

D-brane Instanton Effects in 4D String Vacua

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R.B., M. Schmidt-Sommerfeld (arXiv:0803.1562) , R.B., S. Moster, E. Plauschinn (arXiv:0806.2667)

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(Previous work on world-sheet instantons in Type II and heterotic string theory and for M-brane instantons)

(Dine, Seiberg, Wen, Witten I+II),(Distler, Greene), (Witten), (Becker², Strominger), (Harvey, Moore), (Beasley, Witten), (Green, Gutperle), (Antoniadis, Gava, Narain, Taylor),...

Zero modes

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D-brane instanton effects on $\mathcal{N} = 1$ 4D action

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- Zero mode structure and possible lifting
 - Generic 4 **bosonic** zero modes X_μ and 4 **fermionic** zero modes θ^α and $\bar{\theta}^{\dot{\alpha}}$
 - Due to deformations, complex bosonic zero modes Y_i and fermionic zero modes μ_i^α and $\bar{\mu}_i^{\dot{\alpha}}$
 - For **$E3$ instantons** in Type IIB, these zero modes are counted by $H^{(i,0)}(D)$ $i = 0, 1, 2$.

F-terms

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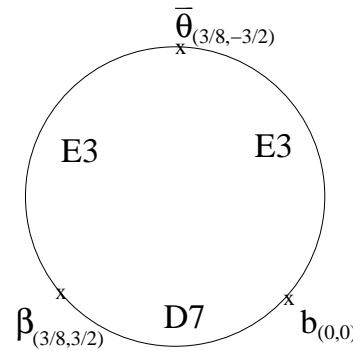
F-terms possible only if

- The two $\bar{\theta}^{\dot{\alpha}}$ zero modes are projected out by $\Omega\sigma$. For this the instanton must be invariant under σ and must be an $O(1)$ instanton (instead of $SP(2)$ or $U(1)$) (Argurio, Bertolini, Ferreti, Lerda, Petersson) , (Ibanez, Schellekens, Uranga) , (Bianchi, Fucito, Morales)

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- The two $\bar{\theta}^{\dot{\alpha}}$ zero modes can be absorbed elsewhere, like for instantons on top of D-brane:



→ fermionic ADHM-constraints (Billo et al., hep-th/0211250) ,
(Petersson)

Charged matter zero modes

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Gauge group

$$\prod_a U(N_a) = \prod_a SU(N_a) \times U(1)_a$$

in general contains **anomalous** $U(1)_a$ symmetries

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Anomaly cancellation via the **4D Green-Schwarz mechanism**

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- Only specific linear combinations of $U(1)$ s are **massless** and remain as unbroken gauge symmetry (like $U(1)_Y$)
- Global $U(1)$ **forbid** some desirable matter **couplings**, e.g. Majorana type **neutrino masses**, $SU(5)$ Yukawa couplings or μ -terms → relation to M-theory on G_2 manifolds(?)

Instanton zero modes

Instanton zero modes

Instanton corrections in string theory can break the axionic shift symmetries and therefore the global $U(1)$ symmetries.

- Additional zero modes charged under $U(1)_a$: Strings between $E3$ and $D7_a$ have DN-boundary conditions in 4D and mixed boundary conditions along $CY_3 \rightarrow$ 1/2 complex fermionic zero mode λ_a ([Ganor, hep-th/9612077](#))

| zero modes | Reps. | number |
|------------------------|---------------------------|-------------------------------------|
| $\lambda_{a,I}$ | $(-1_E, \square_a)$ | $I = 1, \dots, [\Xi \cap \Pi_a]^+$ |
| $\bar{\lambda}_{a,I}$ | $(1_E, \bar{\square}_a)$ | $I = 1, \dots, [\Xi \cap \Pi_a]^-$ |
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Instanton calculus

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D-brane-instantons are described by open strings →
computation of stringy instanton correlation functions should
be possible in **(boundary) conformal field theory**. (Gutperle, Green,
[hep-th/9701093](#)), (Billo et al., [hep-th/0211250](#))

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To compute (rigid) instanton contributions to the charged matter field superpotential

$$W_{np} \simeq \prod_{i=1}^M \Phi_{a_i, b_i} e^{-S_{E2}}.$$

with $\Phi_{a_i, b_i} = \phi_{a_i, b_i} + \theta\psi_{a_i, b_i}$ denoting chiral matter superfields at the intersection of Π_{a_i} with Π_{b_i} (suppress Chan-Paton labels for simplicity).

