e^{-6A} \left[\partial_n \alpha \partial^{\tilde m} \alpha - \frac{1}{2}\delta^m_n (\widetilde{\nabla\alpha})^2 \right] \right\}$$
$$+ \frac{e^{-6A}}{2\kappa_{10}^2} \Box u^I \left\{ S_{In} \partial^{\tilde m}\alpha + \partial_n \alpha S_I^{\tilde m} - \delta^m_n S_{Ip} \partial^{\tilde p}\alpha + 2K_I \left[\partial_n \alpha \partial^{\tilde m}\alpha - \frac{1}{2}\delta^m_n (\widetilde{\nabla\alpha})^2 \right] \right\} \ . \tag{A.39}$$

Giddings, Maharana

Gauge Invariance & Compensators

Previous proposal: (DeWolfe, Giddings)

$$\mathrm{K} = -\log\left(\int e^{-4A}\Omega\wedge\overline{\Omega}\right) \Rightarrow G_{\alpha\overline{\beta}} = -\frac{1}{V_W}\int e^{-4A}\chi_\alpha\wedge\chi_{\overline{\beta}}$$

is not diffeomorphism invariant:

$$\chi \to \chi + d\alpha$$

This turns out to be equivalent to the failure of the metric ansatz in solving the EOM.

Need extra terms proportional to $\partial_\mu S^\alpha$

$$ds^2_{10} \to ds^2_{10} + 2\partial_\mu\partial_\nu S^\alpha e^{2A}K_\alpha(y)dx^\mu dx^\nu + 2e^{2A}B_{\alpha m}(y)\partial_\mu S^\alpha dx^\mu dy^m\,.$$

metric compensators

(Analogously, also flux compensators)

Compensators in E&M

Consider a U(1) gauge field:

$$S = -\frac{1}{4}\int d^{10}x\sqrt{g_{10}}F^{MN}F_{MN}$$

and a family of solutions to $D^M F_{MN} = 0$

parametrized by moduli u^I : $A_M = (A_\mu = 0, A_i(y; u))$

Promoting $u^I \rightarrow u^I(x)$, the kinetic terms give:

$$G_{IJ} = \int d^6y\sqrt{g_6}g^{ij}\frac{\partial A_i}{\partial u^i}\frac{\partial A_j}{\partial u^J}$$

not gauge invariant under $\delta A_i = \partial_i \epsilon$

Compensators in E&M

The error is in assuming that: $A_\mu = 0$

still holds for time-dependent moduli.

This is incorrect because the 10D EOM:

$$D^M F_{M\mu} = 0 \Rightarrow \partial_\mu \partial^i A_i = \partial^i \partial_i A_\mu$$

cannot be solved by: $\partial_\mu A_i \neq 0 \ , \quad A_\mu = 0$

Instead, the time-dependence forces a non-zero:

$$A_\mu = \Omega_I \partial_\mu u^I \ , \quad \partial^i \partial_i \Omega_I = \partial^i \frac{A_i}{\partial u^I}$$

Ω_I : compensator field

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This is incorrect because the 10D EOM:

$$D^M F_{M\mu} = 0 \Rightarrow \partial_\mu \partial^i A_i = \partial^i \partial_i A_\mu$$

Constraint equations:
no second order
time derivatives

cannot be solved by: $\partial_\mu A_i \neq 0 \ , \quad A_\mu = 0$

Instead, the time-dependence forces a non-zero:

$$A_\mu = \Omega_I \partial_\mu u^I \ , \quad \partial^i \partial_i \Omega_I = \partial^i \frac{A_i}{\partial u^I}$$

Ω_I : compensator field

Compensators in E&M

Effect of compensator on dimensionally reduced action:

$$\frac{\partial A_i}{\partial u^I} \to \delta_I A_i \equiv \frac{\partial A_i}{\partial u^I} - \partial_i \Omega_I \text{ so that } \partial^i (\delta_I A_i) = 0$$

Compensator puts $\delta_I A_i$ back into harmonic gauge.

The field space metric is simply:

$$G_{IJ} = \int d^6 y \sqrt{g_6} \; g^{ij} \delta_I A_i \delta_J A_j$$

Natural mathematical definition (Singer): fluctuation $\delta_I A_i$ orthogonal to gauge transformation, w.r.t G_{IJ}

Warped Compactifications

Time-dependence of moduli sources off-diagonal metric:

$$ds^2_{10} = e^{2A(y;u)} g_{\mu\nu}(x) dx^\mu dx^\nu + B^I_j(y) \partial_\mu u^I dx^\mu dy^I + g_{ij}(y;u) dy^i dy^j$$

Compensators put metric back into harmonic gauge.

