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Multi-instanton and string loop corrections in orbifolds

- P. Camara E.D, e-Print: arXiv:0806.3102 [hep-th]
- (also P. Camara, E.D., T. Maillard, G. Pradisi, Nucl.Phys.B795:453-489,2008. e-Print: arXiv:0710.3080 [hep-th])

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1. Motivations

$SO(32)$ heterotic -type I duality (Polchinski,Witten) was explored extensively over the last ten years.

S-duality allows exact computation of E1 instanton effects in type I

- heterotic α' corrections $\to$ E1 instanton corrections
- NS5 effects $\to$ E5 instanton corrections

There are simple examples of dual pairs using Vafa-Witten adiabatic argument : freely-acting orbifolds (see Blumenhagen talk).

Ex. heterotic-type I duality: higher-derivative in $\mathcal{N} =$ 4 SUSY case (Bachas, Kiritsis and coll., Lerche and Stieberger, etc)

$\mathcal{N} = 2$ case discussed in (Antoniadis,Bachas,Fabre,Partouche and Taylor; Bianchi, Morales).

$\mathcal{N} = 1$ models : CDMP, Blumenhagen, Schmidt-Sommerfeld

We will mostly focus today on a standard Z_2 orbifold

- type I side constructed by Bianchi-Sagnotti and Gimon-Polchinski

- heterotic dual pair identified by Berkooz,Leigh,Polchinski,Schwarz, Seiberg,Witten

Our goal : computation of the full one-loop + E1 instanton corrections to the type I effective action .

An important consistency check : comparison of the effective theory Kaplunovsky-Louis formula

$$\frac{4\pi^2}{g^2(\mu)} = Ref_a + \frac{b_a}{4} \ln \frac{M_P^2}{\mu^2}$$
$$+\frac{c_a}{4} K + \frac{C_2(G)}{2} \ln \frac{4\pi^2}{g^2(\mu)} - \sum_r \frac{T_a(r)}{2} \ln det Z_r$$

where $T_a(r)$ is the Dynkin index for the matter representation r with wavefunctions Z_r, and

$$b_a = \sum_r T_a(r) - 3C_2(G) \ , \ c_a = \sum_r T_a(r) - C_2(G)$$

with the one-loop string computation

$$\frac{4\pi^2}{g_a^2(\mu)} = \frac{4\pi^2}{g_{a,0}^2} + \Lambda_a \quad , \text{where}$$

$$\Lambda_a = \frac{b_a}{4} \ln \frac{M_s^2}{\mu^2} + \frac{\Delta_a}{4}$$

Δ_a encodes string threshold corrections from massive string states.

2. The S-dual orbifold pairs

Simplest orbifold $T^4/Z_2 \times T^2$, sixteen fixed points.

Type I side

- One twisted hypermultiplet per fixed point
- Maximal gauge group $U(16)_9 \times U(16)_5$.

Hypers in $(\mathbf{120} + \overline{\mathbf{120}}, \mathbf{1}) + (\mathbf{1}, \mathbf{120} + \overline{\mathbf{120}}) + (\mathbf{16}, \mathbf{16})$

In order to have a perturbative heterotic dual, distribute 1/2 D5 brane per fixed point.

D5 gauge group broken to $U(1)^{16}$. Each $U(1)$ gets mixed with twisted four forms and become massive.

D9 spectrum : hypers in $\mathbf{120} + \overline{\mathbf{120}} + 16 \times \mathbf{16}$.

Heterotic dual

$SO(32)$ compactified to 4d on $T^4/Z_2 \times T^2$. Shift vector on the gauge lattice

$$V = \frac{1}{4}(1, \cdots 1, -3)$$

The gauge group is $U(16)$. Charged matter is in representations :

untwisted : $\mathbf{120} + \overline{\mathbf{120}}$

twisted : $16 \times \mathbf{16}$.