Instanton calculus: Summary

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Probe superpotential by [correlator](#)

$$\langle \Phi_{a_1,b_1} \cdot \dots \cdot \Phi_{a_M,b_M} \rangle_{E2\text{-inst}} = \frac{e^{\frac{\kappa}{2}} Y_{\Phi_{a_1,b_1}, \dots, \Phi_{a_M,b_M}}}{\sqrt{K_{a_1,b_1} \cdot \dots \cdot K_{a_M,b_M}}}$$

$$\begin{aligned} & \langle \Phi_{a_1,b_1}(x_1) \cdot \dots \cdot \Phi_{a_M,b_M}(x_M) \rangle_{E2\text{-inst}} = \\ &= \int d^4x d^2\theta \sum_{\text{conf.}} \Pi_a \left(\prod_{i=1}^{[\Xi \cap \Pi_a]^+} d\lambda_a^i \right) \left(\prod_{i=1}^{[\Xi \cap \Pi_a]^-} d\bar{\lambda}_a^i \right) \\ & \exp(-S_{E2}) \times \exp(Z'_0) \\ & \times \langle \widehat{\Phi}_{a_1,b_1}[\vec{x}_1] \rangle_{\lambda_{a_1}, \bar{\lambda}_{b_1}}^{\text{tree}} \cdot \dots \cdot \langle \widehat{\Phi}_{a_L,b_L}[\vec{x}_L] \rangle_{\lambda_{a_L}, \bar{\lambda}_{b_L}}^{\text{tree}} \times \\ & \prod_k \langle \widehat{\Phi}_{c_k,c_k}[\vec{x}_k] \rangle_{A(E2,D6_{c_k})}^{\text{loop}} \end{aligned}$$

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- Zero mode structure and possible lifting
- CFT instanton calculus (BI, Cvetic, Weigand), (Ibanez, Uranga)
- D-brane instanton corrections to superpotential W
 - Contributions to the moduli field $W \rightarrow$ moduli stabilization (R.B., Moster, Plauschinn)
 - Generation of perturbatively forbidden but phenomenologically desirable matter couplings like Majorana masses for neutrinos, Yukawa couplings for $SU(5)$ models.
 - Computations for global models like toroidal or Gepner model orientifolds (Cvetic, Weigand), (Bianchi, Kirsits), (Ibanez, Schellekens, Uranga)

Achievements

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- D-brane instanton corrections to gauge kinetic function f
 - By S-duality to het. string → test of the D-brane instanton calculus (Bianchi, Morales) , (Camara, Dudas, Maillard, Pradisi)
 - New phenomena from multi-instantons → second part of this talk (R.B., Schmidt-Sommerfeld) , (Grimm) , (R.B., Moster, Plauschinn)

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- Stringy derivation of field theory instanton effects
 - Computation of the AdS superpotential for $\mathcal{N} = 1$ SQCD in a local brane set-up (Akerblom, R.B., Lüst, Plauschinn, Schmidt-Sommerfeld)
 - Generalization for non-compact quiver gauge theories (Florea, Kachru, McGreevy, Saulina), (Argurio, Bertolini, Ferretti, Lerda, Petersson) , (Ibanez, Uranga) , (Krefl) , (Bianchi, Fucito, Morales), ...

