String Dualities and Manifolds with SU(3) Structure

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Motivation

① Jan's talk: fluxes $\leftrightarrow$ manifolds with $SU(3)\big(\times SU(3)\big)$ structure

② Special role in T-duality/Mirror symmetry

③ What about non-perturbative dualities – Heterotic vs. Type IIA?

④ Fluxes on $K3 \longleftrightarrow$ Manifolds with $SU(3) \times SU(3)$ structure

[Curio, Klemm, Körs, Lüst]; [AM, Louis]

Motivation ...

⑤ Non-Abelian gauge symmetries – more natural in Heterotic/Type I

– need additional ingredients (branes, singularities) in type II

– twisted tori/Scherk-Schwarz compactifications

[Kalorer, Myers; Hull et al; Dall'Agata et al]

- is spontaneously broken
- too much susy

☞ This talk – use Scherk-Schwarz compactification in Heterotic – Type IIA duality

☞ Naturally involves M-theory

Plan of the talk

- Brief review of Heterotic/ $K3 \times T^2$ with T^2 fluxes
- 7d manifolds with $SU(3)$ structure
- M-theory compactification
- Duality to Heterotic
- Conclusions

Heterotic/$K3 \times T^2$ with T^2 fluxes

Gauge group in absence of fluxes $U(1)^{n_v+1}$

$$A^0 = g_{\mu 4}\ , \quad A^1 = g_{\mu 5}\ , \quad A^2 = B_{\mu 4}\ , \quad A^3 = B_{\mu 5}\ .$$

Gauge field fluxes on T^2

$$\int_{T^2} F^a = f^a\ ,$$

induce a non-Abelian structure

$$F^0 = dA^0\ , \quad F^1 = dA^1\ ,$$

$$F^2 = dA^2 + f^a A^a \wedge A^1\ , \quad F^3 = dA^3 - f^a A^a \wedge A^0\ , \qquad F^a = dA^a + f^a A^0 \wedge A^1\ ,$$

and gaugings in the vector multiplet sector.

Type IIA dual setup?

✘ Type IIA/CY$_3$ – Abelian vector multiplet sector.

✘ No known fluxes indude gaugings in the vector multiplet sector

Hint: look at the duality in 5d: Heterotic/$K3 \times S^1$ vs M-theory/CY$_3$

Heterotic T^2 fluxes $=$ monodromy of gauge field-scalars around $S^1 \to$ 4d

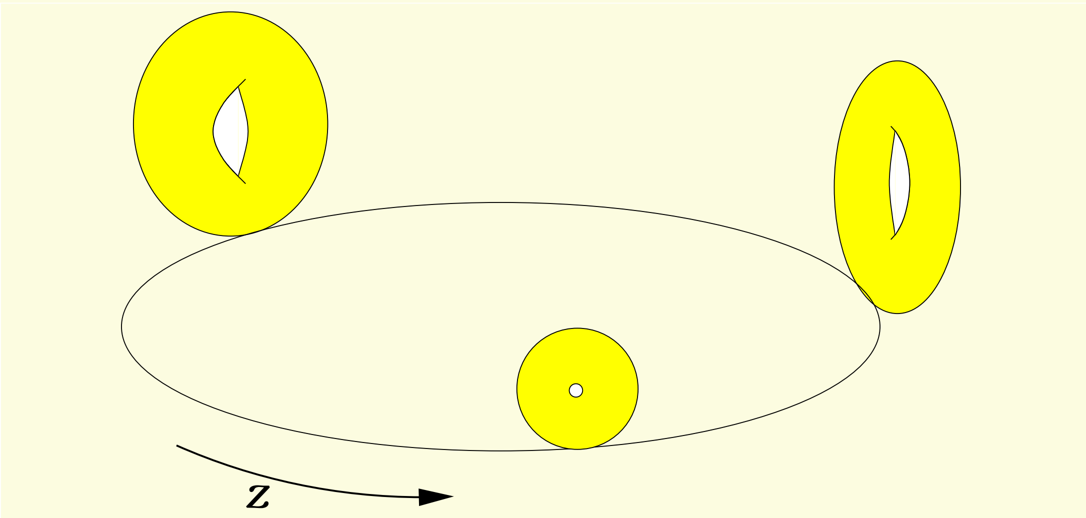
➠ do the same in M-theory

Isometry of the vector moduli space $SO(1,1) \times SO(1, n_v - 2)$

Perform Scherk–Schwarz compactification: twist the vectors multiplets by an element of the isometry group as we go around the circle

Can be done in one step? M-theory on ... 7d manifolds with $SU(3)$ structure.

