Torsional heterotic geometries

Katrin Becker

Texas A&M University

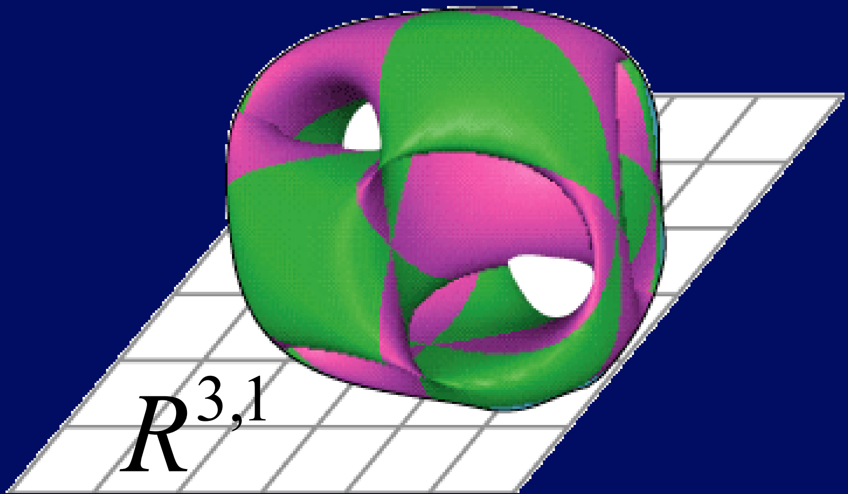
Based on work w/ S. Sethi, to appear.

In our attempts to construct models of particle phenomenology using string theory fluxes play a prominent role. Fluxes

1) stabilize moduli fields of the compact dimensions
2) generate warp factors in the space-time metric
3) break susy without inducing a c.c.,
4) etc etc

Flux backgrounds have mostly been studied in the context of compactifications of type II string theory. In this lecture we will be discussing the construction of flux backgrounds for heterotic strings...

The number of flux backgrounds for heterotic strings is of the same order of magnitude than the number of flux backgrounds in type II theories...



Type IIB:

g_{ij} : Calabi-Yau 3-fold,

H_{NS}, F_{RR}

Heterotic strings:

g_{ij} complex but non-kaehler

H, F_{ij}

Gauge symmetry breaking

$$E_8 \supset SU(4)\times \boxed{SO(10)}$$

$$E_8 \supset SU(5)\times \boxed{SU(5)}$$

GUT

$$E_8 \supset SU(5)\times U(1)\times \boxed{SU(3)\times SU(2)\times U(1)}$$

Warp factors appear in the Einstein frame metric.
Warping is given in terms of the dilaton....

World-sheet description in terms of NS fields only....

Not many solutions are known

Dasgupta, Rajesh, Sethi, 9908088
M. Becker, L-S. Tseng, Yau, 0807.0827
Fernandez, Ivanov, Ugarte, Villacampa 0804.1648