3. Effective action and quantum corrections : type I side

Threshold corrections to gauge couplings depend on moduli of T^2 :

$$T = \frac{\sqrt{G}}{\lambda} + ib \quad , \quad U = \frac{\sqrt{G} + iG_{12}}{G_{22}}$$

One finds (Bachas-Fabre, Antoniadis,Bachas,E.D.)

$$\frac{4\pi^2}{g_a^2(\mu)} \; = \; \frac{4\pi^2}{g_{a,0}^2} \; + \; \Lambda_a \quad \text{where}$$

$$\Lambda_a \; = \; -\frac{1}{4}\tilde{b} \; \ln[\sqrt{G}\mu^2 ReU|\eta(iU)|^4]$$

The effective action is then

$$f_{D9} \;=\; S \;+\; \tilde{b}\,\ln[ReU\eta^2(iU)] + f_{np}$$

$$K \;=\; -\ln(S+\bar{S}) - \ln(U+\bar{U}) - \ln(T+\bar{T}) + \cdots$$

$$-\frac{4\pi}{3}\frac{E(iU,2)}{(T+\bar{T})(S+\bar{S})} + K_{np}$$

where $E(U,k)$ is the Eisenstein series

$$E(U,k) \equiv \frac{1}{\zeta(2k)} \sum_{(j_1,j_2)\neq(0,0)} \frac{(\mathrm{Im}\ U)^k}{|j_1 + j_2 U|^{2k}}$$

Last term in K needs a separate one-loop computation (ABFPT ; Berg,Haack,Kors). We expect E1 instantonic contributions

$$f_{np} \;=\; \sum_{n=1}^{\infty} g_n(U)\, e^{-2\pi n T} \quad , \quad K_{np} \;=\; \sum_{n=1}^{\infty} h_n(U, T{+}\bar{T})\, e^{-2\pi n T} + \text{c.c.}$$

4. Effective action and quantum corrections : heterotic side

Threshold corrections to gauge couplings are given by (Kaplunovsky)

$$\Lambda_a = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2}\frac{i}{4\pi}\frac{1}{|\eta|^4}\sum_{\alpha,\beta=0}^{1/2}\partial_\tau\left(\frac{\vartheta\left[{\alpha \atop \beta}\right]}{\eta}\right)\left(Q_a^2-\frac{1}{4\pi\tau_2}\right)C\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\ ,$$

Q_a is the charge operator of the gauge group; $C\left[{\alpha \atop \beta}\right]$ is the internal six-dimensional partition function.

For the S-dual of the BSGP model we find

$$\Lambda = -\frac{1}{8}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\widehat{Z}_1\widehat{\mathcal{A}}_f$$

with

$$\widehat{\mathcal{A}}_f = -\frac{1}{20\eta^{24}}(D_{10}E_{10} - 48\eta^{24})$$

and

$$\widehat{Z}_1 = \frac{\text{Re } T}{\tau_2} \sum_{n_i,\ell_i} \exp\left[-2\pi T \det(A) - \frac{\pi(\text{Re } T)}{\tau_2(\text{Re } U)} \left| \begin{pmatrix} 1 & iU \end{pmatrix} A \begin{pmatrix} \tau \\ -1 \end{pmatrix} \right|^2 \right] ,$$

$$A = \begin{pmatrix} n_1 & \ell_1 \\ n_2 & \ell_2 \end{pmatrix} ,$$

with n_i and ℓ_i integers. We used methods of Dixon,Kaplunovsky, Louis and Bachas,Kiritsis et al. to evaluate Λ.

We find

$$\Lambda = \frac{\pi}{2}\text{Re }T_1 - 3\log[(\text{Re }U_1)(\text{Re }T_1)\mu^2|\eta(iU_1)|^4] - \frac{\pi}{3}\frac{E(iU_1,2)}{T_1+\bar{T}_1} -$$

$$-\frac{1}{4}\sum_{k>j\geq 0, p>0}\frac{1}{kp}e^{-2\pi pkT_1}\left[\hat{\mathcal{A}}_f(\mathcal{U}) + \frac{1}{\pi kp}\frac{\hat{\mathcal{A}}_K(\mathcal{U})}{T_1+\bar{T}_1}\right] + \text{c.c.}$$

$\hat{\mathcal{A}}_K$ is the almost-holomorphic modular form,

$$\hat{\mathcal{A}}_K = \frac{1}{12\eta^{24}}(\hat{E}_2E_4E_6 + 2E_6^2 + 3E_4^3)$$

