

CKM

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

(mass-ordered)

CKM

PDG parametrization

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$|V_{ud}| \sim |V_{cs}| \sim |V_{tb}| \sim 1$$

$$|V_{us}| \sim |V_{cd}| \sim 0.22$$

$$|V_{cb}| \sim |V_{ts}| \sim 0.04$$

$$|V_{ub}| \sim |V_{td}| \sim 0.005$$



CKM

Wolfenstein parametrization

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda \doteq s_{12}, \quad A\lambda^2 \doteq s_{23}, \quad A\lambda^3(\varrho - i\eta) \doteq s_{13}e^{-i\delta}.$$

CKM

Wolfenstein parametrization

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda + \mathcal{O}(\lambda^7) & A\lambda^3(\varrho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\varrho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 + \mathcal{O}(\lambda^8) \\ A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}) & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\varrho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

$$\bar{\varrho} = \varrho\left(1 - \frac{\lambda^2}{2}\right) + \mathcal{O}(\lambda^4), \quad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right) + \mathcal{O}(\lambda^4).$$

Unitarity of CKM

$$\begin{array}{ll} |V_{us}|(\lambda) \text{ from } K \rightarrow \pi l \nu & \lambda = 0.2253(9) \\ |V_{cb}| (A) \text{ from } B \rightarrow X_c l \nu & A = 0.822(12) \end{array}$$

Unitarity of CKM

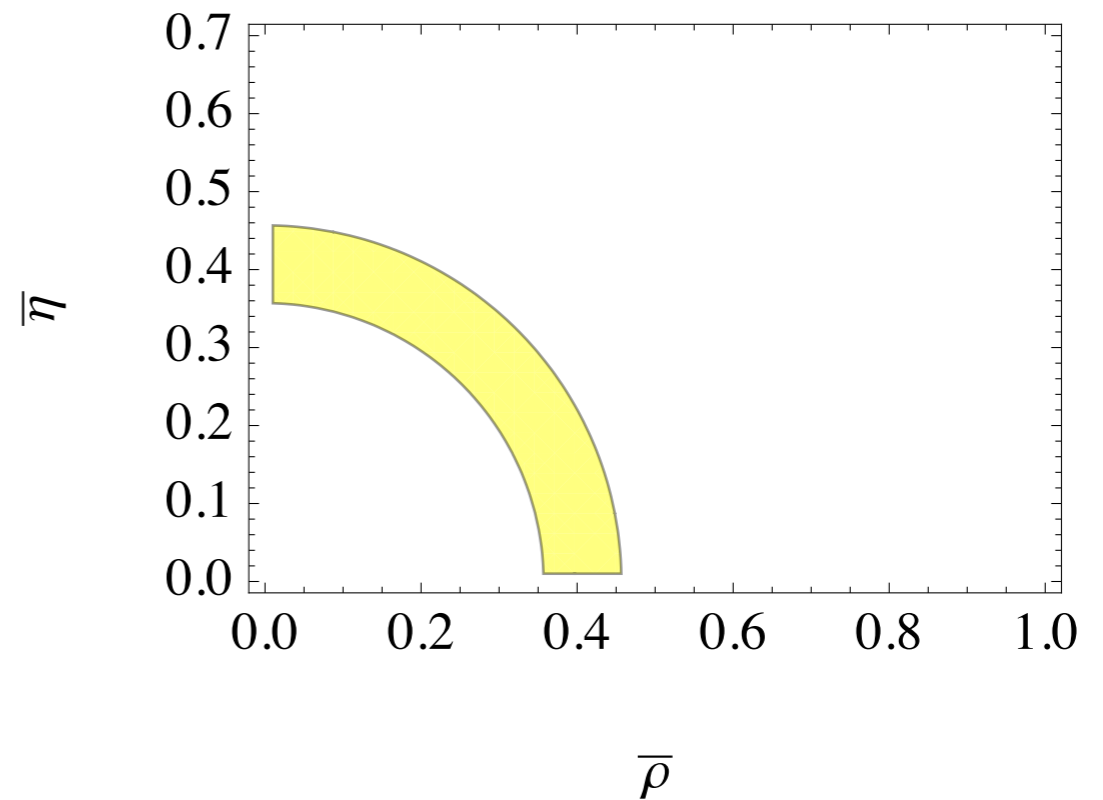
$|V_{us}|(\lambda)$ from $K \rightarrow \pi l \nu$

