

## Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields:  $\chi_k(x)$  - 4 component (spin 1/2), complex,  $m_k$

Majorana condition:

$$C (\bar{\chi}_k(x))^T = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

- Invariant under proper Lorentz transformations.
- Reduces by 2 the number of components in  $\chi_k(x)$ .

Implications:

$$U(1) : \chi_k(x) \rightarrow e^{i\alpha} \chi_k(x) - \text{impossible}$$

- $\chi_k(x)$  cannot absorb phases.
- $Q_{U(1)} = 0 : Q_{\text{el}} = 0, L_l = 0, L = 0, \dots$
- $\chi_k(x)$ : 2 spin states of a spin 1/2 absolutely neutral particle
- $\chi_k \equiv \bar{\chi}_k$

Propagators:  $\Psi(x)$ -Dirac,  $\chi(x)$ -Majorana

$$\langle 0 | T(\Psi_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\Psi_\alpha(x) \Psi_\beta(y)) | 0 \rangle = 0 , \quad \langle 0 | T(\bar{\Psi}_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = 0 .$$

$$\langle 0 | T(\chi_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\chi_\alpha(x) \chi_\beta(y)) | 0 \rangle = -\xi^* S_{\alpha\kappa}^F(x - y) C_{\kappa\beta} ,$$

$$\langle 0 | T(\bar{\chi}_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = \xi C_{\alpha\kappa}^{-1} S_{\kappa\beta}^F(x - y)$$

$$U_{CP} \ \chi(x) \ U_{CP}^{-1} = \eta_{CP} \ \gamma_0 \ \chi(x') , \quad \eta_{CP} = \pm i .$$

Special Properties of the Currents of  $\chi(x)$ -Majorana:

$$\bar{\chi}(x)\gamma_a\chi(x) = 0 : \quad Q_{U(1)} = 0 \quad (Q_{U(1)}(\Psi) \neq 0);$$

Has important implications, e.g. for SUSY DM (neutralino) abundance determination (calculation).

$$\bar{\chi}(x)\sigma_{\alpha\beta}\chi(x) = 0 : \quad \mu_\chi = 0 \quad (\mu_\Psi \neq 0)$$

$$\bar{\chi}(x)\sigma_{\alpha\beta}\gamma_5\chi(x) = 0 : \quad d_\chi = 0 \quad (d_\Psi \neq 0, \text{ if } CP \text{ is not conserved})$$

$\chi(x)$  cannot couple to a real photon (field).

$\chi(x)$  couples to a virtual photon through an anapole moment:

$$(g_{\alpha\beta} q^2 - q_\alpha q_\beta) \gamma_\beta \gamma_5 F_\alpha(q^2).$$

Properties of Currents Formed by  $\chi_1(x)$ ,  $\chi_2(x)$ :  $\chi_2 \rightarrow \chi_1 + \gamma$ ,  $\chi_2 \rightarrow \chi_1 \chi_1 \chi_1$ , etc.

$$\bar{\chi}_1(x) \gamma_\alpha (v - a \gamma_5) \chi_2(x) \quad (\bar{\chi}_1(x) \gamma^\alpha (1 - \gamma_5) \chi_1(x), \dots) :$$

- CP is conserved:  $v = 0$  ( $a = 0$ ) if  $\eta_{1CP} = \eta_{2CP}$  ( $\eta_{1CP} = -\eta_{2CP}$ )
- CP is not conserved:  $v \neq 0$ ,  $a \neq 0$

(Has important implications also, e.g. for SUSY neutralino phenomenology:  
 $e^+ + e^- \rightarrow \chi_1 + \chi_2$ ,  $\chi_2 \rightarrow \chi_1 + l^+ + l^-$ , etc.)

$$\bar{\chi}_1(x) \sigma_{\alpha\beta} (\mu_{12} - d_{12} \gamma_5) \chi_2(x) \quad (F^{\alpha\beta}(x)) :$$

- CP is conserved:  $\mu_{12} = 0$  ( $d_{12} = 0$ ) if  $\eta_{1CP} = \eta_{2CP}$  ( $\eta_{1CP} = -\eta_{2CP}$ )
- CP is not conserved:  $\mu_{12} \neq 0$ ,  $d_{12} \neq 0$

Pontecorvo, 1958:

$$\nu(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}, \quad m_1 \neq m_2 > 0, \quad \eta_{1CP} = -\eta_{2CP}$$

$\chi_{1,2}$  - Majorana, maximal mixing .

Maki, Nakagawa, Sakata, 1962:

$$\nu_{eL}(x) = \Psi_{1L} \cos \theta_C + \Psi_{2L} \sin \theta_C,$$

$$\nu_{\mu L}(x) = -\Psi_{1L} \sin \theta_C + \Psi_{2L} \cos \theta_C,$$

$\Psi_{1,2}$  - Dirac (composite),  $\theta_C$ - the Cabibbo angle .

## Determining the Nature of Massive Neutrinos

## Dirac CP-Nonconservation: $\delta$ in $U_{\text{PMNS}}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP  $\alpha_{21}, \alpha_{31}$

S.M. Bilenky, J. Hosek, S.T.P., 1980;  
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U_{jl}^\dagger$$

$$U = VP : P_j e^{-i(E_j t - p_j x)} P_j^* = e^{-i(E_j t - p_j x)}$$

$P$  - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter:  $\nu_l$  oscillations are not sensitive to the nature of  $\nu_j$ .

$\nu_j$ — Dirac or Majorana particles, fundamental problem

$\nu_j$ —Dirac: conserved lepton charge exists,  $L = L_e + L_\mu + L_\tau$ ,  $\nu_j \neq \bar{\nu}_j$

$\nu_j$ —Majorana: no lepton charge is exactly conserved,  $\nu_j \equiv \bar{\nu}_j$

The observed patterns of  $\nu$ —mixing and of  $\Delta m_{\text{atm}}^2$  and  $\Delta m_{\odot}^2$  can be related to Majorana  $\nu_j$  and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism:  $\nu_j$ — Majorana

Establishing that  $\nu_j$  are Majorana particles would be as important as the discovery of  $\nu$ — oscillations.

If  $\nu_j$  – Majorana particles,  $U_{\text{PMNS}}$  contains (3- $\nu$  mixing)

$\delta$ -Dirac,  $\alpha_{21}$ ,  $\alpha_{31}$  - Majorana physical CPV phases

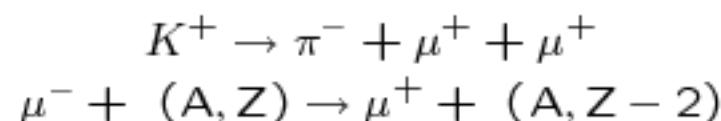
$\nu$ -oscillations  $\nu_l \leftrightarrow \nu_{l'}$ ,  $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ ,  $l, l' = e, \mu, \tau$ ,

- are not sensitive to the nature of  $\nu_j$ ,

S.M. Bilenky et al., 1980;  
P. Langacker et al., 1987

- provide information on  $\Delta m_{jk}^2 = m_j^2 - m_k^2$ , but not on the absolute values of  $\nu_j$  masses.

The Majorana nature of  $\nu_j$  can manifest itself in the existence of  $\Delta L = \pm 2$  processes:



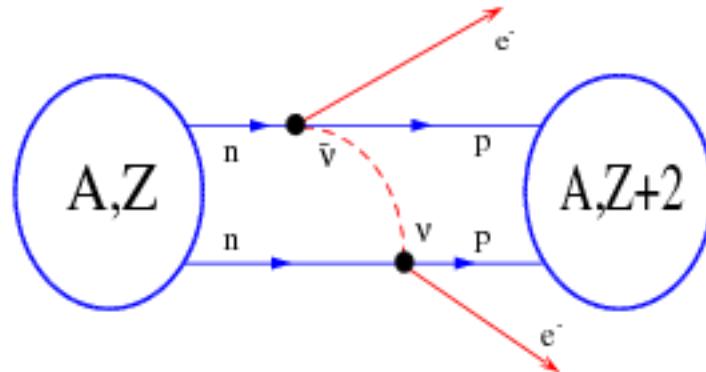
The process most sensitive to the possible Majorana nature of  $\nu_j$  –  $(\beta\beta)_{0\nu}$ -decay



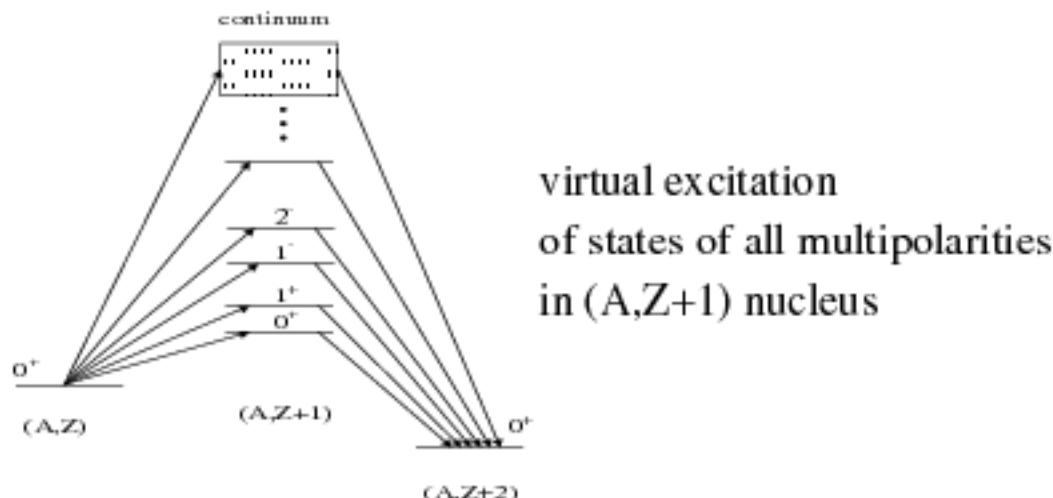
of even-even nuclei,  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{100}\text{Mo}$ ,  $^{116}\text{Cd}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ ,  $^{150}\text{Nd}$ .

