Torsion induced soft-breaking terms

Pablo G. Camara

CPHT, Ecole Polytechnique (Paris)

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Motivation

- One of the most interesting adventures of String Theory is understanding its mechanisms for SUSY-breaking.

- The capability of String Theory to make 4d phenomenological axioms is limited by several issues:

  - The "landscape problem" $\Longrightarrow$ which background??

  - We do not know how to compute string amplitudes in a generic background $\Longrightarrow$ work in the supergravity limit

  - We do not know how to perform dimensional reduction in a generic manifold (non Calabi-Yau)

Motivation

- A particularly interesting setup are GKP compactifications:

1.- The deformation of the moduli space due to the 3-form flux is effectively described in 4d by the GVW superpotential:

$$W = \int_{CY} (F_3 - ie^{-\phi} H_3) \wedge \Omega$$

No scale structure
$$\begin{cases} \sum_{\tilde{i}} K^{\tilde{i}\bar{\tilde{j}}} \partial_{\tilde{i}} K \partial_{\bar{\tilde{j}}} K = 3 \\ K^{\tilde{i}\bar{j}} = 0 \\ \partial_{\tilde{i}} W = 0 \end{cases} \implies V = e^K \sum_{i,j \neq \tilde{i}} K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W}$$

2.- In addition, 3-form fluxes induce μ-terms in the worldvolume of D7-branes:

$$W_{D7_i} \sim (F_{\bar{j}\bar{k}i} + ie^{-\phi} H_{\bar{j}\bar{k}i}) \phi^i \phi^i \qquad i \neq j \neq k$$

3.- Add other effects in the effective theory (non-perturbative effects, loop corrections, anti-branes...)

Phenomenological models (KKLT, LVS,...)

Outline

- Basics of generalized geometry

- No-scale orientifold compactifications with torsion

- Torsion-induced μ-terms

- Moduli dominated soft-terms

- Open-string wavefunctions

- Conclusions

SU(3)-structure

We consider orientifold compact. on SU(3)-structure manifolds:

$$J \wedge \Omega = 0 \ , \qquad J \wedge J \wedge J = -\frac{3i}{4} \, \Omega \wedge \bar{\Omega}$$

- J defines a symplectic structure J_{mn}
- Ω defines a complex structure $I^m{}_n$

$$\Longleftrightarrow \qquad \begin{cases} g_{mn} = J_{mp} \, I^p{}_n \\ \text{global SU(3)-invariant spinor } \eta \end{cases}$$

In general non-integrable structures: **torsion classes**

$$dJ = \frac{3}{2}\mathrm{Im}(\mathcal{W}_1\overline{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \ ,$$

$$d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega \ ,$$

Generalized geometry

- Generalized complex structure:

$$\mathcal{J}: \qquad T_{\mathcal{M}_6 \oplus \mathcal{M}_6^*} \ \to \ T_{\mathcal{M}_6 \oplus \mathcal{M}_6^*} \ , \qquad \mathcal{J}^2 = -1$$

$$\mathcal{J}_- = \begin{pmatrix} I & 0 \\ 0 & -I^T \end{pmatrix} \ , \qquad \mathcal{J}_+ = \begin{pmatrix} 0 & J^{-1} \\ -J & 0 \end{pmatrix}$$

Isomorphism: $\{\mathcal{J}\} \simeq \{O(6,6) \text{ pure spinors}\} \simeq \{(\text{poly-})\text{forms}\}$

$$\Phi_- = e^{i\theta_-} \, \Omega \ , \qquad \Phi_+ = e^{i\theta_+} e^{-iJ}$$

- NSNS 3-form: $\qquad d_H \Phi \equiv e^B d(e^{-B} \Phi)$

- Mukai product: $\qquad \langle A, B \rangle = (-1)^{[(n+1)/2]} A_n \wedge B_{6-n}$

Effective theory

[Benmachiche,Grimm]

$$K = -\log\left[-i\int\langle\Phi_1,\bar{\Phi}_1\rangle\right] - 2\log\left[-i\int\langle\Phi_2,\bar{\Phi}_2\rangle\right] - 2\log\left(e^{-2\phi}\right)$$

$$W = \int\langle\Phi_1, d_H\,\Pi\rangle$$

	Φ_1	Φ_2
IIA	Φ_+	Φ_-
IIB	Φ_-	Φ_+

with $\quad\Pi = \mathcal{C} + i\,e^{-\phi}\,\mathrm{Re}\,\Phi_2$

- In order to perform the integrals we need the internal profile of the fields. That is equivalent to solve the generalized Laplace equation for this background.

[work in progress...]

