Supersymmetry: Lecture 2: The Supersymmetrized Standard Model

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Part I: The Supersymmetrized SM: motivation and structure

before 2012, all fundamental particles we knew had spin 1 or spin 1/2
but we now have the Higgs: it's spin 0
of course spin-0 is the simplest possibility
spin-1 is intuitive too (we all understand vectors)
the world and (QM courses) would have been very different if the particle we know best, the electron, were spin-0
supersymmetry: boson $\leftrightarrow$ fermion
so from a purely theoretical standpoint, supersymmetry would provide an explanation for why we have particles of different spins

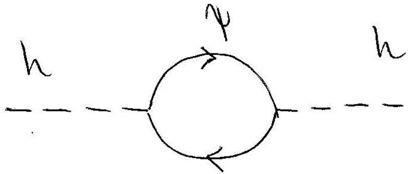
The Higgs and fine tuning:

because the Higgs is spin-0, its mass is quadratically divergent

$$\delta m^2 \propto \Lambda_{UV}^2 \tag{1}$$

unlike
fermions (protected by chiral symmetry)
gauge bosons (protected by gauge symmetry)

Higgs Yukawa coupling: quark (**top**) contribution



also, gauge boson, Higgs loops

practically: we don't care (can calculate anything in QFT, just put in a counter term)
theoretically: believe Λ_{UV} is a concrete physical scale, eg: mass of new fields, scale of new strong interactions
then

$$m^2(\mu) = m^2(\Lambda_{UV}) + \# \Lambda_{UV}^2 \tag{2}$$

$m^2(\Lambda_{UV})$ determined by the full UV theory
$\#$ determined by SM
we know LHS: $m^2 \sim 100^2$ GeV^2
if $\Lambda_{UV} = 10^{18}$ GeV
we need $m^2(\Lambda_{UV}) \sim 10^{36}$ GeV^2
and the 2 terms on RHS tuned to 32 orders of magnitude..
such dramatic tunings are not natural

this is the “fine-tuning”, or “naturalness” problem
in general:
the parameters of the 2 theories must be tuned to $\mathrm{TeV}^2/\Lambda_{UV}^2$

what we saw yesterday:
with supersymmetry (even softly broken):
scalar masses-squared have only log divergences:

$$m^2(\mu) = m^2(\Lambda_{UV}) \left[1 + \# \log \left(\frac{m^2(\Lambda_{UV})}{\Lambda_{UV}^2} \right) \right] \tag{3}$$

just as for fermions!
because:
supersymmetry ties the scalar mass to the fermion mass

the quadratic divergence from fermion loops is cancelled by the quadratic divergence from scalar loops

cutoff only enters in log
$m^2(\Lambda_{UV})$ can be order $(100\ \text{GeV})^2$

this is the main motivation for supersymmetric extensions of the SM
there are other motivations too:
supersymmetry often supplies:
Dark Matter (DM) candidates
new sources of CP violation
and theoretically: extending space time symmetry is appealing
so let's supersymmetrize the SM!

Field content: gauge

each gauge field is now part of a vector supermultiplet
recall

$$A^a_\mu \to (\tilde{\lambda}^a, A^a_\mu) + D^a \tag{4}$$

$$G^a_\mu \to (\tilde{g}^a, G^a_\mu) + D^a \tag{5}$$

physical fields: gluon + gluino

$$W^I_\mu \to (\tilde{w}^I, W^I_\mu) + D^I \tag{6}$$

physical fields: W + wino

$$B_\mu \to (\tilde{b}, B_\mu) + D_Y \tag{7}$$

physical fields: B + bino

Field content: matter

each fermion is now part of a chiral supermultiplet

$$(\phi, \psi) + F \tag{8}$$

we take all SM fermions
q, u^c, d^c, l, e^c
to be L-fermions

$$q \to (\tilde{q}, q) + F_q \quad \text{all transforming as} \ \ (3,2)_{1/6} \tag{9}$$

physical fields: (doublet) quark q + squark $\tilde{q}$

$$u^c \to (\tilde{u}^c, u^c) + F_u \quad \text{all transforming as} \ \ (\bar{3},1)_{-2/3} \tag{10}$$

physical fields: (singlet) up-quark u^c + up squark $\tilde{u}^c$

$$d^c \to (\tilde{d}^c, d^c) + F_d \quad \text{all transforming as} \ \ (\bar{3},1)_{1/3} \tag{11}$$

physical fields: (singlet) down-quark d^c + down squark $\tilde{d}^c$

$$l \to (\tilde{l}, l) + F_l \quad \text{all transforming as } \ (1,2)_{-1/2} \tag{12}$$

physical fields: (doublet) lepton l + slepton $\tilde{l}$

$$e^c \to (\tilde{e}^c, e^c) + F_e \quad \text{all transforming as } \ (1,1)_1 \tag{13}$$

physical fields: (singlet) lepton e^c + slepton $\tilde{e}^c$

with EWSB: the doublets split:

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \tilde{q} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix} \tag{14}$$

$$l = \begin{pmatrix} \nu \\ l \end{pmatrix} \qquad \tilde{l} = \begin{pmatrix} \tilde{\nu} \\ \tilde{l} \end{pmatrix} \tag{15}$$