$2n$  from  $(A, Z)$  exchange a virtual Majorana  $\nu_j$  (via the CC weak interaction) and transform into  $2p$  of  $(A, Z+2)$  and two free  $e^-$ .

## Nuclear $0\nu\beta\beta$ -decay



strong in-medium modification of the basic process  
 $dd \rightarrow ue^-e^-(\bar{\nu}_e\bar{\nu}_e)$



## $(\beta\beta)_{0\nu}$ -Decay Experiments:

- Majorana nature of  $\nu_j$
- Type of  $\nu$ -mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

$^3\text{H}$   $\beta$ -decay, cosmology:  $m_\nu$  (QD, IH)

- CPV due to Majorana CPV phases

$A(\beta\beta)_{0\nu} \sim \langle m \rangle M(A, Z)$ ,  $M(A, Z)$  - NME,

$$\begin{aligned} |\langle m \rangle| &= |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i\alpha_{31}}| \\ &= |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i\alpha_{31}}|, \quad \theta_{12} \equiv \theta_\odot, \theta_{13} \text{- CHOOZ} \end{aligned}$$

$\alpha_{21}, \alpha_{31}$  - the two Majorana CPVP of the PMNS matrix.

**CP-invariance:**  $\alpha_{21} = 0, \pm\pi, \alpha_{31} = 0, \pm\pi$ ;

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of  $\nu_1$  and  $\nu_2$ , and of  $\nu_1$  and  $\nu_3$ .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle \text{ M(A,Z)}, \quad \text{M(A,Z) - NME},$$

$$|\langle m \rangle| \cong \left| \sqrt{\Delta m_{\odot}^2} \sin^2 \theta_{12} e^{i\alpha} + \sqrt{\Delta m_{31}^2} \sin^2 \theta_{13} e^{i\beta_M} \right|, \quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|\langle m \rangle| \cong \sqrt{m_3^2 + \Delta m_{13}^2} |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|\langle m \rangle| \cong m |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

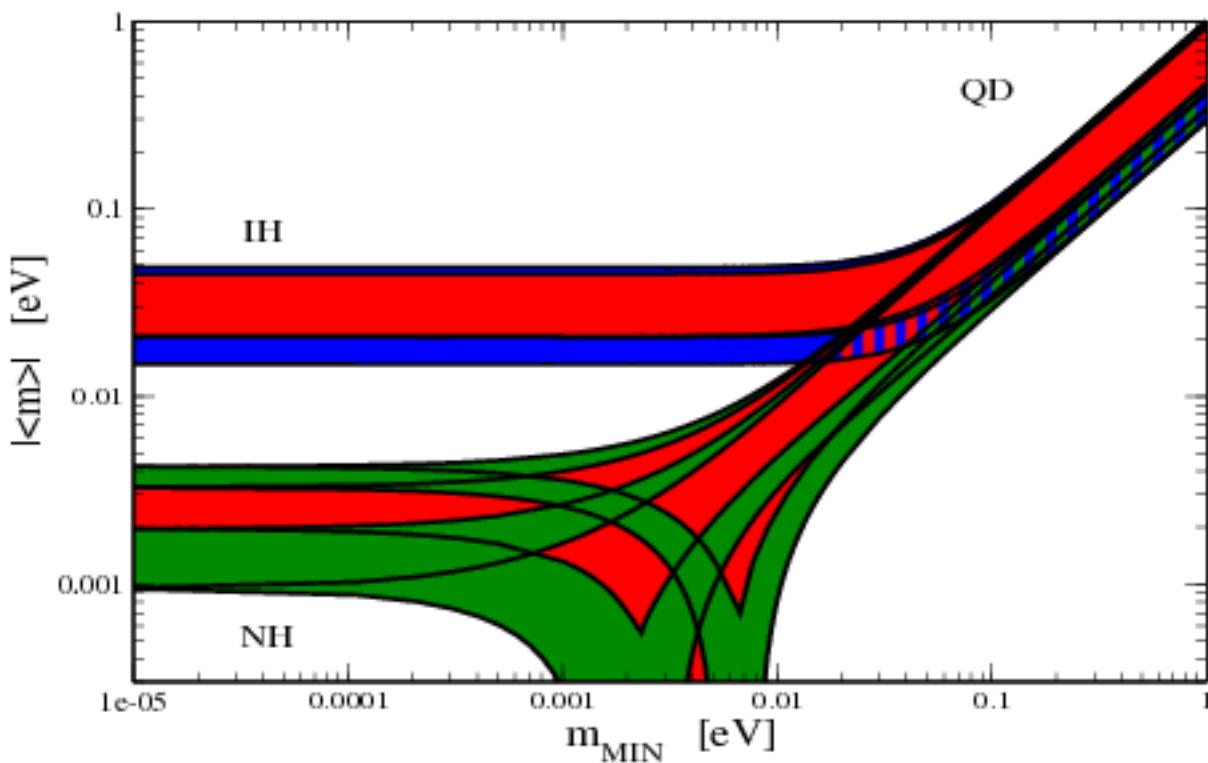
$\theta_{12} \equiv \theta_{\odot}$ ,  $\theta_{13}$ -CHOOZ;  $\alpha \equiv \alpha_{21}$ ,  $\beta_M \equiv \alpha_{31}$ .

**CP-invariance:**  $\alpha = 0, \pm\pi$ ,  $\beta_M = 0, \pm\pi$ ;

$$|\langle m \rangle| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m_{13}^2} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{13}^2} \cong 0.055 \text{ eV, IH};$$

$$m \cos 2\theta_{12} \lesssim |\langle m \rangle| \lesssim m, \quad m \gtrsim 0.10 \text{ eV, QD}.$$



S. Pascoli, PDG, 2012

$$\sin^2 \theta_{13} = 0.0236 \pm 0.0042; \delta = 0.$$

$$1\sigma(\Delta m_{21}^2) = 2.6\%, 1\sigma(\sin^2 \theta_{12}) = 5.4\%, 1\sigma(|\Delta m_{31(23)}^2|) = 3\%.$$

From G.L. Fogli *et al.*, arXiv:1205.5254v3

$2\sigma(|\langle m \rangle|)$  used.

Best sensitivity: GERDA ( $^{76}\text{Ge}$ ), EXO ( $^{136}\text{Xe}$ ), KamLAND-ZEN ( $^{136}\text{Xe}$ ).

Claim for a positive signal at  $> 3\sigma$ :

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|\langle m \rangle| = (0.1 - 0.9) \text{ eV}$  (99.73% C.L.); b.f.v.:  $|\langle m \rangle| = 0.33 \text{ eV}$ .

IGEX  $^{76}\text{Ge}$ :  $|\langle m \rangle| < (0.33 - 1.35) \text{ eV}$  (90% C.L.).

Recent data - NEMO3 ( $^{100}\text{Mo}$ ), CUORICINO ( $^{130}\text{Te}$ ):

$|\langle m \rangle| < (0.45 - 0.96) \text{ eV}$ ,  $|\langle m \rangle| < (0.18 - 0.64) \text{ eV}$  (90% C.L.).