Hard to generalize YM approach. Two strategies:

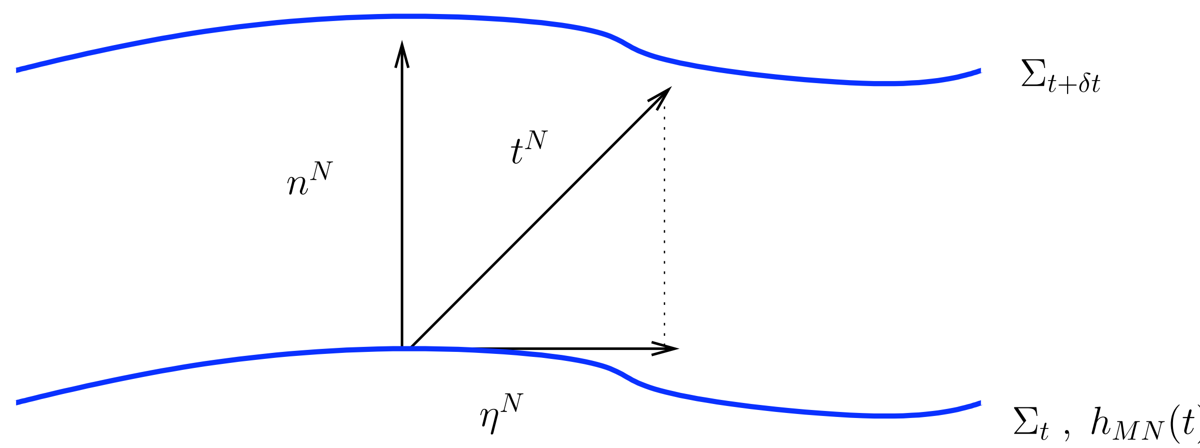
- Lagrangian: gauge-fixed metric ($B^I_j = 0$, compensator gauge), dimensional reduction with 10D constraints.
- Hamiltonian: gauge invariant metric, compensators as Lagrange multipliers enforcing 10D constraints.

Hamiltonian of GR

Split metric into:

h_{MN} space-like piece

η_N tangential shift



Extrinsic curvature: $K_{MN} = \frac{1}{2}(g^{tt})^{1/2}\left(\dot{h}_{MN} - \nabla_M \eta_N - \nabla_N \eta_M\right)$

Canonical momentum: $\pi_{MN} = \frac{\partial \mathcal{L}_{EH}}{\partial \dot{h}_{MN}} = h^{1/2}\left(K_{MN} - h_{MN} K\right)$

Hamiltonian: $\mathcal{H}_G = \sqrt{-g_D}\left(-R^{(D-1)} + h^{-1}\pi^{MN}\pi_{MN} - \frac{1}{D-2}h^{-1}\pi^2\right) - 2\eta_N \nabla_M\left(\pi^{MN}\right)$

$\eta_N =$ Lagrange multipliers enforcing the constraints:

$$\nabla_M\left(\pi^{MN}\right) = 0$$

Kinetic Terms

Here, time-dependence of h_{MN} only implicit through $u^I(x)$

Computing the shift vectors: $\eta^i = B^i_I \dot{u}^I$

Therefore, compensators = Lagrange multipliers of $\mathcal{H}_G$!

The dynamical variables of H define the metric fluctuations:

$$K_{MN} \sim \dot{u}^I \delta_I h_{MN} \equiv \dot{u}^I \frac{\partial h_{MN}}{\partial u^I} - \nabla_M \eta_N - \nabla_N \eta_M$$

$$\pi_{MN} \sim \dot{u}^I \delta_I \overline{h}_{MN} \equiv \dot{u}^I \left(\delta_I h_{MN} - h_{MN} \delta_I h\right)$$

Only effect of compensators is to shift $\partial_I h_{MN} \to \delta_I h_{MN}$

("physical" variation) & enforce constraints: $\nabla^M \left(\delta_I \overline{h}_{MN}\right) = 0$

Kinetic Terms

Kinetic term of Hamiltonian: $\mathcal{H}_{\rm kin}(\dot{u}, \dot{u}) = G_{IJ}(u)\dot{u}^I \dot{u}^J$

$$G_{IJ}(u) = \int d^{D-1}x\sqrt{-g_D}\; g^{tt}\delta_I h^{MN}\delta_J \overline{h}_{MN}$$

The constraints: $\nabla^M\left(\delta_I \overline{h}_{MN}\right) = 0$

imply that physical fluctuations are orthogonal to gauge transformations:

$$\mathcal{H}_{\rm kin}(\nabla\epsilon, \delta h) = 0$$

Equivalently: the constraints minimize G_{IJ} over each gauge orbit.