$|V_{cb}| (A)$ from $B \rightarrow X_c l \nu$

$|V_{ub}|^2 \propto \bar{\rho}^2 + \bar{\eta}^2$ from $B \rightarrow X_u l \nu$

$$\lambda = 0.2253(9)$$

$$A = 0.822(12)$$



Unitarity of CKM

$|V_{us}|(\lambda)$ from $K \rightarrow \pi l \nu$

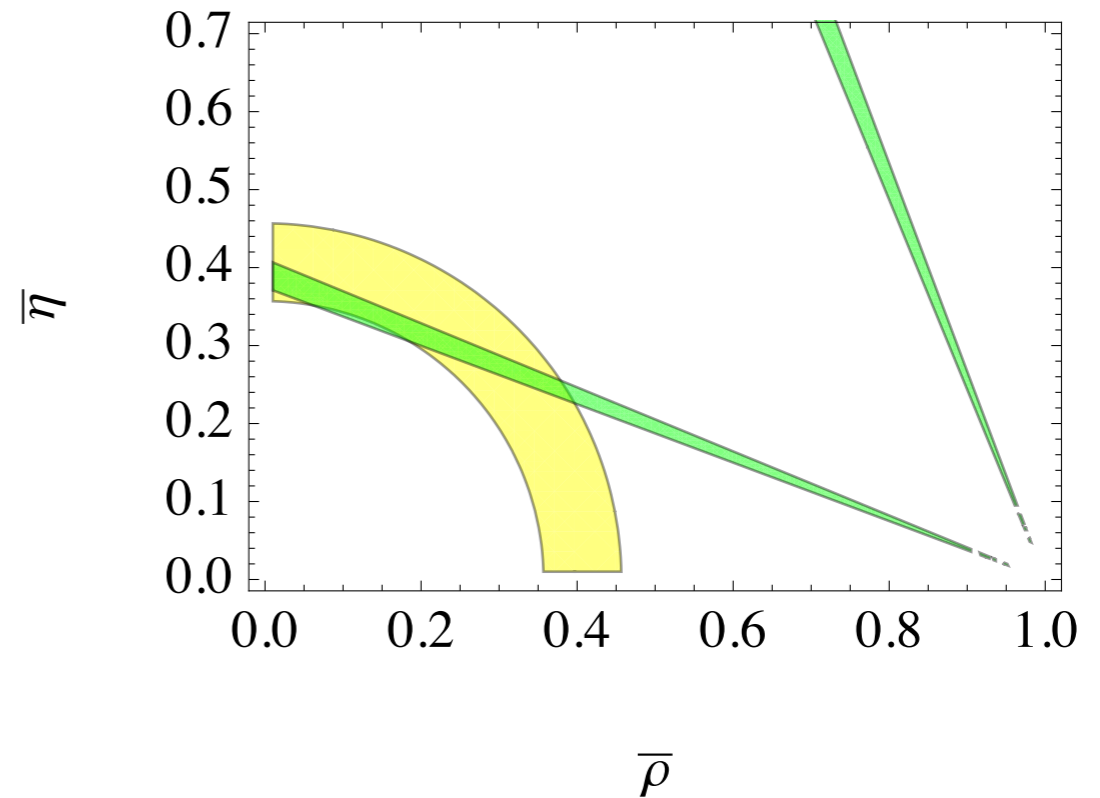
$|V_{cb}| (A)$ from $B \rightarrow X_c l \nu$