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$\tau(^{76}\text{Ge}) = 2.23_{-0.31}^{+0.44} \times 10^{25} \text{ yr at 90\% C.L.}$$

Results from 2012-2013:

$$\tau(^{136}\text{Xe}) > 1.6 \times 10^{25} \text{ yr at 90\% C.L., EXO}$$

$$\tau(^{136}\text{Xe}) > 1.9 \times 10^{25} \text{ yr at 90\% C.L., KamLAND – Zen}$$

$$\tau(^{76}\text{Ge}) > 2.1 \times 10^{25} \text{ yr at 90\% C.L., GERDA.}$$

$$\tau(^{76}\text{Ge}) > 3.0 \times 10^{25} \text{ yr at 90\% C.L., GERDA + IGEX + HdM.}$$

Large number of experiments:  $|\langle m \rangle| \sim (0.01-0.05) \text{ eV}$

CUORE -  $^{130}\text{Te}$ ,

GERDA -  $^{76}\text{Ge}$ ,

KamLAND-ZEN -  $^{136}\text{Xe}$ ;

EXO -  $^{136}\text{Xe}$ ;

SNO+ -  $^{130}\text{Te}$ ;

AMoRE -  $^{100}\text{Mo}$  (S. Korea);

CANDLES -  $^{48}\text{Ca}$ ;

SuperNEMO -  $^{82}\text{Se}, \dots$ ;

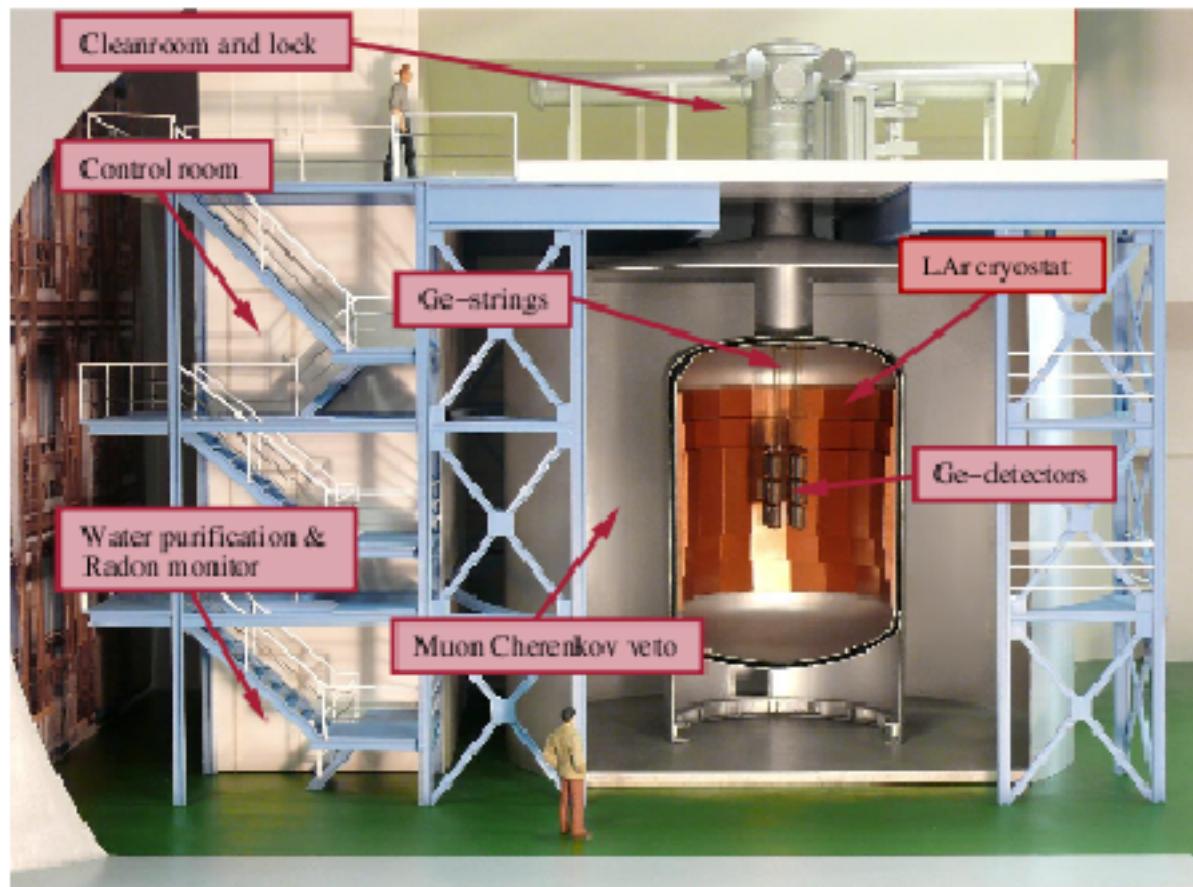
MAJORANA -  $^{76}\text{Ge}$ ;

COBRA -  $^{116}\text{Cd}$ ;

MOON -  $^{100}\text{Mo}$ .



## GERDA: Experimental Setup



GERDA  
VERSITÄT  
NGEN



## Majorana CPV Phases and $|\langle m \rangle|$

CPV can be established provided

- $|\langle m \rangle|$  measured with  $\Delta \lesssim 15\%$  ;
- $\Delta m_{\text{atm}}^2$  (IH) or  $m_0$  (QD) measured with  $\delta \lesssim 10\%$  ;
- $\xi \lesssim 1.5$  ;
- $\alpha_{21}$  (QD): in the interval  $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$ , or  $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$  ;
- $\tan^2 \theta_\odot \gtrsim 0.40$  .

S. Pascoli, S.T.P., W. Rodejohann, 2002

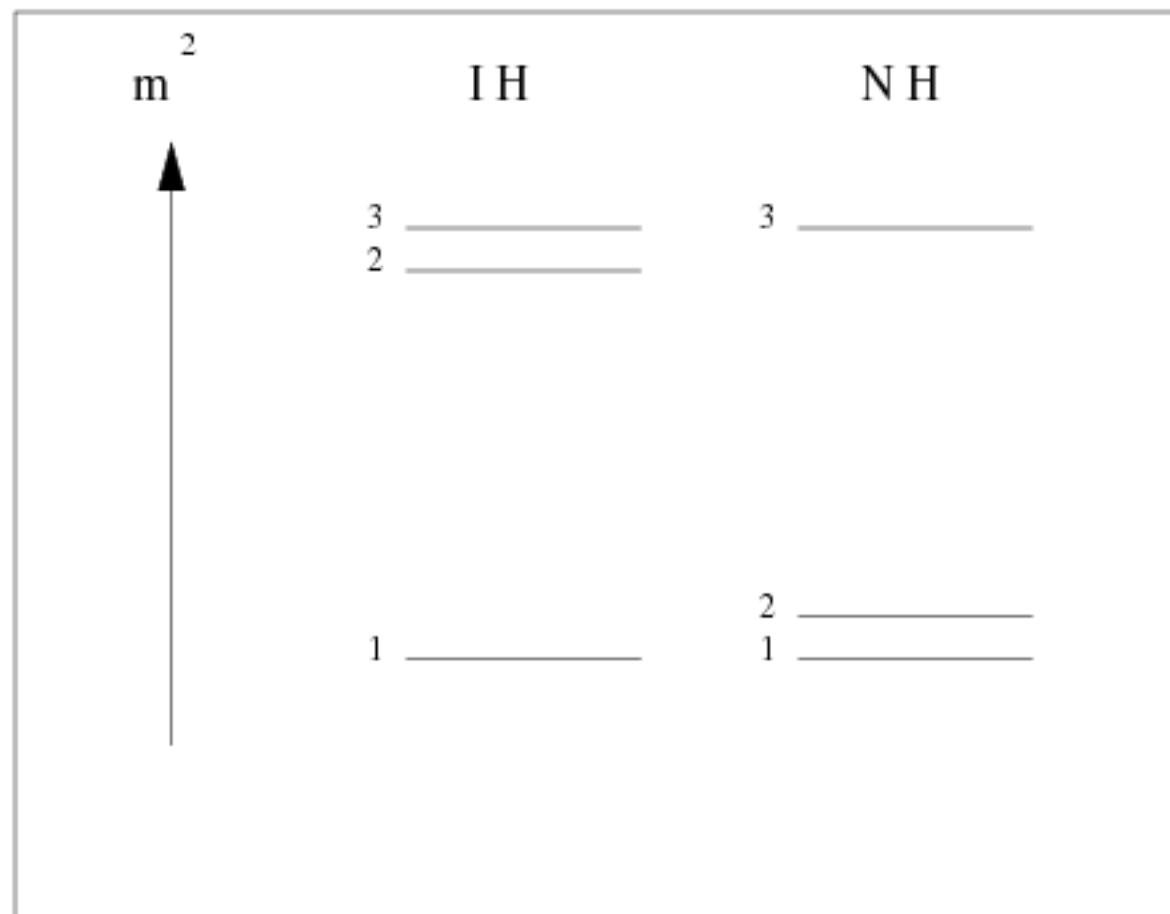
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via  $(\beta\beta)_{0\nu}$ -decay"

V. Barger *et al.*, 2002

## Determining the $\nu$ -Mass Hierarchy ( $\text{sgn}(\Delta m_{\text{atm}}^2)$ )



Our convention for IO:  $m_3 < m_1 < m_2$ .

## Determining the $\nu$ -Mass Hierarchy ( $\text{sgn}(\Delta m_{\text{atm}}^2)$ )

- Reactor  $\bar{\nu}_e$  Oscillations in vacuum (Day Bay II (JUNO), RENO50).
- Atmospheric  $\nu$  experiments: subdominant  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  and  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  oscillations (matter effects) (HK, PINGU (IceCube), INO).
- LBL  $\nu$ -oscillation experiments (T2K, NO $\nu$ A; LBNO, LBNE,  $\nu$ -factory); designed to search also for CP violation.
- ${}^3\text{H}$   $\beta$ -decay Experiments (sensitivity to  $5 \times 10^{-2}$  eV) (NH vs IH).
- $(\beta\beta)_{0\nu}$ -Decay Experiments;  $\nu_j$ - Majorana particles (NH vs IH).
- Cosmology:  $\sum_j m_j$  (NH vs IH).
- Atomic Physics Experiments: RENP.