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- Instantons and fluxes (R.B., Cvetic, Richter, Weigand) , (Garcia-Etxebarria, Marchesano, Uranga) , (Billo, Ferro, Frau, Fucito, Lerda, Morales)
- Phenomenological applications like neutrino masses or supersymmetry breaking (Antusch, Ibanez, Macri) , (Cvetic, Langacker) , (Buican, Franco)

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- Questions

A puzzle about D-instantons

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The exact **superpotential** is expected to have an expansion like (Dine, Seiberg, Wen, Witten)

$$W = W_{\text{tree}} + \sum_{E1-\text{inst.}} \prod_i \Phi_i \ g(\mathcal{U}_I) \ e^{-a^I \mathcal{T}_I}$$

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Similarly, the **gauge coupling** on a D-brane A beyond one-loop receives also instanton corrections from a D-instanton B

$$f_A = f_{A,\text{tree}} + f_{A,1\text{-loop}} + f_{A,\text{NP}}$$

with $f_{A,\text{NP}} \sim \exp(-S_B)$.

Consider the 4D low-energy effective field theory on this D-brane.

A puzzle about D-instanton

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Assume that an **ADS-type superpotential** is dynamically generated by a gauge instanton, Inst_A i.e.

$$\begin{aligned} W_{ADS} &= \frac{1}{\det \Phi \bar{\Phi}} \exp(-f_{A,\text{full}}) \\ &= \frac{1}{\det \Phi \bar{\Phi}} \exp(-f_{A,\text{tree}} - f_{A,\text{1-loop}} - e^{-S_B}). \end{aligned}$$

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By expanding the exponential, it is expected to be a **multi-instanton** correction involving one gauge instanton Inst_A and instantons Inst_B .

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Let us discuss this issue more systematically.

Instanton corrections to f

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For a single instanton to generate a correction to f , the **zero mode structure** must be of a certain type. (related to: (Argurio, Bertolini, Ferretti, Lerda, Petersson), (Bianchi, Fucito, Morales), (Ibanez, Schellekens, Uranga),(Akerblom, Bl, Lüst, Schmidt-Sommerfeld))

It must be an $O(1)$ instanton wrapping a holomorphic curve of genus one (i.e. a 2-torus):

| | |
|-------------------|------------------------|
| x_μ , | position in 4D |
| θ^α , | 2 Goldstino zero modes |
| μ^α , | 2 modulino zero modes |

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Introducing a **graphical** notation, the instanton correction to f can be computed by evaluating the **holomorphic** part of

$$f_a^{E1_r} = \int d^2\theta_r d^2\mu_r \begin{array}{c} \times \diagup \times \diagdown \times \\ \diagup \times \diagdown \times \end{array}_{D9_a \ E1_r} e^{-S_{E1_r}} \exp \left(- \sum_b \begin{array}{c} \bigcirc \bigcirc \\ E1_r \ D9_b \end{array} - \begin{array}{c} \bigcirc \bigcirc \\ E1_r \ O9 \end{array} \right)$$

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Instanton corrections to f

- What builds up in the exponent is the **gauge coupling** on some **fictitious $D5_r$ branes** wrapping the same curve as $E1_r$.
- The gauge coupling in $D5_r$ can by itself receive **instanton correction** from instantons $E1_s$ wrapping a different curve
- By **including** these corrections, one obtains

$$f_a = \int d^2\theta_r d^2\mu_r \text{diag} \left(\begin{array}{c} \text{---} \\ D9_a \\ \text{---} \\ E1_r \end{array} \right) \exp \left(-S_{E1_r} - Z'_0(E1_r) - \sum_s \int d^4x_{rs} d^2\theta_s d^2\mu_s \text{diag} \left(\begin{array}{c} \text{---} \\ E1_r \\ \text{---} \\ E1_s \end{array} \right) e^{-S_{E1_s} - Z'_0(E1_s)} \dots \right),$$

