SUSY 2013

Testing the CKM

Flavor conversion in SM: - only proceeds via 3 CKM angles
- is mediated by c.c. electroweak interaction
- c.c. interactions involve LH fields

Parametrization of CKM

- generation permutation fixed by mass ordering

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

· Strong hierarchical pattern in off-diagonal elements ($|V_{ud}| \sim |V_{cs}| \sim |V_{tb}| \sim 1$)

$|V_{us}| \sim |V_{cd}| \sim 0.22$; $|V_{cb}| \sim |V_{ts}| \sim 4 \times 10^{-2}$, $|V_{ub}| \sim |V_{td}| \sim 5 \times 10^{-3}$

Explicit in Wolfenstein expansion in $\lambda \equiv |V_{us}| \approx 0.22$

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Free parameters: λ, $\underbrace{A, \rho, \eta}_{\mathcal{O}(1)}$ ← phase

Systematic expansion to higher orders possible. (and needed in pheno)

Unitarity of CKM

$$\sum_k V_{ik}^* V_{jk} = \delta_{ij} \quad ; \quad \sum_k V_{ki}^* V_{kj} = \delta_{ij}$$

Phenomenologically most interesting for $i=1,\ j=3$

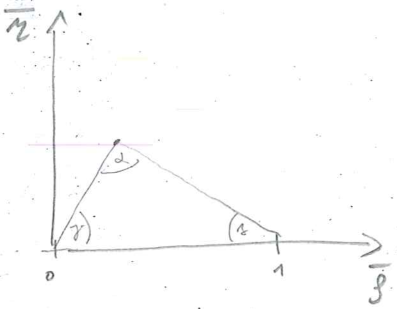
$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Sum of three terms of the same order in λ.

$$\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} + 1 = 0 \iff [\bar\rho + i\bar\eta] + [(1-\bar\rho) - i\bar\eta] + 1 = 0$$

valid to $O(\lambda^5)$:

$$\bar\rho = \rho\left(1 - \frac{\lambda^2}{2}\right) + O(\lambda^4)$$
$$\bar\eta = \eta\left(1 - \frac{\lambda^2}{2}\right) + O(\lambda^4)$$



Defines triangle in a plane

Angles & sides invariant under phase transformation of quark fields $\Rightarrow$ observable quantities

Self consistency of CKM assumption

- $|V_{us}|$ (λ) from $K \to \pi e \nu$ (3‰) $\quad \lambda = 0.2253(9)$
- $|V_{cb}|$ (A) from $B \to X_c e \nu$ (2%) $\quad A = 0.822(12)$
- $|V_{ub}|^2 \propto \bar\rho^2 + \bar\eta^2$ from $B \to X_u e \nu$
- time-dependent CP asymmetry in $B \to \psi K_S$ $\left(S_{\psi K_S} = \sin 2\beta = \frac{2\bar\eta(1-\bar\rho)}{(1-\bar\rho)^2 + \bar\eta^2}\right)$
- rates of $B \to DK$ decay depend on $e^{i\gamma} = \frac{\rho + i\eta}{\rho^2 + \eta^2}$
- rates of $B \to \pi\pi, \rho\pi, \rho\rho$ depend on $\alpha = \pi - \beta - \gamma$
- $\frac{\Delta M_d}{\Delta M_s} \propto \left|\frac{V_{td}}{V_{ts}}\right|^2 = \lambda^2 [(1-\bar\rho)^2 + \bar\eta^2]$
- CPV in $K \to \pi\pi$ (ε_K) depends in a complicated way

Lead to impressive agreement with best fit ranges:

$$\bar\rho = 0.130 \pm 0.024$$

$$\bar\eta = 0.362 \pm 0.014 \leftarrow \text{CKM phase is } \mathcal{O}(1) \; ?$$

$\Rightarrow$ Very likely CPV in flavor changing processes is dominated by CKM phase; KM mechanism of CPV is at work.

Reparametrization invariant measure:

$$\mathrm{Im}\left[V_{ij} V_{kj}^{*} V_{kl} V_{il}^{*}\right] = J_{CKM} \sum_{m,n} \varepsilon_{ikm} \varepsilon_{jln}$$

$$J_{CKM} = \lambda^6 A^2 \eta \simeq \mathcal{O}(10^{-5})$$

$\hookrightarrow$ CPV suppressed by small mixing!