## Reactor $\bar{\nu}_e$ Oscillations in vacuum

$$P_{\text{NO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left( 1 - \cos \frac{\Delta m_A^2 L}{2E_\nu} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_\odot \left( 1 - \cos \frac{\Delta m_\odot^2 L}{2E_\nu} \right) \\ + \sin^2 2\theta_{13} \sin^2 \theta_\odot \sin \frac{\Delta m_\odot^2 L}{4E_\nu} \sin \left( \frac{\Delta m_A^2 L}{2E_\nu} - \frac{\Delta m_\odot^2 L}{4E_\nu} \right),$$

$$P_{\text{IO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left( 1 - \cos \frac{\Delta m_A^2 L}{2E_\nu} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_\odot \left( 1 - \cos \frac{\Delta m_\odot^2 L}{2E_\nu} \right) \\ + \sin^2 2\theta_{13} \cos^2 \theta_\odot \sin \frac{\Delta m_\odot^2 L}{4E_\nu} \sin \left( \frac{\Delta m_A^2 L}{2E_\nu} - \frac{\Delta m_\odot^2 L}{4E_\nu} \right),$$

$\theta_\odot = \theta_{12}$ ,  $\Delta m_\odot^2 = \Delta m_{21}^2 > 0$ ;  $\sin^2 \theta_{12} \leq 0.36$  at  $3\sigma$ ;

$\Delta m_A^2 = \Delta m_{31}^2 > 0$ , NH spectrum,

$\Delta m_A^2 = \Delta m_{23}^2 > 0$ , IH spectrum

The reactor  $\bar{\nu}_e$  detected via

$$\bar{\nu}_e + p \rightarrow e^+ + n.$$

The visible energy of the detected  $e^+$ :

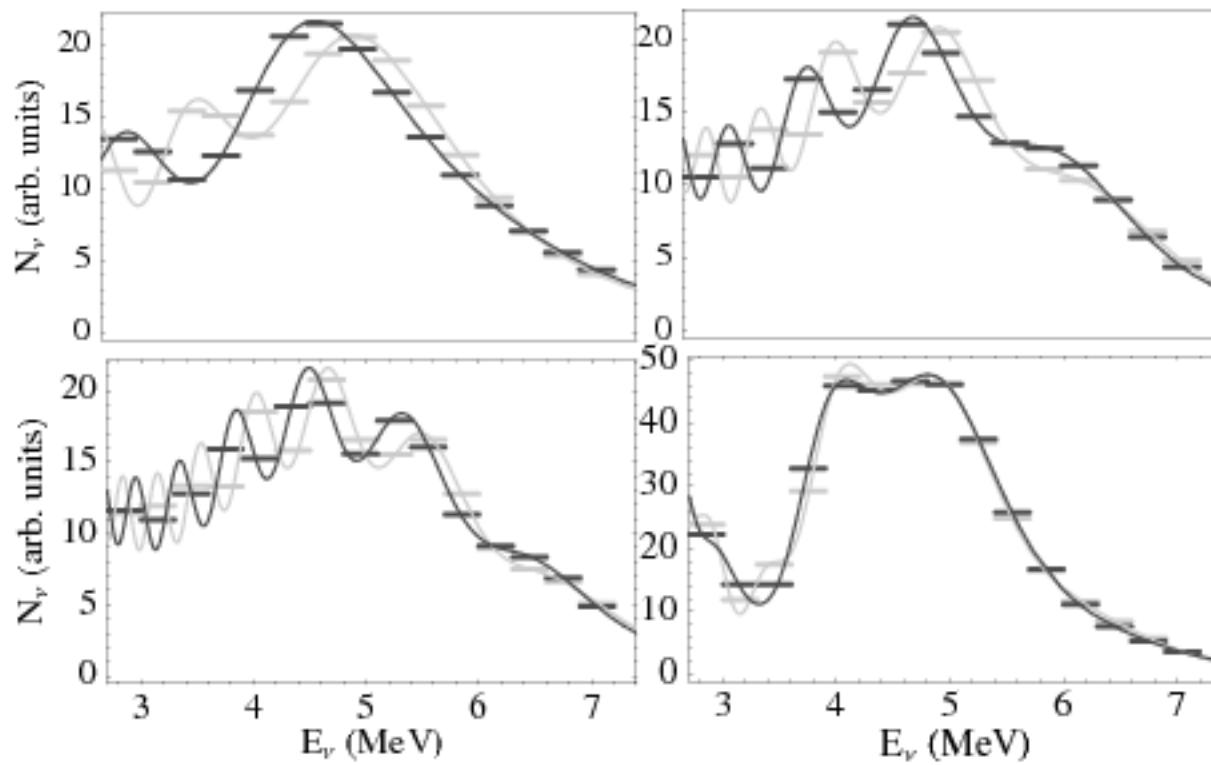
$$E_{vis} = E + m_e - (m_n - m_p) \simeq E - 0.8 \text{ MeV}.$$

The measured event rate spectrum vs.  $L/E_m$ :

$$N(L/E_m) = \int R(E, E_m) \Phi(E) \sigma(\bar{\nu}_e p \rightarrow e^+ n; E) P_{\bar{e}\bar{e}}^{NO(FO)} dE.$$

$$|P_{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) - P_{FO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)| \propto \sin^2 2\theta_{13} \cos 2\theta_{12}$$

$$\cos 2\theta_{12} \cong 0.38; \quad 3\sigma : \quad \cos 2\theta_{12} \geq 0.28; \quad \sin^2 2\theta_{13} \cong 0.09.$$



M. Piai, S.T.P., 2001

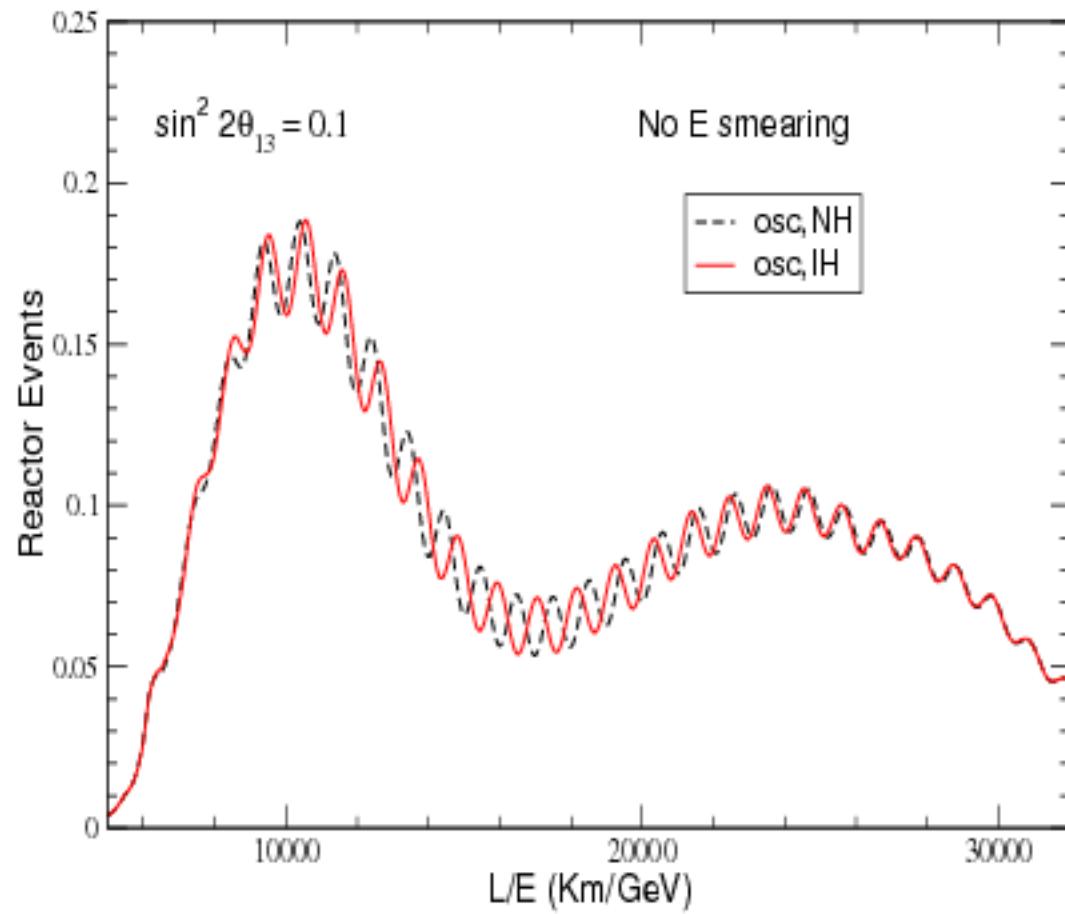
$$\sin^2 \theta_{13} = 0.05, \quad \Delta m_{21}^2 = 2 \times 10^{-4} \text{ eV}^2; \quad \Delta m_A^2 = 1.3; \quad 2.5; \quad 3.5 \times 10^{-3} \text{ eV}^2$$

$$L = 20 \text{ km}, \Delta E_\nu = 0.3 \text{ MeV}.$$

$$\Delta m_{21}^2 = 2 \times 10^{-4} \text{ eV}^2; \quad L = 20 \text{ km};$$

$$\Delta m_{21}^2 = 7.6 \times 10^{-5} \text{ eV}^2; \quad L \cong 53 \text{ km}.$$