Method of Construction

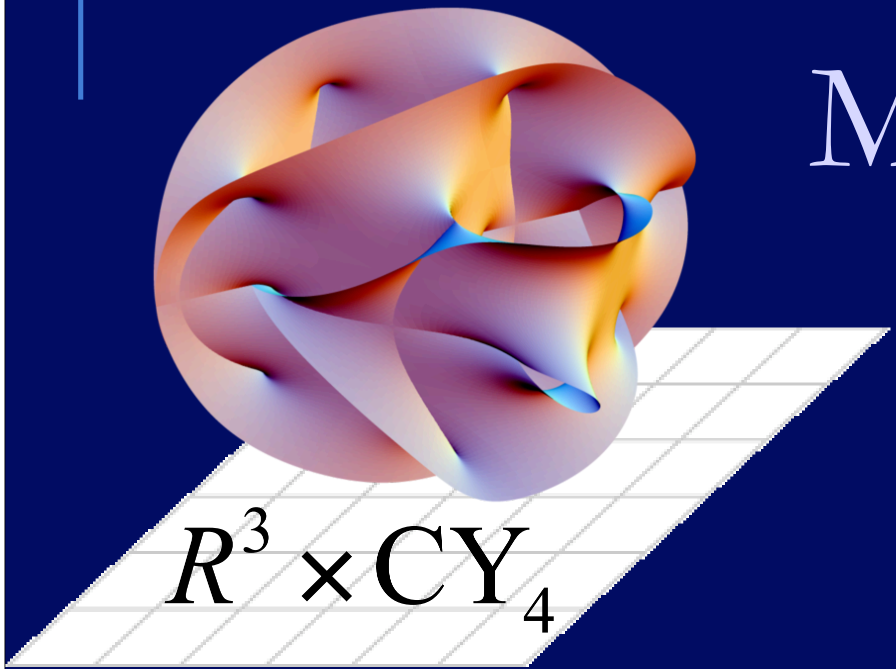
M-theory on a K3 fibered CY4 with flux

String dualities

Heterotic string on torsional space...

A. Sen, ``F-theory and Orientifolds'', hep-th/9605150, ``Orientifold limit of F-theory vacua'', hep-th/9702165.

M-theory



$$\boxed{J \wedge G_{(2,2)} = 0}$$

Kaehler
form of CY4

ds

In the presence of flux the space-time metric is no longer a direct product

$$ds^2 = \Delta(y)^{-1} ds^2_{R^3} + \Delta(y)^{1/2} ds^2_{CY_4}$$

warp factor

coordinates
of the CY4

The warp factor satisfies the differential equation

$$d * d\left(\Delta^{3/2}\right) = -\frac{1}{2} G \wedge G + 4\pi^2 X_8$$

8-form which is quartic in curvature

This is a Laplace eqn with a source and a solution will exist if

Euler ch. of CY4

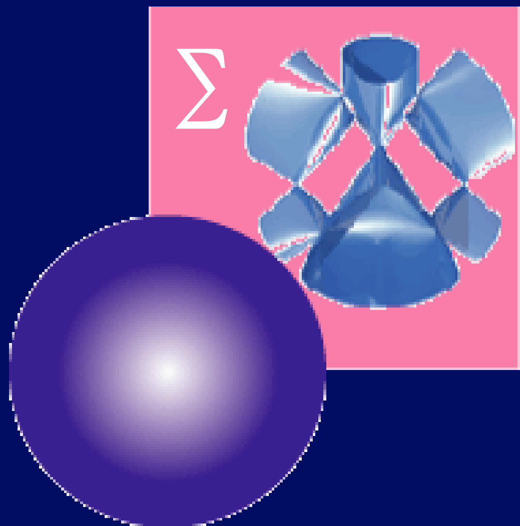
$$\frac{\chi}{24} = \frac{1}{8\pi^2} \int G \wedge G + n$$

number of M2 branes

The data specifying this background are the choice of CY4 and the primitive flux.

Additional restrictions:

1) the CY4 is elliptically fibred so that we can lift the 3d M-theory background to 4d. By shrinking the volume of the fiber to zero this can be converted in a 4d compactification of type IIb.