We have expressed the result in terms of the induced worldvolume complex structure in the $E1$ multi-instantons wrapping the first 2-torus

$$\mathcal{U} = \frac{j+ipU_1}{k}$$

Split between analytic and non-analytic terms ; the corrected Kähler potential and gauge kinetic function are

$$K = -\log(S+\bar{S}) - \sum_{i=1}^{3} \log[(T_i+\bar{T}_i)(U_i+\bar{U}_i)] + \frac{1}{2}\frac{V_{1-loop}+V_{E1}}{S+\bar{S}},$$

$$V_{1-loop} = -\frac{4\pi}{3}\frac{E(iU_1,2)}{T_1+\bar{T}_1} \ ,$$

$$V_{E1} = -\sum_{k>j\geq 0,\ p>0} \frac{e^{-2\pi kpT_1}}{\pi(kp)^2}\left[\frac{\hat{\mathcal{A}}_K(\mathcal{U})}{T_1+\bar{T}_1} - \frac{\pi kp}{2Re\mathcal{U}}\frac{E_{10}(\mathcal{U})}{\eta^{24}(\mathcal{U})}\right] \ + \ c.c. \ ,$$

$$f_{U(16)} = S \ + \frac{\pi T_1}{2} \ - \ 12\log\ \eta(iU_1) - \frac{1}{2}\sum_{k>j\geq 0,\ p>0}\frac{e^{-2\pi kpT_1}}{kp}\mathcal{A}_f(\mathcal{U}) \ ,$$

where the holomorphic modular forms $\mathcal{A}_f$ and $\mathcal{A}_K$, are defined as before, replacing $\hat{E}_2$ by E_2.

5. E1 instantons

The instantonic corrections depend on the moduli of T^2 $\rightarrow$ come from E1 instantons wrapping T^2.

They are of two different types:

$E1$ instantons at orbifold fixed points

They have unitary Chan-Paton factors, $U(r)$, with neutral sector :

- bosonic zero modes x_μ, $y_{1,2}$ and fermionic zero modes $\Theta^{\alpha,a}$, $\Theta^{\dot{\alpha},a}$, with $a = 1, 2$ in the adjoint representation $\mathbf{r}\bar{\mathbf{r}}$.

- bosonic zero modes $y_{3,4,5,6}$ and fermionic ones $\lambda^{\alpha,a}$ in the symmetric representation $\frac{\mathbf{r}(\mathbf{r}+1)}{2} + \frac{\bar{\mathbf{r}}(\bar{\mathbf{r}}+1)}{2}$.

- fermionic zero modes $\tilde{\lambda}^{\dot{\alpha},a}$ in the antisymmetric representation $\frac{\mathbf{r}(\mathbf{r}-1)}{2} + \frac{\bar{\mathbf{r}}(\bar{\mathbf{r}}-1)}{2}$.

• Charged zero modes between the instanton and the 1/2 D5-brane stuck at the singularity:

-bosonic zero modes $\mu^{1,2}$ from the R sector and fermionic zero modes ω^{α} from the NS sector, in the representation $\mathbf{r}_{-1} + \bar{\mathbf{r}}_{1}$.

-bosonic zero modes $\mu'_{1,2}$ in the representation $\mathbf{r}_{+1} + \bar{\mathbf{r}}_{-1}$.

• E1-D9 strings : bosonic zero mode ν in the representation $\mathbf{r}\bar{\mathbf{n}} + \bar{\mathbf{r}}\mathbf{n}$.

$E1$ instantons off the orbifold fixed points

It can be argued that these instantons do not contribute to threshold corrections.

In order the instantons to contribute to the gauge kinetic function, only four fermionic neutral zero modes should be massless (the “goldstinos” ; Akerblom,Blumenhagen, Lust and Schmidt-Sommerfeld) $\rightarrow$ most of the above zero modes should be lifted by interactions.

Possible qualitative picture :

- a $U(1)$ instanton on top of a singularity correspond is a “gauge” instanton for the U(1) gauge theory inside the corresponding half D5-brane. These instantons are analogous to the ones discussed by Petersson, with the extra fermionic zero modes being lifted by couplings involving the D5-branes. They should be responsible for the 1-instanton ($N = 1$) contribution.

• For the N-instanton contribution, when all the instantons are on top of the same singularity, the instanton CP is enhanced to $U(N)$ and only four zero modes survive, with the extra zero modes presumably lifted by interactions with the D5-branes.

(Dedicated works on lifting of zero modes ; Blumenhagen,Cvetic,Richter,Weigand; Garcia-Etxebarria and Uranga)

5. Universality of $\mathcal{N}=2$ instantonic corrections

• We shall try to generalize the instantonic corrections to the effective action in case where the heterotic S-dual description is unknown.

Framework: orbifold compactifications, with orbifold action G containing subgroups G^i leaving unrotated a given complex plane.

