Isometries
and
Approximate Flavor Symmetries
in
Local Models

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- *Continuous Global Symmetries* and Hyperweak Interactions in String Compactifications, arXiv:0805.4037
  C.Burgess, J.Conlon, J.Hung, S.Kom, AM, F.Quevedo
- J.P. Conlon, AM, F.Quevedo, T. Wiseman ....
- Numerical Ricci-flat metrics on K3, hep-th/0506129
  M.Headrick, T.Wiseman

Related Work

- Wave Functions and Yukawa Couplings in Local String Compactifications, arXiv:0807.0789
  J. Conlon, AM , F. Quevedo

Motivation/Introduction

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- Local Models
  - Many features of the physics of the Standard Model can be addressed in the local setting.
  - Can think of various modules for global issue like - cosmology, moduli fixing, SUSY breaking ...
  - Various local metrics/properties of singularities known explicitly.

Yukawas in Local Models

- Yukawa couplings in models involving magnetized D7 branes arise from dimensional reduction of the interaction term

$$\mathcal{L}_{\mathsf{yuk}} = \int d^8x \; \bar{\psi}\Gamma^M [A_M, \psi]$$

- Superpotential involves triple overlaps of solutions to appropriately twisted Dirac and Yang Mills equations on the surface being wrapped by the branes.

Local Models and Yukawas

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- A flavor symmetry can be generated if the local geometry has an isometry.

- Natural mechanism for the symmetry to be broken in a mild fashion.

Wavefunctions

J. Conlon, AM , F. Quevedo

- Wavefunctions for magnetized D7 branes wrapping $\mathbb{P}1 \times \mathbb{P}1$, $\mathbb{P}2$ (with a Fubini Study metric) embedded in a Calabi Yau
  - $\mathbb{P}1$:
$$\psi_{\mathsf{k}} = \frac{z^{\mathsf{k}}}{(1+z\bar{z})^{M/2}} \qquad \mathsf{k} \leq \mathsf{M}$$
  - $\mathbb{P}2$:
$$\psi_{\mathsf{kn}} = \frac{z_1^{\mathsf{k}} z_2^{\mathsf{n}}}{(1+z_1\bar{z}_1+z_2\bar{z}_2)^{M/2}} \qquad \mathsf{k}+\mathsf{n} \leq \mathsf{M}$$

  and complex conjugates.

- There are selection rules for a Yukawa to be non-vanishing, gamma functions
$$Y^{\mathsf{k_1 l_1, k_2 l_2}}_{\mathsf{m_1 n_1 p_1, m_2 n_2 p_2}} \propto \frac{1}{R_1^2} \frac{\Gamma(k_1+l_1)\Gamma(s-k_1-l_1)}{\Gamma(s)}$$
$$\mathsf{s} = \mathsf{m_1} + \mathsf{n_1} + \mathsf{p_1}$$

Isometries and Calabi-Yaus

- Compact Calabi-Yaus, with holonomy group *equal* to $SU(3)$ do not have any continuos isometries.
- Isometries can be present in non-compact geometries
  - Conifold :
    $SU(2) \times SU(2) \times U(1)$.
  - EH_3, Resolution of $\mathbb{C}^3/\mathbb{Z}_3$ singularity :
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- In the limit $\mathcal{V} \to \infty$, symmetry restored.
  Breaking parameter goes as inverse volume, *stable* against quantum corrections.
- The breaking and its effect on Yukawas can be studied explicitly by numerics.