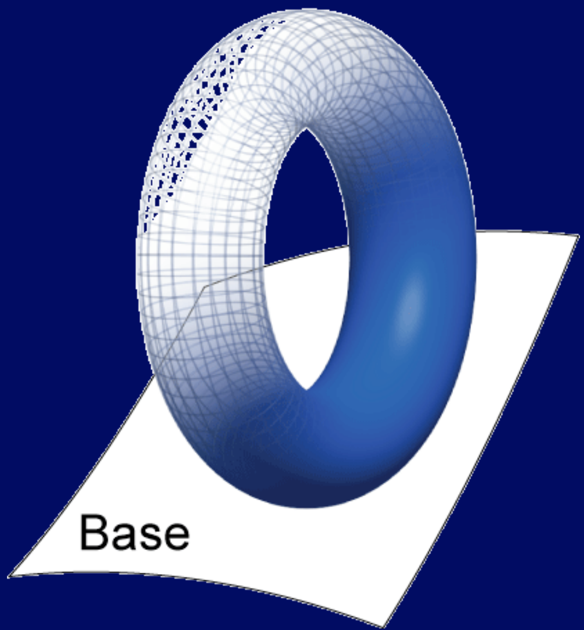


2) the CY4 admits a K3 fibration.
The heterotic string then appears by wrapping the M5 brane on the K3 fibers.
If flux is included the target space of the heterotic string is a torsional space...

3) the CY4 has a non-singular orientifold locus so that the elliptic fibration is locally constant.

The orientifold locus

Type IIB on an elliptic CY3 with constant coupling



$$y^2 = x^3 + f(\vec{u})x + g(\vec{u})$$

base coordinates

Symmetry

I: y $\longrightarrow$ -y

$$\boxed{Z_2 = (-1)^{F_L}\,\Omega\, I}$$

R $\longrightarrow$ 1/R

$$\boxed{Z_2' = \Omega}$$

The metric for an elliptic CY space

In the semi-flat approximation (stringy cosmic string):

$$\boxed{ds^2_{CY} = \underbrace{g_{ij}dy^i dy^j}_{4d\ \ base} + \frac{1}{\tau_2(y)}\,|\,dw_1 - \tau(y)dw_2\,|^2}$$

$$\boxed{\begin{aligned}&\bar{\partial}_{\bar{i}}\tau(y) = 0\\ &R = \partial\bar{\partial}(\log\det g_{i\bar{j}} - \log\tau_2) = 0\end{aligned}}$$

$$I : w_i \longrightarrow -w_i$$

Away from the singular fibers there is a $U(1)\times U(1)$ isometry...

Including flux

In type IIB we can include the fluxes

$$\underbrace{H_3, F_3}_{f_3}, F_5, \tau_B$$

constant

1) Fluxes satisfy the conditions for unbroken susy in type IIB. So for example

$$G_3 = H_3 - \tau_B F_3 \qquad\qquad *G_3 = iG_3$$

2) H_3, F_3 are odd under $(-1)^{F_L}\Omega$. We can decompose differential forms under the action of the ¢$_2$ transformation which inverts the fiber and require

$$f_3 = H^3(M, Z)_-$$

In the presence of flux the type IIB background becomes

$$ds^2 = \Delta^{-1}\eta_{\mu\nu}\,dx^\mu dx^\nu + \Delta ds^2_{CY_3}$$

warp factor

$$ds^2_{CY_3} = g_{ij}dy^i dy^j + \frac{1}{\tau_2}|dw_1 - \tau\, dw_2|^2$$

$$H_3, F_3, F_5, \tau_B$$

We will use this data to make an ansatz for the metric of the torsional heterotic geometry...

Decompose $H_3 = (h_2)_i \wedge \omega^i$

integral harmonic 1-forms of the fiber

Under an $SL(2,Z)$ transformation

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}$$

the 2-form $H_2 = (h_2)_1 - \tau (h_2)_2$ transforms as

$$H_2 \longrightarrow \frac{H_2}{c\tau + d}$$

Locally we can write $H_2 = dA_H$ use this connection to twist the torus fiber of the CY space

The metric of the torsional space is

$$\boxed{ds^2 = \Delta^2(y) g_{ij} dy^i dy^j + \frac{1}{\tau_2(y)} | dw_2 + \tau(y) dw_1 + A_H |^2}$$

The conditions for unbroken susy are a repackaging of the susy conditions on the type IIB side. In particular any elliptic CY3 space gives rise to a torsional space..

$$\tau = \tau(y)$$

We can check some of these backgrounds can be obtained from string duality...

