

Orbifolds, the Standard Model, and Unification

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Based on

arXiv:0705.0294 , arXiv:0706.0217 ,
arXiv:0708.2691 , arXiv:0805.4186

Outline of my Talk

Roughly 3 parts:

- Results of the minilandscape study
15 MSSM models with R-parity
- Are features of unification “generic” in those models?
MSSM @ low energies \leadsto GUT @ high energies?
- How does unification work in detail?
A detailed study of gauge coupling unification

Motivation

What is our motivation in
considering orbifold models
and not e.g. other string
constructions?

The Standard Model

Gauge group:

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

Particle content:

Q	$(\mathbf{3}, \mathbf{2})_{1/3}$	L	$(\mathbf{1}, \mathbf{2})_{-1}$	H	$(\mathbf{1}, \mathbf{2})_1$
\bar{u}	$(\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	\bar{e}	$(\mathbf{1}, \mathbf{1})_2$	\bar{H}	$(\mathbf{1}, \mathbf{2})_{-1}$
\bar{d}	$(\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$\bar{\nu}$	$(\mathbf{1}, \mathbf{1})_0$		

Why are we not happy with SM?

(i) Too many free parameters

Gauge sector: 3 couplings g' , g , g_3	3
Quark sector: 6 masses, 3 mixing angles, 1 CP phase	10
Lepton sector: 6 masses, 3 mixing angles and 1-3 phases	10
Higgs sector: Quartic coupling λ and vev v	2
θ parameter of QCD	1
	26

Why are we not happy with SM?

(ii) Structure of gauge symmetry

Why the product structure $SU(3)_c \times SU(2)_L \times U(1)_Y$?

Why 3 different coupling constants g', g, g_3 ?

(iii) Structure of family multiplets

One family is

$$(\mathbf{3},\mathbf{2})_{1/3} + (\overline{\mathbf{3}},\mathbf{1})_{-4/3} + (\mathbf{1},\mathbf{1})_{-2} + (\overline{\mathbf{3}},\mathbf{1})_{2/3} + (\mathbf{1},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{1})_0$$

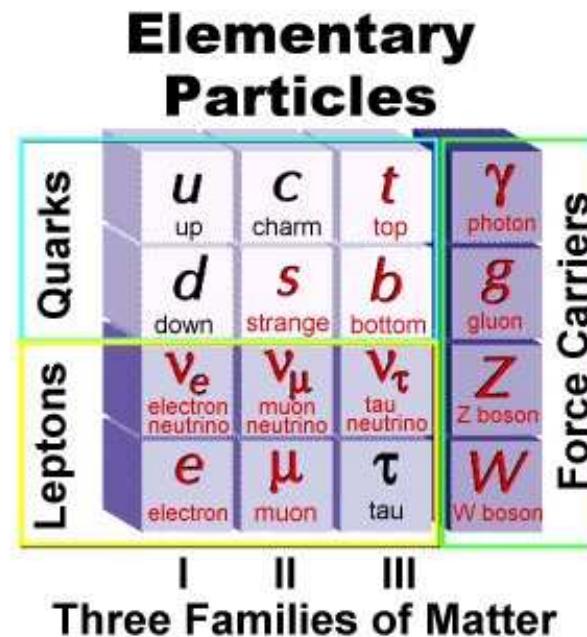
Q \bar{u} \bar{e} \bar{d} L $\bar{\nu}$

Can the particles be reorganized in a single representation?

Why are we not happy with SM?

(iv) Repetition of families

Earth, sun, stars, etc. are built from quarks and leptons of one generation (note though: charm in proton). Why is this pattern for 1 generation replicated 3 times? Horizontal symmetries?



Why are we not happy with SM?

(v) Mass hierarchies and texture of Yukawa couplings

up-quark mass $\sim 2 \times 10^{-3}$ GeV \leftrightarrow top-quark mass ~ 172.3 GeV

Yukawa coupling of top ~ 1 , but why are the other quarks so light?

Minimal mixing in **quark sector**

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 0.97 & 0.22 & 0.00 \\ 0.22 & 0.97 & 0.04 \\ 0.00 & 0.04 & 0.99 \end{pmatrix}$$

Why are we not happy with SM?

(vi) Light neutrinos and texture of Yukawa couplings

Why are neutrinos so light?

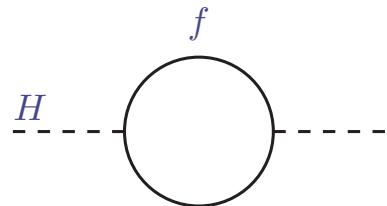
$$\Delta m_\nu^2 \sim 10^{-2} - 10^{-5} \text{ eV}, \quad \sum m_\nu \lesssim 2 \text{ eV}$$

Maximal mixing in **lepton sector**

$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \simeq \begin{pmatrix} 0.8 & 0.5 & 0.0 \\ -0.4 & 0.6 & 0.7 \\ 0.4 & -0.6 & 0.7 \end{pmatrix}$$

Why are we not happy with SM?

(vii) Hierarchy problem



$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda_{\text{UV}}^2 + \dots$$

- Stability of the vacuum
- Requires incredible fine-tuning to set things right

Another way to put it: Why are there 2 fundamental scales at all?

Why are we not happy with SM?

(viii) Dark Matter

23% of our universe is made up of dark matter and the Standard Model offers no candidate particle ...



Why are we not happy with SM?

(ix) Dark Energy

73% of our universe is made up of dark energy



Cosmological constant as calculated from QFT is the worst-predicted quantity in particle physics

Why are we not happy with SM?

(x) Gravity

- Scales relevant in everyday life \leadsto Newton's theory
- Satellites, solar system, etc. \leadsto Still Newton's theory
- Cosmological scales \leadsto Einstein's theory of GR
- Very small scales \leadsto Need quantum theory of gravitation
- Don't know how to quantize gravity and how to unify with SM

(xi) Many other problems

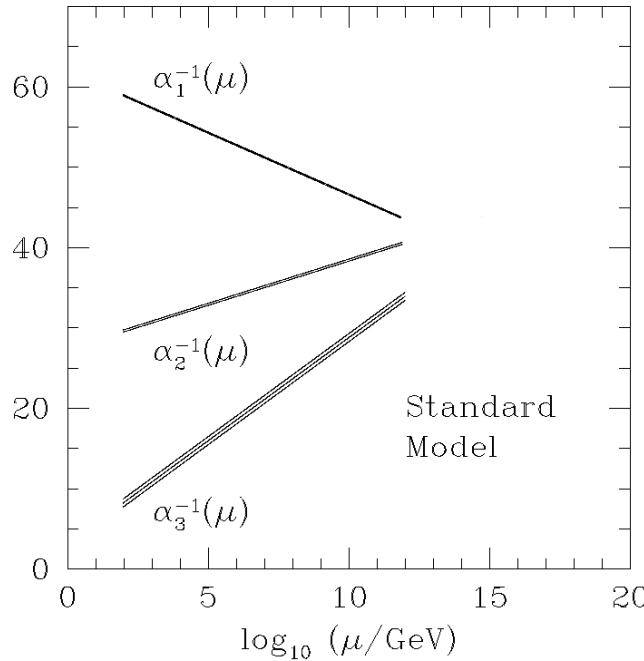
Baryon asymmetry in the universe, charge quantization, ...

Guidelines

Are there hints at physics
beyond the Standard Model
that might guide our model
building efforts?

Hints at Physics Beyond the SM?

figures appropriated from Dienes

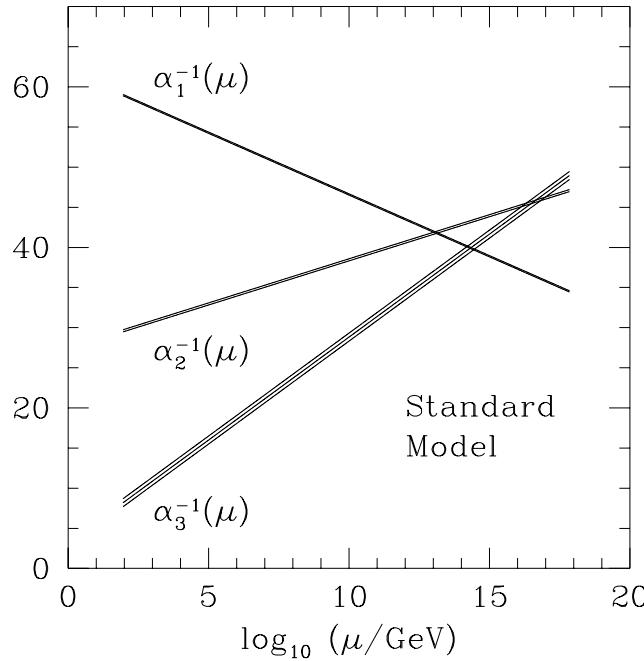


- Running gauge couplings seem to meet at 10^{15} GeV ✓

H. Georgi and S. L. Glashow, "Unity of all elementary particle forces," *Phys. Rev. Lett.* **32** (1974)

Hints at Physics Beyond the SM?

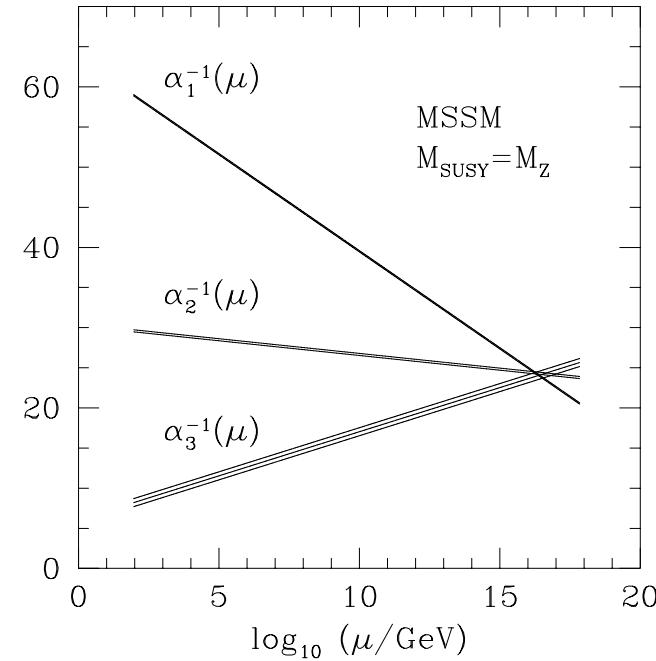
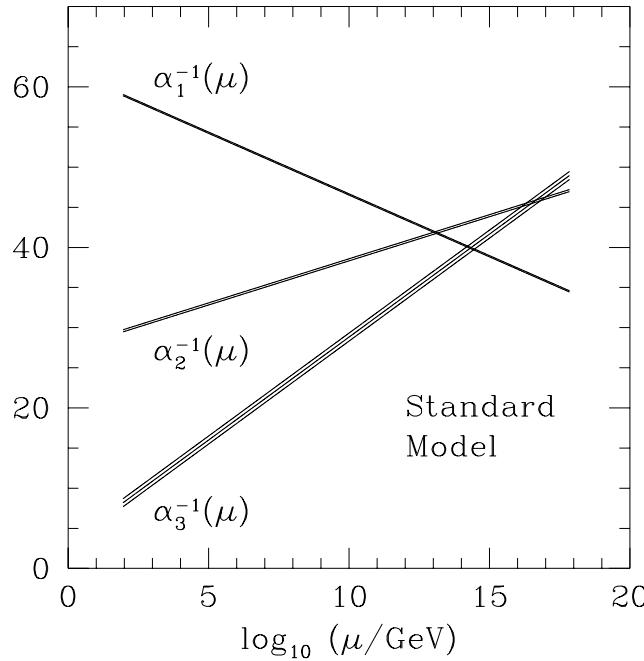
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- Looking more closely, couplings do not unify X
Are we missing something?

Hints at Physics Beyond the SM?

figures appropriated from Dienes

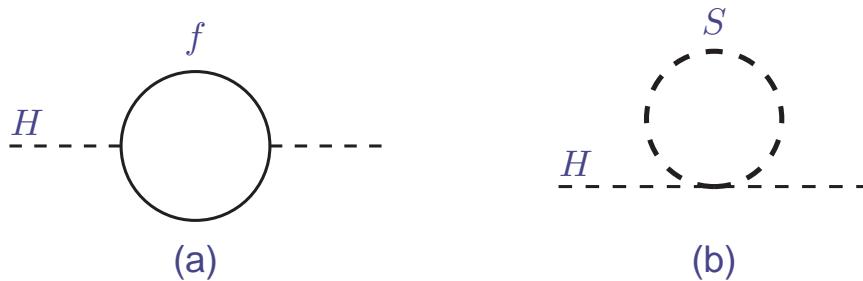


- Supersymmetry helps: Unification at $\sim 3 \times 10^{16}$ GeV ✓
- ~ Strong motivation for GUTS and SUSY!