Interactions: gauge

nothing to do: completely dictated by gauge symmetry + supersymmetry
we wrote the Lagrangian for a general gauge theory in the previous lecture:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{gauge} + \mathcal{D}^\mu \phi_i^* \mathcal{D}_\mu \phi_i + \psi_i^\dagger i\bar{\sigma}^\mu \mathcal{D}_\mu \psi_i \\ &- \sqrt{2} g \left(\phi_i^* \lambda^{aT} T^a \varepsilon \psi_i - \psi_i^\dagger \varepsilon \lambda^{a*} T^a \phi_i\right) - \frac{1}{2} D^a D^a \end{aligned} \quad (16)$$

where

$$D^a = -g \phi_i^\dagger T^a \phi_i \quad (17)$$

applying this to the SM:

$$\psi_i = q_i, u_i^c, d_i^c, l_i, e_i^c \qquad \phi_i = \tilde{q}_i, \tilde{u}_i^c, \tilde{d}_i^c, \tilde{l}_i, \tilde{e}_i^c \quad (18)$$

the covariant derivatives now contain the SU(3), SU(2), U(1) gauge fields
λ^a sums over the SU(3), SU(2), U(1) gauginos

$$\lambda^a \to \tilde{g}^a, \tilde{w}^I, \tilde{b} \tag{19}$$

there are D terms for SU(3), SU(2), U(1)

$$D^a \to D^a, D^I, D_Y \tag{20}$$

and there's of course the pure gauge Lagrangian that I didn't write (we saw it in the previous lecture) let's look at the scalar potential

$$V = \frac{1}{2} D^a D^a + \frac{1}{2} D^I D^I + \frac{1}{2} D_Y D_Y \tag{21}$$

where
for SU(3): (recall $T_{\bar{3}} = -T_3^*$ and we will write things in terms of the fundamental generators)

$$D^a = g_3 \left(\tilde{q}^\dagger T^a \tilde{q} - \tilde{u}^{c\dagger} T^{a*} u^c - \tilde{d}^{c\dagger} T^{a*} u^c \right) \tag{22}$$

similarly for the SU(2) and

$$D_Y = g_Y \sum_i Y_i \tilde{f}_i^\dagger \tilde{f}_i \tag{23}$$

get: 4 scalar interactions with coupling = gauge couplings

note: no freedom (and no new parameter)
but so far: no Higgs
so let's put it in

Field content: Higgs fields

The SM Higgs is a complex scalar, so it must be part of a chiral module

$$H \to (H, \tilde{H}) + F_H \quad \text{all transforming as} \;\; (1,2)_{-1/2} \qquad (24)$$

we immediately see a problem: (in fact, many problems, which are all related)

1) There is a problem with a single Higgs *scalar*:
we want the Higgs (and **only** the Higgs) to get a VEV
but the Higgs is charged under SU(2), U(1)
$\rightarrow$ nonzero D terms:

$$V \sim D^I D^I + D_Y^2 \tag{25}$$

where

$$D^I = g_2 \langle H^\dagger \rangle T^I \langle H \rangle \quad D_Y = g_1 \frac{1}{2} \langle H \rangle^\dagger \langle H \rangle \tag{26}$$

that is: EWSB implies SUSY breaking!

you might think this is good, but it's not (for many reasons)
here's one:
the non-zero D-terms would generate masses for the squarks, sleptons:
consider D_Y for example:

$$D_Y = \frac{1}{2}v^2 + \sum_i Y_i |\tilde{f}_i|^2 \quad (27)$$

where $\tilde{f}$ sums over all squarks, sleptons and Y_i is their hypercharge
consider $V \sim D^2$
some of the squarks will get negative masses-squared of order v^2
this is a disaster: SU(3), EM broken at v!
solution: add a second Higgs scalar, *with opposite charges*
the 2 scalars should then get equal VEVs with all $D = 0$

2) $\tilde{H}$ is a Weyl fermion
if this is all there is, we will have a massless fermion around—the Higgsino
we don't see one
but the problem is worse:

in the presence of massless fermions, gauge symmetries can become anomalous

that is, the gauge symmetry can be broken at the loop level

the SM is amazing: the fermion content is such that there are no anomalies

so far we added scalars (squarks and sleptons, known collectively as sfermions) which are harmless

and gauginos: these are fermions, but they are adjoint fermions, and these don't generate any anomalies (adjoint = real rep)

but the Higgsino $\tilde{H}$ is a massless fermion which is a doublet of SU(2) and charged under $\mathrm{U}(1)_Y$

the simplest way to cancel the anomaly is to add a second Higgsino in the conjugate rep

so we add a second Higgs field

when we consider interactions, we will see other reasons why we must do this so call the SM Higgs H_D and the new Higgs H_U

$$H_D \to (H_D, \tilde{H}_D) + F_{HD} \quad \text{all transforming as} \;\; (1,2)_{-1/2} \tag{28}$$

$$H_U \to (H_U, \tilde{H}_U) + F_{HU} \quad \text{all transforming as} \;\; (1,2)_{1/2} \tag{29}$$

and in the limit of unbroken supersymmetry

$$\langle H_U \rangle = \langle H_D \rangle \tag{30}$$

in the SM: we must add a quartic potential for the Higgs field

$$\lambda (H^\dagger H)^2 \tag{31}$$

here there is some potential: got it for free—from the D terms **a quartic Higgs potential! with quartic coupling =** g_2, g_Y**!**
(but it won't necessarily give mass to the physical Higgs)

