dS vacua and modular inflation in supergravity and string theory

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CERN

dS vacua and modular inflation in supergravity and string theory

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Based on arXiv:0804.1073 and 0805:3290

In collaboration with:
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- Finding string backgrounds giving rise to de Sitter vacua/ modular inflation is important to make contact with pheno/cosmo.
- Although some examples are known, is in general difficult to find explicit realizations

Purpose of this talk: perform a general analysis (from the 4D effective N=1 sugra point of view) on the possibility of

(I) obtaining vacua with broken susy and a non-negative vacuum energy

(II) obtaining a successful model of modular inflation

OUTLINE:

1. de Sitter vacua
   - ★ derivation of the constraints
   - ★ some examples
2. modular inflation
   - ★ derivation of the constraints
   - ★ some examples
3. conclusions

N=1 SUGRA

- From a 4D eff. Lagrangian approach moduli fields are chiral multiplets of an N=1 SUGRA, and in terms of the complex scalar fields ϕ^i in the chiral multiplet

$$\mathcal{L}_{kin} = g_{i\bar{\jmath}} \partial\phi^i \partial\bar{\phi}^{\bar{\jmath}} \qquad \text{and} \qquad V = e^G \left(G_{i\,\bar{\jmath}} G^i G^{\bar{\jmath}} - 3 \right)$$

with

$$G = K(\phi, \bar{\phi}) + \log W(\phi) + \log \bar{W}(\bar{\phi})$$

that is invariant under Kahler transformations

$$(K, W) \rightarrow (K + \Delta + \bar{\Delta}, e^{-\Delta} W)$$

and also $g_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} G$

- Also $G^i = -e^{-G/2} F^i = -F^i / m_{3/2}$ are the order parameters of susy breaking

N=1 SUGRA

- We want to find local minima in which:

$$F^i \neq 0 \text{ and } V \geq 0$$

- The stationary condition ($\nabla_i V = 0$) implies that:

$$e^G\left(G_i + G^k \nabla_i G_k\right) + G_i V = 0$$

$$\nabla_i G_j = G_{ij} - G^k_{ij} G_k$$

- The stability condition requires that the matrix of second derivatives is positive definite:

$$V_{IJ} = \begin{pmatrix} V_{i\bar{j}} & V_{ij} \\ V_{\bar{i}\bar{j}} & V_{\bar{i}j} \end{pmatrix} > 0$$

Find K and W such that these conditions are satisfied!

String Compactifications

- K and W will depend on the details of the compactification
- For W one can have contributions from flux/torsion and/or non-perturbative effects

$$W = W_{flux} + W_{n.p.} = p_{ijk}\phi^i\phi^j\phi^k + A_i e^{-a_i\phi^i}$$

- One could think that its form is generic enough to find dS vacua...
  - but this is not the case
  - there is a necessary condition for the existence of stable dS vacua which is independent of the superpotential!

Constraints on dS vacua

$$V_{i\bar{\jmath}} = e^G \left(G_{i\bar{\jmath}} + \nabla_i G_k \nabla_{\bar{\jmath}} G^k - R_{i\bar{\jmath}m\bar{n}}\, G^m G^{\bar{n}}\right) + \left(G_{i\bar{\jmath}} - G_i G_{\bar{\jmath}}\right) V$$

$$V_{ij} = e^G \left(2\nabla_i G_j + G^k \nabla_i \nabla_j G_k\right) + \left(\nabla_i G_j - G_i G_j\right) V$$

- One could use $\nabla_i \nabla_j G_k$ to tune, for example $V_{ij} = 0$
- Then use $\nabla_i G_j$ to tune the eigenvalues of $V_{i\bar{\jmath}}$ to be positive
- But for $V_{i\bar{\jmath}}$ the projection $G^i \nabla_i G_k$ is fixed by the stationarity condition : $G^k \nabla_i G_k = -G_i + e^{-G} G_i V$

The stability of the mass matrix requires that:

$$\lambda = V_{i\bar{\jmath}} G^i G^{\bar{\jmath}} = e^G \left(2 g_{i\bar{\jmath}} G^i G^{\bar{\jmath}} - R_{i\bar{\jmath}p\bar{q}} G^i G^{\bar{\jmath}} G^p G^q\right) > 0$$

(mass of the sgoldstino non-tachyonic)