S. Dimopoulos, S. Raby, F. Wilczek, "Unification of couplings," *Phys. Today*, **44N10** (1991) 25-33.

Another Motivation for SUSY

- Remember the hierarchy problem



For fermions : $\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda_{\text{UV}}^2 + \dots$

For scalars : $\Delta m_H^2 = \frac{\lambda_S}{16\pi^2} [\Lambda_{\text{UV}}^2 - 2m_S^2 \ln(\Lambda_{\text{UV}}/m_S) + \dots]$

- SUSY correlates 1 fermion to 2 real scalars
~ Quadratic divergencies cancel !
- “Technical” solution to hierarchy problem:
Still 2 mass scales in theory, but energy dependence ‘mild’

Grand Unification

Taking the idea of Grand
Unification seriously . . .

Grand Unification

Assume some grand unified gauge group at 3×10^{16} :

- Big enough to include $SU(3)_c \times SU(2)_L \times U(1)_Y$
- “Minimal” otherwise: Rank 4

All Lie algebras: classical and exceptional

$SU(n-1)$	E_6	G_2
$Sp(2n)$	E_7	F_4
$SO(2n+1)$	E_8	
$SO(2n)$		

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Now look at the minimal ones:

$SU(2)^4$, $SO(5)^2$, $SU(3)^2$, $(G_2)^2$, $SO(8)$, $SO(9)$, $Sp(8)$, F_4 , $SU(5)$

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- Cannot define sensible charge operator

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- Do not contain $SU(3)$ as a subset
- Cannot define sensible charge operator
- No complex representations
- $SU(5)$ is unique candidate !!!

Grand Unification

Minimal $SU(5)$ very predictive, but also very constrained . . .

~ Suffers from some problems and (almost) excluded

We need to introduce some “extra degrees of freedom” into the theory

- Consider next-to-minimal algebras
- Prefer simple Lie algebras, i.e. no direct products
- Must have complex representations for particles and anti-particles

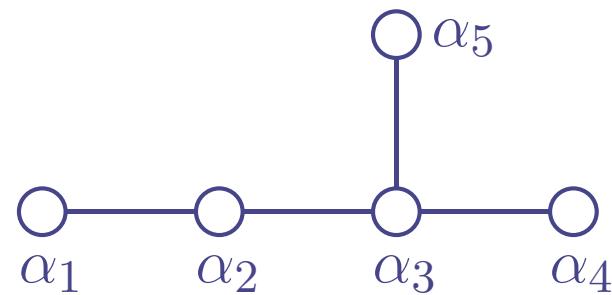
Candidates:

$$SU(n) \text{ for } n \geq 5, \quad SO(4n+2) \text{ for } n \geq 2, \quad E_6$$

Supersymmetric Grand Unification

Assume $\text{SO}(10)$ at fundamental scale $\sim 10^{16}$ GeV

H. Fritzsch and P. Minkowski, "Unified interactions of leptons and hadrons," *Ann. Phys.* **93** (1975)

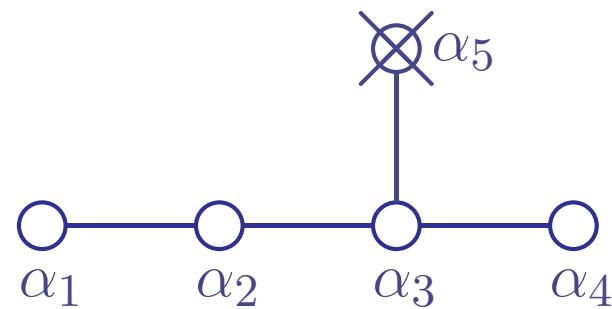


$\text{SO}(10)$

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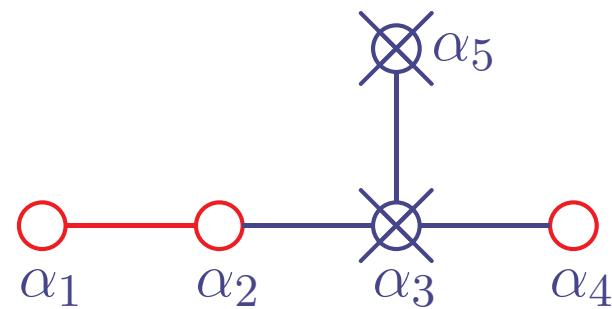


$$\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1)_X$$

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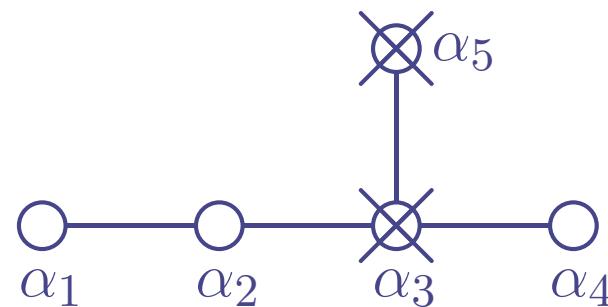


$$\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1)_X \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \times \text{U}(1)_{\text{B-L}}$$

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$$\mathbf{16} \rightarrow \mathbf{10 + \bar{5} + 1}$$

$$\rightarrow (\mathbf{3}, \mathbf{2})_{1/3} + (\mathbf{\bar{3}}, \mathbf{1})_{-4/3} + (\mathbf{1}, \mathbf{1})_{-2} + (\mathbf{\bar{3}}, \mathbf{1})_{2/3} + (\mathbf{1}, \mathbf{2})_{-1} + (\mathbf{1}, \mathbf{1})_0$$

$$Q \qquad \bar{u} \qquad \bar{e} \qquad \bar{d} \qquad L \qquad \bar{\nu}$$

Problems of Grand Unification

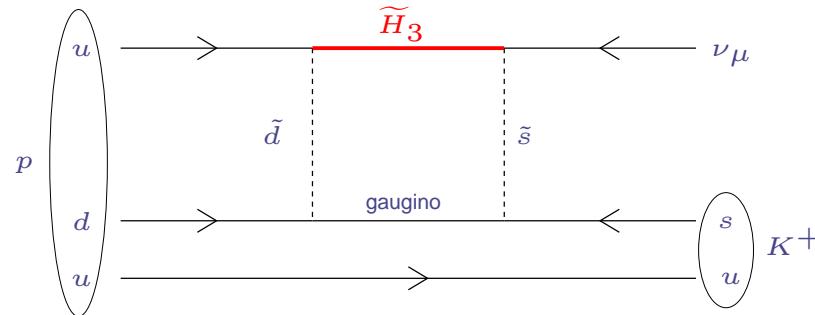
- Large representations required:

24 to break $SU(5)$ \leadsto Many more particles

45 for realistic mass matrices \leadsto Even more particles

- Doublet-triplet splitting problem

$$\mathbf{10} \rightarrow \mathbf{5} + \overline{\mathbf{5}} \rightarrow (\mathbf{1},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{2}) + (\overline{\mathbf{3}},\mathbf{1})$$



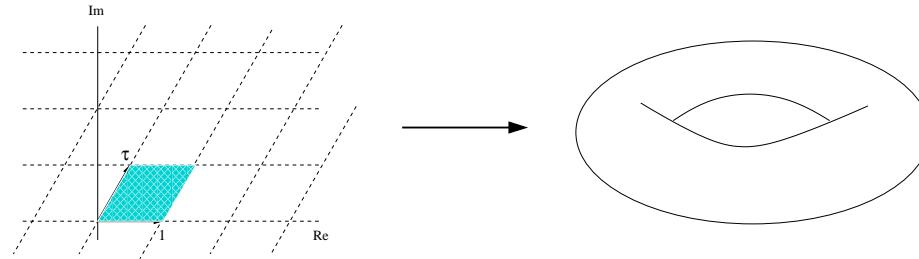
- Both problems can be avoided by extra dimensions !

Why unification in higher dimensions is better . . .

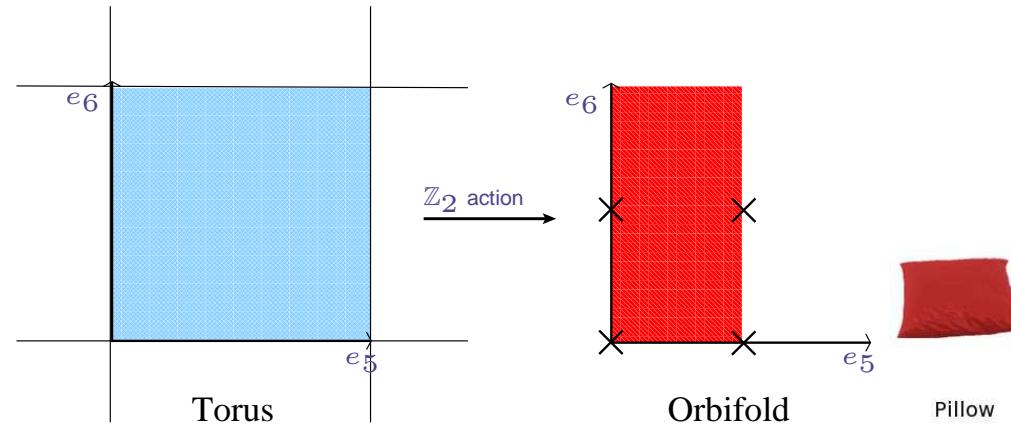
The Idea of an Orbifold

Consider QFT in 6 dimensions: 4 large and 2 small

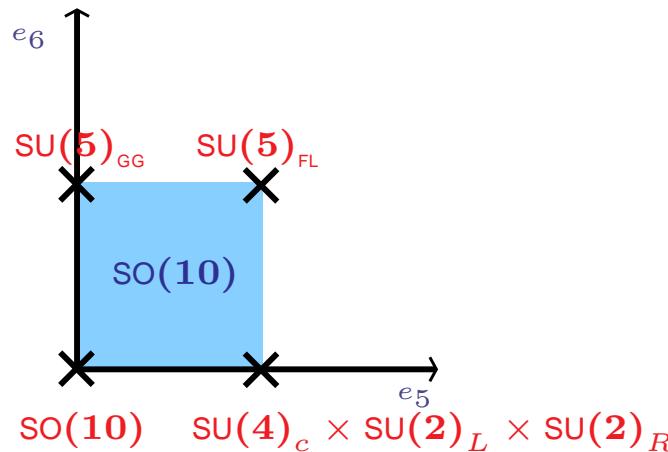
- Compactify 2 dimensions on torus



- Impose e.g. \mathbb{Z}_2 symmetry: $\mathcal{O} = T^2/P, \quad P = \{1, \theta\}$



The Idea of an Orbifold



- 4 gauge groups at fixed points in extra dimension
$$\text{SO}(10) \cap \text{SU}(5) \cap \text{PS} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$$
- Doublet-triplet splitting \leadsto proton stability
- Judicious placement of fields \leadsto Semi-realistic fermion masses

Taking the Step to 10 Dimensions

Some shortcomings of field theory orbifolds:

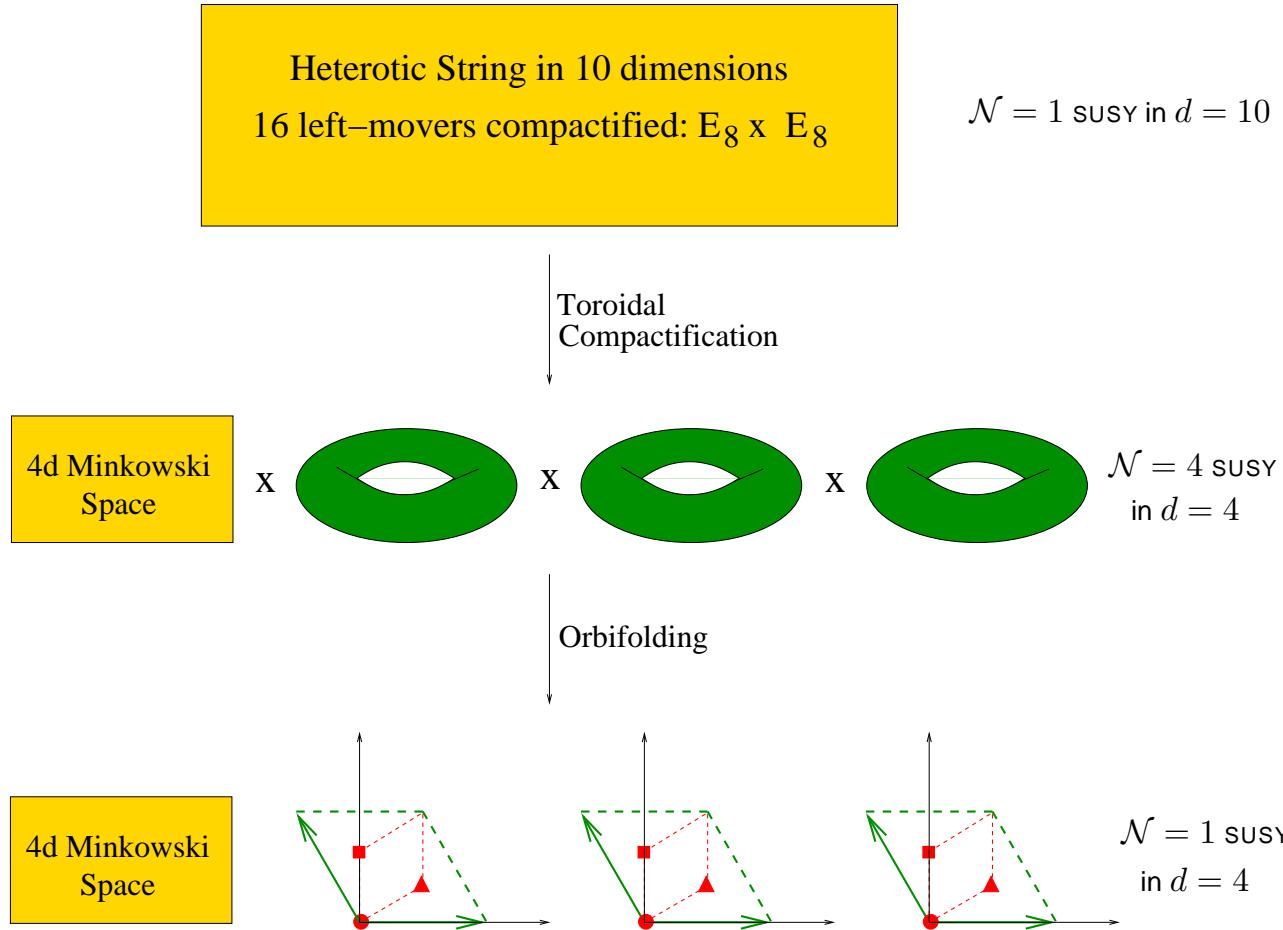
- Non-renormalizable and as such UV-divergent
- Number of dimensions arbitrary
- Symmetry breaking pattern arbitrary
- Placement of fields arbitrary
- In short: No organizing principle

Advantages of stringy orbifolds:

- Finite and does not need renormalization
- Predicts 10 spacetime dimensions
- Symmetry in 10 dimensions (“bulk”) is $E_8 \times E_8$
- Spectrum and localization of particles predicted
- Includes quantum theory of gravity

Orbifolds in 10 Dimensions

Dixon, Harvey, Vafa, Witten, “Strings on Orbifolds,” *Nucl. Phys.* **B261** (1985)



How to choose the Compactification Lattice

- Symmetry must be automorphism of lattice defining T^6
- In 3 dimensions: 219 crystallographic space groups
- In 6 dimensions: 28,927,922 crystallographic space groups
- Demand $P \subset \text{SU}(3) \subset \text{SO}(6)$ for $\mathcal{N} = 1$ SUSY
- Restrict to abelian P for simplicity of construction

How to choose the Compactification Lattice

Point Group	Twist v
\mathbb{Z}_3	(1/3, 1/3, -2/3)
\mathbb{Z}_4	(1/4, 1/4, -1/2)
$\mathbb{Z}_{6-\text{I}}$	(1/6, 1/6, -1/3)
$\mathbb{Z}_{6-\text{II}}$	(1/6, 1/3, -1/2)
\mathbb{Z}_7	(1/7, 2/7, -3/7)
$\mathbb{Z}_{8-\text{I}}$	(1/8, 1/4, -3/8)
$\mathbb{Z}_{8-\text{II}}$	(1/8, 3/8, -1/2)
$\mathbb{Z}_{12-\text{I}}$	(1/12, 1/3, -5/12)
$\mathbb{Z}_{12-\text{II}}$	(1/12, 5/12, -1/2)

Point Group	Twist v_1	Twist v_2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1/2, 0, -1/2)	(0, 1/2, -1/2)
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1/3, 0, -1/3)	(0, 1/3, -1/3)
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{I}}$	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{II}}$	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

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$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{I}}$	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{II}}$	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
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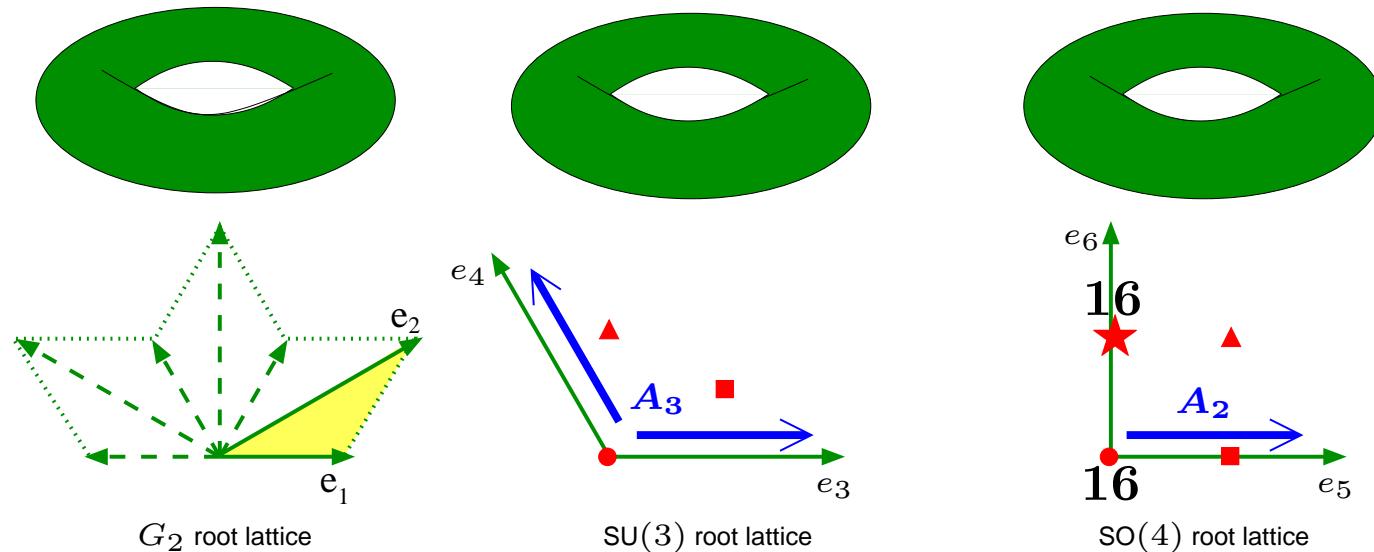
Abelian point groups with $\mathcal{N} = 1$ SUSY in 4 dimensions

Results

The Standard Model from the
heterotic string
(or rather as close as we get)

The \mathbb{Z}_6 -II Orbifold

The geometry of compact space



Gauge embedding

- “Shift” V : 61 choices
 - “Wilson lines” A_3 and A_2 : Thousands of choices
- Need guidelines

Adding Phenomenological Priors

- Grand Unified Group in intermediate step

$$E_8 \xrightarrow{V} SO(10) \times \dots \xrightarrow{A_3} \dots \xrightarrow{A_2} SU(3) \times SU(2) \times U(1)$$

~ leaves us with 15 shifts

- One family in complete multiplet of $SO(10)$

$$\mathbf{16} \xrightarrow{A_3} \dots \xrightarrow{A_5} (\mathbf{3},\mathbf{2})_{1/3} + (\overline{\mathbf{3}},\mathbf{1})_{-4/3} + (\mathbf{1},\mathbf{1})_{-2} + (\overline{\mathbf{3}},\mathbf{1})_{2/3} + (\mathbf{1},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{1})_0 \quad \checkmark$$

~ leaves us with 2 shifts : V_{22} and V_{56}

- Switch on all Wilson lines to scan for Standard Model

A Landscape of Heterotic Orbifold Models

Criterion	$V^{\text{SO}(10),1}$	$V^{\text{SO}(10),2}$	$V^{E_6,1}$	$V^{E_6,2}$
① Inequivalent models with 2 Wilson lines	22,000	7,800	680	1,700
② SM gauge group $\subset \text{SU}(5) \subset \text{SO}(10)$ (or E_6)	3563	1163	27	63
③ 3 net (3, 2)	1170	492	3	32
④ Non-anomalous $U(1)_Y \subset \text{SU}(5)$	528	234	3	22
⑤ Spectrum = 3 generations + vector-like	127	90	3	2
⑥ Vector-like exotics decouple	106	85		
⑦ “Heavy” top	55	32		
⑧ Suitable B-L exists	34	5		
⑨ SM singlets w/ B-L =even/odd	85	8		
⑩ All $U(1)$'s except $U(1)_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

A Typical Spectrum

- Particles w/hypercharge

$3 \times (\mathbf{3}, \mathbf{2})_{1/3}$	$5 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$7 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$16 \times (\mathbf{1}, \mathbf{2})_{-1}$	$45 \times (\mathbf{1}, \mathbf{1})_2$	$129 \times (\mathbf{1}, \mathbf{1})_0$
	$2 \times (\mathbf{3}, \mathbf{1})_{4/3}$	$4 \times (\mathbf{3}, \mathbf{1})_{-2/3}$	$13 \times (\mathbf{1}, \mathbf{2})_1$	$42 \times (\mathbf{1}, \mathbf{1})_{-2}$	

- Vectorlike?

$3 \times (\mathbf{3}, \mathbf{2})_{1/3}$	$3 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$3 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$3 \times (\mathbf{1}, \mathbf{2})_{-1}$	$3 \times (\mathbf{1}, \mathbf{1})_2$	$3 \times (\mathbf{1}, \mathbf{1})_0$
	$2 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$4 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$1 \times (\mathbf{1}, \mathbf{2})_{-1}$	$42 \times (\mathbf{1}, \mathbf{1})_2$	$126 \times (\mathbf{1}, \mathbf{1})_0$
	$2 \times (\mathbf{3}, \mathbf{1})_{4/3}$	$4 \times (\mathbf{3}, \mathbf{1})_{-2/3}$	$1 \times (\mathbf{1}, \mathbf{2})_1$	$42 \times (\mathbf{1}, \mathbf{1})_{-2}$	
			$12 \times (\mathbf{1}, \mathbf{2})_1$		
			$12 \times (\mathbf{1}, \mathbf{2})_{-1}$		

A Typical Spectrum

- Particles w/hypercharge

$3 \times (\mathbf{3}, \mathbf{2})_{1/3}$	$5 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$7 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$16 \times (\mathbf{1}, \mathbf{2})_{-1}$	$45 \times (\mathbf{1}, \mathbf{1})_2$	$129 \times (\mathbf{1}, \mathbf{1})_0$
	$2 \times (\mathbf{3}, \mathbf{1})_{4/3}$	$4 \times (\mathbf{3}, \mathbf{1})_{-2/3}$	$13 \times (\mathbf{1}, \mathbf{2})_1$	$42 \times (\mathbf{1}, \mathbf{1})_{-2}$	

- Vectorlike?