NH – light grey; IH – dark grey



P. Ghoshal, S.T.P., arXiv:1011.1646

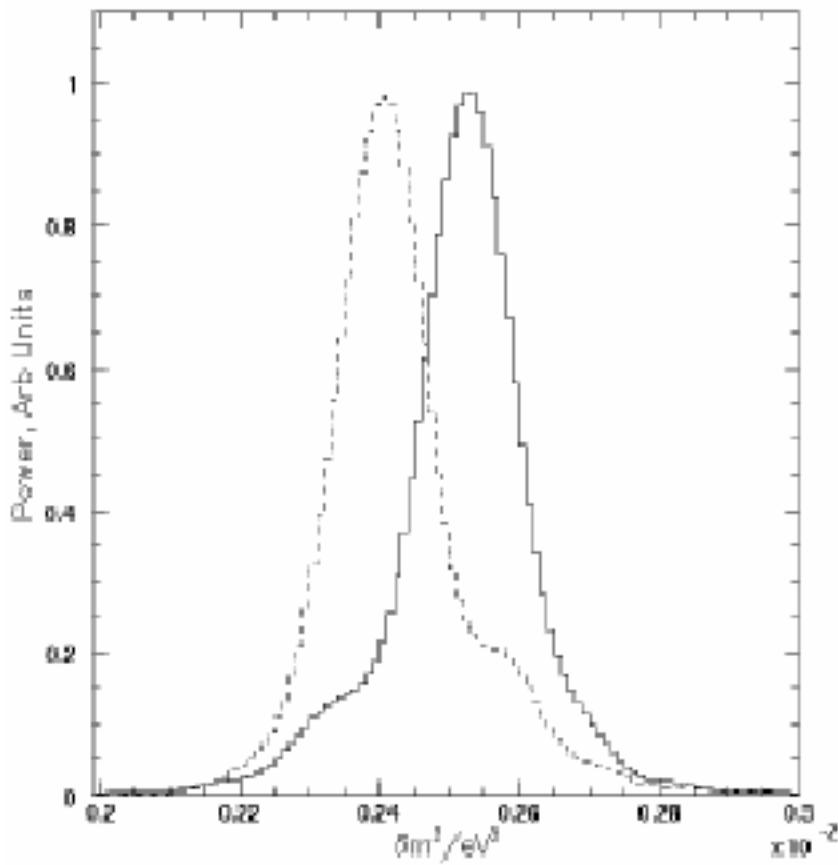
## Fourier Analysis:

$$\text{NO : } \cos^2 \theta_{12} \sin^2 \Delta + \sin^2 \theta_{12} \sin^2(\Delta - \Delta_{21}),$$

$$\text{IO : } \sin^2 \theta_{12} \sin^2 \Delta + \cos^2 \theta_{12} \sin^2(\Delta - \Delta_{21}),$$

$$\Delta \equiv \Delta_{31}(NH) = |\Delta_{32}(IH)|;$$

$$\sin^2 \theta_{12} \cong 0.31, \quad \cos^2 \theta_{12} \cong 0.69.$$



J.Learned et al., 2007

Very challenging; requires:

- energy resolution  $\sigma/E_{\text{vis}} \lesssim 3\%/\sqrt{E_{\text{vis}}}$ ;
- relatively small energy scale uncertainty;
- relatively large statistics ( $\sim (300 - 1000) \text{ kT GW yr}$ );
- relatively small systematic errors;
- subtle optimisations (distance, number of bins, effects of “interfering distant” reactors).

Two experiments planned with  $L \cong 50 \text{ km}$ : Daya Bay II (20 kT), RENO50 (18 kT). Can measure also  $\sin^2 \theta_{12}$ ,  $\Delta m_{21}^2$  and  $|\Delta m_{31}^2|$  with remarkably high precision. Can be used for detection of SN neutrinos as well.

## Atmospheric Neutrino Experiments on $\text{sgn}(\Delta m_{31}^2)$

## Atmospheric $\nu$ experiments

Subdominant  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  and  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s_{23}^2 P_{2\nu}, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c_{23}^2 P_{2\nu},$$
$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s_{23}^4 P_{2\nu} - 2c_{23}^2 s_{23}^2 [1 - \text{Re } (e^{-i\kappa} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))],$$

$P_{2\nu} \equiv P_{2\nu}(\Delta m_{31}^2, \theta_{13}; E, \theta_n; N_e)$ : 2- $\nu$   $\nu_e \rightarrow \nu'_\tau$  oscillations in the Earth,  
 $\nu'_\tau = s_{23} \nu_\mu + c_{23} \nu_\tau$ ;  $\Delta m_{21}^2 \ll |\Delta m_{31(32)}^2|$ ,  $E_\nu \gtrsim 2$  GeV;

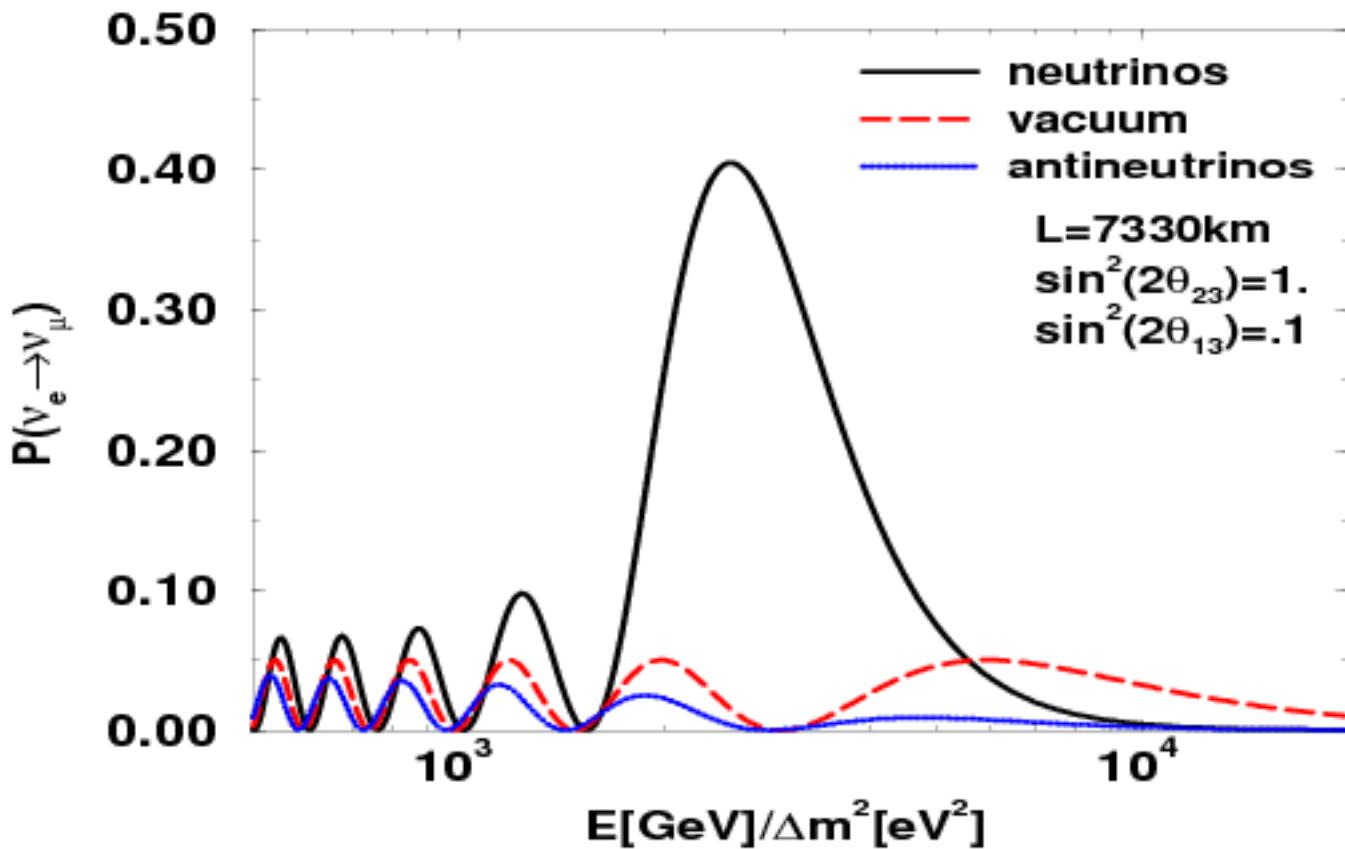
$\kappa$  and  $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$  are known phase and 2- $\nu$  amplitude.