$|V_{ub}|^2 \propto \bar{\rho}^2 + \bar{\eta}^2$ from $B \rightarrow X_u l \nu$

$$S_{\psi K_S} = \sin 2\beta = \frac{2\bar{\eta}(1 - \bar{\rho})}{(1 - \bar{\rho})^2 + \bar{\eta}^2}$$

$$\lambda = 0.2253(9)$$

$$A = 0.822(12)$$



Unitarity of CKM

$|V_{us}|(\lambda)$ from $K \rightarrow \pi l \nu$

$|V_{cb}| (A)$ from $B \rightarrow X_c l \nu$

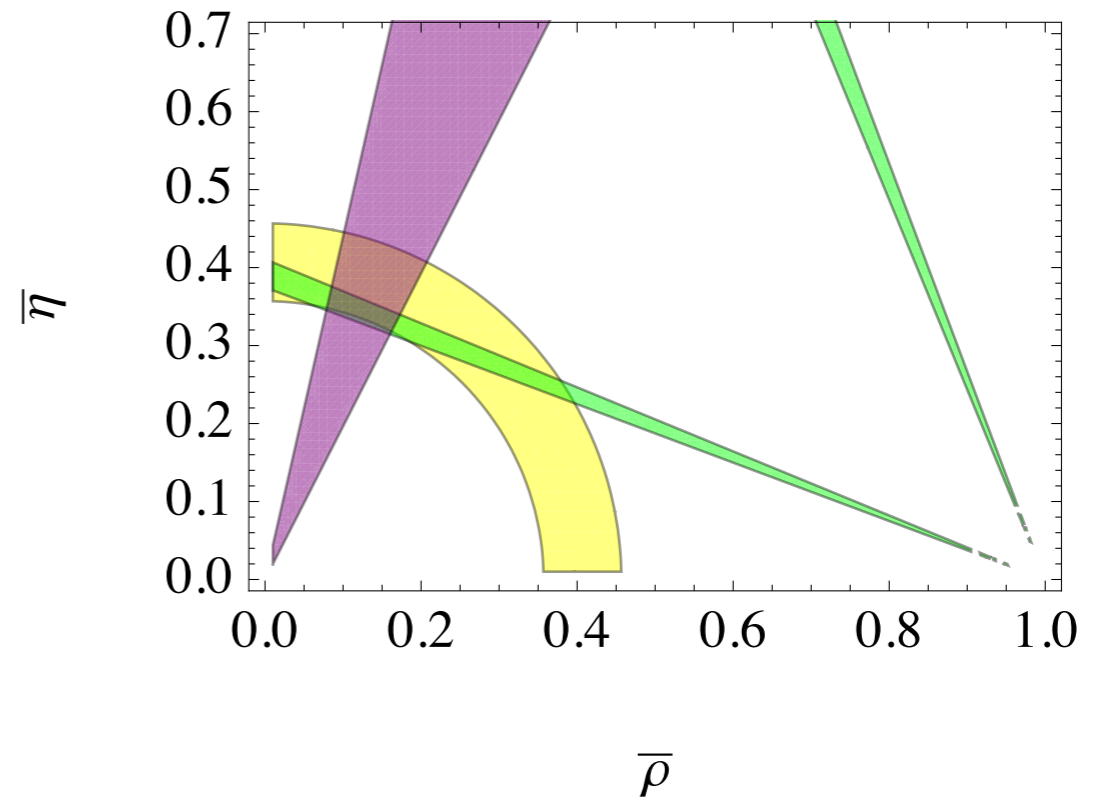
$|V_{ub}|^2 \propto \bar{\rho}^2 + \bar{\eta}^2$ from $B \rightarrow X_u l \nu$

$$S_{\psi K_S} = \sin 2\beta = \frac{2\bar{\eta}(1 - \bar{\rho})}{(1 - \bar{\rho})^2 + \bar{\eta}^2}$$

$$e^{i\gamma} = \frac{\bar{\rho} + i\bar{\eta}}{\bar{\rho}^2 + \bar{\eta}^2} \quad (B \rightarrow D \text{ } K \text{ rates})$$

$$\lambda = 0.2253(9)$$

$$A = 0.822(12)$$



Unitarity of CKM

$$|V_{us}|(\lambda) \text{ from } K \rightarrow \pi l \nu$$

$$|V_{cb}| (A) \text{ from } B \rightarrow X_c l \nu$$

$$|V_{ub}|^2 \propto \bar{\rho}^2 + \bar{\eta}^2 \text{ from } B \rightarrow X_u l \nu$$

