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FLAVOR & CPV

What is flavor?

- In SM: basic constituents of matter - excitations of fermionic fields (spin = 1/2)

  matter flavors = several copies of same gauge representation

- ~~Under~~ Under unbroken SM gauge group $SU(3)_c \times U(1)_{EM}$
  - up-type quarks: $(3)_{2/3}$: u, c, t
  - down-type quarks: $(3)_{-1/3}$: d, s, b
  - charged leptons: $(1)_{-1}$: e, μ, τ
  - neutrinos: $(1)_0$: ν_1, ν_2, ν_3

  $\longleftrightarrow$ differ only in mass

- ordinary matter essentially 1'st gen.:
  - u, d bound within protons & neutrons
  - electrons (forming atoms)
  - "electron neutrinos" (admixture of $\nu_{1,2,3}$) produced in fusion reactions inside stars

- 2'nd & 3'rd gen. particles produced in high-energy collisions
  - decay via weak interactions into 1'st gen particles

One of Big open questions in fundamental physics

SUSY 2013

- Why 3 almost identical replicas of quarks & leptons?
- Which is the origin of different masses?

"Flavor physics" refers to interactions that distinguish between flavors.

Within SM: weak & Yukawa interaction

"Flavor parameters" that carry flavor indices.

Within SM: 9 masses of charged fermions

4 mixing parameters (3 angles + phase) - $W^+ \bar{u}^i d^j$ interaction

(Adding Majorana mass terms for neutrinos: 3 ν masses

6 mixing parameters (3 angles + 3 phases)

$W^+ \bar{\nu}^i e^j$ interaction)

"Flavor universal" interactions with couplings proportional to the identity in flavor space (also weak interactions in interaction basis)

Within SM: QCD & QED ; also "flavor blind". $\propto I^{ij}$

"Flavor diagonal" interactions with couplings that are diagonal (in mass basis), but not neccessarily universal

Within SM: Yukawa interaction of the Higgs boson (in the mass basis)

ICTP

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"flavor changing" processes where initial & final flavor-numbers are different (number of particles with certain flavor − number of anti-particles of the same flavor).

- "FC charged current": both up- and down-type quark flavors or both charged lepton & neutrino flavors are involved: $\mu^- \to e^- \bar{\nu}_i \nu_j$, $K^- \to \mu^- \bar{\nu}_i$ $(s\bar{u} \to \mu^- \bar{\nu}_i)$

  within SM: 1 W exchange @ tree level $(A \propto G_F)$

- "FC neutral currents": involve either up- or down-type flavors but not both; and/or either charged lepton flavors or neutrino flavors but not both: $\mu^- \to e^- \gamma$, $K_L \to \mu^+ \mu^-$ $(s\bar{d} \to \mu^+\mu^-)$

- within SM: occur at higher orders in weak expansion (loops) often highly suppressed

"flavor violation" (later)

Why is flavor physics interesting?

- can discover NP or probe it before it is directly observed in high-energy experiments
- smallness of $\dfrac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu^+\nu)}$ → prediction of charm quark

- $\Delta m_K = m_{K_L} - m_{K_S}$ $\rightarrow$ prediction of charm quark mass
- $\Delta m_B = m_{B_H^0} - m_{B_L^0}$ $\rightarrow$ prediction of top quark mass
- observation of $K_L \rightarrow \pi^+\pi^-$ (ϵ_K) $\rightarrow$ prediction of 3'rd generation

CPV

- within SM: single CPV parameter in flavor changing processes

Baryogenesis requires new CPV sources!

EW hierarchy problem (quadratic sensitivity of EW scale to UV physics)

- requires NP $\lesssim$ TeV
- if generic flavor structure $\Rightarrow$ FCNCs orders of magnitude above observed rates

$\Rightarrow$ flavor probes NP scales $\gtrsim \mathcal{O}(10^5\,\text{TeV})$

"NP flavor puzzle" NP @ TeV needs to exhibit approximate flavor symmetries

SM flavor parameters

- hierarchical : $m_u \ll m_c \ll m_t$
- most are small : $m_{f \neq t} \ll m_{W,t}$

"SM flavor puzzle"

Points to some unknown flavor dynamics?