Torsional solutions via duality

M-theory on elliptic CY4, with $G_{(2,2)}$ → IIB on CY3 orientifold with flux → Type I on torsional space → Heterotic string on torsional space

$$\boxed{ds^2_{het} = \Delta(y)^2 g_{ij} dy^i dy^j + \frac{1}{\tau_2} \left| dw_2 + \tau\, dw_1 + \Theta \right|^2}$$

warp factor of type IIB solution

base $F_i,\ i = 0, 1, 2, 4.$

$$\Theta = (B^{NS}_{w_2 y_i} + \tau\, B^{NS}_{w_1 y_i}) dy^i$$

The Bianchi identity

The equation determining the warp factor in type IIB arises from

$$\underbrace{d * F_5 = H_3 \wedge F_3}_{dH} + \rho$$

$$\int_{D7} C_4 \wedge \mathrm{tr}(R \wedge R)$$

T-duality maps

$$*F_5 \longrightarrow *_T F_7 = H,$$

$$F_3 \longrightarrow H,$$

$$H_3 \longrightarrow \partial g$$

type I flux

...giving rise to the type I/het Bianchi identity

$$\boxed{dH = \mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F}$$

Using $F_5 = (1 + *) d\Delta^{-2} \wedge dx^{0123}$ the type IIB Bianchi identity becomes the differential equation for the warp factor

$$d * d\Delta^{-2} = H_3 \wedge F_3 + \delta^{(2)}\ tr\ R \wedge R$$

which we can solve in the large radius expansion, which means for a large base and a large fiber. So the existence of a solution is guaranteed.

In the large radius limit on the het side, which corresponds to large base and small fiber in type IIB, this is a non-linear differential equation for the dilaton (of Monge-Ampere type if the base is K3).

When the fiber is twisted the topology of the space changes.
A caricature is a 3-torus with NS-flux

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2$$
$$B_{NS} = N x_1 dx_2 \wedge dx_3$$
$$x_i \sim x_i + 1, \ i = 1, 2, 3$$

$$ds^2 = dx_1^2 + dx_2^2 + (dx_3 - N x_1 dx_2)^2$$
$$x_i \sim x_i + 1, \ i = 2, 3$$
$$x_1 \sim x_1 + 1, \ x_3 \sim x_3 + N x_2$$
$$\omega = d(dx_3 - N x_1 dx_2) = -N dx_1 \wedge dx_2$$

Using the same type of argument one can show that
the twisting changes the betti numbers of the torsional space
compared to the CY3 we started with. Topology change is
the `mirror' statement to moduli stabilization in type IIB.
Some fields are lifted from the low-energy effective
action once flux is included....

Take the oldest example of a torsional background (i.e. a K3 base)

$$ds^2 = e^{2\phi} ds^2_{K3} + |\, dz + \alpha \,|^2$$

$$\omega = d(dz + \alpha)$$

$$J_{K3} \wedge \omega = 0$$

On this space the gauge fields can be of the form

$$F_{U(1)} = p\omega_1 + q\omega_2$$

$$\omega = \omega_1 + i\omega_2$$

tensored with any stable bundle on K3. In particular one can choose an SU(5) bundle on K3 times the U(1) gauge field breaking E8 to the standard model. Higgsing the hypercharge is avoided because of the topology change.

Generalized solutions

A further generalization is possible

$$\boxed{ds^2_{het} = \rho_2(y)\left[\Delta(y)^2 g_{ij} dy^i dy^j + \frac{1}{\tau_2(y)} | dw_2 + \tau(y) dw_1 + \Theta |^2 \right]}$$

$$\bar{\partial}_{\bar{i}} \rho(y) = \bar{\partial}_{\bar{i}} \tau(y) = 0$$

Here $\rho = \rho_1 + i\rho_2$ and ρ_1 is the B field in the two fiber directions. The low energy effective action is invariant under

$$SL(2,Z)_\tau \times SL(2,Z)_\rho$$

The End