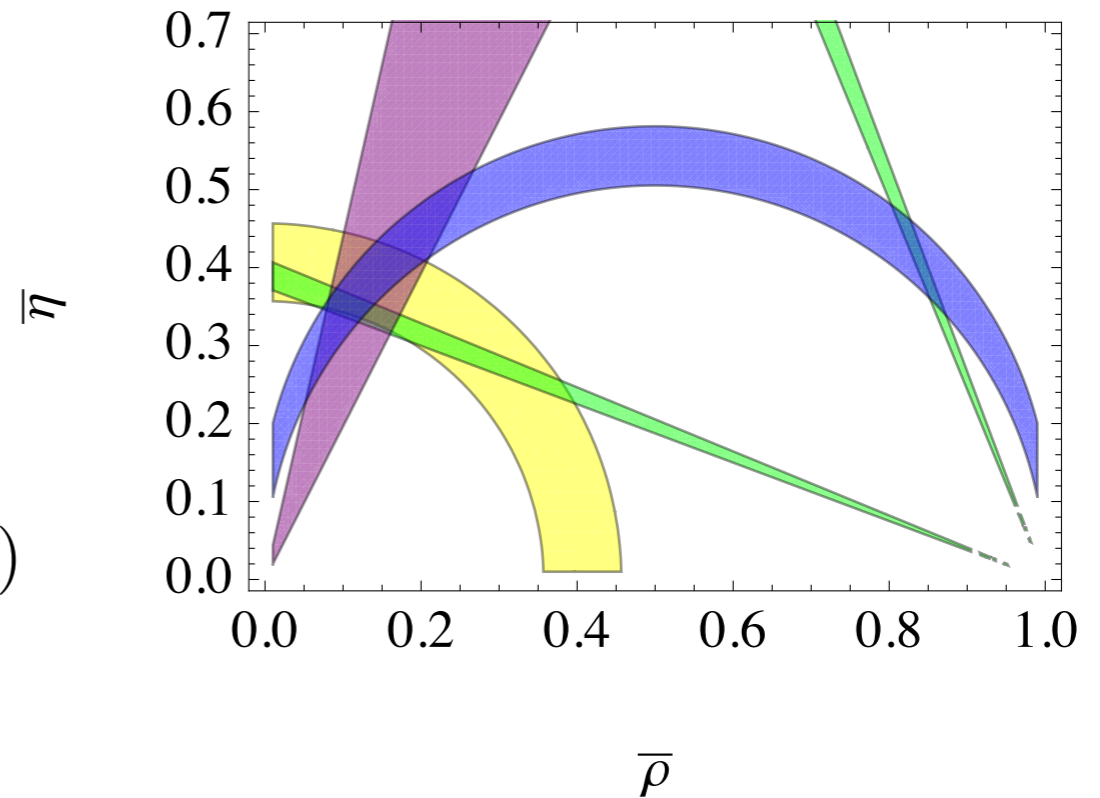
$$S_{\psi K_S} = \sin 2\beta = \frac{2\bar{\eta}(1 - \bar{\rho})}{(1 - \bar{\rho})^2 + \bar{\eta}^2}$$

$$e^{i\gamma} = \frac{\bar{\rho} + i\bar{\eta}}{\bar{\rho}^2 + \bar{\eta}^2} \quad (B \rightarrow D \text{ } K \text{ rates})$$

$$\alpha = \pi - \beta - \gamma \quad (B \rightarrow \pi\pi, \rho\pi, \rho\rho \text{ rates})$$

$$\lambda = 0.2253(9)$$

$$A = 0.822(12)$$



Unitarity of CKM

$$|V_{us}|(\lambda) \text{ from } K \rightarrow \pi \ell \nu$$

$$|V_{cb}| (A) \text{ from } B \rightarrow X_c \ell \nu$$

$$|V_{ub}|^2 \propto \bar{\rho}^2 + \bar{\eta}^2 \text{ from } B \rightarrow X_u \ell \nu$$

$$S_{\psi K_S} = \sin 2\beta = \frac{2\bar{\eta}(1 - \bar{\rho})}{(1 - \bar{\rho})^2 + \bar{\eta}^2}$$