Jarlskog determinant in SM

$$J = J_{CKM} \prod_{i>j} \frac{m_i^2 - m_j^2}{v^2} = \mathcal{O}(10^{-22})$$

further suppressed by mass hierarchies (small 1st gen masses)

Closer look at CPV in neutral meson mixing

For simplicity focus on B_d $(B^0 \sim \bar{b} d, \ \bar{B}^0 \sim b \bar{d})$

$$CP|B^0\rangle = e^{i\xi_B}|\bar{B}^0\rangle$$
$$CP|\bar{B}^0\rangle = e^{-i\xi_B}|B^0\rangle$$

Time evolution

$$|\psi(0)\rangle = a(0)|B^0\rangle + b(0)|\bar{B}^0\rangle$$
$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\bar{B}^0\rangle + \underbrace{c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \dots}_{B,\bar{B}\text{ decay products}}$$

If only interested in $a(t), b(t)$: (oscillation & decay)

$$\mathcal{H} = M + i\frac{\Gamma}{2}, \quad M, \Gamma - \text{time independent, Hermitian, } 2\times 2 \text{ mat.}$$

M: dispersive (off-shell intermediate states)

Γ: absorptive (on-shell intermediate states)

$$i\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t)\end{pmatrix} = \mathcal{H}\begin{pmatrix} a(t) \\ b(t)\end{pmatrix}$$

Eigenvectors:

$$|B_{L,H}\rangle = p_{L,H}|B^0\rangle \pm q_{L,H}|\bar{B}^0\rangle$$

with $|p_{L,H}|^2 + |q_{L,H}|^2 = 1$

if CPT: $M_{11} = M_{22}, \ \Gamma_{11} = \Gamma_{22} \Rightarrow p_L = p_H = p$, $q_L = q_H = q$

if CP: $\mathrm{Arg}(M_{12}) = \mathrm{Arg}(\Gamma_{12}) \Rightarrow \left|\frac{q}{p}\right| = 1$

Exercise: check!

Define: $m = \frac{m_L + m_H}{2}$ $\qquad \Gamma = \frac{\Gamma_L + \Gamma_H}{2}$

$\Delta m = m_H - m_L$ $\qquad \Delta\Gamma = \Gamma_H - \Gamma_L$

$\left(\text{also} \quad x = \frac{\Delta m}{\Gamma} \quad , \; y = \frac{\Delta\Gamma}{2\Gamma}\right)$

time evolution

$|B^0(t)\rangle$ pure $|B^0\rangle$ at $t=0$

$|\overline{B^0}(t)\rangle$ pure $|\overline{B^0}\rangle$ at $t=0$

$$|B^0(t)\rangle = g_+(t)|B^0\rangle - \frac{q}{p}\, g_-(t)\, |\overline{B^0}\rangle$$

$$|\overline{B^0}(t)\rangle = g_+(t)|\overline{B^0}\rangle - \frac{q}{p}\, g_-(t)\, |B^0\rangle$$

$$g_\pm(t) = \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right)$$

decay to final state f after time t

decay amplitudes: $\langle f | H | B^0\rangle = A_f$ $\qquad \langle f | H | \overline{B^0}\rangle = \bar{A}_f$

$\langle \bar f | H | B^0\rangle = A_{\bar f}$ $\qquad \langle \bar f | H | \overline{B^0}\rangle = \bar{A}_{\bar f}$

time dependent decay rate:

$$\frac{d\Gamma\left(|B^0(0)\rangle \to |f(t)\rangle\right)}{dt} = N_0\, e^{-\Gamma t}\, |A_f|^2 \;\times$$

$\uparrow$ flux normalization

$$\times \left\{ \frac{1+|\lambda_f|^2}{2}\cosh\frac{\Delta\Gamma t}{2} + \frac{1-|\lambda_f|^2}{2}\cos \Delta m t + \mathrm{Re}\,\lambda_f \sinh\frac{\Delta\Gamma t}{2} - \mathrm{Im}\,\lambda_f \sin \Delta m t \right.$$

$$\frac{d\Gamma(|\bar{B}^0(0)\rangle \to |f(t)\rangle)}{dt} = N_0 e^{-\Gamma t} |\bar{A}_f|^2 \times$$

$$\left\{ \frac{1+|\bar{\lambda}_f|^2}{2} \cosh\frac{\Delta\Gamma t}{2} + \frac{1-|\bar{\lambda}_f|^2}{2} \cos\Delta m t + \mathrm{Re}\,\bar{\lambda}_f \sinh\frac{\Delta\Gamma t}{2} - \mathrm{Im}\,\bar{\lambda}_f \sin\Delta m t \right\}$$

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f} \qquad \bar{\lambda}_f \equiv \frac{p}{q}\frac{A_f}{\bar{A}_f} = \frac{1}{\lambda_f}$$

And analogously for decays to $\bar{f}$.