Yukawa couplings

In the SM we have Higgs-fermion-fermion Yukawa couplings

consider the down-quark Yukawa first

$$y_D H_D q^T \varepsilon d^c \quad (\text{Higgs } - \text{ quark} - \text{ quark}) \tag{32}$$

with supersymmetry, this must be accompanied by

$$+y_D \left(\tilde{q} \tilde{H}_D^T \varepsilon d^c + \tilde{d}^c \tilde{H}_D^T \varepsilon q\right) \quad (\text{squark } - \text{ Higgsino} - \text{ quark})$$

all coming from the superpotential

$$W_D = y_D H_D q d^c \tag{33}$$

similarly for the lepton Yukawa:

$$\begin{aligned} W_l &= y_l H_D l e^c \rightarrow & (34) \\ \mathcal{L}_l &= y_l (H_D l^T \varepsilon e^c + \tilde{l} \tilde{H}_D^T \varepsilon e^c + \tilde{e}^c \tilde{H}_D^T \varepsilon l + \mathrm{hc}) & (35) \\ & \quad \mathrm{Higgs - lepton - lepton} & \\ &+ \quad \mathrm{slepton - Higgsino - lepton} & (36) \end{aligned}$$

what about the up Yukawa?
need

$$(\mathrm{Higgs})q^T \varepsilon u^c \tag{37}$$

this coupling must come from a superpotential

$$(\mathrm{Higgs})qu^c \tag{38}$$

in the SM (Higgs)$= H_D^\dagger$
but the superpotential is **holomorphic** : no daggers allowed
this is the 4th reason why we needed a second Higgs field with the opposite charges (but they are all the same reason really)

$$W_U = y_U H_U q u^c \rightarrow \tag{39}$$

$$\mathcal{L}_U = y_U(H_U q^T \varepsilon u^c + \tilde{q}\tilde{H}_U^T \varepsilon u^c + \tilde{u}^c \tilde{H}_U^T \varepsilon q) + \mathrm{hc} \tag{40}$$

you can see what's going on:
holomorphy makes a scalar field "behave like a fermion":
in a supersymmetric theory, the interactions of scalar fields are controlled by the superpotential, which is holomorphic
for a fermion to get mass you need an LR coupling
so starting from an L fermion you need an R fermion
or another L fermion with the opposite charge(s)
for a scalar ϕ to get mass in a non-supersymmetric theory: you don't need anything else (just use ϕ^*)
not so in a susy theory
because you cant use ϕ^*, must have another scalar with the opposite charge(s)

Summary: The Higgs Yukawas

we have 2 Higgs fields H_U and H_D
the SM Yukawa couplings come from the superpotential

$$W = y_U H_U q u^c + y_D H_D q d^c + y_l H_D l e^c \tag{41}$$

but note: no freedom (and no new parameter)

R-symmetry

Also note: we have a $U(1)_R$ symmetry:
let's take:
gaugino$= -1$
sfermions$= 1$
Higgsinos$= 1$
(all others neutral)
The Lagrangian is invariant

to recap:
we wrote down the Supersymmetric Standard Model
gauge bosons + gauginos (spin 1/2)
fermions + sfermions (spin 0)
2 Higgses + 2 Higgsinos (spin 1/2)
the interactions are all dictated by SM + SUSY:
the new ones are:
gauge-boson - scalar - scalar
gauge-boson - gauge-boson - scalar - scalar
gaugino-sfermion-fermion
gauge-boson Higgsino Higgsino
4-scalar (all gauge invariant contributions)
all these have couplings = gauge couplings
in particular: a 4-Higgs coupling: quartic Higgs potential

Yukawa part:
Higgsino-quark-squark
coupling = SM Yukawa
consistent with the $U(1)_R$ symmetry
$\rightarrow$ in each of the interactions: the new superpartners appear in pairs!
this is important both for the LHC
and for DM

Implications

no quadratic divergence in Higgs mass:

each quark contribution canceled by L, R squarks
the top loop canceled by L, R stops

similarly: Higgs self coupling (from D term) canceled by Higgsino

each gauge boson contribution canceled by gaugino

Implications

but we now have:
massless gluinos
a wino degenerate with the W
a selectron degenerate with the electron etc
supersymmetry must be broken:
somehow the gluino, wino, slectron etc should get mass
it would be nice if the SSM broke supersymmetry spontanously
(after all we have lots of scalars with a complicated potential)
but no such luck
so we must add more fields and interactions that break supersymmetry
these new fields must couple to the SM fields in order to generate masses for the superpartners

The supersymmetrized standard model with supersymmetry-breaking superpartner masses

General structure

SB ——— SSM

SB = new fields and interactions such that supersymmetry is spontaneously broken
$\rightarrow$ in SB: mass splittings between bosons-fermions of the same multiplet