Constraints on dS vacua

- The stability condition can be rewritten as follows:

$$\lambda = e^G \Big(-\frac{2}{3} G^i G_i (G^i G_i - 3) + \sigma \Big) > 0$$

sign depends on rescalings of G^i

sign does not depend on rescalings of G^i

where

$$\sigma = \Big[\frac{1}{3}(g_{i\bar{\jmath}}\, g_{m\bar{n}} + g_{i\bar{n}}\, g_{m\bar{\jmath}}) - R_{i\bar{\jmath}m\bar{n}}\Big] G^i G^{\bar{\jmath}} G^m G^{\bar{n}}$$

- Thus this condition can be rewritten as follows:

$$\hat{\sigma} = \frac{\sigma}{(G^i G_i)^2} = \frac{2}{3} - \mathcal{R}_f > 0 \quad \Longrightarrow \quad \mathcal{R}_f > \frac{2}{3}$$

where
$$\begin{cases} \mathcal{R}_f = R_{i\bar{\jmath}p\bar{q}} f^i f^{\bar{\jmath}} f^p f^{\bar{q}} & \text{(sectional curvature)} \\ f^i = \dfrac{G^i}{\sqrt{G^k G_k}} & \text{(unit vector in the } G^i \text{ direction)} \end{cases}$$

Simple Examples

- $K = X\bar{X}$ $\begin{cases} \sigma = \frac{2}{3}(G^X G_X)^2 > 0 \quad (R_X = 0) \\ \text{always possible to obtain dS vacua} \end{cases}$

- $K = -n\,\mathrm{Log}(T+\bar{T})$ $\begin{cases} \sigma = \frac{2}{3n}(n-3)(G^T G_T)^2, \; (R_T = \frac{2}{n}) \\ n > 3 \;\; \text{to obtain dS vacua} \end{cases}$

- $K = -n\,\mathrm{Log}(T+\bar{T}) + X\bar{X}$ $\begin{cases} \text{dS vacua always possible} \\ \text{aligning } G^i \text{ with } G^X \end{cases}$

No-Scale Models

- We can particularize our condition for no-scale models

$$K^i K_i = 3$$

- From the no-scale condition it follows that:

$$\sigma(K^i) = \partial_i \sigma(K^i) = 0$$

  - the direction $G_i \propto K_i$ corresponds to a family of stationary points of σ with $\sigma = 0$

Therefore there are two possibilities:

$G^i = K^i$ is a maximum $\longrightarrow$ dS vacua NOT POSSIBLE

$G^i = K^i$ is not a maximum $\longrightarrow$ dS vacua POSSIBLE (depending on W)

Heterotic Models

- We consider a class of models which arises in compactifications of the heterotic string on Calabi-Yau threefolds
- The Kahler potential for the Kahler moduli is:

$$K = -\log \mathcal{V}\ , \qquad \text{with} \qquad \mathcal{V} = \frac{1}{3!}\, d_{ijk}\, (T^i + \bar{T}^i)(T^j + \bar{T}^j)(T^k + \bar{T}^k)$$

so that we get

$$\sigma = -\frac{4}{3}\left(G^i G_i\right)^2 + e^{2K} G^i G^j d_{ijp} g^{pq} d_{qmn} G^{\bar{m}} G^{\bar{n}}$$

- Example 1: K3-fibrations with a large P_1 base

$$K = -\log(T_1 + \bar{T}_1) - \log\left(d_{1ab}(T_a + \bar{T}_a)(T_b + \bar{T}_b)\right)$$

$$\boxed{\sigma \leq -(2G^1 G_1 - G^a G_a)^2 \leq 0}$$

Heterotic Models

- Example 2: generic two field model. One can compute:

$$\sigma \leq -\frac{1}{24}\, e^{4K}\, \frac{\Delta}{(\det g)^3}\, |C|^2$$

where

$$\Delta = -27\Big(d_{111}^2 d_{222}^2 - 3\, d_{112}^2 d_{122}^2 + 4\, d_{111} d_{122}^3 + 4\, d_{112}^3 d_{222} - 6\, d_{111} d_{112} d_{122} d_{222}\Big)$$

is the discriminant of the cubic polynomial defined by the volume

- Also $C = 0$ for $G^i = K^i$

$\sigma > 0$ for $\Delta < 0$!