$3 \times (\mathbf{3}, \mathbf{2})_{1/3}$	$3 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$3 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$3 \times (\mathbf{1}, \mathbf{2})_{-1}$	$3 \times (\mathbf{1}, \mathbf{1})_2$	$3 \times (\mathbf{1}, \mathbf{1})_0$
	$2 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$4 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$1 \times (\mathbf{1}, \mathbf{2})_{-1}$	$42 \times (\mathbf{1}, \mathbf{1})_2$	$126 \times (\mathbf{1}, \mathbf{1})_0$
	$2 \times (\mathbf{3}, \mathbf{1})_{4/3}$	$4 \times (\mathbf{3}, \mathbf{1})_{-2/3}$	$1 \times (\mathbf{1}, \mathbf{2})_1$	$42 \times (\mathbf{1}, \mathbf{1})_{-2}$	
			$12 \times (\mathbf{1}, \mathbf{2})_1$		
			$12 \times (\mathbf{1}, \mathbf{2})_{-1}$		

Choice of Vacuum

- Choice of vacuum:

$$\{\tilde{s}_i\} = \{\chi_1, \chi_2, \chi_3, \chi_4, h_1, h_2, h_3, h_4, h_5, h_6, h_9, h_{10}, s_1^0, s_4^0, s_5^0, s_6^0, s_9^0, s_{11}^0, s_{13}^0, s_{15}^0, s_{16}^0, s_{17}^0, s_{18}^0, s_{20}^0, s_{21}^0, s_{22}^0, s_{23}^0, s_{25}^0, s_{26}^0, s_{27}^0, s_{30}^0, s_{31}^0\}$$

- Terms of arbitrary order appear in superpotential, one example is:

$$\phi \bar{\phi} \tilde{s}_1 \tilde{s}_2 \dots \tilde{s}_n$$

Consider terms up to order 6 in singlets, i.e. $n = 6$: **286,781 terms**

- Check that exotics decouple
- Mass matrices: Collect terms quadratic in field \times singlets

$$Y_u = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 & 1 \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & \tilde{s}^5 & 0 \\ \tilde{s}^5 & 0 & 0 \\ 0 & \tilde{s}^6 & 0 \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^5 & 0 & 0 \\ \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}.$$

F- and *D*-Flatness

- Theory supersymmetric \leftrightarrow Minimum of potential is 0

$$V = \sum F_i^* F_i + \sum D^a D^a$$

\leadsto Unbroken supersymmetry is tantamount to $F_i = D^a = 0$

- *D*-terms for U(1)'s

$$D^a = \sum \phi_i^* T^a \phi_i = \sum q_i |\phi_i|^2$$

- *F*-terms

$$F_{\phi_i} = \frac{\partial W}{\partial \phi_i} \Big|_{\phi_i = \langle \phi_i \rangle}$$

Under certain conditions, $F = 0$ implies $D = 0$

- 32 polynomial equations up to order 5 \leadsto Works



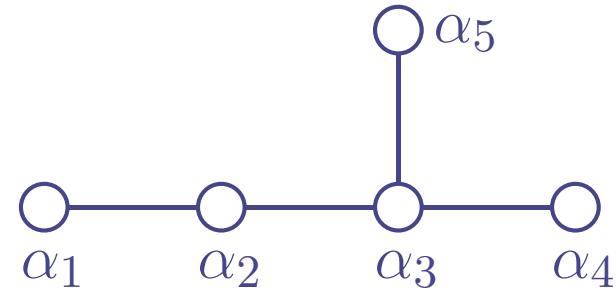
The landscape revisited

Criterion	$V^{\text{SO}(10),1}$	$V^{\text{SO}(10),2}$	$V^{E_6,1}$	$V^{E_6,2}$
① Inequivalent models with 2 Wilson lines	22,000	7,800	680	1,700
② SM gauge group $\subset \text{SU}(5) \subset \text{SO}(10)$ (or E_6)	3563	1163	27	63
③ 3 net (3, 2)	1170	492	3	32
④ Non-anomalous $U(1)_Y \subset \text{SU}(5)$	528	234	3	22
⑤ Spectrum = 3 generations + vector-like	127	90	3	2
⑥ Vector-like exotics decouple	106	85		
⑦ “Heavy” top	55	32		
⑧ Suitable B-L exists	34	5		
⑨ SM singlets w/ B-L =even/odd	85	8		
⑩ All $U(1)$'s except $U(1)_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

Generalizations

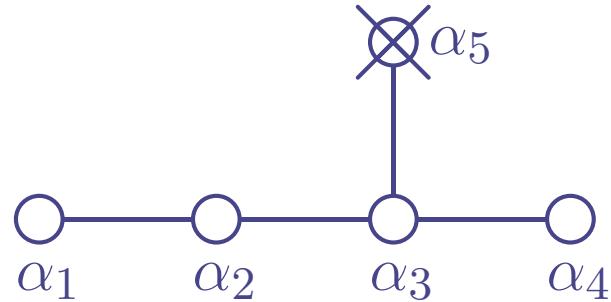
- What if we do not restrict ourselves to hypercharge from $\text{SO}(10)$?
More models?
- 3 generations + no chiral exotics implies $\text{SO}(10)$ structure? Can string theory “prove” GUTs?

Hypercharge from GUTs



$\text{SO}(10)$

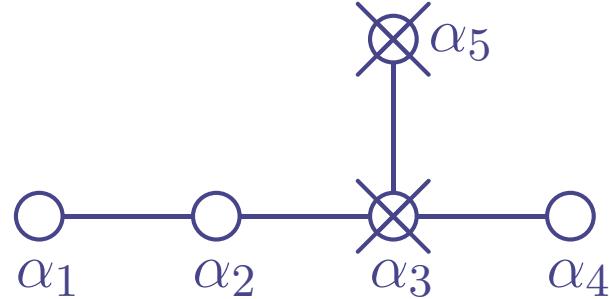
Hypercharge from GUTs



$$\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1)_X$$

$$\text{U}(1)_X = 4\alpha_5^* = 4 \sum_{j=1}^5 (A_{\text{SO}(10)}^{-1})_{5j} \alpha_j = (2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4 + 5\alpha_5)$$

Hypercharge from GUTs



$$\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1)_X \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \times \text{U}(1)_X$$

$$\text{U}(1)_X = 4\alpha_5^* = 4 \sum_{j=1}^5 (A_{\text{SO}(10)}^{-1})_{5j} \alpha_j = (2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4 + 5\alpha_5)$$

$$\text{U}(1)_Y = \frac{5}{3}\alpha_3^* = \frac{5}{3} \sum_{j=1}^4 (A_{\text{SU}(5)}^{-1})_{3j} \alpha_j = \frac{1}{3}(2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4)$$

The General Case

$$\mathbf{SU(3) \times SU(2) \times U(1)^9}$$

$3 \times (\mathbf{3}, \mathbf{2})$	$12 \times (\bar{\mathbf{3}}, \mathbf{1})$	$29 \times (\mathbf{1}, \mathbf{2})$
	$6 \times (\mathbf{3}, \mathbf{1})$	$216 \times (\mathbf{1}, \mathbf{1})$

Ansatz : $\mathbf{U(1)_Y} = x_1 \mathbf{U(1)_1} + x_2 \mathbf{U(1)_2} + \dots + x_9 \mathbf{U(1)_9}$

Too many possibilities : $\binom{3}{3} \times \binom{12}{6} \times \binom{6}{3} \times \binom{29}{5} \times \binom{5}{4} \times \binom{216}{3} \times \binom{213}{3} > 3 \times 10^{22}$ ↗

- Choose left-handed quark doublets,
- right-handed quark singlets and distinguish up- and down-type,
- 3 leptons, 2 Higgses and distinguish leptons and up-type Higgs,
- right-handed electrons and right-handed neutrinos.

The Method

- ① Choose $U(1)$ directions to lie in $E_8 \times E_8'$ lattice
- ② Most general ansatz:

$$Y = x_1 \mathbf{U}_1 + x_2 \mathbf{U}_2 + \dots + x_n \mathbf{U}_n$$

- ③ Choose $3 \times (3, 2)$ and $6 \times (\bar{3}, 1)$ as Q, \bar{u}, \bar{d} , respectively
- ④ Prescribe hypercharge assignments on Q, \bar{u}, \bar{d}
- ⑤ For all other hypercharges and vector-likeness:
Necessary, but not sufficient equations

$$\sum_{(3,2),(\bar{3},2)} Y = 1, \quad \sum_{(3,1),(\bar{3},1)} Y = -2, \quad \sum_{(1,2)} Y = -3, \quad \sum_{(1,1)} Y = 6$$

The Method

⑥ Demand that hypercharge be non-anomalous

$$(x_1 U_1 + x_2 U_2 + \dots + x_n U_n) \cdot \mathbf{U}_{1A} = 0$$

⑦ Cubic and quintic equations

$$\sum_{(\mathbf{3},\mathbf{2}), (\overline{\mathbf{3}},\mathbf{2})} Y^3 = \frac{1}{9}, \quad \sum_{(\mathbf{3},\mathbf{1}), (\overline{\mathbf{3}},\mathbf{1})} Y^3 = -\frac{56}{9}, \quad \sum_{(\mathbf{1},\mathbf{2})} Y^3 = -3, \quad \sum_{(\mathbf{1},\mathbf{1})} Y^3 = 24$$

$$\sum_{(\mathbf{3},\mathbf{2}), (\overline{\mathbf{3}},\mathbf{2})} Y^5 = \frac{1}{81}, \quad \sum_{(\mathbf{3},\mathbf{1}), (\overline{\mathbf{3}},\mathbf{1})} Y^5 = -\frac{992}{81}, \quad \sum_{(\mathbf{1},\mathbf{2})} Y^5 = -3, \quad \sum_{(\mathbf{1},\mathbf{1})} Y^5 = 96$$

⑧ Solve system of equations

↪ Gröbner basis methods

An Example

Linear, cubic and quintic equations

$$f_0 = 36 + 8x_3 + 10x_7 - 4x_2 + 4x_1 + 64x_6 - 8x_5 + 6x_0 - 4x_4$$

$$f_1 = 36x_3 + 90x_7 + 36x_2 - 72x_1 + 252x_6 - 72x_5 + 18x_0 - 36x_4$$

$$f_2 = 54 + 10x_3 + 10x_7 - 8x_2 + 10x_1 + 82x_6 - 8x_5 + 8x_0 - 4x_4$$

$$f_3 = -108 + 16x_3 + 70x_7 + 52x_2 - 92x_1 + 88x_6 - 56x_5 + 2x_0 - 28x_4$$

$$f_4 = -6 + 6x_2 - 6x_1 - 6x_6$$

$$f_5 = -6 - x_3 - x_2 - 6x_6 - x_0$$

$$f_6 = -6 - x_3 - x_2 - 6x_6 - x_0$$

$$f_7 = 24 - 6x_3 - 2x_7 - 2x_1 + 4x_6 + 4x_5 + 4x_0 + 2x_4$$

$$f_8 = -12 + 6x_3 - 12x_8 - 2x_7 - 2x_1 + 4x_6 - 2x_5 + 4x_0 - 4x_4$$

$$f_9 = -12 - x_3 - 6x_7 - x_2 + 5x_0$$

$$f_{10} = 24 - x_3 + 6x_7 - x_2 + 12x_6 - x_0$$

$$f_{11} = -12 - x_3 - 6x_7 - x_2 + 5x_0$$

$$f_{12} = 24 - x_3 + 6x_7 - x_2 + 12x_6 - x_0$$

$$f_{13} = -648 + 2(-x_3 - x_2 - 6x_6 - x_0)^3 + (6x_2 - 6x_1 - 6x_6)^3$$

$$f_{14} = 36288 + (6x_7 + 6x_1 + 12x_6 - 6x_0)^3 + 2(2x_3 + 2x_2 + 12x_6 + 2x_0)^3 + (2x_4 - 6x_3 - 2x_7 - 2x_1 + 4x_6 + 4x_5 + 4x_0)^3 + (4x_4 - 6x_3 + 12x_8 + 2x_7 + 2x_1 - 4x_6 + 2x_5 - 4x_0)^3 + 2(2x_4 - 2x_3 + 12x_8 - 2x_7 + 4x_2 - 2x_1 + 4x_6 - 2x_5 + 2x_0)^3 + 2(-x_3 - 6x_7 - x_2 + 5x_0)^3 + (-x_3 - 2x_2 - 12x_6 - 2x_0)^3 + (6x_7 - 6x_2 + 12x_6)^3 + 2(-x_3 + 6x_7 - x_2 + 12x_6 - x_0)^3 + 2(-2x_4 + 6x_3 + 2x_7 + 2x_1 - 4x_6 - 4x_5 - 4x_0)^3 + 2(-4x_4 + 6x_3 - 12x_8 - 2x_7 - 2x_1 + 4x_6 - 2x_5 + 4x_0)^3 + (-2x_4 + 2x_3 - 12x_8 + 2x_7 - 2x_2 + 2x_1 + 2x_6 + 2x_5 - 2x_0)^3$$

$$f_{15} = 17496 + (-4x_4 + 6x_3 - 12x_8 - 2x_7 + 4x_1 - 2x_6 - 2x_5 - 2x_0)^3 + (-2x_4 + 6x_3 + 2x_7 - 4x_1 + 2x_6 - 4x_5 + 2x_0)^3 + 2(-2x_4 + 3x_3 + 2x_7 - 3x_2 + 2x_1 + 2x_6 - 4x_5 - x_0)^3 + (-2x_4 + 2x_3 - 12x_8 + 2x_7 + 2x_2 + 2x_1 + 2x_6 + 2x_5 - 2x_0)^3 + (-2x_3 + 6x_7 - 2x_2 + 6x_6 - 2x_0)^3 + (-6x_7 + 6x_1 - 6x_6)^3 + 4(-6x_4 + 3x_3 + 3x_8 + 3x_7 + 3x_2 + 12x_6 - 6x_5)^3 + 2(-x_3 - x_2 + 18x_6 + 5x_0)^3 + 2(4x_4 - 6x_3 + 12x_8 + 2x_7 - 4x_1 + 2x_6 + 2x_5 + 2x_0)^3 + 2(2x_4 - 6x_3 - 2x_7 + 2x_6 + 2x_5 - 2x_0)^3 + 2(-2x_4 + 3x_3 - 12x_8 + 2x_7 - 3x_2 + 2x_1 + 2x_6 + 2x_5 - x_0)^3 + 2(2x_4 - 2x_3 + 12x_8 - 2x_7 - 2x_2 - 2x_1 - 2x_6 - 2x_5 + 2x_0)^3 + 2(2x_3 - 6x_7 + 2x_2 - 6x_6 + 2x_0)^3 + 2(4x_4 + 3x_3 - 3x_8 - x_7 - 3x_2 + 2x_1 + 8x_6 + 2x_5 + 2x_0)^3 + (6x_7 - 6x_1 + 6x_6)^3 + 2(6x_4 - 3x_3 - 3x_8 - 3x_7 - 3x_2 - 12x_6 + 6x_5)^3 + (6x_7 + 6x_2 + 6x_6 - 6x_0)^3$$