——— = some coupling(s) between SSM fields and SB fields
$\rightarrow$ mass splitting between SM fields and their superpartners

the couplings ——— mediate the breaking
this is what determines the supersymmetry-breaking terms in the SSM
(and leads to different experimental signatures)

the supersymmetry-breaking terms: what do we expect?
remember: any term is allowed unless a symmetry prevents it
now that we broke supersymmetry, new supersymmetry breaking terms are allowed
matter sector: sfermions get mass
(fermions don't: protected by chiral symmetry)

gauge sector: gauginos get mass
(gauge bosons don't: protected by gauge symmetry)

Higgs sector: Higgses get mass
(Higgsinos don't: protected by chiral symmetry
so this isn't so good and we have to do something about it)

in addition: there are trilinear scalar terms that can appear:
Higgs-squark-squark Higgs-slepton-slepton
(allowed by gauge symmetry, and supersymmetry is no longer there to forbid them)

So the supersymmetry-breaking part of the SSM Lagrangian is:

$$
\begin{aligned}
\mathcal{L}_{soft} &= -\frac{1}{2}[\tilde{m}_3\tilde{g}^T\varepsilon\tilde{g} + \tilde{m}_2\tilde{w}^T\varepsilon\tilde{w} + \tilde{m}_1\tilde{b}^T\varepsilon\tilde{b}] \\
&- \tilde{q}^*\tilde{m}_q^2\tilde{q} - \tilde{u}^{c*}\tilde{m}_{uR}^2\tilde{u}^c - \tilde{d}^{c*}\tilde{m}_{dR}^2\tilde{d}^c \\
&- \tilde{l}^*\tilde{m}_l^2\tilde{l} - \tilde{e}^{c*}\tilde{m}_{eR}^2\tilde{e}^c \\
&- H_U^* m_{H_U}^2 H_U - H_D^* m_{H_D}^2 H_U \\
&- H_U\tilde{q}^* A_U\tilde{u}^c - H_D\tilde{q}^* A_U\tilde{d}^c - H_D\tilde{l}^* A_l\tilde{e}^c \\
&- B\mu H_U H_D
\end{aligned}
\tag{42}
$$

- the last line: a quadratic term for the Higgs scalars
- the line before last: new trilinear scalar interactions
  when the Higgses get VEVs these too will turn into sfermion mass terms (mixing L and R scalars)
- m_q^2 etc are 3×3 matrices in generation space
  so are the A-terms (A_U etc)

the values of the (supersymmetry breaking) parameters are determined by the SB theory and (mainly) the mediation

you sometimes hear people criticize supersymmetric extensions of the SM for having a hundred or so new parameters (the parameters of $\mathcal{L}_{soft}$)

but as we said: these are all determined by the SB and the mediation

often: very few new parameters

also remember:
the parameters of $\mathcal{L}_{soft}$ are the only freedom we have
and where all the interesting physics lies:
they determine the spectrum of squarks, sleptons
these in turn determine the way supesymmetry manifests itself in Nature
= experimental signatures

R-parity

The gaugino masses and A-terms break the $U(1)_R$ symmetry
but there's something left: a Z_2
this is R-parity:
under R-parity: gauginos, sfermions, Higgsinos: odd
all SM fields: even
so: supersymmetrizing the SM (without adding any new interactions)
we have a new parity
$\rightarrow$ **the lightest superpartner is stable**

the mu-term: a supersymmetric Higgs, Higgsino mass

before we go on, let's discuss one remaining problem:
we have 2 massless Higgsinos in the theory
(can't get mass by supersymmetry-breaking)
so must also include a supersymmetric mass term:

$$W = \mu H_U H_D \tag{43}$$

Mediating the breaking

what can mediate supersymmetry breaking?
what is the coupling ——— ?
anything:
gauge interactions → Gauge Mediated Supersymmetry Breaking (GMSB)
Planck-suppressed interactions → Anomaly Mediated Supersymmetry Breaking (AMSB), mSUGRA
even Yukawa-like interactions

Gauge interactions

gauge interactions are the ones we know best
so gauge mediation gives full, concrete (and often calculable)
supersymmetric extensions of the SM
so let's start with this

we can start with a toy example to illustrate how things work
we saw the O'Raifeartaigh model

$$W = \phi\,(\phi_1^2 - f) + m\phi_1\phi_2 \tag{44}$$

supersymmetry is sponatneously broken
recall: the spectrum contains a supermultiplet
with supersymmetry-breaking mass splittings:
a fermion of mass m
scalars of masses-squared $m^2 + 2f$, $m^2 - 2f$,
let's complicate the model slightly

$$W = \phi\,(\phi_{+1}\phi_{1-} - f) + m\phi_{+1}\phi_{2-} + m\phi_{-1}\phi_{2+} \tag{45}$$

now the model has a $U(1)$ symmetry

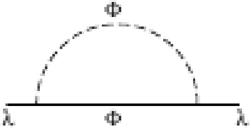
you can show that supersymmetry is still broken
again we will have supermultiplets with supersymmetry breaking splittings
but now let's assume that the $U(1)$ symmetry is hypercharge
a squark is charged under hypercharge: so it couples to these split supermultiplets
a squark mass will be generated!
this isn't a very good model (gauginos don't get mass)
but it gives you an idea of how things work