7d manifolds with $SU(3)$ structure



Twist $H^2(CY_3)$ as we go around the circle $\omega_i \longrightarrow \gamma_i^j \omega_j$

Consistency condition: $\gamma_i^j \in$ U-duality group $\Gamma(\mathbf{Z}) = SO(1, n_v - 2, \mathbf{Z})$

7D manifolds with $SU(3)$ structure

Spread monodromy evenly over the circle

$$\omega_i(z+\epsilon) = \omega_i(z) + \epsilon M_i^j \omega_j(z)\ , \quad M\ -\ \text{constant}\ , \quad \gamma_i^j = (e^M)_i^j$$

ω_i - harmonic on CY_3 slices

$$\boxed{\mathbf{d\omega_i = M_i^j \omega_j \wedge dz}}$$

Consistency condition on the CY_3 intersection numbers $\mathcal{K}_{ijk}$

$$0 = \int_{X_7} d(\omega_i \wedge \omega_j \wedge \omega_k) \quad \Leftrightarrow \quad \mathcal{K}_{ijl} M_k^l + \mathcal{K}_{jkl} M_i^l + \mathcal{K}_{kil} M_j^l = 0$$

M-theory compactifications

$$\hat{C}_3 = C_3 + B \wedge (dz - A^0) + A^i \wedge \omega_i + b^i \omega_i \wedge (dz - A^0) + \ldots$$

Gauge transformations:

$$\begin{aligned} z &\rightarrow z + \epsilon \,, \quad \omega_i \rightarrow \omega_i + \epsilon M_i^j \omega_j \,, \quad A^0 \rightarrow A^0 + d\epsilon \,, \\ A^i &\rightarrow A^i - \epsilon M_j^i A^j \,, \quad b^i \rightarrow b^i - \epsilon M_j^i b^j \,, \quad v^i \rightarrow v^i - \epsilon M_j^i v^j \,, \end{aligned}$$

$\hat{C}_3 \rightarrow \hat{C}_3 + d(\eta^i \omega_i)$ - 11d gauge symmetry

$$A^0 \rightarrow A^0 \,, \quad A^i \rightarrow A^i + d\eta^i + M_j^i \eta^j A^0 \,, \quad v^i \rightarrow v^i \,, \quad b^i \rightarrow b^i + M_j^i \eta^j \,.$$

M-theory compactifications – 4d results

$N = 2$ supergravity + vector multiplets

$$S_V = \int_{M_4} \Big[\tfrac{1}{4}\mathrm{Im}\mathcal{N}_{IJ}F^I \wedge *F^J + \tfrac{1}{4}\mathrm{Re}\mathcal{N}_{IJ}F^I \wedge F^J - \tfrac{1}{6}M_i^l\mathcal{K}_{jkl}A^i \wedge A^j \wedge dA^k$$
$$-g_{ij}Dx^i \wedge *D\bar{x}^j - \tfrac{1}{\mathcal{K}}v^i v^j M_i^k M_j^l g_{kl}\Big] \,,$$

Non-vanishing structure constants:

$$\boxed{\mathbf{f_{i0}^j = -M_i^j}}$$

Gauged isometries:

$$\boxed{\mathbf{k_0^i = -x^k M_k^i}} \quad \text{and} \quad \boxed{\mathbf{k_i^j = M_i^j}}$$

Duality to Heterotic/$K3 \times T^2$

$CY_3 = K3$ fibered over a $\mathbf{P_1}$ base.

Limit Vol$(\mathbf{P_1}) \sim$ large ➞ non-vanishing intersection numbers

$$\mathcal{K}_{123} = -1 \ , \qquad \mathcal{K}_{1ab} = 2\delta_{ab} \ , \quad a, b = 4, \ldots, h^{1,1} = n_v \ .$$

Solve constraint $M^l_{(i}\mathcal{K}_{jk)l} = 0$:

$$M_2^2 = m_2 \ , \quad M_a^2 = m_a \ , \quad M_3^3 = m_3 \ , \quad M_a^3 = \tilde{m}_a \ , \quad M_a^b = -M_b^a = m_a^b \ ,$$

$$M_2^a = \tfrac{1}{2}\tilde{m}_a \ , \quad M_3^a = \tfrac{1}{2}m_a \ , \quad M_a^a = -\tfrac{1}{2}M_1^1 = \tfrac{1}{2}(m_2 + m_3) \ .$$

Duality to Heterotic/$K3 \times T^2$

Claim: only $\tilde{m}_a \neq 0$ reproduce the heterotic compactification with gauge field fluxes on T^2.

Result: Low energy action obtained from heterotic string compactifications on $K3 \times T^2$ with gauge field fluxes on T^2 is the same as M-theory compactifications on 7d manifolds with $SU(3)$ structure obtained from fibering ($K3$ fibered) CY_3 over a circle with only $\tilde{m}_a$ twisting parameters non-vanishing.

Obs1: Subtlety with electric-magnetic duality in 4d – crucial role played by generalized Chern-Simons term on the M-theory side

Obs2: More parameters on M-theory side eg m_a or m_a^b – seem to be related to $\tilde{m}_a$ by various T-dualities → expect to encounter them in various T-fold generalizations on the heterotic side.

Mass spectrum

Puzzling issue: Why M-theory?

IIA string coupling $=$ dilaton $\in$ hypermultiplet – does not appear in the potential
$\rightarrow$ the potential does not force the dilaton in a specific regime!

Twisting from 5d to 4d $\rightarrow$ masses of order M/R_z – need to be small

In type IIA: $R_{11} \sim R_z \gg 1$

Moreover, we want to ignore CY_3 KK states with mass of order $1/R_{CY}$

Hence: $\boxed{R_z \gg R_{CY} \Longrightarrow \text{M-theory regime}}$

Conclusions

✔ Heterotic – type IIA duality: naturally involves M-theory for T^2-fluxes

✔ 7d manifolds with $SU(3)$ structure – $CY_3 \times S^1$

✔ twist $H^2(CY_3)$ over S^1 – twist parameters $=$ heterotic T^2 fluxes

✔ non-Abelian structure in M-theory

✔ M-theory regime – natural due to the masses of the fields we decide to keep