NO:  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  matter enhanced,  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  - suppressed

IO:  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  matter enhanced,  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  - suppressed

No charge identification (SK, HK, IceCube-PINGU); event rate (DIS regime):  
[ $2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)$ ]/3

## Earth matter effect in $\nu_\mu \rightarrow \nu_e$ , $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (MSW)



$$\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2, E^{\text{res}} = 6.25 \text{ GeV}; P^{3\nu} = \sin^2 \theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}; N_e^{\text{res}} \cong 2.3 \text{ cm}^{-3} N_A; L_m^{\text{res}} = L^{\nu} / \sin 2\theta_{13} \cong 6250 / 0.32 \text{ km}; 2\pi L / L_m \cong 0.75\pi (\neq \pi).$$

Hyper Kamiokande (10SK), INO, IceCube-PINGU;

Iron Magnetised detector: INO

INO: 50 or 100 kt (in India);  $\nu_\mu$  and  $\bar{\nu}_\mu$  induced events detected ( $\mu^+$  and  $\mu^-$ );  
not designed to detect  $\nu_e$  and  $\bar{\nu}_e$  induced events.

IceCube at the South Pole: PINGU (?)

PINGU: 50SK;  $\nu_\mu$  and  $\bar{\nu}_\mu$  induced events detected ( $\mu^+$  and  $\mu^-$ , no  $\mu$  charge identification); Challenge:  $E_\nu \gtrsim 2$  GeV (?)

Water-Cerenkov detector: Hyper Kamiokande (10SK)

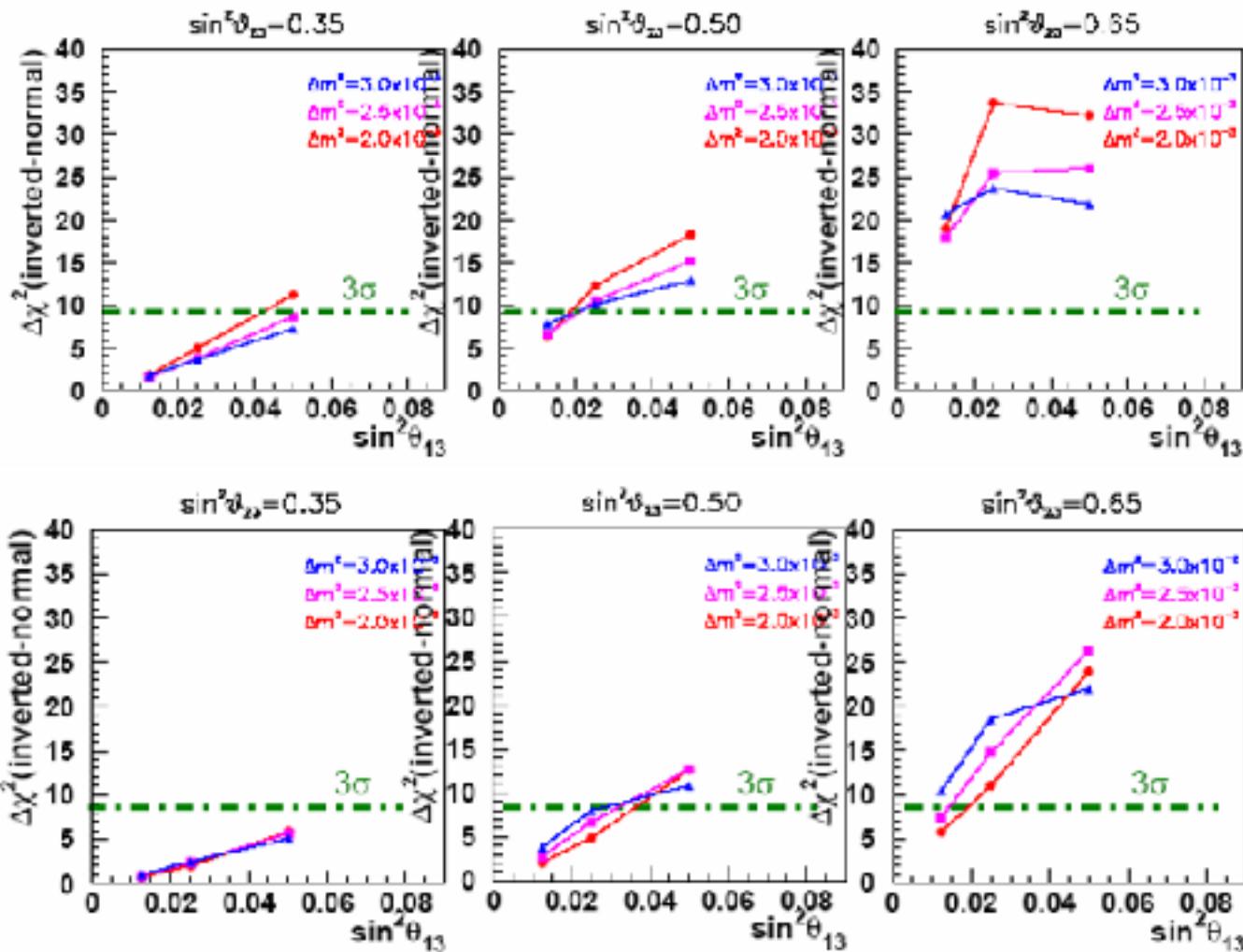
Sensitivity depends critically on  $\theta_{23}$ , the “true” hierarchy.

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003

$$P(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

No charge identification (SK, HK, IceCube-PINGU);  
event rate (DIS regime):

$$[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$$

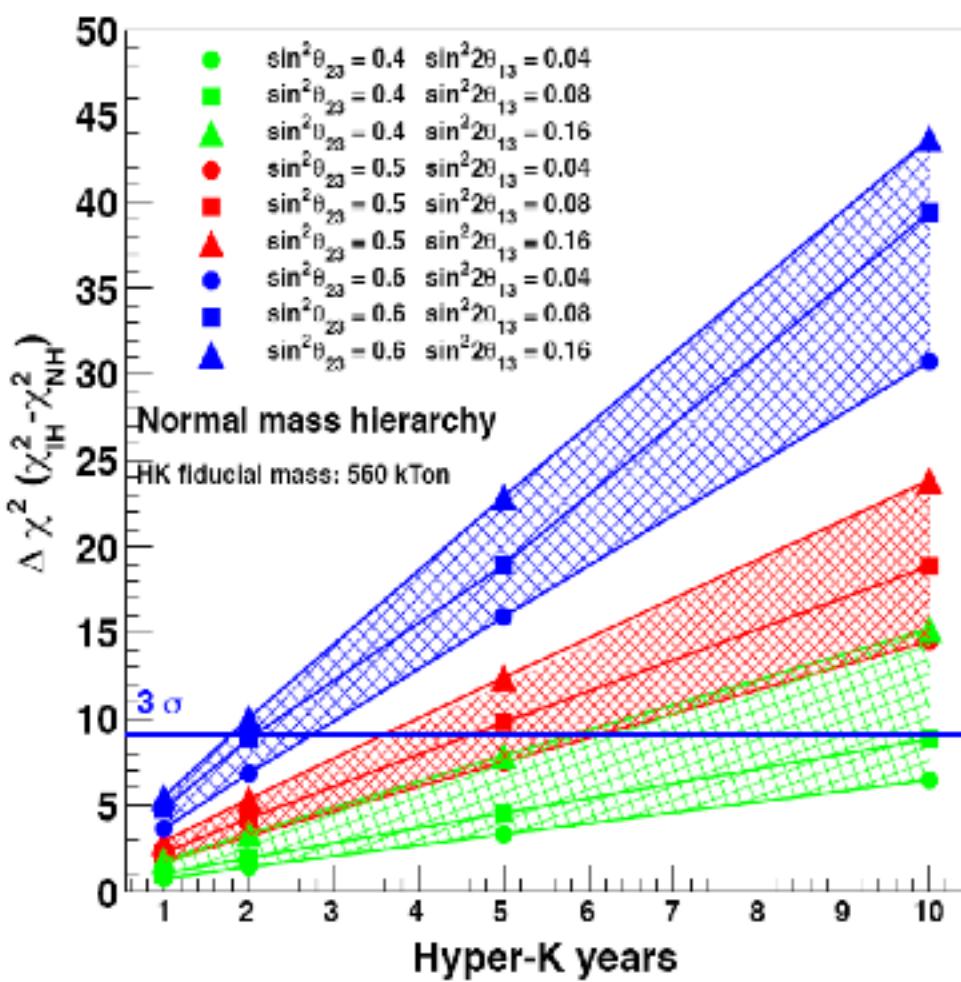


Water-Cerenkov detector, 1.8 MTy (HK = 10SK)

Critical dependence on  $\theta_{23}$ , "true hierarchy".