Flavor in SM

- Any (local) QFT model ($\mathcal{L} = ?$) specified by
  (i) symmetries + pattern of spontaneous breaking
  (ii) representations of fermions & scalars

- SM Lagrangian:

(i) $G^{SM}_{\substack{local \\ (gauge)}} = SU(3)_c \times SU(2)_L \times U(1)_Y$

$G^{SM}_{local} \rightarrow SU(3)_c \times U(1)_{EM}$

(ii) $Q_L^i \; (3,2)_{1/6}$ $\quad u_R^i \; (3,1)_{2/3}$ $\quad D_R^i \; (3,1)_{-1/3}$ $\quad L_L^i \; (1,2)_{-1/2}$ $\quad E_R^i \; (1,1)_{-1}$

$i = 1,2,3$

$\phi \; (1,2)_{1/2}$: $\quad \langle \phi^0 \rangle = \frac{v}{\sqrt{2}} \simeq 174 \text{ GeV}$

$$\mathcal{L}_{SM} = \underbrace{\mathcal{L}^{SM}_{\substack{kinetic \\ (gauge)}}}_{} + \underbrace{\mathcal{L}^{SM}_{Higgs}}_{} + \underbrace{\mathcal{L}^{SM}_{Yukawa}}_{}$$

simple & symmetric
completely specified
by G^{SM}_{local} + fermion reps.
3 params. (g, g', g_s)

EWSB
2 params

SM flavor dynamics
flavor parameters

Interaction basis

$$\mathcal{L}^{SM}_{\text{kinetic}} = \sum_{i,j=1,\dots,3} \sum_{\psi = Q_L \dots E_R} \bar{\psi}^i \, i \not{D} \, \delta^{ij} \psi^j - \frac{1}{4} \sum_{a=1,\dots,8} G^a_{\mu\nu} G^{a,\mu\nu} - \frac{1}{4} \sum_{a=1,\dots,3} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Flavor universal
CP conserving

$$\left(D^\mu = \partial^\mu + i g_s G^\mu_a L_a + i g W^\mu_b T_b + i g' B^\mu Y \right)$$

$L_a =$ $SU(3)_c$ generators $(0, \frac{1}{2}\lambda_a, \dots)$

$T_a =$ $SU(2)_L$ generators $(0, \frac{1}{2}\sigma_b, \dots)$

$Y =$ $U(1)_Y$ charges

$$\mathcal{L}^{SM}_{\text{Higgs}} = \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

also CP conserving
(and $SO(4)$ symmetric - custodial symmetry)

$\mathcal{L}^{SM}_{\text{kinetic}}$ ($+ \mathcal{L}^{SM}_{\text{Higgs}}$) have large global flavor symmetry corresponding to independent unitary rotations in flavor space of 5 fermionic fields

$$G^{SM}_{\text{flavor}} = SU(3)^3_q \times SU(3)^2_e \quad [\times \, U(1)^5]$$

$$SU(3)^3_q = SU(3)_Q \times SU(3)_U \times SU(3)_D$$

$$SU(3)^2_e = SU(3)_L \times SU(3)_E$$

$$U(1)^5 = U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{PQ} \times U(1)_E$$

$U(1)_Y$: gauged, broken by $\langle \phi^0 \rangle$

$U(1)_{PQ}$: Higgs & D_R, E_R have opposite charges

Exercise: compute embedding of $U(1)^5$ into $U(3)^5$

$$-\mathcal{L}^{SM}_{\text{Yukawa}} = Y_d^{ij}\, \overline{Q}_L^i\, \phi\, D_R^j + Y_u^{ij}\, \overline{Q}_L^i\, \tilde{\phi}\, u_R^j + Y_e^{ij}\, \overline{L}^i\, \phi\, E_R^j + \text{h.c.}$$

$$\tilde{\phi}^i = \epsilon^{ij} \phi^j \quad \text{or} \quad \tilde{\phi} = i \sigma_2 \phi$$

in general flavor dependent ($Y_f \not\propto I$) & CPV

when $Y_f \neq 0$:

$U(1)_E$ broken by $Y_e \neq 0$ & $\text{rk}(Y_e Y_e^\dagger) = 3$

$U(1)_{PQ}$ broken by $Y_u \cdot Y_D \neq 0 \qquad Y_u \cdot Y_e \neq 0$

$SU(3)_Q \times SU(3)_u$ broken $\underset{(U(1)_u \times U(1)_c \times U(1)_t)}{\text{completely}}$ by $Y_u \not\propto_{u} I$ (up to Z_3)

$SU(3)_Q \times SU(3)_D$ broken $\underset{(U(1)_d \times U(1)_s \times U(1)_b)}{\text{completely}}$ by $Y_D \not\propto I$ (up to Z_3)

$\} \; [Y_u, Y_d] \neq 0$

$SU(3)_L \times SU(3)_E \to U(1)_e \times U(1)_\mu \times U(1)_\tau$ (containing also $U(1)_L$)

$$G^{SM}_{\text{global}}(Y_f \neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$$

"Flavor physics" : interactions (parameters) which break $SU(3)_q^3 \times SU(3)_e^2$ are flavor violating.

Spurion analysis : $Y_u \sim (3, \overline{3}, 1)_{SU(3)_q^3} \qquad Y_D \sim (3, 1, \overline{3})_{SU(3)_q^3}$

$Y_E \sim (3, \overline{3})_{SU(3)_e^2}$

useful for parameter counting, identification of suppression factors and idea of MFV

In the following concentrate on the quark sector.

