Compactifications on Generalized Geometries

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Introduction

Phenomenological models in string theory:

- space-time background

$$M_{10} = M_4 \times \mathbf{Y_6}$$

- $N = 1$ (spontaneously broken) supersymmetry

realized as

➪ **Heterotic string** with $\mathbf{Y_6}$: **Calabi-Yau threefold/orbifold**

➪ **Type II/I** with space-time filling D-branes and orientifold-planes

$\mathbf{Y_6}$: **generalized Calabi-Yau orientifold**

(**with background fluxes**)

Problems:

- mechanism for spontaneous supersymmetry breaking
- stabilization of moduli
- “realistic” models of particle physics
- model for inflation
- cosmological constant

they might all be (un-) related

Purpose of this talk:

discuss string compactifications on generalized geometries

Compactification:

Space-time background: $\mathbf{M}_{10} = \mathbf{M_4} \times \mathbf{Y_6}$

Lorentz-group: $\mathbf{SO(1,9)} \rightarrow \mathbf{SO(1,3)} \times \mathbf{SO(6)}$

10D Supercharge: $\mathbf{16} \rightarrow (\mathbf{2},\mathbf{4}) \oplus (\bar{\mathbf{2}},\bar{\mathbf{4}})$

Impose:

1. existence of 4D supercharge $\Rightarrow$ existence of globally def. spinor η

   $\Rightarrow$ $\mathbf{Y_6}$ has reduced structure group $\mathbf{SU(3)}$

$$\mathbf{SO(6)} \rightarrow \mathbf{SU(3)} \quad \text{s.t.} \quad \mathbf{4} \rightarrow \mathbf{3} + \mathbf{1}$$

2. background preserves supersymmetry

$$\delta\Psi = \nabla\eta + (\gamma \cdot \mathbf{F})\,\eta + \ldots \; = \; 0 \;, \qquad \mathbf{F} = \text{background fluxes}$$

- $\nabla\eta \; = \; 0 \quad \Rightarrow \mathbf{Y_6}$ is **Calabi-Yau manifold**
- here: study manifolds with $SU(3)$-structure, i.e. $\nabla\eta \; \neq \mathbf{0}$

Manifolds with $SU(3)$ structure: [Gray,Hervella,Salamon,Chiossi,Hitchin,...]

characterized by two tensors $\mathbf{J}, \mathbf{\Omega}$ (follows from existence of η)

➪ $(1,1)$-form

$$\mathbf{J_{mn}} = \eta^\dagger \gamma_{[\mathbf{m}}\gamma_{\mathbf{n}]}\eta \ , \qquad \mathbf{dJ \neq 0}$$

➪ $(3,0)$-form

$$\mathbf{\Omega_{mnp}} = \eta^\dagger \gamma_{[\mathbf{m}}\gamma_{\mathbf{n}}\gamma_{\mathbf{p}]}\eta \ , \qquad \mathbf{d\Omega \neq 0}$$

Remarks:

- $\mathbf{dJ}, \mathbf{d\Omega}$ $\sim$ (intrinsic) torsion of Y_6
- **Calabi-Yau:** $\nabla\eta = \mathbf{0} \quad \Rightarrow \quad \mathbf{dJ = d\Omega = 0}$

  $\Rightarrow$ torsion parameterizes the deviation from Calabi-Yau

Manifolds with $SU(3) \times SU(3)$ structure:

[Jescheck,Witt; Grana,Minasian,Petrini,Tomasiello; Grana,Waldram,JL]

In type II string theory one can be slightly more general:

choose different spinors $\eta^{\mathbf{1}}, \eta^{\mathbf{2}}$ for the two gravitini $\Psi^{1,2}$

each η def. $SU(3)$-structure $\Rightarrow$ together: $\mathbf{SU(3)} \times \mathbf{SU(3)}$-structure

(characterized by pair $\mathbf{J^{1,2}}, \mathbf{\Omega^{1,2}}$)

Hitchin: embed in $\mathbf{SU(3)} \times \mathbf{SU(3)}$ in $\mathbf{O(6,6)}$ acting on $T \oplus T^*$

$\Rightarrow$ structure characterized by two pure spinors $\mathbf{\Phi}^+, \mathbf{\Phi}^-$ of $\mathbf{O(6,6)}$

$$\mathbf{\Phi}^+ = \mathbf{e^B} \eta^{\mathbf{1}}_+ \otimes \bar{\eta}^{\mathbf{2}}_+ \simeq \sum \mathbf{\Phi}^+_{\text{even}} \ , \qquad \mathbf{\Phi}^- = \mathbf{e^B} \eta^{\mathbf{1}}_+ \otimes \bar{\eta}^{\mathbf{2}}_- \simeq \sum \mathbf{\Phi}^+_{\text{odd}} \ ,$$