T. Kajita et al., 2004

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003



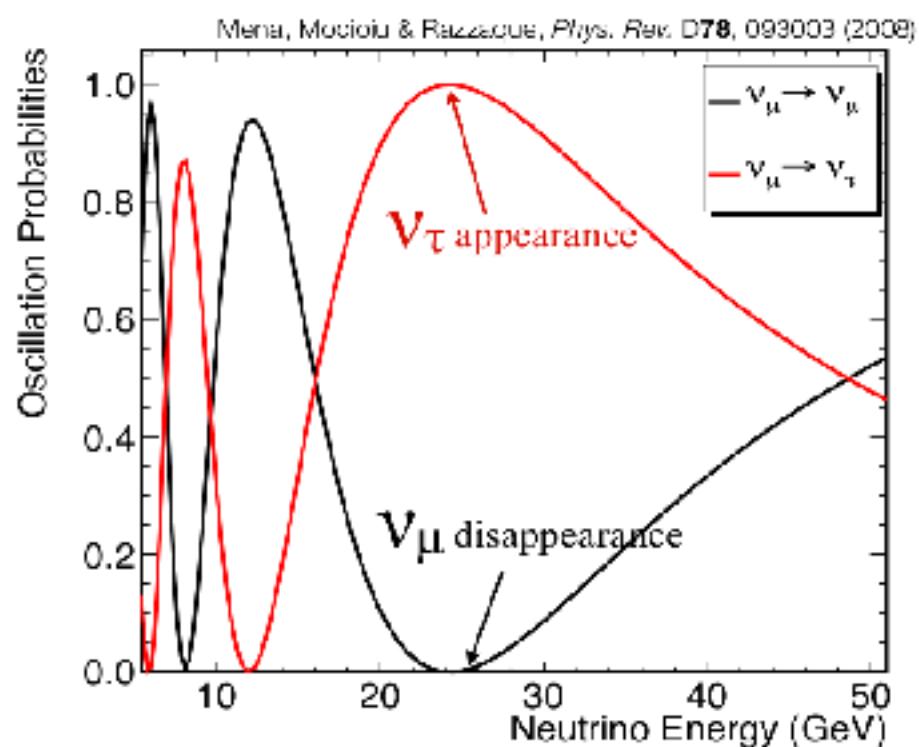
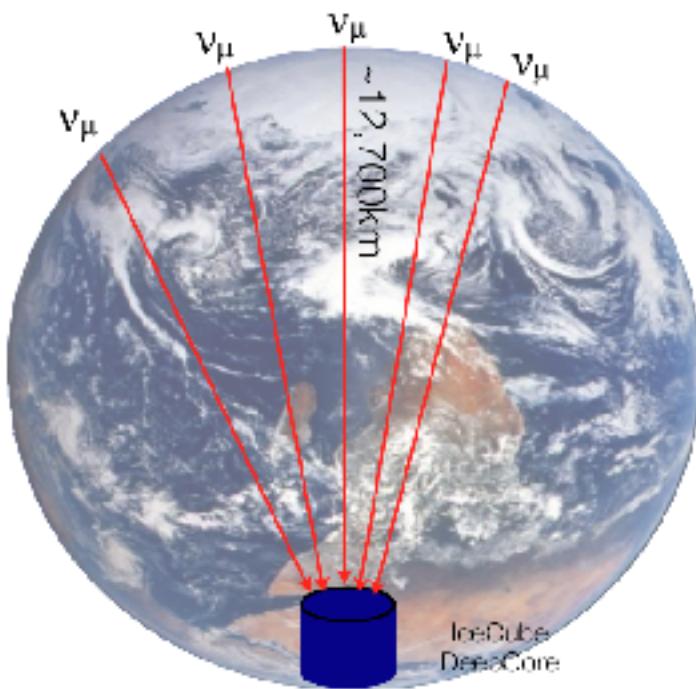
Sensitivity to the neutrino mass hierarchy from HK atmospheric neutrino data.  $\theta_{23}$  and  $\theta_{13}$  are assumed to be known as indicated in the figure.

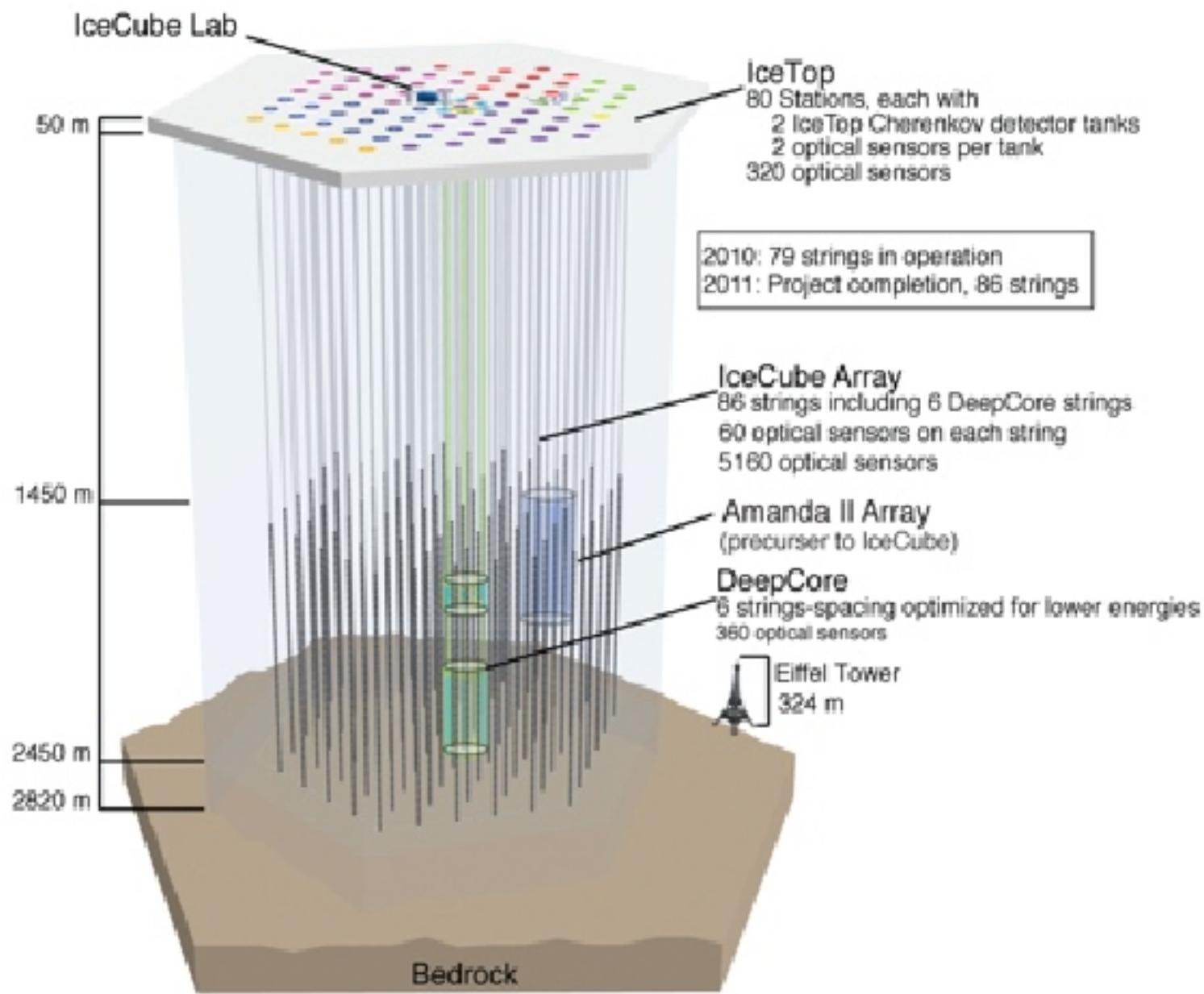
K. Abe et al. [Letter of intent: Hyper-Kamiokande Experiment], arXiv:1109.3262.