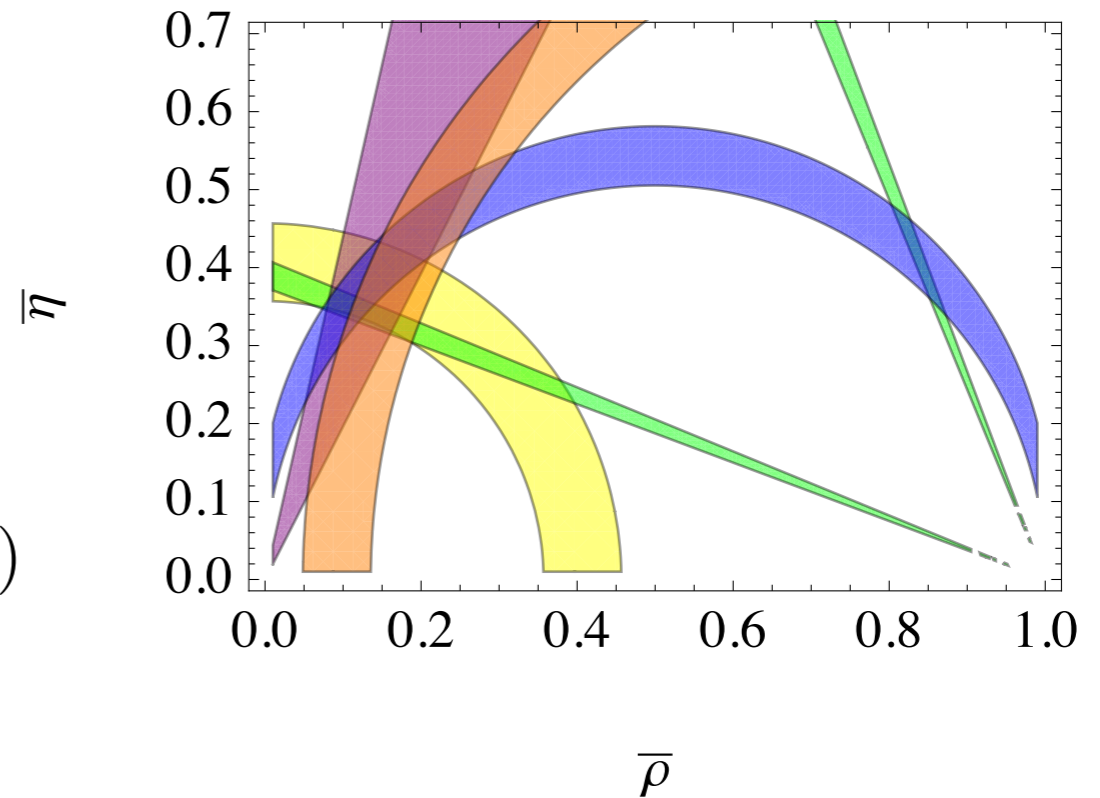
$$e^{i\gamma} = \frac{\bar{\rho} + i\bar{\eta}}{\bar{\rho}^2 + \bar{\eta}^2} \quad (B \rightarrow D \text{ } K \text{ rates})$$

$$\alpha = \pi - \beta - \gamma \quad (B \rightarrow \pi\pi, \rho\pi, \rho\rho \text{ rates})$$

$$\frac{\Delta m_d}{\Delta m_s} \propto \left| \frac{V_{td}}{V_{ts}} \right|^2 = \lambda^2 [(1 - \bar{\rho})^2 + \bar{\eta}^2]$$

$$\lambda = 0.2253(9)$$

$$A = 0.822(12)$$



Unitarity of CKM

$$|V_{us}|(\lambda) \text{ from } K \rightarrow \pi \ell \nu$$

$$|V_{cb}| (A) \text{ from } B \rightarrow X_c \ell \nu$$

$$|V_{ub}|^2 \propto \bar{\rho}^2 + \bar{\eta}^2 \text{ from } B \rightarrow X_u \ell \nu$$

$$S_{\psi K_S} = \sin 2\beta = \frac{2\bar{\eta}(1 - \bar{\rho})}{(1 - \bar{\rho})^2 + \bar{\eta}^2}$$

$$e^{i\gamma} = \frac{\bar{\rho} + i\bar{\eta}}{\bar{\rho}^2 + \bar{\eta}^2} \quad (B \rightarrow D \text{ } K \text{ rates})$$