An Example

$$\begin{aligned}
f_{16} = & -139968 + (-12x_4 + 4x_3 - 6x_8 + 4x_2 - 6x_5 - 2x_0)^3 + (4x_4 - 4x_3 + 12x_8 - 4x_7 + 2x_2 - 4x_1 - 4x_6 + 2x_5 + 4x_0)^3 + 2(-2x_3 + 4x_2 - 6x_1 + 4x_0)^3 + 2(-x_3 - x_2 - 6x_1 + 5x_0)^3 + 2(-x_3 - 7x_2 + 6x_1 - x_0)^3 + 10(3x_3 - 6x_8 + \\
& 3x_2 - 6x_1 + 3x_0)^3 + (2x_4 - 6x_3 + 4x_7 - 2x_1 + 4x_6 + 4x_5 - 2x_0)^3 + (4x_4 - 6x_3 + 12x_8 - 4x_7 + 2x_1 - 4x_6 + 2x_5 + 2x_0)^3 + (2x_3 + 2x_2 - 6x_1 + 2x_0)^3 + (12x_4 - 2x_3 + 6x_8 + 4x_2 + 6x_5 - 2x_0)^3 + 5(6x_4 - 12x_8 + 6x_5)^3 + 4(6x_4 - \\
& 3x_3 - 3x_8 - 3x_7 + 3x_2 - 6x_1 + 6x_6 + 6x_5 + 6x_0)^3 + 2(-4x_4 + x_3 + 3x_8 + x_7 + x_2 - 2x_1 + 10x_6 - 2x_5 + 2x_0)^3 + 10(-4x_4 + x_3 - 3x_8 + x_7 + x_2 - 2x_1 + 10x_6 - 2x_5 + 2x_0)^3 + 2(-12x_4 + 3x_3 - 6x_8 + 3x_2 - 6x_1 - 6x_5 + 3x_0)^3 + \\
& 2(2x_4 - 5x_3 - 8x_7 + x_2 - 2x_1 - 8x_6 + 4x_5 + 5x_0)^3 + (-2x_4 + 2x_3 - 12x_8 + 2x_7 + 2x_2 + 2x_1 - 16x_6 + 2x_5 - 8x_0)^3 + 10(-4x_4 + 4x_3 - 6x_8 + 4x_7 - 2x_2 + 4x_1 + 4x_6 - 2x_5 - 4x_0)^3 + 2(2x_4 + x_3 + 12x_8 + 4x_7 + x_2 - 2x_1 + 4x_6 - \\
& 2x_5 - x_0)^3 + 2(-2x_4 - 3x_3 + 15x_8 - x_7 + 3x_2 + 2x_1 - 10x_6 - 4x_5 - 4x_0)^3 + 2(3x_3 - 3x_2 + 6x_1 - 3x_0)^3 + 2(-4x_3 - 12x_8 - 4x_2 + 6x_5 + 2x_0)^3 + 2(-6x_4 + 5x_3 + 3x_8 + 3x_7 - x_2 - 6x_6 - 6x_5 - 4x_0)^3 + 2(4x_4 + 3x_3 - 3x_8 - x_7 + \\
& 3x_2 + 2x_1 - 10x_6 + 2x_5 - 4x_0)^3 + 4(-6x_4 + 3x_3 + 3x_8 - 3x_7 - 3x_2 + 6x_1 + 6x_6 - 6x_5)^3 + 10(-2x_4 + 2x_3 + 6x_8 - 4x_7 + 2x_2 - 4x_1 - 4x_6 - 4x_5 + 4x_0)^3 + 2(6x_4 - 3x_3 - 3x_8 + 3x_7 + 3x_2 - 6x_6 + 6x_5 - 6x_0)^3 + 2(-6x_4 - x_3 - \\
& 9x_8 + 3x_7 - x_2 - 6x_6 - 4x_0)^3 + 2(6x_4 - x_3 + 9x_8 + 3x_7 - x_2 - 6x_6 - 4x_0)^3 + 2(2x_3 + 12x_8 - 4x_2 - 6x_5 + 2x_0)^3 + 2(8x_4 - 5x_3 + 9x_8 - 5x_7 + x_2 - 2x_1 - 14x_6 + 4x_5 + 2x_0)^3 + 2(-8x_4 + 3x_3 - 9x_8 - x_7 + 3x_2 + 2x_1 - 10x_6 - \\
& 4x_5 - 4x_0)^3 + 2(-10x_4 + x_3 + 6x_8 + 4x_7 + x_2 - 2x_1 + 4x_6 - 8x_5 - x_0)^3 + (2x_4 + 12x_8 + 4x_7 + 6x_2 - 2x_1 + 4x_6 - 2x_5 - 2x_0)^3 + 2(-2x_4 + 2x_3 - 4x_7 + 2x_2 - 4x_1 - 4x_6 - 4x_5 + 4x_0)^3 + 2(-2x_4 - 12x_8 + 2x_7 + 2x_1 + 20x_6 + \\
& 2x_5 + 2x_0)^3 + 2(2x_4 + 12x_8 + 4x_7 - 6x_2 + 4x_1 + 4x_6 - 2x_5 - 2x_0)^3 + 2(-2x_4 + 3x_3 - 12x_8 - 4x_7 + 3x_2 + 2x_1 - 4x_6 + 2x_5 - x_0)^3 + (2x_4 - 2x_3 + 12x_8 - 8x_7 - 2x_2 + 4x_1 - 8x_6 - 2x_5 + 2x_0)^3 + 4(-x_3 + 5x_2 - x_0)^3 + 2(4x_4 + \\
& 3x_3 - 3x_8 - x_7 - 3x_2 - 4x_1 - 10x_6 + 2x_5 + 2x_0)^3 + 2(12x_4 - 4x_3 + 6x_8 - 4x_2 + 6x_5 + 2x_0)^3 + 10(2x_4 + x_3 + 6x_8 + 4x_7 + x_2 - 2x_1 + 4x_6 - 2x_5 - x_0)^3 + 2(-4x_4 + 4x_3 - 12x_8 + 4x_7 - 2x_2 + 4x_1 + 4x_6 - 2x_5 - 4x_0)^3 + 2(-4x_4 + \\
& 7x_3 - 12x_8 + 4x_7 + x_2 - 2x_1 + 4x_6 - 2x_5 - x_0)^3 + (2x_3 - 4x_2 + 6x_1 - 4x_0)^3 + 2(2x_4 + 7x_3 - 3x_8 + x_7 + x_2 - 2x_1 + 10x_6 - 2x_5 + 2x_0)^3 + 10(-3x_3 + 6x_8 - 3x_2 + 6x_1 - 3x_0)^3 + 2(-2x_4 + 6x_3 - 4x_7 + 2x_1 - 4x_6 - 4x_5 + 2x_0)^3 + \\
& 2(-4x_4 + 6x_3 - 12x_8 + 4x_7 - 2x_1 + 4x_6 - 2x_5 - 2x_0)^3 + 2(-x_3 + 6x_7 - x_2 - 12x_6 - 7x_0)^3 + 2(-2x_3 - 2x_2 + 6x_1 - 2x_0)^3 + 2(-12x_4 + 2x_3 - 6x_8 - 4x_2 - 6x_5 + 2x_0)^3 + 4(2x_4 - 5x_3 + 4x_7 + x_2 - 2x_1 + 4x_6 + 4x_5 - x_0)^3 + \\
& 2(-6x_4 + 3x_3 + 3x_8 + 3x_7 - 3x_2 + 6x_1 - 6x_6 - 6x_5 - 6x_0)^3 + (-6x_2 - 6x_1 + 6x_0)^3 + 2(6x_4 - x_3 - 3x_8 + 3x_7 - x_2 - 6x_6 + 6x_5 - 4x_0)^3 + 2(12x_4 - 3x_3 + 6x_8 - 3x_2 + 6x_1 + 6x_5 - 3x_0)^3 + 4(-x_3 - x_2 - x_0)^3 + 2(2x_4 - 2x_3 + \\
& 12x_8 - 2x_7 - 2x_2 - 2x_1 + 16x_6 - 2x_5 + 8x_0)^3 + 5(4x_4 - 4x_3 + 6x_8 - 4x_7 + 2x_2 - 4x_1 - 4x_6 + 2x_5 + 4x_0)^3 + (-6x_4 - 6x_3 + 18x_8 - 6x_5)^3 + 4(-3x_3 + 3x_2 - 6x_1 + 3x_0)^3 + (4x_3 + 12x_8 + 4x_2 - 6x_5 - 2x_0)^3 + 2(-2x_4 + 3x_3 - \\
& 4x_7 + 3x_2 + 2x_1 - 4x_6 - 4x_5 - x_0)^3 + 2(6x_4 - 3x_3 - 3x_8 + 3x_7 + 3x_2 - 6x_1 - 6x_6 + 6x_5)^3 + 5(2x_4 - 2x_3 - 6x_8 + 4x_7 - 2x_2 + 4x_1 + 4x_6 + 4x_5 - 4x_0)^3 + 2(-6x_4 + 3x_3 + 3x_8 - 3x_7 - 3x_2 + 6x_6 - 6x_5 + 6x_0)^3 + 2(-x_3 - 6x_7 - \\
& x_2 - 24x_6 - x_0)^3 + (-2x_3 - 12x_8 + 4x_2 + 6x_5 - 2x_0)^3 + 2(-6x_4 - x_3 - 9x_8 - 3x_7 + 5x_2 + 6x_6 + 2x_0)^3 + 2(6x_4 - x_3 + 9x_8 - 3x_7 + 5x_2 + 6x_6 + 2x_0)^3 + (6x_7 - 12x_6 - 6x_0)^3 + 2(4x_4 + 3x_3 - 3x_8 + 5x_7 + 3x_2 - 4x_1 + 14x_6 + \\
& 2x_5 + 2x_0)^3 + 2(-2x_4 - 12x_8 - 4x_7 - 6x_2 + 2x_1 - 4x_6 + 2x_5 + 2x_0)^3 + (2x_4 - 2x_3 + 4x_7 - 2x_2 + 4x_1 + 4x_6 + 4x_5 - 4x_0)^3 + (2x_4 + 12x_8 - 2x_7 - 2x_1 - 20x_6 - 2x_5 - 2x_0)^3 + 2(8x_4 - 5x_3 + 9x_8 + x_7 + x_2 - 2x_1 + 10x_6 + 4x_5 + \\
& 2x_0)^3 + 5(-6x_3 + 6x_8)^3 + (-2x_4 - 12x_8 - 4x_7 + 6x_2 - 4x_1 - 4x_6 + 2x_5 + 2x_0)^3 + 2(-2x_4 + 2x_3 - 12x_8 + 8x_7 + 2x_2 - 4x_1 + 8x_6 + 2x_5 - 2x_0)^3
\end{aligned}$$

$$f_{17} = -(6x_2 - 6x_1 - 6x_6)^5 + 2(-x_3 - x_2 - 6x_6 - x_0)^5$$

$$\begin{aligned}
f_{18} = & 23141376 + 2(-x_3 - 6x_7 - x_2 + 5x_0)^5 + 2(2x_4 - 2x_3 + 12x_8 - 2x_7 + 4x_2 - 2x_1 + 4x_6 - 2x_5 + 2x_0)^5 + (4x_4 - 6x_3 + 12x_8 + 2x_7 + 2x_1 - 4x_6 + 2x_5 - 4x_0)^5 + (-2x_3 - 2x_2 - 12x_6 - 2x_0)^5 + 2(-x_3 + 6x_7 - x_2 + 12x_6 - \\
& x_0)^5 + (2x_4 - 6x_3 - 2x_7 - 2x_1 + 4x_6 + 4x_5 + 4x_0)^5 + (6x_7 + 6x_1 + 12x_6 - 6x_0)^5 + (6x_7 - 6x_2 + 12x_6)^5 + (-2x_4 + 2x_3 - 12x_8 + 2x_7 - 4x_2 + 2x_1 - 4x_6 + 2x_5 - 2x_0)^5 + 2(-4x_4 + 6x_3 - 12x_8 - 2x_7 - 2x_1 + 4x_6 - 2x_5 + 4x_0)^5 + \\
& 2(2x_3 + 2x_2 + 12x_6 + 2x_0)^5 + 2(-2x_4 + 6x_3 + 2x_7 + 2x_1 - 4x_6 - 4x_5 - 4x_0)^5
\end{aligned}$$