(for example for $d_{112} = d_{122} = 0$)

Orientifold Models

- We consider now orientifold compactifications of type IIB
- The Kahler potential for the Kahler moduli is:

$$K = -2 \log \mathcal{V} \,, \qquad \text{with} \qquad \mathcal{V} = \frac{1}{48}\, d^{ijk} v_i v_j v_k$$

where now the Kahler moduli are defined in an implicit way:

$$T^i + \bar{T}^i = \frac{1}{8}\, d^{ijk} v_j v_k$$

- Example 1: K3-fibrations with a large P_1 base $\Rightarrow$ $\sigma \leq 0$

- Example 2: two field model $\Rightarrow$ $\sigma \leq \dfrac{\Delta}{24}\, e^{-4K}\, (\det g)^3\, |C|^2$

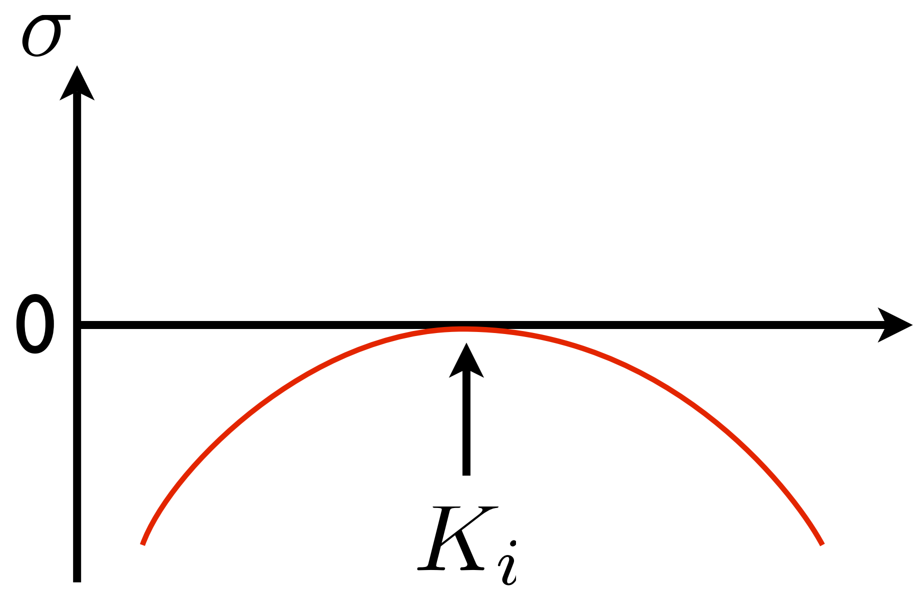
$\sigma > 0$ for $\Delta > 0$!
(for example for $d^{111} = d^{222} = 0$)

we get the opposite sign!

Subleading corrections

Subleading corrections to the Kahler potential may improve the situation when

$\sigma \leq 0$



Example: α' corrections $K = -\log(\mathcal{V} + \xi)$

breaks the no-scale condition !

$$\sigma \simeq 120 \frac{\xi}{\mathcal{V}} \left(1 + \frac{V}{3\, m_{3/2}^2} \right)^2$$

One can get $\sigma > 0$ depending on the sign of ξ

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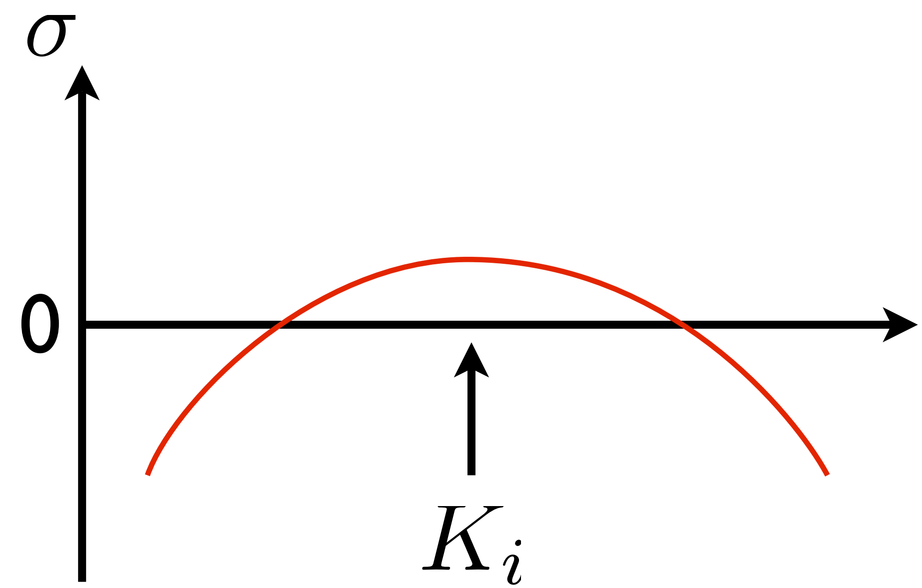