Applications: Warped Compactifications

Conformal Calabi-Yau background:

$$ds^2_{10} = e^{2A(y;u)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y;u)}\tilde{g}_{mn}(y;u)dy^m dy^n$$

Constraint equations:

$$(1)\ \delta A = \frac{1}{8}\delta\tilde{g} \leftrightarrow \text{Invariance of } V_W = \int d^6y\sqrt{\tilde{g}_6}e^{-4A}$$

$$(2)\ \tilde{\nabla}^{\tilde{m}}(\delta\tilde{g}_{mn} - \frac{1}{2}\tilde{g}_{mn}\delta\tilde{g}) = 4\partial^{\tilde{m}}A\delta\tilde{g}_{mn}$$

$$\leftrightarrow \text{``Warped'' Harmonic Gauge Condition}$$

Warped moduli space metric:

$$G_{IJ}(u) = \frac{1}{4V_W}\int d^6y\sqrt{\tilde{g}_6}\ e^{-4A}\tilde{g}^{ik}\tilde{g}^{jl}\delta_I\tilde{g}_{ij}\delta_J\tilde{g}_{kl}$$

Properties of Moduli Space Metric

- Metric fluctuations are orthogonal to gauge transformation w.r.t. G_{IJ}.
- Warp factor appears in inner product. Metric fluctuations no longer in harmonic gauge.
- Expression differs from the conjectured form:

$$G_{\alpha\overline{\beta}} = -\frac{1}{V_W}\int e^{-4A}\chi_\alpha \wedge \chi_{\overline{\beta}}$$

χ_α are harmonic forms of the underlying CY.

Warped Deformed Conifold

- Compute the field space metric for the complex moduli S in the deformed conifold
- Klebanov-Strassler solution:

$$ds^2_{10} = \frac{|S|^{2/3}}{(g_s N\alpha')} \; I(\tau)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + (g_s N\alpha') \; I(\tau)^{1/2} \left[\frac{1}{3K(\tau}\right)^2 \left(d\tau^2 + (g^5)^2\right)$$

$$+ K(\tau)\cosh^2\left(\frac{\tau}{2}\right)\left((g^3)^2 + (g^4)^2\right) + K(\tau)\sinh^2\left(\frac{\tau}{2}\right)\left((g^1)^2 + (g^2)^2\right)\Big]$$

where $\quad e^{-4A(\tau)} = \dfrac{(g_s N\alpha')^2}{|S|^{4/3}} \; I(\tau)$

- Note 6D metric independent of S, which only enters the 4D redshift factor.

Warped Deformed Conifold

- Internal metric fluctuations are completely determined by compensators!

$$\delta_S g_{ij} = -\nabla_i \eta_j - \nabla_j \eta_i$$

- The field space metric then becomes:

$$G_{S\overline{S}} = -\frac{1}{2V_W}\left(\int \prod_i g^i\right) \sqrt{g_6}\; e^{2A}\eta_i \delta_S \overline{g}^{i\tau}|_{\tau=0}^{\tau=\tau_\Lambda}$$

- Solving compensator equations near IR end:

$$G_{S\overline{S}} = \frac{k}{V_W}\frac{(g_s N \alpha')^2}{|S|^{4/3}}$$

Same qualitative feature as DG, but differs by order one coefficient.

Warped EFT: Summary

- Many subtle issues need to be taken into account for strong warping - all important and coupled.
- Calculate warping and KK corrections to 4D EFT, Kahler potential differs from previous proposals.
- Future direction: universal Kahler modulus in strong warping. Important for many phenomenological & cosmological applications.

D7-branes

* Moduli Stabilization — Kachru, Kallosh, Linde, Trivedi
* Vacuum energy uplifting — Burgess, Kallosh, Quevedo
* Brane Inflation:
    * Brane-antibrane — Baumann et al; ...
    * D3-D7 — Haack, Kallosh, Linde, Lust, Zaggerman; ...
* Multi-field effects — Chen, Gong, GS
* SUSY D7 in warped deformed conifold

Chen, Ouyang, GS

THANKS