An Example

$$\begin{aligned}
f_{19} = & 5668704 + (-2x_4 + 2x_3 - 12x_8 + 2x_7 + 2x_2 + 2x_1 + 2x_6 + 2x_5 - 2x_0)^5 + 2(2x_3 - 6x_7 + 2x_2 - 6x_6 + 2x_0)^5 + (6x_7 - 6x_1 + 6x_6)^5 + 2(-x_3 - x_2 + 18x_6 + 5x_0)^5 + (6x_7 + 6x_2 + 6x_6 - 6x_0)^5 + (-2x_4 + 6x_3 + 2x_7 - 4x_1 + \\
& 2x_6 - 4x_5 + 2x_0)^5 + 2(6x_4 - 3x_3 - 3x_8 - 3x_7 - 3x_2 - 12x_6 + 6x_5)^5 + (-4x_4 + 6x_3 - 12x_8 - 2x_7 + 4x_1 - 2x_6 - 2x_5 - 2x_0)^5 + 2(-2x_4 + 3x_3 + 2x_7 - 3x_2 + 2x_1 + 2x_6 - 4x_5 - x_0)^5 + 2(2x_4 - 2x_3 + 12x_8 - 2x_7 - 2x_2 - 2x_1 - \\
& 2x_6 - 2x_5 + 2x_0)^5 + (-2x_3 + 6x_7 - 2x_2 + 6x_6 - 2x_0)^5 + 2(4x_4 + 3x_3 - 3x_8 - x_7 - 3x_2 + 2x_1 + 8x_6 + 2x_5 + 2x_0)^5 + (-6x_7 + 6x_1 - 6x_6)^5 + 2(-2x_4 + 3x_3 - 12x_8 + 2x_7 - 3x_2 + 2x_1 + 2x_6 + 2x_5 - x_0)^5 + 2(2x_4 - 6x_3 - 2x_7 + \\
& 4x_1 - 2x_6 + 4x_5 - 2x_0)^5 + 4(-6x_4 + 3x_3 + 3x_8 + 3x_7 + 3x_2 + 12x_6 - 6x_5)^5 + 2(4x_4 - 6x_3 + 12x_8 + 2x_7 - 4x_1 + 2x_6 + 2x_5 + 2x_0)^5 \\
f_{20} = & -181398528 + 2(4x_4 + 3x_3 - 3x_8 - x_7 + 3x_2 + 2x_1 - 10x_6 + 2x_5 - 4x_0)^5 + 2(-x_3 - 6x_7 - x_2 - 24x_6 - x_0)^5 + 10(3x_3 - 6x_8 + 3x_2 - 6x_1 + 3x_0)^5 + (2x_4 + 12x_8 + 4x_7 + 6x_2 - 2x_1 + 4x_6 - 2x_5 - 2x_0)^5 + 2(-2x_4 - 12x_8 + \\
& 2x_7 + 2x_1 + 20x_6 + 2x_5 + 2x_0)^5 + 2(-12x_4 + 3x_3 - 6x_8 + 3x_2 - 6x_1 - 6x_5 + 3x_0)^5 + 2(6x_4 - x_3 + 9x_8 + 3x_7 - x_2 - 6x_6 - 4x_0)^5 + 2(-6x_4 - x_3 - 9x_8 + 3x_7 - x_2 - 6x_6 - 4x_0)^5 + 2(6x_4 - x_3 - 3x_8 + 3x_7 - x_2 - 6x_6 + 6x_5 - \\
& 4x_0)^5 + 4(-x_3 + 5x_2 - x_0)^5 + 2(3x_3 - 3x_2 + 6x_1 - 3x_0)^5 + (-12x_4 + 4x_3 - 6x_8 + 4x_2 - 6x_5 - 2x_0)^5 + 2(-2x_4 + 2x_3 - 4x_7 + 2x_2 - 4x_1 - 4x_6 - 4x_5 + 4x_0)^5 + 2(2x_4 + 12x_8 + 4x_7 - 6x_2 + 4x_1 + 4x_6 - 2x_5 - 2x_0)^5 + 10(-2x_4 + \\
& 2x_3 + 6x_8 - 4x_7 + 2x_2 - 4x_1 - 4x_6 - 4x_5 + 4x_0)^5 + 2(-10x_4 + x_3 + 6x_8 + 4x_7 + x_2 - 2x_1 + 4x_6 - 8x_5 - x_0)^5 + 2(8x_4 - 5x_3 + 9x_8 - 5x_7 + x_2 - 2x_1 - 14x_6 + 4x_5 + 2x_0)^5 + 2(-8x_4 + 3x_3 - 9x_8 - x_7 + 3x_2 + 2x_1 - 10x_6 - 4x_5 - \\
& 4x_0)^5 + 2(4x_4 + 3x_3 - 3x_8 - x_7 - 3x_2 - 4x_1 - 10x_6 + 2x_5 + 2x_0)^5 + 2(-2x_4 + 3x_3 - 12x_8 - 4x_7 + 3x_2 + 2x_1 - 4x_6 + 2x_5 - x_0)^5 + (2x_4 - 2x_3 + 12x_8 - 8x_7 - 2x_2 + 4x_1 - 8x_6 - 2x_5 + 2x_0)^5 + 4(-6x_4 + 3x_3 + 3x_8 - 3x_7 - 3x_2 + \\
& 6x_1 + 6x_6 - 6x_5)^5 + (12x_4 - 2x_3 + 6x_8 + 4x_2 + 6x_5 - 2x_0)^5 + (4x_4 - 4x_3 + 12x_8 - 4x_7 + 2x_2 - 4x_1 - 4x_6 + 2x_5 + 4x_0)^5 + 2(-x_3 - 7x_2 + 6x_1 - x_0)^5 + (4x_4 - 6x_3 + 12x_8 - 4x_7 + 2x_1 - 4x_6 + 2x_5 + 2x_0)^5 + 2(2x_4 - 5x_3 - 8x_7 + \\
& x_2 - 2x_1 - 8x_6 + 4x_5 + 5x_0)^5 + 2(-x_3 - x_2 - 6x_1 + 5x_0)^5 + 2(-2x_3 + 4x_2 - 6x_1 + 4x_0)^5 + (4x_3 + 12x_8 + 4x_2 - 6x_5 - 2x_0)^5 + (2x_4 - 6x_3 + 4x_7 - 2x_1 + 4x_6 + 4x_5 - 2x_0)^5 + 4(6x_4 - 3x_3 - 3x_8 - 3x_7 + 3x_2 - 6x_1 + 6x_6 + 6x_5 + \\
& 6x_0)^5 + 10(-4x_4 + x_3 - 3x_8 + x_7 + x_2 - 2x_1 + 10x_6 - 2x_5 + 2x_0)^5 + 2(-4x_4 + x_3 + 3x_8 + x_7 + x_2 - 2x_1 + 10x_6 - 2x_5 + 2x_0)^5 + (-2x_3 - 12x_8 + 4x_2 + 6x_5 - 2x_0)^5 + 2(-2x_4 - 3x_3 + 15x_8 - x_7 + 3x_2 + 2x_1 - 10x_6 - 4x_5 - \\
& 4x_0)^5 + 10(-4x_4 + 4x_3 - 6x_8 + 4x_7 - 2x_2 + 4x_1 + 4x_6 - 2x_5 - 4x_0)^5 + 2(2x_4 + x_3 + 12x_8 + 4x_7 + x_2 - 2x_1 + 4x_6 - 2x_5 - x_0)^5 + 2(-x_3 + 6x_7 - x_2 - 12x_6 - 7x_0)^5 + (-6x_4 - 6x_3 + 18x_8 - 6x_5)^5 + 2(-6x_4 + 3x_3 + 3x_8 - 3x_7 - \\
& 3x_2 + 6x_6 - 6x_5 + 6x_0)^5 + (-2x_4 + 2x_3 - 12x_8 + 2x_7 + 2x_2 + 2x_1 - 16x_6 + 2x_5 - 8x_0)^5 + (2x_3 + 2x_2 - 6x_1 + 2x_0)^5 + 10(-3x_3 + 6x_8 - 3x_2 + 6x_1 - 3x_0)^5 + 2(-2x_4 - 12x_8 - 4x_7 - 6x_2 + 2x_1 - 4x_6 + 2x_5 + 2x_0)^5 + (2x_4 + \\
& 12x_8 - 2x_7 - 2x_1 - 20x_6 - 2x_5 - 2x_0)^5 + 2(12x_4 - 3x_3 + 6x_8 - 3x_2 + 6x_1 + 6x_5 - 3x_0)^5 + 4(-3x_3 + 3x_2 - 6x_1 + 3x_0)^5 + 2(12x_4 - 4x_3 + 6x_8 - 4x_2 + 6x_5 + 2x_0)^5 + (2x_4 - 2x_3 + 4x_7 - 2x_2 + 4x_1 + 4x_6 + 4x_5 - 4x_0)^5 + (-2x_4 - \\
& 12x_8 - 4x_7 + 6x_2 - 4x_1 - 4x_6 + 2x_5 + 2x_0)^5 + 5(2x_4 - 2x_3 - 6x_8 + 4x_7 - 2x_2 + 4x_1 + 4x_6 + 4x_5 - 4x_0)^5 + 2(4x_4 + 3x_3 - 3x_8 + 5x_7 + 3x_2 - 4x_1 + 14x_6 + 2x_5 + 2x_0)^5 + 2(-2x_4 + 3x_3 - 4x_7 + 3x_2 + 2x_1 - 4x_6 - 4x_5 - x_0)^5 + \\
& 2(6x_4 - x_3 + 9x_8 - 3x_7 + 5x_2 + 6x_6 + 2x_0)^5 + 2(-6x_4 - x_3 - 9x_8 - 3x_7 + 5x_2 + 6x_6 + 2x_0)^5 + 2(-2x_4 + 2x_3 - 12x_8 + 8x_7 + 2x_2 - 4x_1 + 8x_6 + 2x_5 - 2x_0)^5 + 5(-6x_3 + 6x_8)^5 + 2(8x_4 - 5x_3 + 9x_8 + x_7 + x_2 - 2x_1 + 10x_6 + \\
& 4x_5 + 2x_0)^5 + 2(6x_4 - 3x_3 - 3x_8 + 3x_7 + 3x_2 - 6x_1 - 6x_6 + 6x_5)^5 + 2(-12x_4 + 2x_3 - 6x_8 - 4x_2 - 6x_5 + 2x_0)^5 + 10(2x_4 + x_3 + 6x_8 + 4x_7 + x_2 - 2x_1 + 4x_6 - 2x_5 - x_0)^5 + 2(-4x_4 + 4x_3 - 12x_8 + 4x_7 - 2x_2 + 4x_1 + 4x_6 - 2x_5 - \\
& 4x_0)^5 + 2(-4x_4 + 7x_3 - 12x_8 + 4x_7 + x_2 - 2x_1 + 4x_6 - 2x_5 - x_0)^5 + 2(-4x_4 + 6x_3 - 12x_8 + 4x_7 - 2x_1 + 4x_6 - 2x_5 - 2x_0)^5 + 4(2x_4 - 5x_3 + 4x_7 + x_2 - 2x_1 + 4x_6 + 4x_5 - x_0)^5 + (-6x_2 - 6x_1 + 6x_0)^5 + 5(6x_4 - 12x_8 + 6x_5)^5 + \\
& (2x_3 - 4x_2 + 6x_1 - 4x_0)^5 + 2(2x_4 + 7x_3 - 3x_8 + x_7 + x_2 - 2x_1 + 10x_6 - 2x_5 + 2x_0)^5 + (6x_7 - 12x_6 - 6x_0)^5 + 2(-4x_3 - 12x_8 - 4x_2 + 6x_5 + 2x_0)^5 + 2(-2x_4 + 6x_3 - 4x_7 + 2x_1 - 4x_6 - 4x_5 + 2x_0)^5 + 4(-x_3 - x_2 - x_0)^5 + \\
& 2(-6x_4 + 3x_3 + 3x_8 + 3x_7 - 3x_2 + 6x_1 - 6x_6 - 6x_5 - 6x_0)^5 + 2(2x_4 - 2x_3 + 12x_8 - 2x_7 - 2x_2 - 2x_1 + 16x_6 - 2x_5 + 8x_0)^5 + 2(-2x_3 - 2x_2 + 6x_1 - 2x_0)^5
\end{aligned}$$