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For a single $E1_s$ instanton, expanding the exponential gives

$$f_a = \int d^2\theta_r d^2\mu_r \begin{array}{c} \times \\ \text{---} \\ \text{---} \\ \times \end{array} \begin{array}{c} \times \\ \text{---} \\ \text{---} \\ \times \end{array} e^{-S_{E1r}} e^{-Z'_0(E1r)} \times$$

$$\left[\sum_{n=0}^{\infty} \int d^{4n}x_{rs} d^{2n}\theta_s d^{2n}\mu_s \frac{(-1)^n}{n!} \left(\begin{array}{c} \times \\ \text{---} \\ \text{---} \\ \times \end{array} \right)^n e^{-nS_{E1s}} e^{-nZ'_0(E1s)} \right]$$

$$= \int d^2\theta_r d^2\mu_r \begin{array}{c} \times \\ \text{---} \\ \text{---} \\ \times \end{array} \begin{array}{c} \times \\ \text{---} \\ \text{---} \\ \times \end{array} e^{-S_{E1r}} e^{-Z'_0(E1r)} +$$

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$$e^{-Z'_0(E1r)-Z'_0(E1s)} e^{-S_{E1r}-S_{E1s}} +$$

revealing the multi-instanton nature of these iterated instanton corrections (see also: (Bl, Cvetiv, Richter, Weigand), (Garcia-Etxebarria, Marchesano, Uranga), (Cvetic, Richter, Weigand))

Heterotic - Type I S-dual pair

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Consider the Ω orientifold of Type IIB on the shift $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold (Camara, Dudas, Maillard, Pradisi)

$$\Theta : \begin{cases} z_1 \rightarrow -z_1 \\ z_2 \rightarrow -z_2 + \frac{1}{2} \\ z_3 \rightarrow z_3 + \frac{1}{2} \end{cases} \quad \Theta' : \begin{cases} z_1 \rightarrow z_1 + \frac{1}{2} \\ z_2 \rightarrow -z_2 \\ z_3 \rightarrow -z_3 + \frac{1}{2} \end{cases} \quad \Theta'' : \begin{cases} z_1 \rightarrow -z_1 + \frac{1}{2} \\ z_2 \rightarrow z_2 + \frac{1}{2} \\ z_3 \rightarrow -z_3 . \end{cases}$$

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- One gets only one *O9 plane*, whose tadpole can be canceled by 32 *D9*-branes yielding the gauge group *SO(32)* with no massless matter.

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- One gets only one *O9 plane*, whose tadpole can be canceled by 32 *D9*-branes yielding the gauge group *SO(32)* with no massless matter.
- On the dual *heterotic* side the shift symmetry act in an *asymmetric* way

$$X_L \rightarrow X_L + \frac{\pi R}{2} + \frac{\pi \alpha'}{2R}, \quad X_R \rightarrow X_R + \frac{\pi R}{2} - \frac{\pi \alpha'}{2R},$$

Heterotic thresholds

Heterotic thresholds

One can compute the one-loop (in g_s) threshold corrections

$$\begin{aligned}\Lambda &= \int_{\mathcal{F}} \frac{d^2\tau}{4\tau_2} \mathcal{B}(\tau) \\ &= \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \sum_{i=1}^3 \left(\frac{1}{\eta^2 \vartheta_2^2} \hat{Z}_i \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \frac{1}{\eta^2 \vartheta_4^2} \hat{Z}_i \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \frac{i}{\eta^2 \vartheta_3^2} \hat{Z}_i \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \\ &\quad \sum_{a,b} \left(\frac{\vartheta_b^{[a]}}{\eta} \right)^{16} \left(\frac{\vartheta_b^{[a]''}}{\vartheta_b^{[a]}} + \frac{\pi}{\tau_2} \right)\end{aligned}$$

with $(a, b) \in \{(0, 0), (0, 1/2), (1/2, 0)\}$ originating from the $\widehat{SO}(32)_1$ left-moving current algebra.

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The questions we would like to answer are:

- Can we **quantitatively** reproduce the **holomorphic** part of the heterotic result on the **Type I** side in terms of **E_1 -brane instanton** corrections?