Counting SM quark flavor parameters

Above described flavor symmetry breaking pattern is useful in counting the number of physical ~~par~~ flavor parameters in the theory:

- Theory with global symmetry group G_f with N_{total} generators
- Adding interaction with $N_{general}$ parameters and breaking $G_f \Rightarrow H_f$ with $N_{total} - N_{broken}$ generators
- Then the N_{broken} generators can be used to rotate away N_{broken} number of symmetry breaking parameters.
- Remaining physical parameters: $N_{phys} = N_{general} - N_{broken}$

Example: magnetic field $(B_x, B_y, B_z) = \vec{B}$

$$SO(3) \to SO(2)$$

$$\theta_{xz}\ \theta_{yz}\quad \vec{B} = (0, 0, B_z')$$

single physical parameter

Breaking of $U(3)_Q \times U(3)_U \times U(3)_D \to U(1)_B$: $N_{total} = 3\times(3+i6)$

by Y_u, Y_D : $2\times(9+i9)$

$N_{broken} = N_{total} - 1i$

$= 9 + i17$

$$\boxed{N_{phys} = 9 + i1}$$

6 quark masses
3 mixing angles

1 CPV phase

Discrete symmetries of SM

Any local Lorentz invariant QFT conserves CPT (thus also SM)

$\Rightarrow$ T violation = CP violation

No reason a priori for C, P & CP to be related to flavor physics, but in SM (in Nature) this is so.

In SM C & P violated maximally:

- C & P change chirality of fermion fields
- In SM left-handed & right handed fields have different gauge representations

$\Rightarrow$ Independently of the values of SM parameters C, P maximal.

In SM CP violation depends on parameters (Yukawas):

- Hermiticity of $\mathcal{L}$: $$Y_{ij}\,\bar\psi_{Li}\,\phi\,\psi_{Rj} + Y_{ij}^*\,\bar\psi_{Rj}\,\phi^\dagger\,\psi_{Li}$$
$$\Downarrow \text{CP}$$
$$Y_{ij}\,\bar\psi_{Rj}\,\phi^\dagger\psi_{Li} + Y_{ij}^*\,\bar\psi_{Li}\,\phi\,\psi_{Rj}$$
CP symmetric if $Y_{ij} = Y_{ij}^*$

- More precisely: $$\mathrm{Im}\left(\det[Y^dY^{d\dagger}, Y^uY^{u\dagger}]\right) \neq 0$$ $\equiv J$ (Jarlskog invariant)

Exercise: check whether SM gauge-kinetic terms are CP conserving

- Single scalar doublet has CP conserving potential (Not necessarily true for more scalars)

Exercise: check that $J = 0$ in 2-gen SM

The mass basis

Upon replacement $\mathrm{Re}(\phi^0) \to \frac{v + h^0}{\sqrt{2}}$ Yukawa interactions give rise to fermion mass matrices

$$M_q = \frac{v}{\sqrt{2}} Y_q$$

The mass basis corresponds, by definition, to diagonal mass matrices.

Unitary transformation between any two bases:

$$Q_L \to V_Q Q_L \,, \quad U_R \to V_U U_R \quad D_R \to V_D D_R$$ leaves gauge-kinetic term invariant.

$$Y_u \to V_Q Y_u V_U^+ \qquad Y_d \to V_Q Y_d V_D^+$$

Diagonalisation of M_q requires bi-unitary transformation

$$V_Q^u M_u V_U^+ = M_u^{diag} = \frac{v}{\sqrt{2}} \lambda_u \quad ; \quad \lambda_u = \mathrm{diag}(y_u, y_c, y_t)$$

$$V_Q^d M_d V_D^+ = M_d^{diag} = \frac{v}{\sqrt{2}} \lambda_d \quad ; \quad \lambda_d = \mathrm{diag}(y_d, y_s, y_b)$$

V_U, V_D unphysical (gauge-kinetic term invariant)

$V_Q^u \neq V_Q^d$

Since $[M_U, M_D] \neq 0$: $V_Q^u V_Q^{d\,+} \equiv V_{(CKM)} \neq \mathbb{1}$

SM flavor Lagrangian

$$\mathcal{L}_m^F = \left(\bar{q}^i \not{D} q^j \delta_{ij} \right)_{NC} + \frac{g}{\sqrt{2}} \overline{u_L^i} \gamma^\mu V_{(CKM)}^{ij} d_L^j W_\mu^+ +$$

$$+ \overline{u_L^i} \lambda_u^{ij} u_R^j \left(\frac{v + h^0}{\sqrt{2}} \right) + \overline{d_L^i} \lambda_d^{ij} d_R^j \left(\frac{v + h^0}{\sqrt{2}} \right) + \text{h.c.}$$

$$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} \quad ; \quad NC = \text{neutral current (gluons, } \gamma, Z)$$

Exercise: Show that NC are diagonal (there are no FCNCs at this order; also for Higgs interaction)

Exercise: Show that in absence of neutrino masses, there is no mixing in the leptonic sector