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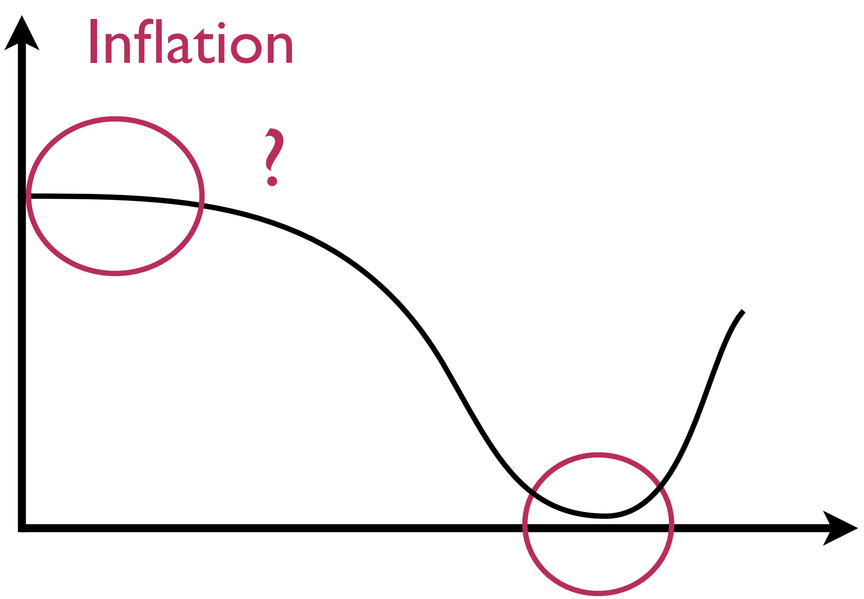
Summarising: In a given model characterised by K

If:

$$\sigma(G^i) > 0$$

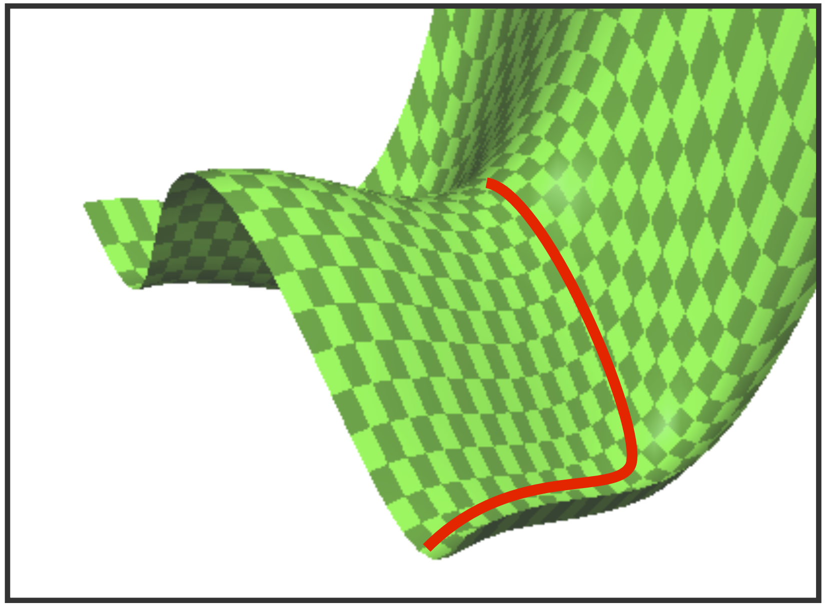
then, it is always possible to build dS vacua provided that there is enough freedom to tune W

★ Now, can we say something about slow-roll inflation ?