- terms proportional to $|A_f|^2$, $|\bar{A}_f|^2$ – decay without net oscillation
- — // — $|\lambda_f|^2$, $|\bar{\lambda}_f|^2$ – decays following net oscillation
- — // — $\sin\Delta m t$, $\sinh\frac{\Delta\Gamma t}{2}$ – interference between these
- CPV in this interference possible only if $\mathrm{Im}\,\lambda_f \neq 0$

can be observed in neutral B decays to CP eigenstates f_{CP}

$$A_{f_{CP}}(t) \equiv \frac{\frac{d\Gamma}{dt}(\bar{B}^0(0) \to f_{CP}(t)) - \frac{d\Gamma}{dt}(B^0(0) \to f_{CP}(t))}{\frac{d\Gamma}{dt}(\bar{B}^0(0) \to f_{CP}(t)) + \frac{d\Gamma}{dt}(B^0(0) \to f_{CP}(t))}$$

In B (and B_s) system: $\Delta\Gamma \ll \Delta m$, $\left|\frac{q}{p}\right| \simeq 1$

$$A_f(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t)$$

$$S_f \equiv \frac{2\,\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2} \qquad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$$

Phases in decay amplitudes

$B \to f$: A_f $\updownarrow CP$

$\bar{B} \to \bar{f}$: $\bar{A}_{\bar{f}}$

- complex parameters in $\mathcal{L}$, appear complex conjugated after CP
  => opposite signs in A_f and $\bar{A}_{\bar{f}}$
  "weak phases" (CP odd) - due to W exchanges in SM
  single amplitude phases convention dependent
  difference between phases of different amplitudes are physical (convention independent)

- on-shell intermediate states in scattering or decay amplitudes (poles & cuts in scattering amplitude sheet)
  Even if $\mathcal{L}$ is real => independent of CP
  => same signs in A_f and $\bar{A}_{\bar{f}}$ (CP even)
  "strong phases" - due to strong rescattering in SM
  again only relative phases between amplitudes are physical

$$\Rightarrow \quad A_f = |a_1| e^{i(\delta_1 + \phi_1)} + |a_2| e^{i(\delta_2 + \phi_2)} + \dots$$

$$\bar{A}_{\bar{f}} = |a_1| e^{i(\delta_1 - \phi_1)} + |a_2| e^{i(\delta_2 - \phi_2)} + \dots$$

$a_1, a_2 \dots$ - contribution to amplitude with different phases

$\delta_1, \delta_2 \dots$ - strong phases

$\phi_1, \phi_2 \dots$ - weak phases

CPV : $B \to \psi K_S$

- to good approximation, just single weak decay amplitude, to CP-eigenstate

$$A_f = |a_f|\, e^{i(\delta_f + \phi_f)}$$
$$\bar{A}_f = |a_f|\, e^{i(\delta_f - \phi_f)}\, \eta_f$$

$$\lambda_f = \eta_f \frac{q}{p} e^{-2i\phi_f}$$

- $|\Gamma_{12}| \ll |M_{12}|$: $\mathcal{O}(\delta_F^2)$ LD effect, suppressed by small CKM elements

Experimentally verified by measured $\Delta\Gamma$, or …

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \simeq e^{2i\xi_B}$$

$$\lambda_f = \eta_f\, e^{i(\xi_B - 2\phi_f)}$$

$$A_{f_{CP}}(t) = \eta_f \sin(\xi_B - 2\phi_f)$$

In SM:

$$\xi_B = -\mathrm{Arg}(M_{12}) = -\mathrm{Arg}\left[(V_{tb}^* V_{td})^2\right] = -\mathrm{Arg}\left[\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right]$$

$$-e^{-2i\phi_f} = \frac{\bar{A}^{(B)}_{\psi K_S}}{A^{(B)}_{\psi K_S}} = -\frac{V_{cb} V_{cs}^*\, a_T + \dots}{V_{cb}^* V_{cs}\, a_T + \dots}$$

suppressed by small CKM elements

$$e^{i\xi_K} = -\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}$$

$K-\bar{K}$ oscillation forming K_S

$$\lambda^{(B)}_{\psi K_S} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} = -e^{-2i\beta}$$

$$S^{(B)}_{\psi K_S} = \sin 2\beta \qquad (C_{\psi K_S} = 0)$$

CPV in interference between mixing & decay amplitudes

to accuracy $\sim 1\%$!