- For the time being, we expand as if we were in a Calabi-Yau and take the lowest mode to be constant

⤷ good for small warping

No-scale vacua with torsion

- We take the following strategy:

$$W = \int \langle \Phi_1, d_H \Pi \rangle$$

1.- Decompose $d_H \Pi$ in SU(3) representations

$$d_H \Pi = G^+ + G^- \quad , \quad *_6 \lambda[G^\pm] = \pm i G^\pm \ ,$$

IIB: $\quad G^+ = \dfrac{3}{2} G^+_{(1)} \overline{\Omega} + G^+_{(3)} \wedge J + G^+_{(6)}$

IIA: $\quad G^+ = G^+_{(1)} e^{iJ} + G^+_{mn} \gamma^m e^{-iJ} \gamma^n + G^+_m \gamma^m \, \bar{\Omega} + \tilde{G}^+_m \Omega \gamma^m$

and similar expressions for $G^- \dots$

2.- Compute the F-terms

3.- Translate the no-scale condition into conditions on the SU(3) reps.

4.- Complete the solutions with warping

No-scale vacua with torsion

2 types of no-scale solutions

- $d_H\Pi$ is an ISD poly-form
  (e.g. GKP)

  $\Longrightarrow$ breaking mediated (at tree-level)
  by hypermultiplets

  [also Lawrence et al.]

- $d_H\Pi$ is a real poly-form
  (only NSNS-flux)

  $\Longrightarrow$ breaking mediated (at tree-level) by vector
  multiplets (1/3) and hypermultiplets (2/3)

- Necessary conditions, but not sufficient $\Longrightarrow$ **constraints on the manifold**

- Example 1. IIB-O9/O5, $d_H \Pi$ ISD

- $\mathcal{M}_6$ a fibration of a complex 2-cycle Σ_2 over a 4d base B

$$J = J_B + J_{\Sigma_2} \ , \quad dJ_{\Sigma_2} \neq 0 \ , \quad dJ_B \wedge \Omega = 0$$

- No-scale condition $\implies G^- = 0 \implies$
$$\begin{cases} \mathcal{W}_1 = e^\phi F_{(1)} \ , \\ \mathcal{W}_2 = 2\mathcal{W}_1(J_B - 2J_{\Sigma_2}) \ , \\ \mathcal{W}_3 = -e^\phi *_6 F_{(6)} \ , \\ \mathcal{W}_5^* = \dfrac{i}{2}e^\phi F_{(3)} = -\bar{\partial}A = -\dfrac{1}{2}\bar{\partial}\phi \end{cases}$$

- E.g. compactification on nilmanifold (0, 0, 45 - 15 - 42, 0, 0, 0) and

$$g_s F_3 = -\frac{t}{8u^3}[(1-3iu)\Omega+(1-iu)(z^1\wedge z^2\wedge \bar{z}^3+z^1\wedge \bar{z}^2\wedge z^3+\bar{z}^1\wedge z^2\wedge z^3)]+e^{2A}*_4 d(e^{-4A})+c.c$$

- Example 2. IIA-O6, $d_H \Pi$ real

 - $\mathcal{M}_6$ a trivial fibration of a complex 2-cycle Σ_2 over a 4d base B

$$J = J_{\mathcal{B}} + J_{\Sigma_2} \ , \quad dJ_{\Sigma_2} = 0 \ , \quad K_{U_k} = -\log(U + U^*) + K'_{U_{\tilde{k}}} \ , \quad K'^{\tilde{k}\bar{\tilde{p}}} K'_{\tilde{k}} K'_{\bar{\tilde{p}}} = 2$$

- No-scale condition $\quad \Longrightarrow \quad$
$$\begin{cases} G^+ \ + \ G^- \ \text{real} \\[2ex] G^\pm_{mn}\big|_{\mathcal{B}} = 0 \ , \quad \displaystyle\int J_{\mathcal{B}} \wedge d(\alpha_0 + \frac{\text{Re } U}{\text{Re } S}\beta_U) = 0 \end{cases}$$

- E.g. compactification on algebraic solvmanifold $(0, 64, 45, 0, 34, 42)$

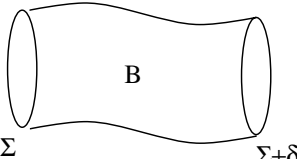- In type IIB these are always **non-geometric compactifications**

Torsion induced μ-terms

- Dp-branes wrapping (generalized) calibrated cycles: [Koerber,Martucci]

- D5-branes wrapping complex 2-cycles
- D6-branes wrapping special lagrangian 3-cycles

F-term condition $\implies$ $W = \int_{\mathcal{B}} P_{\mathcal{B}}[e^{3A-\phi}\Phi_1] \wedge e^{\tilde{\mathcal{F}}}$

"easy" for lowest mode in adjoints

Torsion induces μ-terms in the worldvolume. SUSY-breaking in the bulk gets manifest as soft-breaking terms, open-string moduli fixing.