An Example

Solve system of polynomial equations

Gröbner basis

$$g_1 = x_7 + 2$$

$$g_2 = 3x_6 + x_7 + 5$$

$$g_3 = x_5 - 3x_6 - x_7 - 2x_8 + 1$$

$$g_4 = 4x_3 - 2x_4 - x_5 - 3x_6 - x_7 - 6x_8 - 11$$

$$g_5 = x_2 + x_3 - 10x_6 - 4x_7 - 18$$

$$g_6 = x_1 - x_2 + x_6 + 1$$

$$g_7 = x_0 + x_2 + x_3 - 12x_6 - 6x_7 - 24$$

$$g_8 = x_4^2 + 8x_4x_8 + 16x_8^2$$

$$g_9 = 35x_4x_8^4 + 115x_8^5 + 144x_4x_8^3 + 456x_8^4 + 144x_4x_8^2 + 432x_8^3$$

$$g_{10} = 875x_8^6 + 4800x_8^5 - 1440x_4x_8^3 + 2160x_8^4 - 3456x_4x_8^2 - 10368x_8^3$$

Solution

$$x_1 = x_2 = x_3 = x_4 = x_5 = x_9 = 0, \quad x_6 = -6, \quad x_7 = -1, \quad x_8 = -2$$

$$x_1 = x_2 = x_3 = x_4 = 0, \quad x_5 = \frac{48}{5}, \quad x_6 = -\frac{54}{5}, \quad x_7 = -1, \quad x_8 = -2, \quad x_9 = -\frac{12}{5}$$

An Example

Spectrum

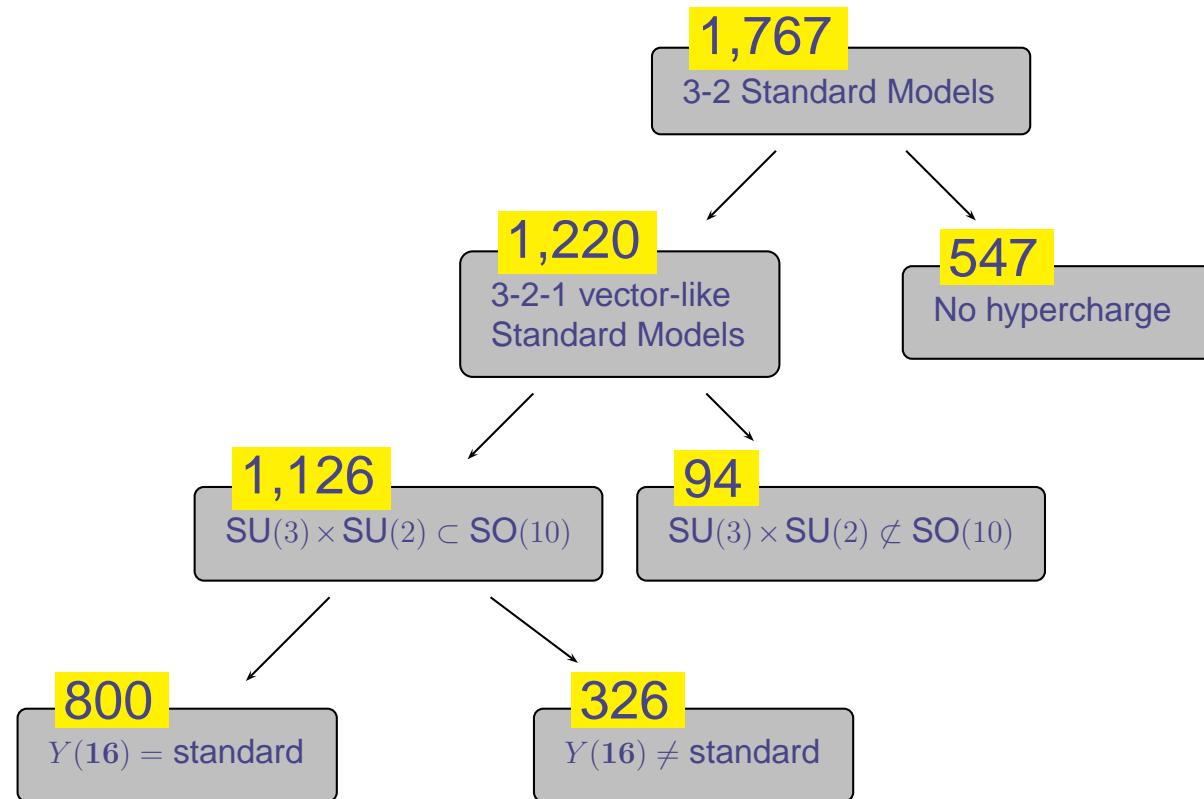
$$Y_1 = \left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{5}{6}, -\frac{5}{6}, \frac{5}{6}, 0, -2, 0, 0, 0, 0, 0, 2 \right)$$

$3 \times (\mathbf{3}, \mathbf{2})_{1/3}$	$5 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$7 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$16 \times (\mathbf{1}, \mathbf{2})_{-1}$	$45 \times (\mathbf{1}, \mathbf{1})_2$	$129 \times (\mathbf{1}, \mathbf{1})_0$
	$2 \times (\mathbf{3}, \mathbf{1})_{4/3}$	$4 \times (\mathbf{3}, \mathbf{1})_{-2/3}$	$13 \times (\mathbf{1}, \mathbf{2})_1$	$42 \times (\mathbf{1}, \mathbf{1})_{-2}$	

$$Y_2 = \left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{5}{6}, -\frac{5}{6}, \frac{5}{6}, 0, -2, -\frac{4}{5}, -\frac{4}{5}, -\frac{4}{5}, -\frac{4}{5}, -\frac{4}{5}, -2 \right)$$

$3 \times (\mathbf{3}, \mathbf{2})_{1/3}$	$5 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$16 \times (\mathbf{1}, \mathbf{2})_{-1}$	$30 \times (\mathbf{1}, \mathbf{1})_2$	$15 \times (\mathbf{1}, \mathbf{1})_{6/5}$
	$2 \times (\mathbf{3}, \mathbf{1})_{4/3}$	$13 \times (\mathbf{1}, \mathbf{2})_1$	$27 \times (\mathbf{1}, \mathbf{1})_{-2}$	$15 \times (\mathbf{1}, \mathbf{1})_{-6/5}$
	$7 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$		$79 \times (\mathbf{1}, \mathbf{1})_0$	$25 \times (\mathbf{1}, \mathbf{1})_{4/5}$
	$4 \times (\mathbf{3}, \mathbf{1})_{-2/3}$			$25 \times (\mathbf{1}, \mathbf{1})_{-4/5}$

The Results

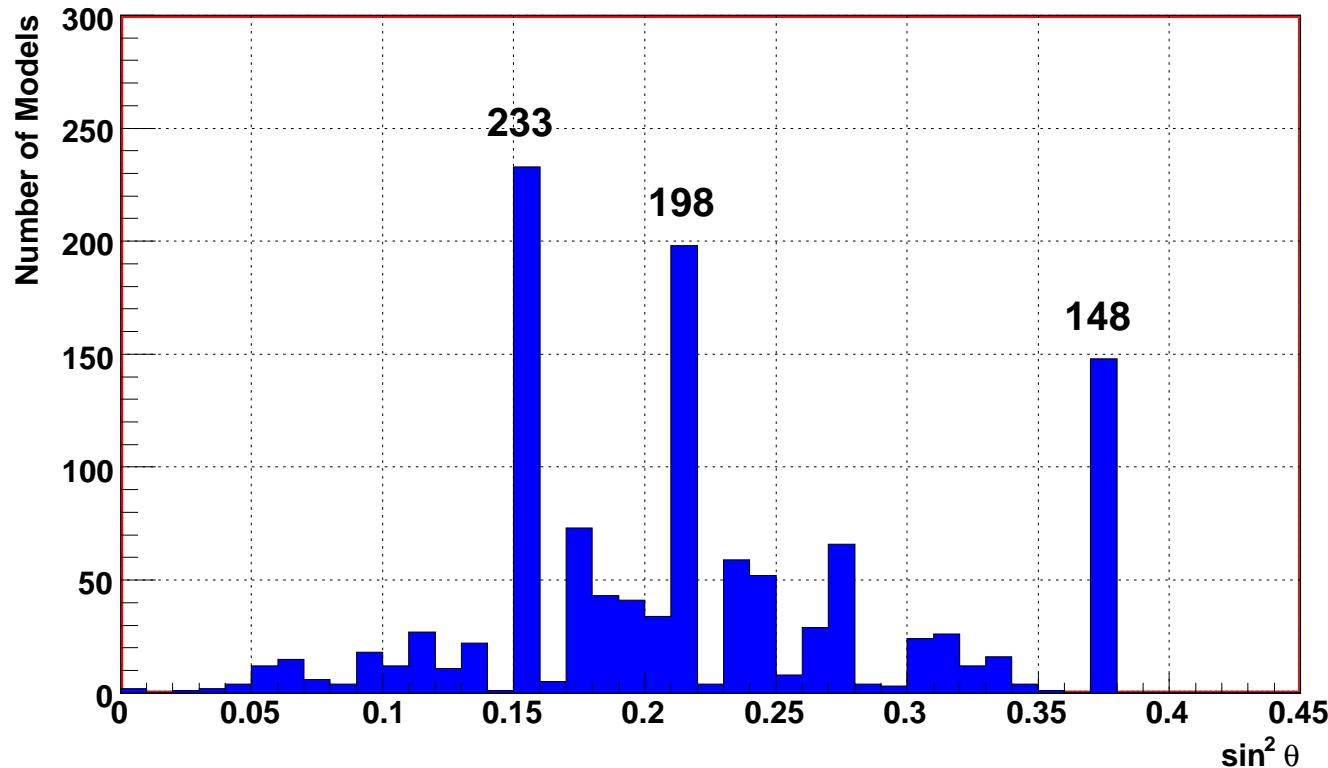


The number of 1,220 models (or 800) is to be compared w/
127 of the minilandscape search ...

The Weinberg Angle

Calculate Weinberg angle @ GUT scale:

$$\sin^2 \theta_w = \frac{g'^2}{g^2 + g'^2} = \frac{1}{1+C^2} \quad \text{with} \quad C^2 = \frac{1}{2} \|Y\|^2$$



Do Orbifolds support the idea of Unification?

Let us collect the facts:

- 3-2 Standard Models
 $\sim 1,767$
- Standard Model, no chiral exotics, $U(1)_Y$ from anywhere
 $\sim 1,220$ or 69%
- Standard Model, no chiral exotics, $U(1)_Y$ from anywhere, $SU_3 \times SU_2 \subset SO_{10}$
 $\sim 1,126$ or 64%
- Standard Model, no chiral exotics, $U(1)_Y$ from anywhere, $SU_3 \times SU_2 \subset SO_{10}$,
 $Y(16)$ standard
 ~ 800 or 45%
- Standard Model, no chiral exotics, $U(1)_Y$ from anywhere, $\sin^2 \theta$ standard
 ~ 148 or 8%
- Standard Model, no chiral exotics, with $U(1)_Y \subset SO(10)$
 ~ 127 or 7%

Tentative conclusions: (i) Standard Model quantum numbers easy to get, (ii) $\sin^2 \theta$ forces unification upon us, (iii) this might be different for shifts w/o $SO(10)$, (iv) need landscape study à la Dienes, Lennek, Gmeiner, Honecker, Schellekens

Unification

So how does unification in
orbifold models work in detail?