CPV in B_s mixing

golden channel $B_s \to \psi \phi$

- admixture of different CP eigenstates (VV) $\Rightarrow$ need angular analysis
- B_s oscillates faster than B_d

$$\frac{\Delta M_s}{\Delta M_d} \sim \frac{|M_{12}^s|}{|M_{12}^d|} \propto \left|\frac{V_{ts}}{V_{td}}\right|^2 \sim 30$$

- $\Delta\Gamma_s$ effects cannot be neglected compared to ΔM_s in the time evolution

In SM: $$\lambda_{\psi\phi}^{(B_s)} = - e^{i\underbrace{(\delta_{B_s} - 2\phi_{\psi\phi})}}$$

$$\left[S_{\psi\phi}^{(B_s)}\right]_{SM} = 2\,\mathrm{Arg}\,\underbrace{\frac{V_{tb}^* V_{ts}}{V_{cb}^* V_{cs}}}_{\beta_s} = 0.04(1)$$

Very small!

$$S_{\psi\phi}^{(B_s)} = 0.08(20)$$

Exercise: Show that if $B \to \pi\pi$ is dominated by single (tree) amplitudes, then $S_{\pi\pi} = \sin 2\alpha$

CPV in B decays to CP conjugate states

- interesting if $B^0 \to \bar{f}$ and $\overline{B^0} \to f$ are forbidden

$|A_f| = |\bar{A}_{\bar{f}}|$ and $|A_{\bar{f}}| = |\bar{A}_f| = 0$

$$\frac{\frac{d\Gamma}{dt}\left(\overline{B^0}(0) \to f(t)\right) - \frac{d\Gamma}{dt}\left(B^0(0) \to \bar{f}(t)\right)}{\frac{d\Gamma}{dt}\left(\overline{B^0}(0) \to f(t)\right) + \frac{d\Gamma}{dt}\left(B^0(0) \to \bar{f}(t)\right)} = \frac{\left|\frac{p}{q}\right|^2 - \left|\frac{q}{p}\right|^2}{\left|\frac{p}{q}\right|^2 + \left|\frac{q}{p}\right|^2}$$

$$(|\Gamma_{12}| \ll |M_{12}|) \simeq \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right)$$

Time independent measurement.

CPV in mixing – "indirect CPV"

Example: $$a_{SL}^{(d)} = \frac{\Gamma(\overline{B^0} \to X e^+ \nu) - \Gamma(B^0 \to X e^- \nu)}{\Gamma(\overline{B^0} \to X e^+ \nu) + \Gamma(B^0 \to X e^- \nu)}$$

In SM: $a_{SL}^{(d)} = 3.3(7) \times 10^{-3}$

CPV in charged B decays

Interesting in the case of $B^\pm \to DK^\pm$, since $D-\bar{D}$ oscillations allow for interference of two tree-level dominated decay amplitudes

$$B^- \to D^0 K^- : \quad b \to c\bar{u}s ;$$
$$B^- \to \bar{D}^0 K^- : \quad b \to \bar{c}us ;$$

particularly transparent in CP D-decay eigenstates

$$D^0 \to f_{CP} : \quad c \to d\bar{d}u,\ s\bar{s}u$$
$$\bar{D}^0 \to f_{CP} : \quad \bar{c} \to d\bar{d}\bar{u},\ s\bar{s}\bar{u}$$

In SM

$$\frac{A(D\to f)K}{A(\bar{D}\to f)K} = \frac{V_{cb}^* V_{us}\, a_{DK}}{V_{ub}^* V_{cs}\, a_{\bar{D}K}} e^{i(\delta_{DK}-\delta_{\bar{D}K})}\, \eta_f \frac{V_{cd} V_{ud}^*}{V_{cd}^* V_{ud}} \frac{a_{Df}}{a_{\bar{D}f}} e^{i(\delta_f - \delta_{\bar{f}})}$$

$$\simeq \eta_f\, r_B\, e^{i(\delta_B - \gamma)} \quad ; \quad \gamma \equiv \mathrm{Arg}\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right) \simeq 70^\circ$$

Several decay rates

$$A(B^- \to f_+ K^-) = A_0 \times \left[1 + r_B\, e^{i(\delta_B - \gamma)}\right]$$
$$A(B^- \to f_- K^-) = A_0 \times \left[1 - r_B\, e^{i(\delta_B - \gamma)}\right]$$
$$A(B^+ \to f_+ K^+) = A_0 \times \left[1 + r_B\, e^{i(\delta_B + \gamma)}\right]$$
$$A(B^+ \to f_- K^+) = A_0 \times \left[1 - r_B\, e^{i(\delta_B + \gamma)}\right]$$

CPV in decay ("direct CPV")

can be used to extract 3 hadronic parameters (A_0, r_B, δ_B) and γ.

- theoretically extremely clean (CPV in $D-\bar{D}$ mixing negligible)
- experimentally advantageous to have large r_B, large δ_B
- possible also for non CP D-decay eigenstates