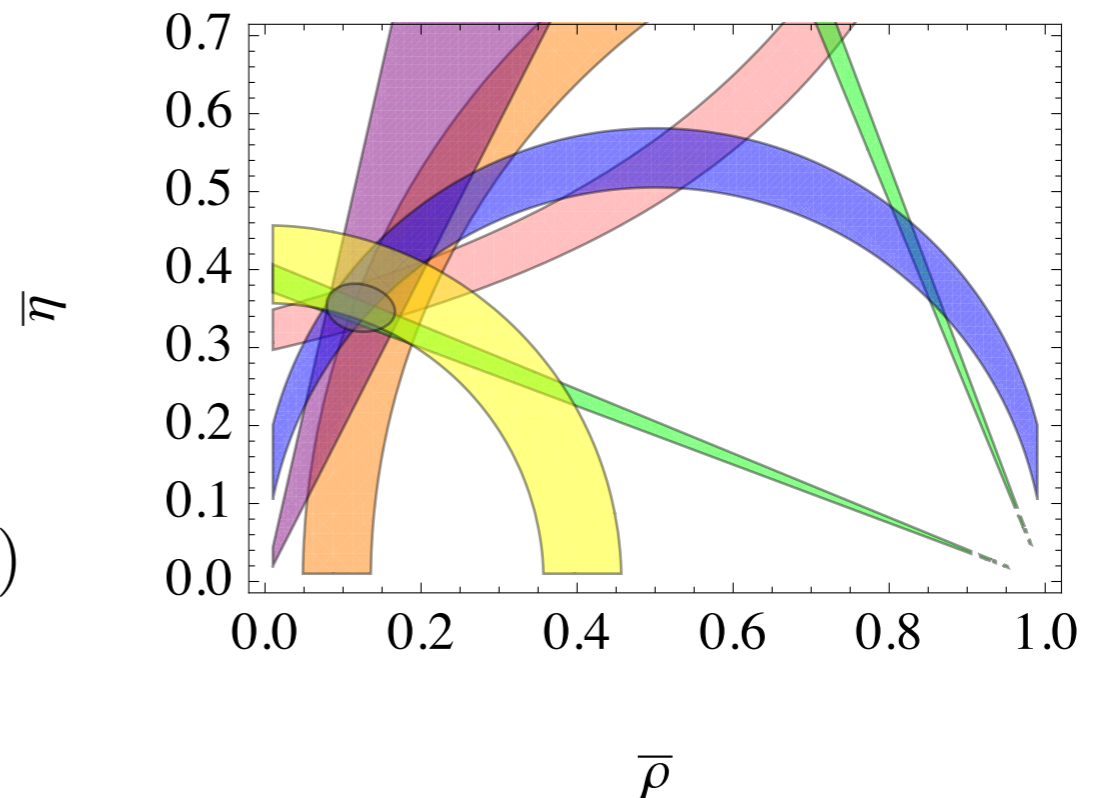
$$\alpha = \pi - \beta - \gamma \quad (B \rightarrow \pi\pi, \rho\pi, \rho\rho \text{ rates})$$

$$\frac{\Delta m_d}{\Delta m_s} \propto \left| \frac{V_{td}}{V_{ts}} \right|^2 = \lambda^2 [(1 - \bar{\rho})^2 + \bar{\eta}^2]$$

$$\epsilon_K \quad (\text{CPV in } K \rightarrow \pi\pi)$$

$$\lambda = 0.2253(9)$$

$$A = 0.822(12)$$



$$\bar{\rho} = 0.130 \pm 0.024$$

$$\bar{\eta} = 0.362 \pm 0.014$$