$\mathbf{SU(3)}$ structure ($\eta^{\mathbf{1}} = \eta^{\mathbf{2}}$): $\qquad \mathbf{\Phi}^+ = \mathbf{e^{B+iJ}} \ , \qquad \mathbf{\Phi}^- = \mathbf{e^B \Omega} \ ,$

Low energy effective action:

$$\mathcal{S} = \int_{\mathbf{M_4}} \tfrac{\mathbf{1}}{\mathbf{2}}\mathbf{R} - \mathbf{g_{ab}}(\mathbf{z})\,\mathbf{D}_\mu \mathbf{z^a} \mathbf{D}^\mu \mathbf{z^b} - \mathbf{V}(\mathbf{z}) + \ldots$$

➪ Type II string theory: $\mathcal{S}$ is $N = 2$ gauged supergravity

(before orientifolding)

➪ $\mathbf{z^a}$: coordinates of scalar manifold $\mathcal{M}$

- correspond to deformations of $\mathbf{B}, \mathbf{J}, \mathbf{\Omega}$ or $\mathbf{\Phi}^+, \mathbf{\Phi}^-$
- scalars from RR-sector

➪ $N = 2$ constraint: $\mathcal{M} = \mathcal{M}_{\mathrm{SK}} \times \mathcal{M}_{\mathrm{QK}}$

$$\text{type IIA}: \qquad \mathcal{M}_{\mathrm{SK}} = \mathcal{M}_{\mathbf{\Phi}^+}\ , \qquad \mathcal{M}_{\mathrm{QK}} \supset \mathcal{M}_{\mathbf{\Phi}^-}$$

$$\text{type IIB}: \qquad \mathcal{M}_{\mathrm{SK}} = \mathcal{M}_{\mathbf{\Phi}^-}\ , \qquad \mathcal{M}_{\mathrm{QK}} \supset \mathcal{M}_{\mathbf{\Phi}^+}$$

Metric $\mathbf{g_{ab}}$: special Kähler metric on $\mathcal{M} = \mathcal{M}_{\Phi^+} \times \mathcal{M}_{\Phi^-}$

[Hitchin,Graña,Gurrieri,Micu,Waldram,JL,...]

$$\mathbf{e^{-K_{\Phi^+}}} = \int_{\mathbf{Y}} \langle \mathbf{\Phi^+}, \overline{\mathbf{\Phi}}^+ \rangle = \mathbf{X^A \bar{F}_A - \bar{X}^A F_A}$$

$$= \int_{\mathbf{Y}} \mathbf{J \wedge J \wedge J}\,, \qquad \text{for} \qquad \mathbf{\Phi^+ = e^{B+iJ}}\,,$$

$$\mathbf{e^{-K_{\Phi^-}}} = \int_{\mathbf{Y}} \langle \mathbf{\Phi^-}, \overline{\mathbf{\Phi}}^- \rangle = \mathbf{Z^I} \bar{\mathcal{F}}_{\mathbf{I}} - \mathbf{\bar{Z}^I} \mathcal{F}_{\mathbf{I}}$$

$$= \int_{\mathbf{Y}} \mathbf{\Omega_3 \wedge \overline{\Omega}_3}\,, \qquad \text{for} \qquad \mathbf{\Phi^- = \Omega_3}$$

where $\langle \Phi^+, \overline{\Phi}^+ \rangle = \Phi_0^+ \wedge \overline{\Phi}_6^+ - \Phi_2^+ \wedge \overline{\Phi}_4^+ + \Phi_4^+ \wedge \overline{\Phi}_2^+ - \Phi_6^+ \wedge \overline{\Phi}_0^+$, etc.

and $F_A(X)$, $\mathcal{F}_I(Z)$ are $N = 2$ prepotentials

➪ $e^{-\mathbf{K}}$ is quartic invariant of $O(6,6)$ (Hitchin functional)

➪ for $SU(3)$ same expression as in Calabi-Yau compactifications

Potential: is derived from Killing prepotential (or superpotential) $\vec{P}$

➪ IIA

$$\mathbf{P^1} + \mathbf{iP^2} = e^{\frac{1}{2}\mathbf{K}_{\mathbf{\Phi}}^{-} + \phi^{(4)}} \int_{Y_6} \langle \mathbf{\Phi}^{+}, d\mathbf{\Phi}^{-} \rangle \,, \qquad \mathbf{P^3} = -e^{2\phi^{(4)}} \int_{Y_6} \langle \mathbf{\Phi}^{+}, F_{\mathsf{A}} \rangle$$

$$F \equiv \sum_{\text{RR-forms}} F^{\text{RR}}$$

➪ IIB

$$\mathbf{P^1} + \mathbf{iP^2} = e^{\frac{1}{2}\mathbf{K}_{\mathbf{\Phi}}^{+} + \phi^{(4)}} \int_{Y_6} \langle \mathbf{\Phi}^{-}, d\mathbf{\Phi}^{+} \rangle \,, \qquad \mathbf{P^3} = e^{2\phi^{(4)}} \int_{Y_6} \langle \mathbf{\Phi}^{-}, F_{\mathsf{B}} \rangle$$