The simplest gauge mediation models: Minimal Gauge Mediation

suppose we have a supersymmetry-breaking model with
chiral supermultiplets Φ_i and $\bar{\Phi}_i$, $i = 1, 2, 3$
such that
the fermions ψ_{Φ_i} and $\psi_{\bar{\Phi}_i}$ combine into a Dirac fermion of mass M
the scalars have masses-squared $M^2 \pm F$
$(F < M^2)$
now identify i as an SU(3) color index

so Φ is a 3 of SU(3), $\bar{\Phi}$ is a $\bar{3}$ of SU(3)
and these fields have supersymmetry-breaking masses

the gluino talks to the Φ's directly
$\rightarrow$ gets mass at one loop



the squarks talk to the gluino
$\rightarrow$ the squarks get mass at two loops

so we have
a gluino mass at one loop:

$$m_{\tilde{g}} = \# \frac{\alpha}{4\pi} \frac{F}{M} + \mathcal{O}(F^2/M^2) \tag{46}$$

a squark mass-squared at two loops:

$$m_{\tilde{q}} = \# \frac{\alpha^2}{(4\pi)^2} \frac{F^2}{M^2} + \mathcal{O}(F^4/M^6) \tag{47}$$

the numbers are group theory factors
we can infer this very simply:

- loop factor
- the masses should vanish as $F \to 0$
- the masses should vanish as $M \to \infty$

this is very elegant

- soft masses are determined by gauge couplings
- the squark matrices are flavor-blind ($\propto 1_{3\times3}$ in flavor space)
- gluino masses $\sim$ squark masses
- the only new parameter is F/M (a scale) (almost)
  if want soft masses around TeV, $F/M \sim 100$ TeV

the new fields Φ are the *messengers* of susy breaking in order to give masses to everything we need messenger field charged under SU(3), SU(2), U(1)

eg, N_5 copies of $(3,1)_{-1/3} + (\bar{3},1)_{1/3}$ and $(1,2)_{-1/2} + (1,2)_{1/2}$
(filling up a $5 + \bar{5}$ of SU(5))
parameters:
N_5 (number of messengers)
F/M (overall scale)

and there's running: the soft masses are generated at the messenger scale$\sim M$

to calculate them at the TeV we need to include RGE effects

so the messenger scale M is also important the gravitino mass $m_{3/2} = F_{eff}/M_P$

where F_{eff} is the the dominant F term

so

$$m_{3/2} \geq \frac{F}{M_P} \sim 100 \text{ TeV} \frac{M}{M_P} \tag{48}$$

for a low messenger scale, the gravitino can be very light (eV)

this is just a simple toy model: gauge mediation can in principle have a very different structure
the only defining feature is that the soft masses are generated by the SM gauge interactions
so there are a few generic features:

- colored superpartners (gluinos, squarks) are heavier than non-colored (EW gauginos, sleptons..) by a factor

$$\frac{\alpha_3}{\alpha_2} \quad \text{or} \quad \frac{\alpha_3}{\alpha_1} \tag{49}$$

- in particular: gaugino masses scale as

$$\alpha_3 : \alpha_2 : \alpha_1 \tag{50}$$

  the bino is lightest
- no A terms at M
- a light gravitino

Gravity Mediation

with gauge mediation, we had to do some real work:
add new fields, make sure they get some supersymmetry-breaking masses
but supersymmetry breaking is one place where we expect a free lunch:
imagine we have, in addition to the SM, some SB fields
eg, the O'Raifeartaigh model
since supersymmetry is a space-time symmetry, the SM fields should know this automatically
we would expect soft terms to be generated, suppressed by M_P
this is known as "gravity mediation"
we will discuss first the purest form of gravity mediation: anomaly mediation
and then what's commonly referred to as gravity mediation

Anomaly mediation

so we imagine supersymmtery is broken by some fields that have no coupling to the SM (the hidden sector)
the gravitino gets mass $m_{3/2}$ (a **scale**)

would the SSM "know" about supersymmtery breaking?
yes: at the quantum level, it's not scale-invariant:
all the couplings (gauge, Yukawa) run—they are scale dependent
the beta functions are nonzero
so **all** the soft terms are generated

gaugino masses:

$$m_{1/2} = b\frac{\alpha}{4\pi}\, m_{3/2} \tag{51}$$

α is the appropriate coupling
b is the beta-function coefficient

so for SU(3) $b = 3$, for SU(2) $b = -1$ and for U(1) $b = -33/5$

sfermions get masses proportional to their anomalous dimensions:

$$m_0^2 \sim \frac{1}{16\pi^2}(y^4 - y^2 g^2 + b g^4)\, m_{3/2}^2 \tag{52}$$

for the first and second generation sfermions, we can neglect the Yukawas so

$$m_0^2 \sim \frac{g^4}{16\pi^2}\, b\, m_{3/2}^2 \tag{53}$$

A terms are generated too, proportional to the beta functions of the appropriate Yukawa

this is amazing: these contributions are **always there**
everything determined by SM couplings
one new parameter: the gravitino mass

too good to be true..
while SU(3) is ASF $b_3 > 0$, SU(2), U(1) are not: $b_2, b_1 < 0$
so the sleptons are tachyonic
there are various fixes to this..

but the gaugino masses are fairly robust:
putting in the numbers:

$$m_{\tilde{w}} : m_{\tilde{b}} : m_{\tilde{g}} : m_{3/2} \sim 1 : 3.3 : 10 : 370 \qquad (54)$$

wino(s) are lightest!