Numerical Metrics on K3 - Headrick and Wiseman

- One parameter family of K3
  - Start with a square $T4$ of volume b^4, orbifold by $\mathbb{Z}2$.
  - Blow up sixteen the fixed points, to a $\mathbb{P}1$ of volume πa^2. (blow up parameter is the same for all of the singularities).
  - Scan the Kahler cone within this parameter space,

$$\alpha = \frac{4\pi a^2}{b^2} \qquad 0 < \alpha < 1$$

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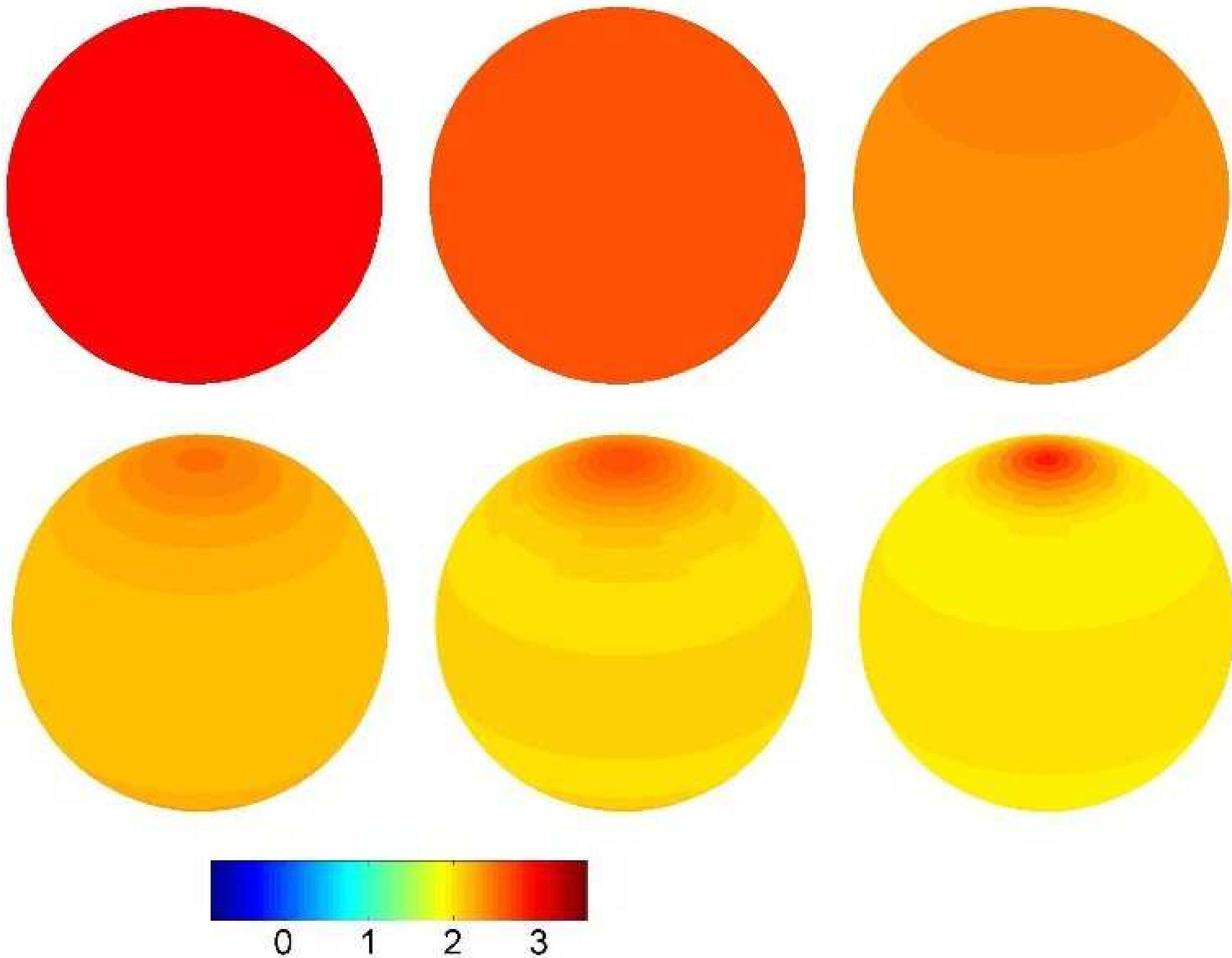
- Orbifold Limit and Eguchi-Hanson Geometry

$$ds^2 = \frac{dr^2}{f(r)} + \frac{r^2 f(r)}{4}(d\psi + \cos\theta d\phi)^2 + \frac{r^2}{4}(d\theta^2 + \sin^2\theta d\phi^2)$$

$f(r) = 1 - (\frac{a}{r})^4$, $r > a$,
  - $SO(3) \times U(1)$ isometry
  - minimal area two sphere at $r = a$ has a round metric.