The “factor 20” problem

- Weakly coupled heterotic relates the gauge coupling with Newton’s constant:

$$G_N = \frac{1}{8} \alpha_{\text{string}} \alpha'$$

- For

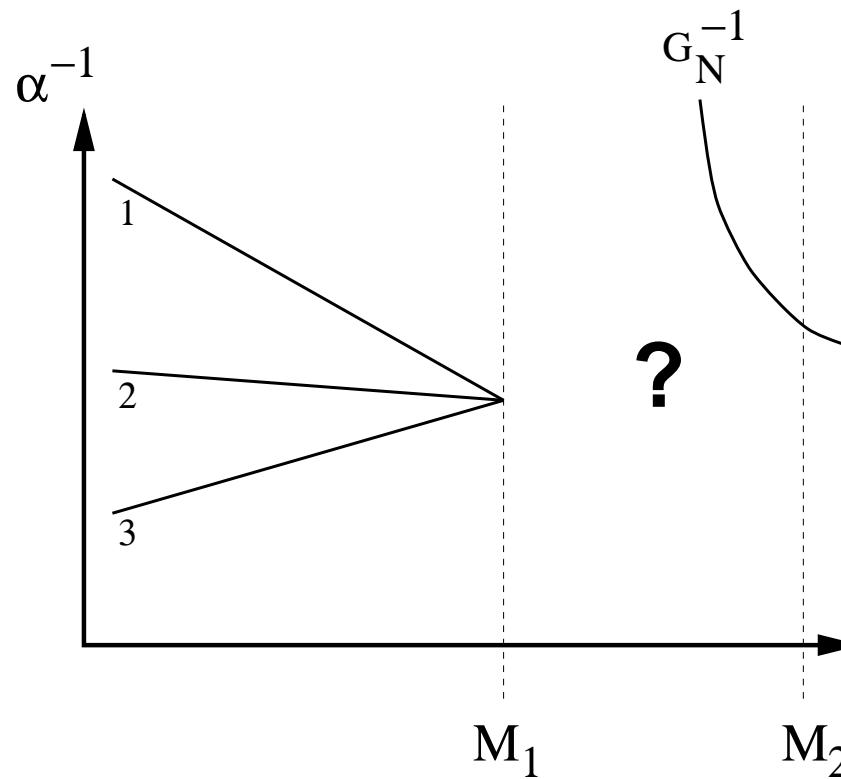
$$M_s = \frac{1}{\sqrt{\alpha'}}, \quad \alpha_{\text{string}} \simeq \alpha_{\text{GUT}} \simeq \frac{1}{24}, \quad M_s \simeq M_{\text{GUT}} \simeq 10^{16} \text{ GeV}$$

Newton’s constant is off for a factor of 400!

The “factor 20” problem

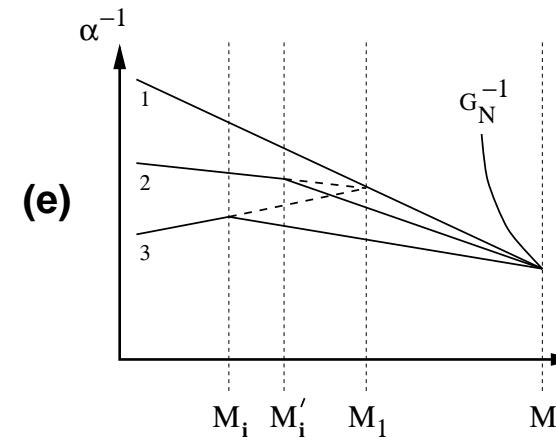
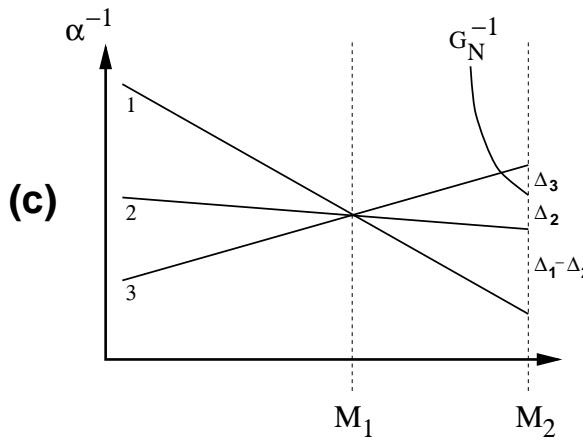
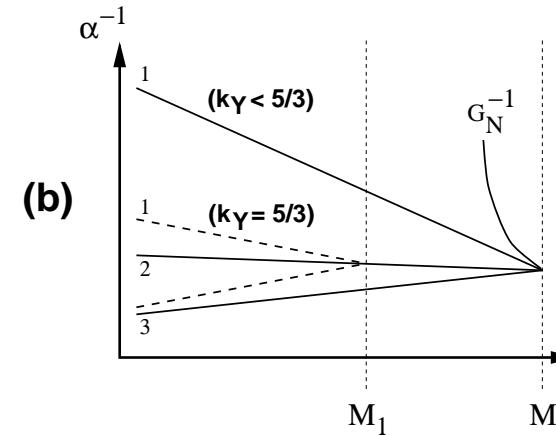
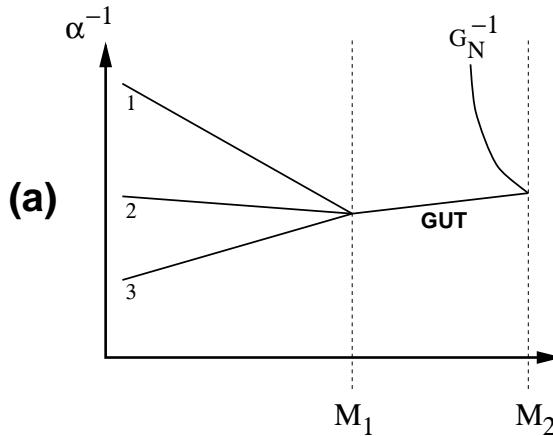
figure appropriated from Dienes

- Another way to put it: Gauge coupling unification (from experiment) is incompatible with stringy unification



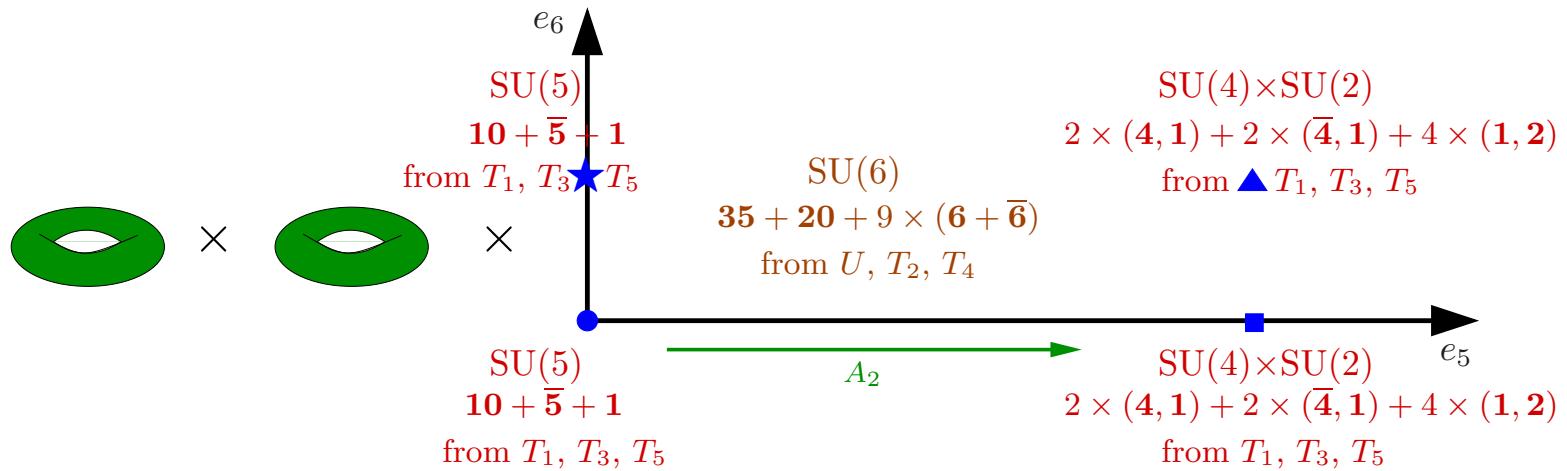
Various approaches to this problem

figures appropriated from Dienes



Anisotropic compactifications

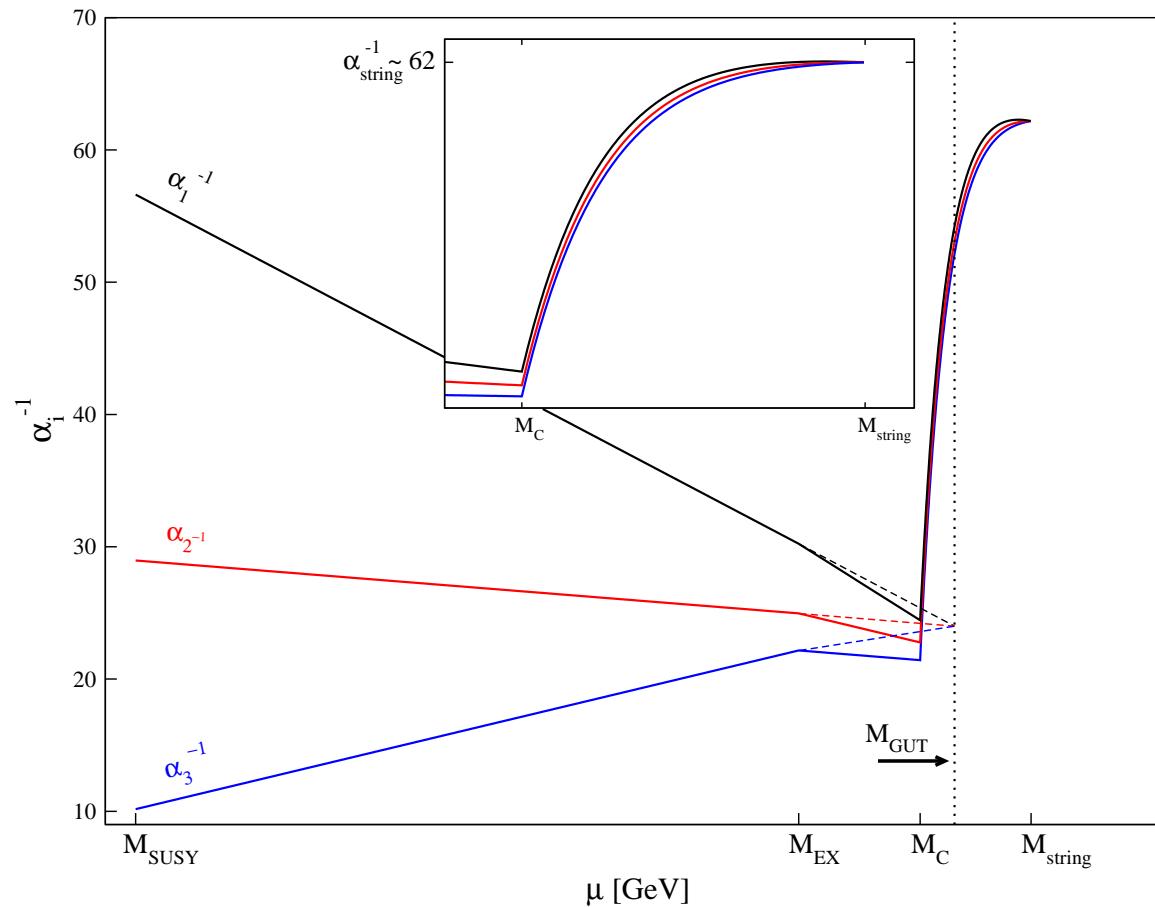
- Assume 1 torus $\sim M_{\text{GUT}}$ is larger than the others $\sim M_s$



- Give mass to vectorlike exotics at $M_C \lesssim M_{\text{GUT}}$
- Can the Kaluza-Klein tower of these states account for corrections to the RGEs such that all couplings unify?

Anisotropic compactifications

- Sadly, this simple (and natural) idea does not work
- But introducing another scale M_{EX} , unification works!



Conclusions

- Taking “string phenomenology” seriously means (i) reproducing the key features of the SM, (ii) extending the SM in a way so as to eliminate/mitigate its shortcomings
- String theory, in particular heterotic compactifications, can reproduce many key features of the Standard Model
- Models presented come closer to MSSM than any other string construction so far, but it is till a far way to go to reproduce what the phenomenologists and experimenters call the MSSM
- Unification is not “generic” in string compactifications, but can be “accommodated”
- Take unification as “experimental input” or guiding principle to limit the proliferation of vacua and deforest the landscape