- Twisted tori are a good laboratory:

$$de^a = \frac{1}{2} f^a_{bc} e^b \wedge e^c \ , \quad f^a_{[bc} f^g_{d]a} = 0$$

Torsion induced μ-terms

	D9	D5$_1$	D5$_2$	D5$_3$
$u_1\mu_{11}$	$\tilde{f}^1_{2\bar{3}}$	0	$\tilde{f}^3_{1\bar{2}}$	$\tilde{f}^2_{\bar{3}1}$
$u_2\mu_{22}$	$\tilde{f}^2_{\bar{3}1}$	$\tilde{f}^3_{\bar{1}2}$	0	$\tilde{f}^1_{2\bar{3}}$
$u_3\mu_{33}$	$\tilde{f}^3_{1\bar{2}}$	$\tilde{f}^2_{3\bar{1}}$	$\tilde{f}^1_{\bar{2}3}$	0

	D6$_0$	D6$_1$
$t_1\mu_{11}$	$\frac{1}{2u_1}(T_2 f^{\hat{2}}_{\hat{1}3} - T_3 f^{\hat{3}}_{\hat{1}2} - 2T_1 f^1_{23})$	$\frac{1}{2s}(T_2 f^2_{\hat{3}\hat{1}} + T_3 f^3_{\hat{1}\hat{2}} - T_1 f^1_{\hat{2}\hat{3}})$
$t_2\mu_{22}$	$\frac{1}{2u_2}(-T_1 f^1_{\hat{2}3} + T_3 f^3_{\hat{2}1} + 2T_2 f^2_{31})$	$\frac{-1}{2u_3}(2T_3 f^3_{12} + T_1 f^1_{2\hat{3}} + T_2 f^2_{1\hat{3}})$
$t_3\mu_{33}$	$\frac{1}{2u_3}(T_1 f^{\hat{1}}_{\hat{3}2} - T_2 f^{\hat{2}}_{\hat{3}1} + 2T_3 f^3_{12})$	$\frac{1}{2u_2}(2T_2 f^2_{13} + T_1 f^1_{3\hat{2}} + T_3 f^3_{1\hat{2}})$

	D6$_2$	D6$_3$
$t_1\mu_{11}$	$\frac{1}{2u_3}(2T_3 f^3_{21} + T_1 f^1_{2\hat{3}} + T_2 f^2_{1\hat{3}})$	$\frac{-1}{2u_2}(2T_2 f^2_{31} + T_3 f^3_{1\hat{2}} + T_1 f^1_{3\hat{2}})$
$t_2\mu_{22}$	$\frac{1}{2s}(T_1 f^1_{2\hat{3}} + T_2 f^2_{1\hat{3}} + T_3 f^3_{1\hat{2}})$	$\frac{1}{2u_1}(2T_1 f^1_{32} + T_3 f^3_{2\hat{1}} + T_2 f^2_{3\hat{1}})$
$t_3\mu_{33}$	$\frac{-1}{2u_1}(2T_1 f^1_{23} + T_2 f^2_{3\hat{1}} + T_3 f^3_{2\hat{1}})$	$\frac{1}{2s}(T_1 f^1_{2\hat{3}} - T_2 f^2_{1\hat{3}} - T_3 f^3_{1\hat{2}})$

- Most of the adjoints *a priori* can be lifted by the torsion!

- Good matching with the T-dual result in vacua where a T-dual description is available.

- Non-holomorphic terms are due to terms in the Kahler potential contributing through the Giudice-Masiero mechanism

Moduli dominated soft-terms

We can compute the induced soft-terms for pure moduli mediation:

$$K(M, \bar{M}, \phi, \bar{\phi}) = \hat{K}(M, \bar{M}) + Z_{i\bar{j}}(M, \bar{M})\, \phi^i \bar{\phi}^{\bar{j}} + \frac{1}{2}\left(H_{ij}(M, \bar{M})\, \phi^i \phi^j + \text{h.c.}\right) + \dots \, ,$$

$$W(M, \phi) = W(M) + \frac{1}{2}\, \tilde{\mu}_{ij}(M)\, \phi^i \phi^j + \frac{1}{6}\, \tilde{Y}_{ijk}(M)\, \phi^i \phi^j \phi^k + \dots \, .$$

$$V = e^K \left(\sum_{i,j} K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3|W|^2 \right)$$