The contribution of these sectors to threshold corrections :

$$\Lambda_a = -\frac{1}{8}\sum_i \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \widehat{Z}_i \widehat{\mathcal{A}}^a_{f,i} \ .$$

The gauge group is given by a product $G = \prod_a G_a$.

• T_i, U_i are moduli of the corresponding unrotated plane

• $\widehat{\mathcal{A}}^a_{f,i} \sim M^a_i/\eta^{24}$, with M^a_i an almost holomorphic modular form of degree 24.

By imposing the absence of tachyons in the spectrum, we obtain

$$\widehat{\mathcal{A}}^a_{f,i} = 2b^a_i + \frac{\gamma_i}{20\eta^{24}} \left[D_{10}E_{10} - 528\eta^{24} \right] ,$$

where b^a_i is the β-function coefficient of the $\mathcal{N} = 2$ gauge theory associated to a would-be $T^6/\mathbb{G}^i$ orbifold and γ_i is a model dependent (but gauge group independent) coefficient.

We get

$$\Lambda_a = \sum_i \Bigg\{ \frac{\pi(b_i^a + 6\gamma_i)}{12} \mathrm{Re}\ T_i \ + \ \frac{\pi\gamma_i}{3} \frac{E(iU_i, 2)}{T_i + \bar{T}_i} \ - $$
$$ - \frac{1}{4} \left(\sum_{k>j\geq 0, p>0} \frac{1}{kp} e^{-2\pi pk T_i} \left[\widehat{\mathcal{A}}^a_{f,i}(\mathcal{U}_i) - \frac{\gamma_i}{\pi kp} \frac{\widehat{\mathcal{A}}_K(\mathcal{U}_i)}{T_i + \bar{T}_i} \right] \ + \ \text{c.c.} \right) \ - $$
$$ - \ \frac{b_i^a}{4} \Big(\log|\eta(iU_i)|^4 + \log[(\mathrm{Re}\ U_i)(\mathrm{Re}\ T_i)\mu^2] \Big) \Bigg\} \ ,$$

The Kähler potential and gauge kinetic functs are

$$K = -\log(S+\bar{S}) - \sum_i \left\{ \log[(T_i+\bar{T}_i)(U_i+\bar{U}_i)] + \frac{1}{2}\frac{V^i_{1-loop}+V^i_{E1}}{S+\bar{S}} \right\} ,$$

$$V^i_{1-loop} = \frac{4\pi\gamma_i}{3}\frac{E(iU_i,2)}{T_i+\bar{T}_i} ,$$

$$V^i_{E1} = \frac{\gamma_i}{\pi}\sum_{k>j\geq 0,\ p>0}\frac{e^{-2\pi kpT_i}}{(kp)^2}\left[\frac{\widehat{\mathcal{A}}_K(\mathcal{U}_i)}{T_i+\bar{T}_i} - \frac{2ikp}{\mathcal{U}_i-\bar{\mathcal{U}}_i}\frac{E_{10}(\mathcal{U}_i)}{\eta^{24}(\mathcal{U}_i)}\right] + \text{c.c.} ,$$

$$f_a = S + \sum_i \left\{ \frac{\pi(b_i^a+6\gamma_i)T_i}{12} - b_i^a \log\ \eta(iU_i) - \sum_{k,j,p}\frac{e^{-2\pi kpT_i}}{2kp}\mathcal{A}^a_{f,i}(\mathcal{U}_i) \right\} .$$

γ_i are model-dependent.

Conclusions

• We computed E1 instanton corrections in the $T^4/Z_2 \times T^2$ type I orbifold model :

holomorphic corrections $\rightarrow f$.

non-holomorphic corrections $\rightarrow K$.

We expect similar results for the superpotential. Suppose S, U moduli are stabilized and there are field-theory (E5) nonperturbative effects on D9 branes (gaugino condensation, racetrack). Then

$$W_{np} \; = \; Ae^{-B(f+f_{np})} = A' \sum_{n=1}^{\infty} d_n e^{-2\pi n T}$$

• When combined with suitable fluxes and/or E5 effects, they generate moduli stabilization and SUSY breaking, changes in the soft spectra.

• Similar to the perturbative threshold corrections , the E1 instantonic corrections to the gauge kinetic functions from $N = 2$ sectors have some universal properties. Similar results therefore for $N = 1$ models with $N = 2$ sectors : Z_6, Z_6', etc.

• S-dual of stringy instantonic effects in type I could teach us more about the heterotic string dynamics.

• The instantonic corrections to f and K deserve more phenomenological studies.