Modular Inflation

Consider a model with several complex scalars ϕ^i spanning a space with a metric $g_{i\bar{j}}$. Then, for a given potential V :



Slow-roll dynamics:

$$v_i = \frac{\nabla_i V}{V}$$

$$N = \frac{1}{V} \begin{pmatrix} \nabla^i \nabla_j V & \nabla^i \nabla_{\bar{j}} V \\ \nabla^{\bar{i}} \nabla_j V & \nabla^{\bar{i}} \nabla_{\bar{j}} V \end{pmatrix}$$

Multi-field slow-roll conditions:

$$\epsilon = \frac{\nabla^i V \nabla_i V}{V^2} \ll 1 \quad \& \quad \eta = \min \text{ eigenvalue} \{N\} \ll 1$$

One can say much about ϵ and $\eta = \min \text{eigenvalue}\,\{N\}$

$$\epsilon = \frac{\nabla^i V \nabla_i V}{V^2} \qquad N = \frac{1}{V}\begin{pmatrix} \nabla^i \nabla_j V & \nabla^i \nabla_{\bar{j}} V \\ \nabla^{\bar{i}} \nabla_j V & \nabla^{\bar{i}} \nabla_{\bar{j}} V \end{pmatrix}$$

(1) ϵ : can be made arbitrarily small by tuning $G^j \nabla_i G_j$

(2) $\nabla_i \nabla_j V$: can be adjusted as desired by tuning $\nabla_i \nabla_j G_k$

(3) $\nabla_i \nabla_{\bar{j}} V$: Most of its eigenvalues can be made arbitrarily large & positive by adjusting $\nabla_i G_j$

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One exception: Projection of $\nabla_i \nabla_{\bar{j}} V$ along G_i is restricted by K !

Constraints on Modular Inflation

- Note that for any given unit vector $u_I = \sum_k c_{(k)} \omega^I_{(k)}$ we get that:

$$u_I N^I_J u^J = \sum_k |c_{(k)}|^2 \underbrace{\lambda_{(k)}}_{\text{eigenvalues of N}} \geq \min\{\lambda_{(k)}\} \equiv \eta$$

- We can get a bound on η projecting N into the direction

$$u_I = \left(e^{-i\alpha} f_i, e^{i\alpha} f_{\bar{\imath}}\right) \quad \text{where} \quad f_i = \frac{G_i}{\sqrt{G^k G_k}}$$

- Doing this we get

$$\eta \leq \frac{\nabla_i \nabla_{\bar{\jmath}} V}{V} f^i f^{\bar{\jmath}} + \mathrm{Re}\left\{ e^{2i\alpha} \frac{\nabla_i \nabla_j V}{V} f^i f^j \right\}$$

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- Doing this we get

$$\eta \leq \frac{\nabla_i \nabla_{\bar{\jmath}} V}{V} f^i f^{\bar{\jmath}}$$

Constraints on Modular Inflation

- This projection is given by:

$$\frac{\nabla_i \nabla_{\bar{\jmath}} V}{V} f^i f^{\bar{\jmath}} = -\frac{2}{3} + \frac{4}{\sqrt{3}} \frac{1}{\sqrt{1+\gamma}} \mathrm{Re}\left\{ \frac{\nabla_i V}{V} f^i \right\} + \frac{\gamma}{1+\gamma} \frac{\nabla^i V \nabla_i V}{V^2} + \frac{1+\gamma}{\gamma} \hat{\sigma}(f^i)$$

$\leq \sqrt{\epsilon}$ $\qquad\qquad$ ϵ

where

$$\gamma = \frac{H^2}{m_{3/2}^2} \qquad \hat{\sigma}(f^i) = \frac{2}{3} - R(f^i) \qquad R(f^i) = R_{i\bar{\jmath}p\bar{q}}\, f^i f^{\bar{\jmath}} f^p f^{\bar{q}}$$

(holomorphic sectional curvature)

- Then we get that:

$$\eta \leq \eta_{\max} \equiv -\frac{2}{3} + \frac{4}{\sqrt{3}} \frac{1}{\sqrt{1+\gamma}} \sqrt{\epsilon} + \frac{\gamma}{1+\gamma} \epsilon + \frac{1+\gamma}{\gamma} \hat{\sigma}(f^i)$$

Thus, to have successful slow-roll inflation, a given model requires a Kahler geometry satisfying the condition:

$$\hat{\sigma}(f^i) \gtrsim \frac{2}{3}\frac{\gamma}{1+\gamma}$$

- This condition implies a strong restriction on the Kahler potential
- The condition for getting a scale of inflation much bigger than the gravitino scale is more difficult to realise
- If this condition is satisfied one still needs to tune W to adjust η to its appropriate value