Heterotic thresholds

The questions we would like to answer are:

- Can we quantitatively reproduce the holomorphic part of the heterotic result on the Type I side in terms of *E1-brane instanton* corrections?
- Are there poly-instanton contributions on the Type I dual side and, if so, are they also included in the heterotic dual?

Heterotic thresholds

Heterotic thresholds

Following (Dixon, Kaplunovsky, Louis), by unfolding the integral over \mathcal{F} one gets (see also: (Bachas, Fabre, Kiritsis, Obers, Vanhove))

$$\Lambda(\vec{\mathcal{U}}, \vec{\mathcal{T}}) = \sum_{i=1}^3 \Lambda(\mathcal{U}_i) \frac{1}{\det(A)} e^{2\pi i \det(A) \mathcal{T}^{(i)}}.$$

For the holomorphic prefactor $\Lambda(\mathcal{U})$ we obtain

$$\begin{aligned} \Lambda(\mathcal{U}) &= \mathcal{A} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(\frac{j + p\mathcal{U}}{k} \right) = \\ &\quad \frac{(-1)^k}{\eta^2 \vartheta_2^2} \sum_{a,b} \left(\frac{\vartheta_b^{[a]}}{\eta} \right)^{16} \left(\frac{\vartheta_b^{[a]\prime\prime}}{\vartheta_b^{[a]}} \right) \left(\frac{j + p\mathcal{U}}{k} \right) \end{aligned}$$

for $k \in \mathbb{Z}$ and $j \in \mathbb{Z} + \frac{1}{2}$, and similarly for $\mathcal{A} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\mathcal{A} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

One-instanton sector

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Leading order instantons: $\det(A) = 1/2$ and only one orbit

$$A = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{so that}$$

One-instanton sector

Leading order instantons: $\det(A) = 1/2$ and only one orbit

$$A = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{so that}$$

$$\begin{aligned} \Lambda_1(\mathcal{U}, \mathcal{T}) &= 2\mathcal{A} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (2\mathcal{U}) e^{\pi i \mathcal{T}} \\ &= \frac{2\pi^2}{3} \left[\frac{e^{\pi i \mathcal{T}}}{\eta^4(\mathcal{U})} \left(\frac{\vartheta_3}{\eta}(2\mathcal{U}) \right)^{16} (E_2 + \vartheta_2^4 - \vartheta_4^4) (2\mathcal{U}) + \right. \\ &\quad \frac{e^{\pi i \mathcal{T}}}{\eta^4(\mathcal{U})} \left(\frac{\vartheta_4}{\eta}(2\mathcal{U}) \right)^{16} (E_2 - \vartheta_2^4 - \vartheta_3^4) (2\mathcal{U}) + \\ &\quad \left. \frac{e^{\pi i \mathcal{T}}}{\eta^4(\mathcal{U})} \left(\frac{\vartheta_2}{\eta}(2\mathcal{U}) \right)^{16} (E_2 + \vartheta_3^4 + \vartheta_4^4) (2\mathcal{U}) \right] \end{aligned}$$

Type I E1-instantons

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For each T^2 , we find **three** candidate $O(1)$ instantons with discrete Wilson lines