Note:

- NS 3-form flux H included
- gauged $N = 2$ supergravity with charged fields in hypermultiplets
- as expected $\mathbf{P}$ depends on torsion and flux

'Formal aspects'

- Mirror symmetry for generalized geometries corresponds to

$$\mathbf{\Phi}^{+}(\mathbf{Y}) = \mathbf{\Phi}^{-}(\tilde{\mathbf{Y}}) \qquad \mathbf{\Phi}^{-}(\mathbf{Y}) = \mathbf{\Phi}^{+}(\tilde{\mathbf{Y}})$$

  manifest in the large volume (supergravity) limit.

- Non-perturbative dualities with flux/torsion

  Het. on K3 $\times$ $\mathbf{T^2}$ with flux $\quad\leftrightarrow\quad$ **IIA on $\mathbf{SU(3) \times SU(3)}$**

  can be embedded for **M-Theory on 7d-$SU(3)$-manifold**

  [Aharony,Berkooz,Micu,JL, see Micu's talk]

- non-geometric backgrounds [Hull; Shelton,Wecht,Taylor,. . .]
  can be included into $SU(3) \times SU(3)$-structure formalism

‘Formal aspects’

- Heterotic string compactified on $SU(3)$-structure manifolds

  [Becker,Becker,. . . ; Lüst, . . . ; Gurrieri,Lukas,Micu; Benmachiche,Martinez,JL]

- type II compactifications on $SU(2)$-structure manifolds

  $\Rightarrow$ gauged $N = 4$ supergravity in the same formalism

  [Spanjaard, Triendl, JL]

- heterotic compactifications on $SU(2)$-structure manifolds

  [Martinez,Micu,JL]

- RR-scalars can be incorporated into geometrical description

  $\Rightarrow$ E_7-covariant formulation of $N = 2$ backgrounds

  [Graña,Waldram,JL]

Orientifolds

[Benmachiche,Grimm,JL, Shelton,Taylor,Wecht, Micu,Palti,Tasinato, ...]

Orientifold projection: $N = 2 \longrightarrow N = 1$

- truncates spectrum
- chooses complex structure on field space and selects a Kähler subspace in $\mathcal{M}_{\mathrm{QK}}$
- repackages $\mathbf{P}$ into $\mathbf{W\&D}$
  - large volume/large complex structure limit:

$$\mathbf{W} \;\sim\; \kappa_{\mathbf{abc}}\phi^{\mathbf{a}}\phi^{\mathbf{b}}\phi^{\mathbf{c}}$$

IIA: W cubic in Kähler moduli, linear in c.s. moduli

IIB: W cubic in c.s. moduli, linear in Kähler moduli

  - W depends on both type of moduli

$N = 1$ de Sitter backgrounds [Covi,Gomez-Reino,Gross,Palma,Scrucca,JL]

motivation: relevant for **early** and **late** universe

problem: sGoldstino is potentially tachyonic

mass is entirely determined by Kähler geometry:

$$\frac{\mathbf{\Lambda}}{\mathbf{m^2_{3/2} M^2_{\mathrm{PL}}}} \gg \mathbf{1} \qquad \Rightarrow \qquad \mathbf{Sec} \leq 0$$

$$\frac{\mathbf{\Lambda}}{\mathbf{m^2_{3/2} M^2_{\mathrm{PL}}}} \to \mathbf{0} \qquad \Rightarrow \qquad \mathbf{Sec} \leq \frac{2}{3}$$

sectional curvature: $\mathbf{Sec} := |\mathbf{G}|^{-4}\, \mathbf{R_{i\bar{j}m\bar{n}} G^i G^{\bar{j}} G^m G^{\bar{n}}}$

Summary

➪ discussed backgrounds with $SU(3) \times SU(3)$ structure

- NS scalar manifold is product of special geometries
  $\mathbf{K} = \mathbf{K}_{\mathbf{\Phi}^+} + \mathbf{K}_{\mathbf{\Phi}^-}$ is given by Hitchin functionals
  and independent of torsion
- (super)potential depends on torsion and background fluxes
- generalized mirror symmetry intact
- non-geometric backgrounds can be included into formalism

➪ **phenomenological questions:**

- **stabilization of moduli**
- **inclusion of D-branes**
- **supersymmetry breaking**
- **inflationary scenarios**