(the gravitino is roughly a loop factor heavier than the SM superpartners)

Gravity mediation: mediation by Planck suppressed operators

return to our basic setup

SSM: the supersymmetric standard model

SB: new fields and interactions that break supersymmetry (the "hidden sector")

generically, we expect higher-dimension operators (suppressed by M_P) that couple the SB fields and the SSM fields

supersymmetry breaking $\leftrightarrow$ non-zero F terms (or D terms) for the SB fields

so these will generate supersymmetry-breaking terms in the SSM sfermion masses from

$$\frac{|F|^2}{M_P^2}\tilde{f}^\dagger \tilde{f} \tag{55}$$

gaugino masses from

$$\frac{|F|}{M_P}\lambda^T \varepsilon \lambda \tag{56}$$

you can think of these as mediated by tree-level exchange of Planck-scale fields unlike the previous two schemes, here we don't know the order-one coefficients: consider eg the doublet-squarks

$$c_{ij}\,\frac{|F|^2}{M_P^2}\tilde{q}_i^\dagger\tilde{q}_j \tag{57}$$

so

$$(m^2_{\tilde{q}})_{ij} = c_{ij}\,m_0^2 \quad \text{where} \quad m_0 \equiv \frac{|F|}{M_P} \tag{58}$$

and c_{ij} are order-one numbers
in "minimal sugra", or the cMSSM one **assumes**

$$c_{ij} = \delta_{ij} \tag{59}$$

it is not easy to justify this: the Yukawas are presumably generated at this high scale, so there are flavor-dependent couplings in the theory

all this is at the high scale (where the soft masses are generated)
running to low scales:

$$\frac{d}{dt} m_{1/2} \propto \frac{\alpha}{4\pi} m_{1/2} \tag{60}$$

starting from a common gaugino mass at the GUT scale one finds at low energies:
the gaugino masses scale as

$$\alpha_3 : \alpha_2 : \alpha_1 \tag{61}$$

as in gauge mediation
(bino lightest)
the gravitino mass?
of order the superpartner masses

Other possibilities

these are a few possibilities but by no means an exhaustive list
example: Flavored Gauge Mediation:
in minimal gauge mediation: messenger fields
$(1,2)_{1/2}$ and $(1,2)_{-1/2}$
same charges as H_U and H_D
so in principle: superpotential couplings of the messengers to matter fields
new (calculable) contributions to soft terms

Implications

EWSB and the Higgs mass

The MSSM Higgs spectrum

In the SSM: H_U and H_D:

$$\langle H_U \rangle = \begin{pmatrix} v_U \\ 0 \end{pmatrix} \qquad \langle H_D \rangle = \begin{pmatrix} 0 \\ v_D \end{pmatrix} \tag{62}$$

let's start in the SUSY limit (and no mu term)

$$D = 0 \quad \longrightarrow \qquad v_U = v_D \tag{63}$$

count scalars:
8 real dofs
3 eaten by $W^\pm$, Z

consider the heavy Z supermultiplet:

- a heavy gauge boson (3 polariztions, or dof's)
- must have a Dirac fermion (4 dof's)
- need one more real scalar: coming from the Higgs fields

similarly for $W^{\pm}$ so 3 real scalars "join" the heavy $W^{\pm}$, Z supermultiplets
usually called $H^{\pm}$ and H; with masses M_W, M_Z
2 neutral fields remain:
(2 because must form the complex scalar of a chiral supermultiplet)
h (real part: CP even) and A (imaginary part: CP odd)
NO POTENTIAL for h

NO POTENTIAL for h : not surprising
we haven't added any Higgs superpotential so only quartic is from V_D
but along D-flat direction: physical Higgs is massless
Higgs mass must come from supersymmetry breaking !

EWSB

fortunately supersymmetry is broken—we have soft terms
The Higgs potential comes from the following sources:
quadratic terms:
A. the mu term: $W = \mu H_U H_D$

$$\delta V = |\mu|^2 |H_U|^2 + |\mu|^2 |H_D|^2 \quad (64)$$

B. the Higgs soft masses:

$$\delta V = \tilde{m}^2_{H_U} |H_U|^2 + \tilde{m}^2_{H_D} |H_D|^2 \quad (65)$$

so need $m^2_{H_U} < 0$ and/or $m^2_{H_U} < 0$
C. the $B\mu$ term:

$$\delta V = B\mu H_U H_D + \mathrm{hc} \quad (66)$$

quartic terms:

$$\delta V = \frac{1}{2} g_2^2 D^I D^I + \frac{1}{2} g_1^2 D_Y D_Y \tag{67}$$

where

$$D^I = H_U^\dagger \tau^I H_U - H_D^\dagger \tau^{I*} H_D \tag{68}$$

and

$$D_Y = \sum_i Y_i \tilde{f}_i^\dagger \tilde{f}_i + \frac{1}{2}(H_U^\dagger H_u - H_D^\dagger H_D) \tag{69}$$