Log (base10) of the Ricci scalar of on minimal area $\mathbb{P}1$ for

$$\alpha = 0.13, 0.28, 0.50, 0.79, 0.85, 0.92$$

- Numerical studies to obtain quantitative understanding of the hierarchy generated
  - nature of singularity
  - moduli dependence

Nature of Symmetries

- Banks and Dixon : No continuous exact global symmetries arise in string compactifications
- In string theory symmetries arise from the world sheet. World sheet currrent genrates the conserved charge

$$Q = \frac{1}{2\pi i}\int [dz j(z) - d\bar{z}\bar{j}(z)]$$

- The operators

$$j\bar{\partial}X^{\mu}\text{exp}(ik.X), \qquad \bar{j}\partial X^{\mu}\text{exp}(ik.X)$$

have conformal dimension (1,1) for $k^2 = 0$.

- Thus one can write vertex operator for gauge bosons

$$V = \int d^2z \; j\bar{\partial}X^{\mu}\text{exp}(ik.X), \qquad \int d^2z \; \bar{j}\partial X^{\mu}\text{exp}(ik.X)$$

which couple to the currents.

- What happens when we put in the open string sector ?
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- Open string vertex operators involve integration over the boundary.
- $\partial X^{\mu}\text{exp}(ik.X)$ has dimensions (1,0) for $k^2 = 0$ no room to insert currents.
- Any current associated with a worldsheet charge must be gauged in the closed string sector.

Approximate Global Symmetry at Low Energies

- Two scales for KK modes
  - Scale associated with the local geometry, $\frac{1}{R_s}$
  - Scale associated with the bulk, $\frac{1}{R_b}$
- Isometries are restored in the limit of $R_b \to \infty$, with R_s held fixed.
- In this limit bulk KK modes become massless, the symmetry is gauged by the bulk KK gauge bosons.

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  - Symmetry broken.
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- Symmetry is gauged in the higher dimensional effective field theory.
- In the four dimensional low energy effective action we do have a continous (approximate) global symmetry.

Large Volume Models

- A class of compactifications in IIB string theory with all moduli fixed.
- Underlying Calabi-Yau has $h_{21} > h_{11}$, atleast one blow up mode.
- For $W_0 = \mathcal{O}(1)$, Exponentially large volume,

$$\mathcal{V} \sim e^{a/g_s}$$

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- Standard Model is necessarily a local construction, with $D7$ branes wrapping local cycles.
- Susy broken predominantly F term of the volume modulus,

$$m_{3/2} = \frac{W_0}{\mathcal{V}}$$

Hierarchy problem solved by $\mathcal{V} \approx 10^{15}$, i.e $\mathrm{M_s} \approx 10^{11}$GeV

- Soft masses,

$$\frac{M_{\text{soft}}}{M_{KK}} \sim \frac{1}{\mathcal{V}^{1/3}} \ll 1$$

Thus the can safely integrate out the KK gauge bosons, leaving a (approximate) global symmetry in low energies.

- Size of the standard model cycle (R_s) set by SM couplings. Recall R_b was fixed to get the right SUSY breaking scale.

$$\frac{R_s}{R_b} \approx 0.01$$

- Thus for *Large Volume Models* the mechanism relates the electroweak hierarchy to the hierarchy in fermion masses.

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- The symmetry will be gauged, by the bulk Kaluza-Klein modes. For models in which the $M_{\mathrm{soft}} \ll M_{KK}$, Kaluza-Klein modes can be integrated out. The MSSM will possess an approximate continuous golbal symmetry (possibly non-abelian).

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- For *Large Volume Models*, mechanism correlates the weak scale with the hierarchy in fermion masses.
- Dependence on nature of singularity, moduli to be explored numerically.