Thus, to have successful slow-roll inflation, a given model requires a Kahler geometry satisfying the condition:

$$\hat{\sigma}(f^i) \gtrsim \frac{2}{3}\frac{\gamma}{1+\gamma} = \begin{cases} 2/3 & \text{if} \quad \gamma \gg 1 \quad (i.e.\ m_{3/2} \ll H) \\ 0 & \text{if} \quad \gamma \ll 1 \quad (i.e.\ m_{3/2} \gg H) \end{cases}$$

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Simple Examples

- $K = X\bar{X}$ $\longrightarrow$ $R(f^i) = 0$ $\longrightarrow$ $\hat{\sigma}(f^i) = 2/3$

Here the condition can be satisfied for arbitrary γ

- $K = -n\log(T+T)$ $\longrightarrow$ $R(f^i) = 2/n$

$\longrightarrow$ $\hat{\sigma}(f^i) = \frac{2}{3}\left(1 - \frac{3}{n}\right)$

The condition can only be satisfied for $n \gtrsim 3(1+\gamma)$

- $K = -n\log(T+\bar{T}) + X\bar{X}$

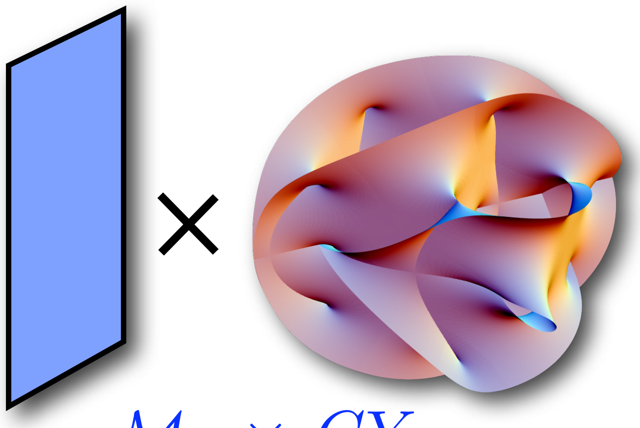
One may align the f^i in the X direction to satisfy our condition.

- $K = -n\log(T+\bar{T} - X\bar{X})$ $\longrightarrow$ $R(f^i) = 2/n$

This case is identical to the one given by $K = -n\log(T+\bar{T})$

CY String Models

✦ Let's apply this to the Kahler moduli sector in models emerging as CY compactifications of string theory



$\mathcal{M}_4 \times CY_3$

- No-scale property $K^i K_i = 3$
- Kahler geometry restricted
- Hence $\hat{\sigma}(f^i)$ restricted
- $k^i = K^i/\sqrt{3}$

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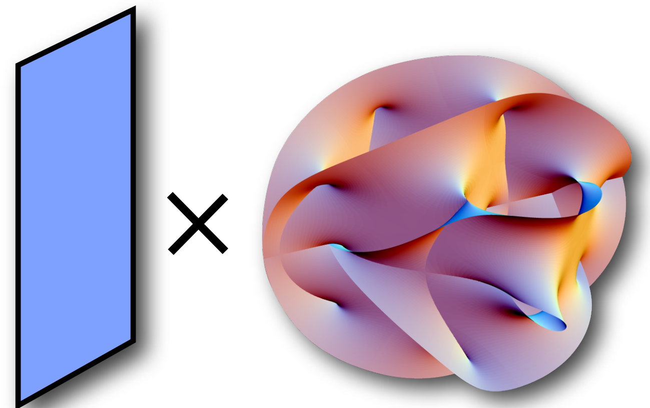

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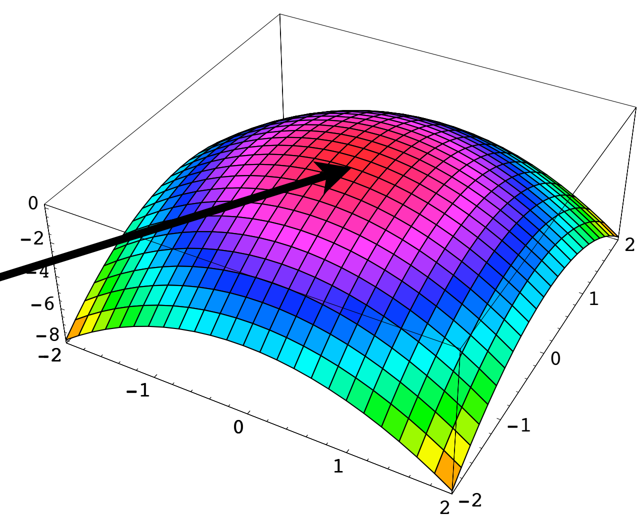