	$D9$	$D5_1$	$D5_2$	$D5_3$
μ_{11}	0	0	$4e^{\hat{K}/2}\tilde{f}^3_{1\bar{2}}t_3$	0
μ_{22}	0	$4e^{\hat{K}/2}\tilde{f}^3_{\bar{1}2}t_3$	0	0
μ_{33}	$4e^{\hat{K}/2}\tilde{f}^3_{1\bar{2}}t_3$	0	0	0
$m^2_{1\bar{1}}$	0	0	$|\mu_{33}|^2 + |m_{3/2}|^2$	0
$m^2_{2\bar{2}}$	0	$|\mu_{22}|^2 + |m_{3/2}|^2$	0	0
$m^2_{3\bar{3}}$	$|\mu_{11}|^2 + |m_{3/2}|^2$	0	0	0
B_{11}	0	0	$2\mu_{33}\bar{m}_{3/2}$	0
B_{22}	0	$2\mu_{22}\bar{m}_{3/2}$	0	0
B_{33}	$2\mu_{11}\bar{m}_{3/2}$	0	0	0
A_{123}	$g_{D9}m_{3/2}$	$g_{D5_1}m_{3/2}$	$g_{D5_2}m_{3/2}$	0
$C_{1\bar{2}\bar{3}}$	0	0	$\mu_{33}\, g_{D5_2}$	0
$C_{\bar{1}2\bar{3}}$	0	$\mu_{22}\, g_{D5_1}$	0	0
$C_{\bar{1}\bar{2}3}$	$\mu_{11}\, g_{D9}$	0	0	0

Table 3: Torsion induced soft parameters for $D9$, $D5_1$, $D5_2$ and $D5_3$-branes, in a no-scale vacuum of a factorizable twisted torus with W independent of S, T_1, T_2, and $D_M W = 0$ for the remaining moduli. The gauge coupling constants are $g_{D9} = (S + \bar{S})^{-1/2}$ and $g_{D5_k} = (T^k + \bar{T}^k)^{-1/2}$, and we have set $M_{Pl} = 1$.

Moduli dominated soft-terms

	$D6_0$	$D6_1$	$D6_2$	$D6_3$
μ_{11}	$2e^{\frac{\hat{K}}{2}}(t_2 f_{\hat{1}3}^{\hat{2}} - t_3 f_{\hat{1}2}^{\hat{3}})$	$2e^{\frac{\hat{K}}{2}}(t_2 f_{\hat{3}\hat{1}}^{2} + t_3 f_{\hat{1}\hat{2}}^{3})$	0	0
μ_{22}	0	0	$2e^{\frac{\hat{K}}{2}}(t_2 f_{\hat{1}3}^{2} + t_3 f_{\hat{1}\hat{2}}^{3})$	$2e^{\frac{\hat{K}}{2}}(t_2 f_{\hat{3}\hat{1}}^{\hat{2}} + t_3 f_{\hat{2}\hat{1}}^{\hat{3}})$
μ_{33}	0	0	$2e^{\frac{\hat{K}}{2}}(t_2 f_{\hat{1}3}^{\hat{2}} + t_3 f_{\hat{1}2}^{\hat{3}})$	$2e^{\frac{\hat{K}}{2}}(t_2 f_{\hat{3}\hat{1}}^{2} - t_3 f_{\hat{1}\hat{2}}^{3})$
$m^2_{1\bar{1}}$	$|\mu_{11}|^2$	$|\mu_{11}|^2$	$-|m_{3/2}|^2$	$-|m_{3/2}|^2$
$m^2_{2\bar{2}}$	0	0	$|\mu_{22}|^2 + |m_{3/2}|^2$	$|\mu_{22}|^2 + |m_{3/2}|^2$
$m^2_{3\bar{3}}$	0	0	$|\mu_{33}|^2 + |m_{3/2}|^2$	$|\mu_{33}|^2 + |m_{3/2}|^2$
B_{11}	0	0	0	0
B_{22}	0	0	$2\mu_{22}\,\bar{m}_{3/2}$	$2\mu_{22}\,\bar{m}_{3/2}$
B_{33}	0	0	$2\mu_{33}\,\bar{m}_{3/2}$	$2\mu_{33}\,\bar{m}_{3/2}$
A_{123}	0	0	$g_{D6_2}\,m_{3/2}$	$g_{D6_3}\,m_{3/2}$
$C_{1\bar{2}\bar{3}}$	$\mu_{11}\,g_{D6_0}$	$\mu_{11}\,g_{D6_1}$	0	0
$C_{\bar{1}2\bar{3}}$	0	0	$\mu_{22}\,g_{D6_2}$	$\mu_{22}\,g_{D6_3}$
$C_{\bar{1}\bar{2}3}$	0	0	$\mu_{33}\,g_{D6_2}$	$\mu_{33}\,g_{D6_3}$

Table 4: Torsion induced soft parameters for $D6_M$-branes, in a no-scale vacuum of a factorizable twisted torus with W independent of T_1, U_2, U_3. The gauge coupling constants are $g_{D6_0} = (S + \bar{S})^{-1/2}$ and $g_{D6_k} = (U^k + \bar{U}^k)^{-1/2}$, and we have set $M_{Pl} = 1$.