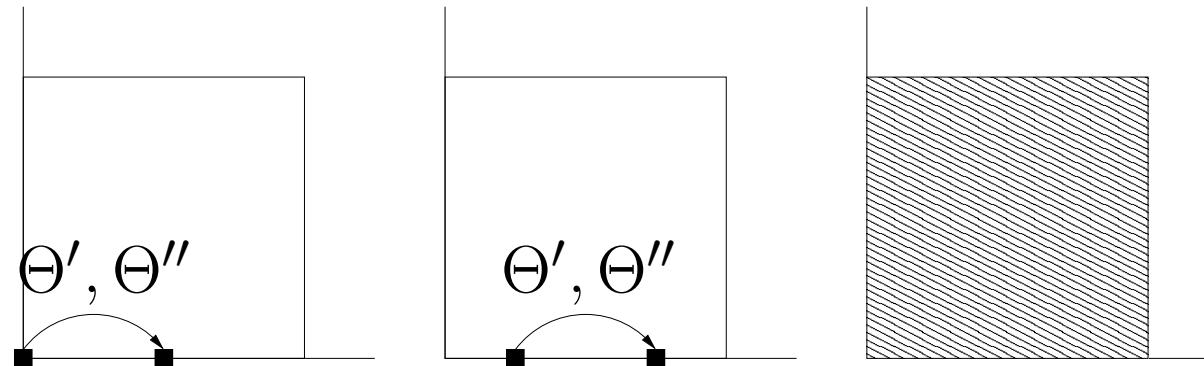
$$E1_{(0,\frac{1}{2})}^{(i)}, \quad E1_{(1,0)}^{(i)}, \quad E1_{(1,\frac{1}{2})}^{(i)} \quad i = 1, 2, 3 .$$

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To have precisely the two modulino zero modes μ^α , i.e. they must be rigid along the other two T^2 factors transverse to the instantons.



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$$\text{Diagram} = -16 \log \left(\frac{\vartheta_j}{\eta} (2U) \right)$$


where the prefactor of 16 originates from the 32 *D9-branes*.

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Multiple wrapped instantons

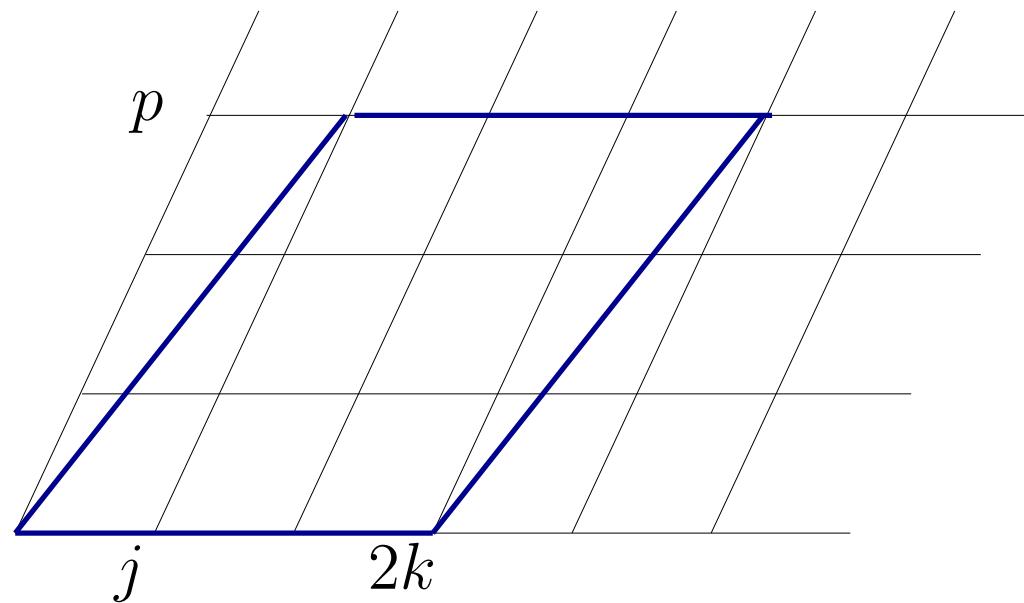
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For $k \in \mathbb{Z} + \frac{1}{2}$ and $j, p \in \mathbb{Z}$ they correspond to the instanton