parameters: 2 VEVs:
trade for:
1. $\sqrt{v_U^2 + v_D^2}$: determined by W mass to be 246 GeV
2. $\tan\beta \equiv v_U/v_D$

requiring a minimum of the potential determines:

$$B\mu = \frac{1}{2}(m_{H_U}^2 + m_{H_D}^2 + 2\mu^2)\sin 2\beta \quad (70)$$

$$\mu^2 = \frac{m_{H_D}^2 - m_{H_U}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{M_Z^2}{2} \quad (71)$$

so for given $m_{H_U}^2$, $m_{H_D}^2$: $B\mu$ and μ determined
free parameters: $\tan\beta$, sign(μ)

scalar spectrum:

$$\begin{aligned}
H^{\pm} \quad &: \quad M_W^2 + M_A^2 \qquad\qquad (\mathrm{SUSY}: M_W^2) \\
H^0 \quad &: \quad \frac{1}{2}(M_Z^2 + M_A^2) + \frac{1}{2}\sqrt{(M_Z^2 + M_A^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta} \\
& \qquad\qquad\qquad\qquad\qquad\qquad (\mathrm{SUSY}: M_Z^2) \\
A^0 \quad &: \quad M_A^2 = B\mu(\cot\beta + \tan\beta) \qquad (\mathrm{SUSY}: 0)
\end{aligned} \tag{72}$$

for the light Higgs (SUSY:=0)

$$m_h^2 = \frac{1}{2}(M_Z^2 + M_A^2) - \frac{1}{2}\sqrt{(M_Z^2 + M_A^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta} \tag{73}$$

PREDICTION:

$$m_h \leq m_Z |\cos 2\beta| \leq M_Z \tag{74}$$

The measurement of the Higgs mass provides the first quantitaive test of the Minimal Supersymmetric Standard Model

[saturated for $M_A^2 \gg M_Z^2$: the DECOUPLING LIMIT]

does it fail?
the result (73) is at tree-level
there are large radiative corrections from stop masses
(will see why soon)
in the decoupling limit

$$m_h^2 \sim m_Z^2 \cos^2 2\beta + \frac{3m_t^2}{4\pi^2 v^2} \left[\log \frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2}\right] \tag{75}$$

where

$$\begin{aligned} X_t &= A_t - \mu \cot\beta \quad \text{the LR stop mixing} \\ M_S &= \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \quad \text{the average stop mass} \end{aligned}$$

can raise Higgs mass to around 130 GeV

for 126 GeV need: heavy stops and/or large stop A terms
fine-tuning !

at best, stops at 1.5-2TeV

at worst: minimal gauge mediation: no A-terms at messenger scale
stops around 8-10 TeV (and other squarks close)

so: Higgs mass is a stronger constraint than direct searches

caveat: can easily add a quartic potential for the Higgs (see next slide) let's compare this to SM (part I: quartic): not so bad

SM: added a quartic Higgs potential to get the Higgs mass
here we didn't have to: D-terms give a quartic potential
but no new parameter: $\lambda = g$

could add a quartic interaction a la the SM:
must add at least one new field:
a SM singlet S: with

$$W = \lambda S H_U H_D \quad \rightarrow \quad V = \lambda^2 (|H_U|^2 |H_D|^2 + \ldots) \tag{76}$$

aka the "NMSSM" Next to Minimal SSM

compare to SM (part II: quadratic): much more beautiful

SM: EWSB by hand: put in a negative mass-squared

MSSM: a dynamical origin:
recall: we needed $\tilde{m}^2_{H_U} < 0$ or $\tilde{m}^2_{H_D} < 0$
this happens (almost) automatically in SUSY theories:
the RGEs drive the Higgs mass-squared negative!
(through Yukawa coupling to stop)

dynamical origin of EWSB !

suppose we start with $\tilde{m}^2_{H_U} > 0$ at the supersymmetry breaking scale

$$\frac{d}{dt} m^2_{H_U} \sim + \frac{g^2}{16\pi^2}\, m^2_{1/2} - \frac{y_t^2}{16\pi^2}\, \tilde{m}_t^2 \tag{77}$$

a large negative contribution because of

1. the large Yukawa (compared to SU(2), U(1) coupling)
2. the stop is colored (color factor $= 3$)

NOTE: many scalars in MSSM but Higgs is special:
it's an SU(3) singlet: so no large $(+)$ contribution from gluino
it does have an order-1 Yukawa (to the colored stop)
so the Higgs develops a VEV

Recap: EWSB and Higgs

putting aside (..) the 125 GeV Higgs mass:

supersymmetry gives a very beautiful picture:

the MSSM (SSM + soft terms): only log divergence
the quadratic divergence in the Higgs mass-squared cancelled by superpartners at $\tilde{m}$
(tuning $\sim M_Z^2/\tilde{m}^2$)
$\rightarrow$ **the hierarchy between the EWSB scale and the Planck/GUT scale is stabilized**

furthermore:

starting with $\tilde{m}^2_{H_U} > 0$ in the UV:
the running (from stop) drives it negative
electroweak symmetry is broken: proportional to $\tilde{m}$

and finally:

with a SB sector that breaks supersymmetry dynamically:
the supersymmetry breaking scale is exponetially suppressed: $\tilde{m}$ can naturally be around the TeV
the correct hierarchy between the EWSB scale and the Planck/GUT scale is generated!