Open-string wavefunctions

Consider e.g. D9-branes. The 4d spectrum of fermionic open-string deformations is given by solving the generalized internal Dirac operator. This can be obtained from the 10d gaugino action,

$$\displaystyle{\not{D}}_6\psi \equiv \gamma^a(\hat{\partial}_a + \tfrac{1}{4}\omega_a^{bc}\gamma_{bc} + A_a)\psi + \frac{e^{\phi/2}}{24}\gamma^{abc}F_{abc}\psi = \lambda\gamma_{(7)}\psi$$

The flux enter in 3 ways

- Mass-gap
- Twisted derivatives $\quad [\hat{\partial}_a, \hat{\partial}_b] = -f_{ab}^c\hat{\partial}_c$
- Warping

The lowest uncharged mode is indeed constant (up to warping):

$$\hat{\partial}_a(\text{const.}) = 0 \;\Rightarrow\; \lambda = \text{mass gap} = e^{K/2}\;\mu\text{-term}$$

We can address the problem with the tools of non-commutative harmonic analysis.
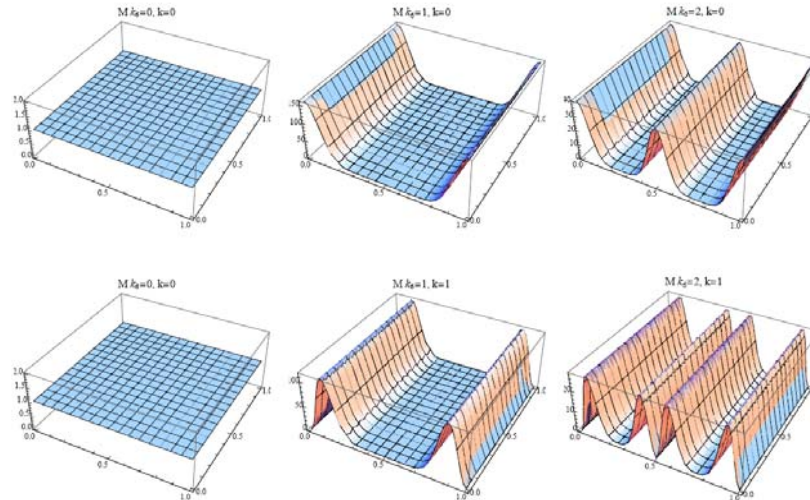
Open-string wavefunctions

E.g. consider compactification in $\underbrace{\text{(5d Heisenberg manifold)}}_{\text{fibration } S^1 \text{ over } T^4} \times S^1 + \text{RR flux}$

Two types of modes: same than in a T^6

- not excited along the fiber $\lambda^2 = \dfrac{4\pi^2}{t^1\tau_1^1}[k_4^2 + k_1^2(\tau_1^1)^2] + \dfrac{4\pi^2}{t^2\tau_1^2}[k_5^2 + k_2^2(\tau_1^2)^2] + \dfrac{4\pi^2\tau_1^3}{t^3}k_3^2$

- excited along the fiber $\lambda^2 = \dfrac{4\pi M|k_6|}{(t_1 t_2)^{1/2}}(n+1) + \dfrac{4\pi^2}{t^3\tau_1^3}[k_6^2 + k_3^2(\tau_1^3)^2]$

Conclusions

- Generalized geometry provide us with a good framework for exploring no-scale solutions in general SU(3)-structure compactifications. These could be used as 'starting point' of phenomenological scenarios.

- Two types of no-scale vacua, characterized by an ISD poly-form (e.g GKP, breaking by hypers) or by a real poly-form (no RR flux, breaking by mixture of hypers and vectors).

- We computed the torsion-induced effective μ-terms for the adjoints inside D5, D6 and D9-branes, for twisted tori. The pattern results very rich, a priori allowing for the lifting of many of the adjoints.

- Uncharged light modes have a constant internal profile, turning the dimensional reduction easy. For other modes, one has to solve the internal generalized Dirac and Laplace operators.