Two-instanton sector

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However, what about the various Type I [poly two-instanton](#) contributions

$$E1_3^{i,k} - E1_4^{i',k'},$$

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For the first one, we get

$$\begin{aligned} \Lambda_{34} &= \int d^4x_{rs} d^2\theta_r d^2\theta_s d^2\mu_r d^2\mu_s \\ &\left(\begin{array}{c} \text{Diagram: Two horizontal lines, each with two circles. The left line has } D9 \text{ at bottom, } E1_3 \text{ at top-left, } E1_3 \text{ at top-right, } E1_4 \text{ at bottom. The right line has } E1_4 \text{ at top-left, } E1_3 \text{ at top-right, } E1_4 \text{ at bottom. Circles have 'x' marks.} \\ D9 \quad E1_3 \quad E1_3 \quad E1_4 + D9 \quad E1_4 \quad E1_4 \quad E1_3 \end{array} \right) \\ &\exp \left(- \begin{array}{c} \text{Diagram: Two horizontal lines, each with two circles. The left line has } E1_3 \text{ at bottom, } D9 \text{ at top-left, } D9 \text{ at top-right. The right line has } O9 \text{ at top-left, } O9 \text{ at top-right. Circles have 'x' marks.} \\ E1_3 \quad D9 - E1_3 \quad O9 \end{array} \right) \exp \left(- \begin{array}{c} \text{Diagram: Two horizontal lines, each with two circles. The left line has } E1_4 \text{ at bottom, } D9 \text{ at top-left, } D9 \text{ at top-right. The right line has } O9 \text{ at top-left, } O9 \text{ at top-right. Circles have 'x' marks.} \\ E1_4 \quad D9 - E1_4 \quad O9 \end{array} \right) e^{2\pi i \mathcal{T}} \end{aligned}$$

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Collecting all terms, we find

$$\Lambda_{34} = \frac{\pi^4 \kappa}{3} e^{2\pi i \mathcal{T}} \frac{\vartheta_4^4(2\mathcal{U})}{\eta^8(\mathcal{U})} \left(\frac{\vartheta_3 \vartheta_4}{\eta^2}(2\mathcal{U}) \right)^{16} (2E_2 - \vartheta_3^4 - \vartheta_4^4)(2\mathcal{U}) ,$$

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Is this poly-instanton included in the heterotic thresholds?

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Similarly, we find the poly instanton $E1_2^{i,k} - E1_4^{i',k'}$ in the heterotic amplitude and the sector $E1_2^{i,k} - E1_3^{i',k'}$ is **vanishing** due to extra zero modes.

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To settle this issue, consider the poly three-instanton sector.

Three-instanton sector

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For the three instanton amplitude we eventually find

$$\begin{aligned}\Lambda_{234} &= -\kappa \pi^4 \frac{\vartheta_4^8(2\mathcal{U})}{\eta^{12}(\mathcal{U})} \left(\frac{\vartheta_2 \vartheta_3 \vartheta_4}{\eta^3}(2\mathcal{U}) \right)^{16} \sum_{r=2,3,4} \frac{\vartheta_r''}{\vartheta_r}(2\mathcal{U}) e^{\frac{3\pi i \mathcal{T}}{2}} \\ &= \kappa \frac{\pi^6}{2^{18}} \frac{E_2}{\vartheta_2^2 \vartheta_3^2}(2\mathcal{U}) e^{\frac{3\pi i \mathcal{T}}{2}},\end{aligned}$$

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This term is **not present** on the heterotic side!

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- What effects do these pure **stringy poly instanton** corrections might have?

Applications Moduli Stabilization

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(BI, Plauschinn, Moster)

Consider Type IIB orientifolds of Calabi-Yau manifolds with O_7 and O_3 planes. Generalize the KKLT resp. LARGE volume scenario (Conlon, Quevedo, Cicoli):

$$\mathcal{K} = -2 \ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) - \ln\left(S + \overline{S}\right) + \mathcal{K}_{\text{CS}}$$

with $\hat{\xi} = \xi/g_s^{3/2}$ and

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Stabilize complex structure and dilaton with G_3 form flux with $W = 0$. Not massively suppressed (DeWolfe, Giryavets, Kachru, Taylor).

Moduli Stabilization

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Consider the racetrack- multi-instanton superpotential:

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