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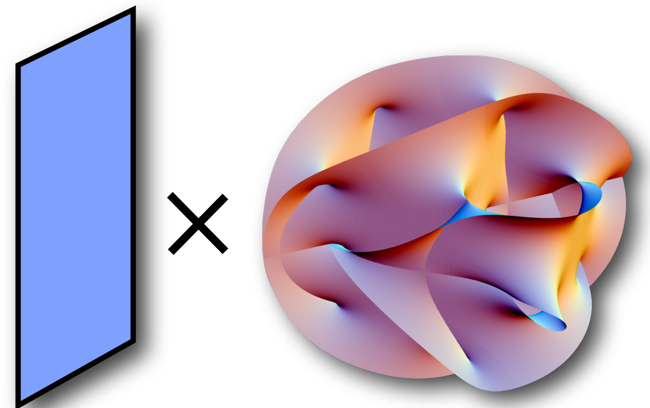

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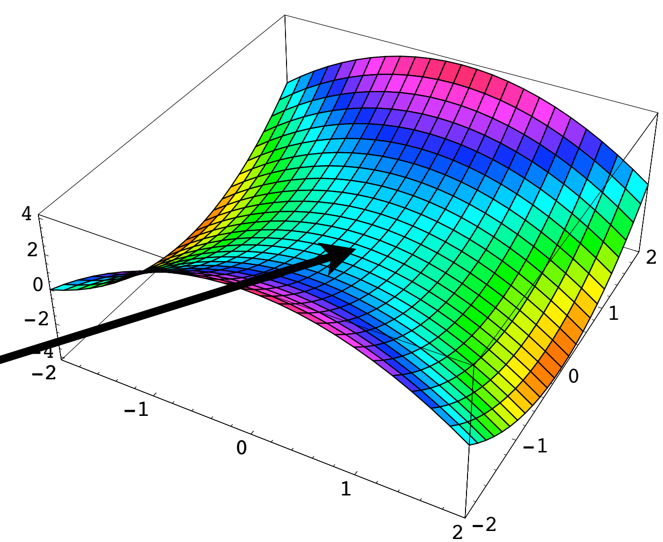

CY String Models

- K3-fibrations: no inflation possible
- Generic two-field models: inflation ok for arbitrary γ (depending on the sign of Δ !)
  - There are direction for which $\hat{\sigma}(f^i) > 2/3$ so models with $H \gg m_{3/2}$ are possible
- Subleading corrections can improve the situation, but...

$$K^i K_i \simeq 3 + \mathcal{O}(\delta) \quad \Longrightarrow \quad \hat{\sigma}(k^i) \simeq \mathcal{O}(\delta)$$

so only models with γ small are possible

$$\gamma \lesssim \mathcal{O}(|\delta|) \quad \Longrightarrow \quad H \ll m_{3/2}$$

CONCLUSIONS I

- In general, stable dS vacua with broken susy are only granted in models where a non-vanishing $F^i = m_{3/2} G^i$ exists such that:

$$\boxed{\sigma(G^i) > 0 \quad \longrightarrow \quad \mathcal{R}_f < \frac{2}{3}}$$

  - This condition is necessary and sufficient for a generic enough W

- For large-volume string compactifications $G_i \propto K_i$ corresponds to a family of stationary points of σ with $\sigma = 0$

- If these turn out to be maxima...

  - no vacua, unless subleading corrections are taken into account!

- For two field CY models there can be vacua, depending on the value of Δ !

CONCLUSIONS II

- The problem of obtaining slow-roll inflation in string theory is closely related to the characterisation of dS vacua:

$$\boxed{\hat{\sigma}(f^i) \gtrsim \frac{2}{3}\frac{\gamma}{1+\gamma} \quad \Longrightarrow \quad \mathcal{R}_f < \frac{2}{3}\frac{1}{1+\gamma}}$$

  - Models admitting dS vacua are good models to accommodate inflation as well !

- The condition to realise slow-roll inflation becomes stronger as the parameter $\gamma = H^2/m_{3/2}^2$ grows

- For no-scale models subleading corrections can provide with models of inflation, but with $H \ll m_{3/2}$

- For two field CY models, models of inflation with $H \gg m_{3/2}$ can be build depending on the sign of Δ !