with $m_h = 126$ GeV:
Minimal SSM is stretched: need heavy stops: tuning is worse
more practically: discovery becomes more of a challenge

now that we understand supersymmetry breaking and EWSB
let's turn to the superpartner spectrum

Neutralino spectrum

we have 4 neutral 2-component spinors: two gauginos and 2 Higgsinos

$$\tilde{b}\,, \quad \tilde{W}^0\,, \quad \tilde{H}_D^0\,, \quad \tilde{H}_U^0 \tag{78}$$

with the mass matrix

$$\begin{pmatrix} M_1 & 0 & -g_1 v_D/\sqrt{2} & g_1 v_U/\sqrt{2} \\ 0 & M_2 & g_2 v_D/\sqrt{2} & -g_2 v_U/\sqrt{2} \\ -g_1 v_D/\sqrt{2} & g_2 v_D/\sqrt{2} & 0 & \mu \\ g_1 v_U/\sqrt{2} & -g_2 v_U/\sqrt{2} & \mu & 0 \end{pmatrix} \tag{79}$$

4 neutralinos $\tilde{\chi}^0$ $i = 1, \ldots, 4$
similarly: 2 charginos (charged Higgsino+wino) $\tilde{\chi}_i^\pm$ $i = 1, 2$

Sfermion spectrum

consider eg up squarks
6 complex scalars: $\tilde{u}_{Li} \quad \tilde{u}_{Ra}$
6×6 mass-squared matrix:

$$\begin{pmatrix} m^2_{LL} & m^2_{LR} \\ m^{2\dagger}_{LR} & m^2_{RR} \end{pmatrix} \tag{80}$$

consider $m^2_{U,LL}$: gets contributions from:

1. the SSM Yukawa (supersymmetric)
2. the SUSY breaking mass-squared
3. the D-term (because $D \sim v_U^2 - v_D^2 + \tilde{q}^\dagger T q + \cdots$) (supersymmetry breaking)

$$m^2_{U,LL} = m_u^\dagger m_u + \tilde{m}_q^2 + D_U 1_{3\times 3} \tag{81}$$

consider m^2_{LR}: gets contributions from:

1. the A term (susy breaking)
2. the μ term:

$$\left|\frac{\partial W}{\partial H_D}\right|^2 \quad \to \quad \frac{\partial W}{\partial H_D} = \mu H_U + y_U q u^c \tag{82}$$

so

$$m^2_{U,LR} = v_U \left(A^*_U - y_U \mu \cot\beta\right) \tag{83}$$

Flavor structure

in quark mass basis (up, charm, top):

- up squark mass matrix
- bino - u_{Li} - $\tilde{u}_{Lj}$ interaction
- bino - u_{Ri} - $\tilde{u}_{Rj}$ interaction
- ...

for a generic up squark mass matrix:
physical parameters: 6 masses + mixings

similarly for 6 down squarks, 6 charged sleptons

(3 sneutrinos: LL only)

Flavor structure

neglect for simplicity LR: and consider 3 L up squarks:

- up squark mass matrix $m^2_{U,LL}$ (3×3)
- bino - u_{Li} - $\tilde{u}_{Lj}$ interaction

working in **quark mass** basis:

$bino - u_{Li} - \tilde{u}_{Li}$ interaction: defines 3 flavor eigenstates:
$\tilde{u}_L$, $\tilde{c}_L$, $\tilde{t}_L$

but if we're interested in LHC production: want squark mass eigenstates

so: diagonalize $m^2_{U,LL}$ to get 3 mass eigenstates $\tilde{u}_{L,a}$ with $a = 1, 2, 3$

now the bino-quark-squark interaction is not diagonal:

$$K_{ia}\ bino - u_{Li} - \tilde{u}_{La}$$

we get mixings between the different generations: each squark (mass state) is a composition of the 3 flavor states

are sfermions degenerate? is $m^2_{U,LL} \propto 1$?
that depends on the mediation of supersymmetry
but remember: we don't understand fermion masses
their structure is very strange, probably hinting towards a fundamental theory of flavor
if there is such a theory, it will also control the structure of $m^2_{U,LL}$ and the other sfermion mass-matrices

R-parity violating couplings

so far, we generalized the SM gauge and Yukawa interactions
but we can add *new* Yukawa like interactions

$$W = \lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \lambda''_{ijk} u^c_i d^c_j d^c_k \tag{84}$$

(these the only terms we can add: nothing else is gauge invariant)

- $\rightarrow\ \tilde{l}_l - l_j - e^c_k$ etc: break R-parity
- the first 2 terms break lepton number, the 3rd breaks baryon number
  if they are all there: get proton decay !!
- further constraints from flavor-violation: very roughly: only couplings that involve the third generation can be substantial