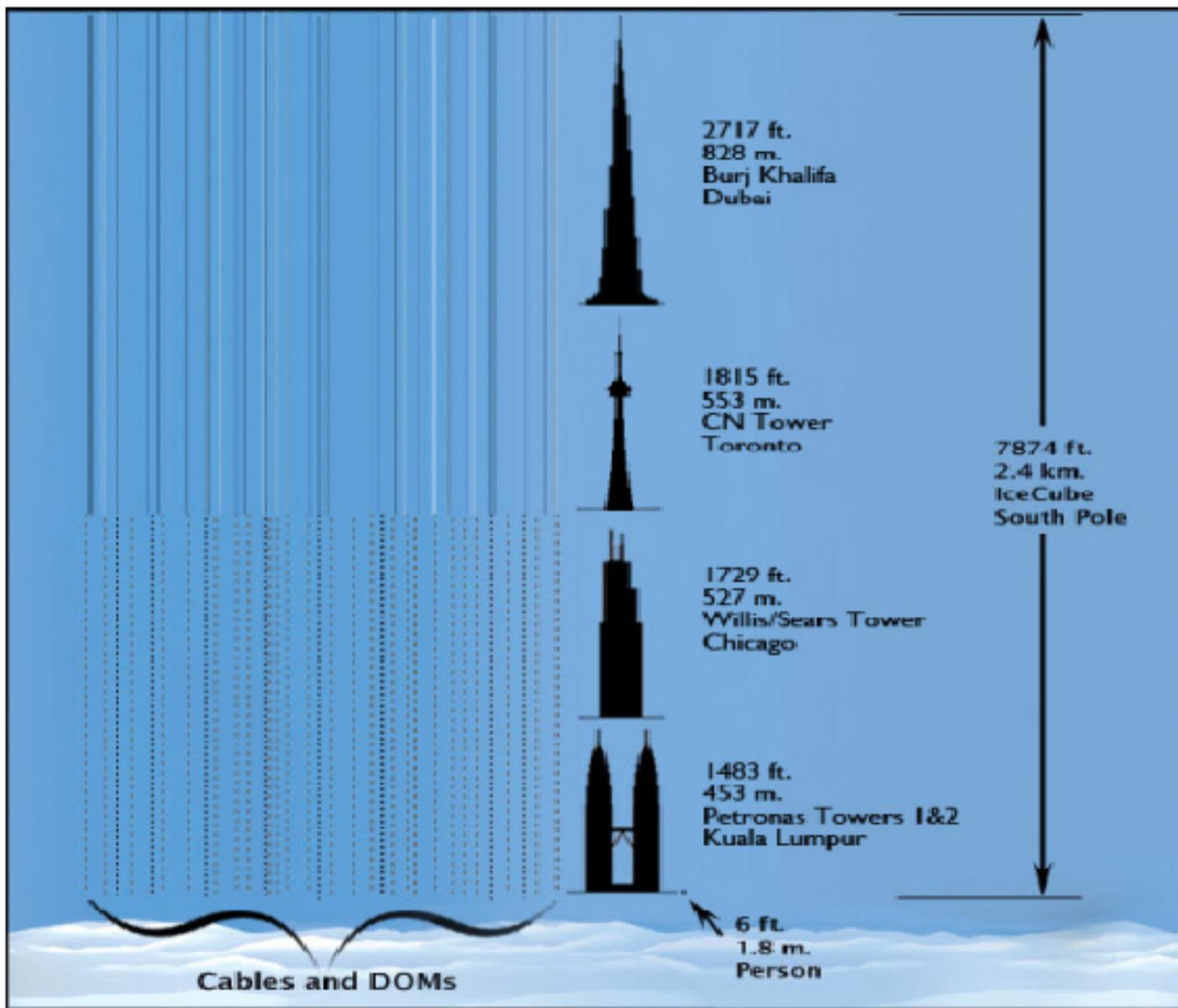
# Neutrino Oscillation Source

- Oscillation
- IceCube-DeepCore Physics
- PINGU
- Beyond

- Northern Hemisphere  $\nu_\mu$  oscillating over one earth radii produces  $\nu_\mu$  ( $\nu_\tau$ ) oscillation minimum(maximum) at  $\sim 25$  GeV
  - Covers all possible terrestrial baselines
  - "Beam" is free and never turns off



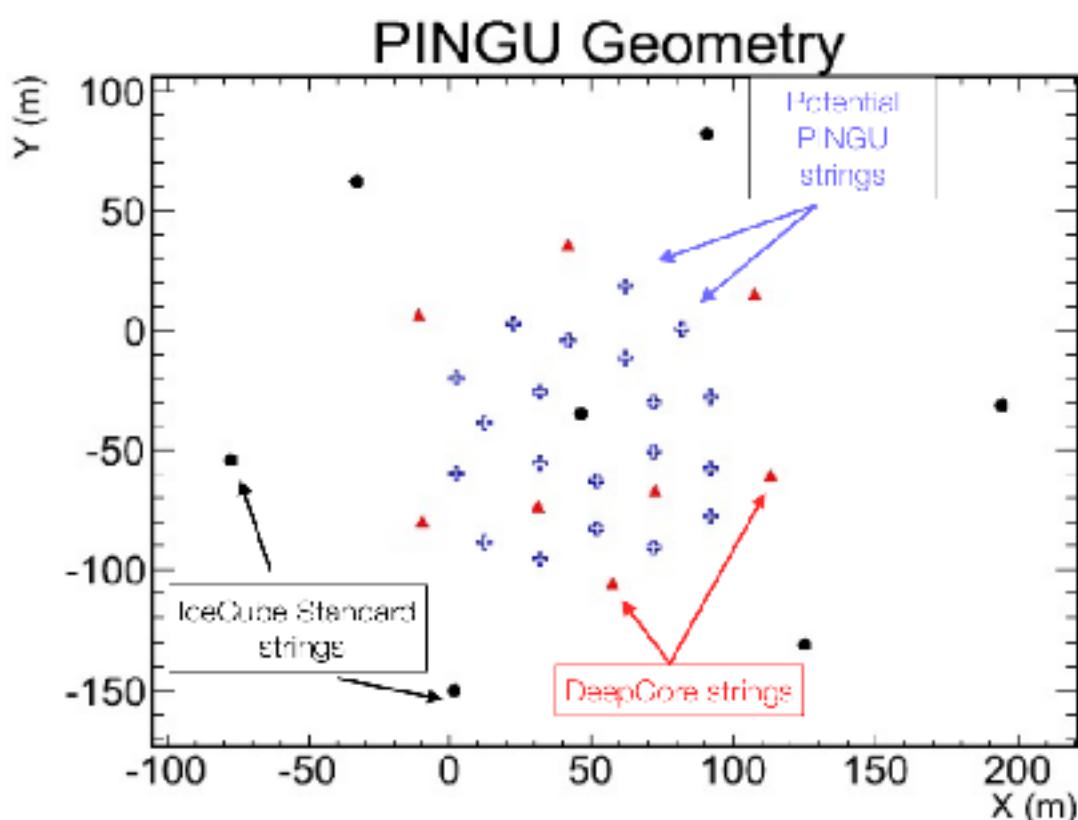




# PINGU: Possible Geometry

- Oscillation
- IceCube-DeepCore Physics
- PINGU
- Beyond

- ~20 strings within DeepCore volume w/ short string-string spacing
  - IC-IC: 125m
  - DC-DC: ~80m
  - PINGU-PINGU: <= 26m
- Shorter DOM-DOM spacing
  - IC-IC: 17m
  - DC-DC: 7m
  - PINGU-PINGU: <= 5m
- R & D for future water/ice cerenkov





# Future LBL Neutrino Oscillation Experiments on $\text{sgn}(\Delta m_{31}^2)$ (the Hierarchy) and CP Violation

## Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the background of electrons, protons and neutrinos, which generates an effective potential in the neutrino Hamiltonian:  $H = H_{vac} + V_{eff}$ .

This modifies the neutrino mixing since the eigenstates and the eigenvalues of  $H_{vac}$  and of  $H = H_{vac} + V_{eff}$  are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$\sin^2 2\theta_{13}^m$ ,  $\Delta M_{31}^2$  depend on the matter potential  
 $V_{eff} = \sqrt{2} G_F N_e$ ,

For antineutrinos  $V_{eff}$  has the opposite sign:

$$V_{eff} = -\sqrt{2} G_F N_e.$$

$\Delta m_{31}^2 > 0$  (NO):  $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  matter enhanced,  
 $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  - suppressed

$\Delta m_{31}^2 < 0$  (IO):  $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$  matter enhanced,  
 $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$  - suppressed

Up to 2nd order in the two small parameters  $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$  and  $\sin^2 \theta_{13} \ll 1$ :

$$P_m^{3\nu \text{ man}}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{\text{man}} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

## LBL Oscillation Experiments NO $\nu$ A, LBNE, LBNO

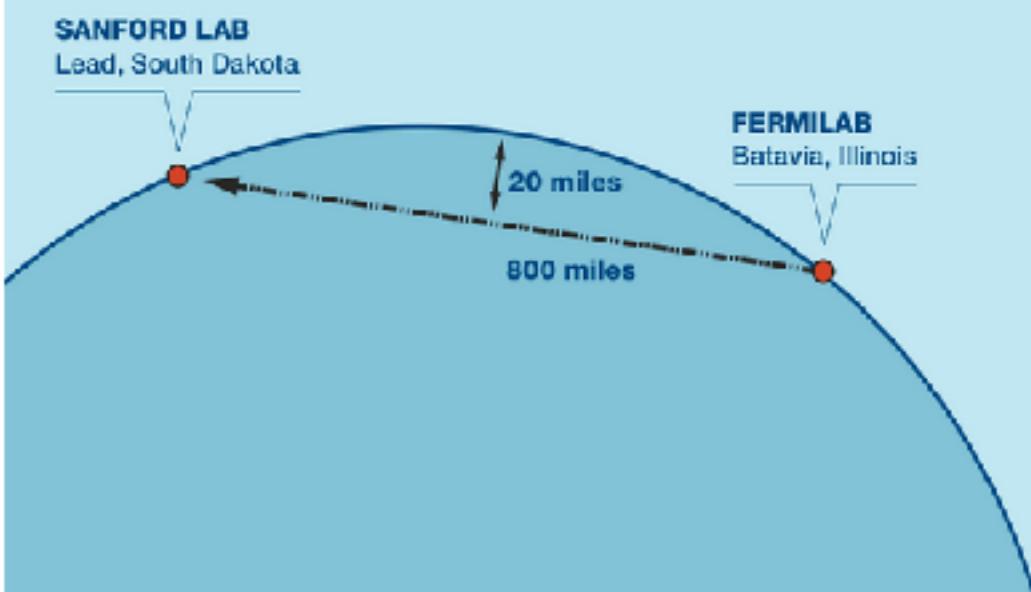
**NO $\nu$ A:** Fermilab - site in Minnesota; off-axis  $\nu$  beam,  $E = 2$  GeV,  $L \cong 810$  km, 14 kt liquid scintillator; 2013.

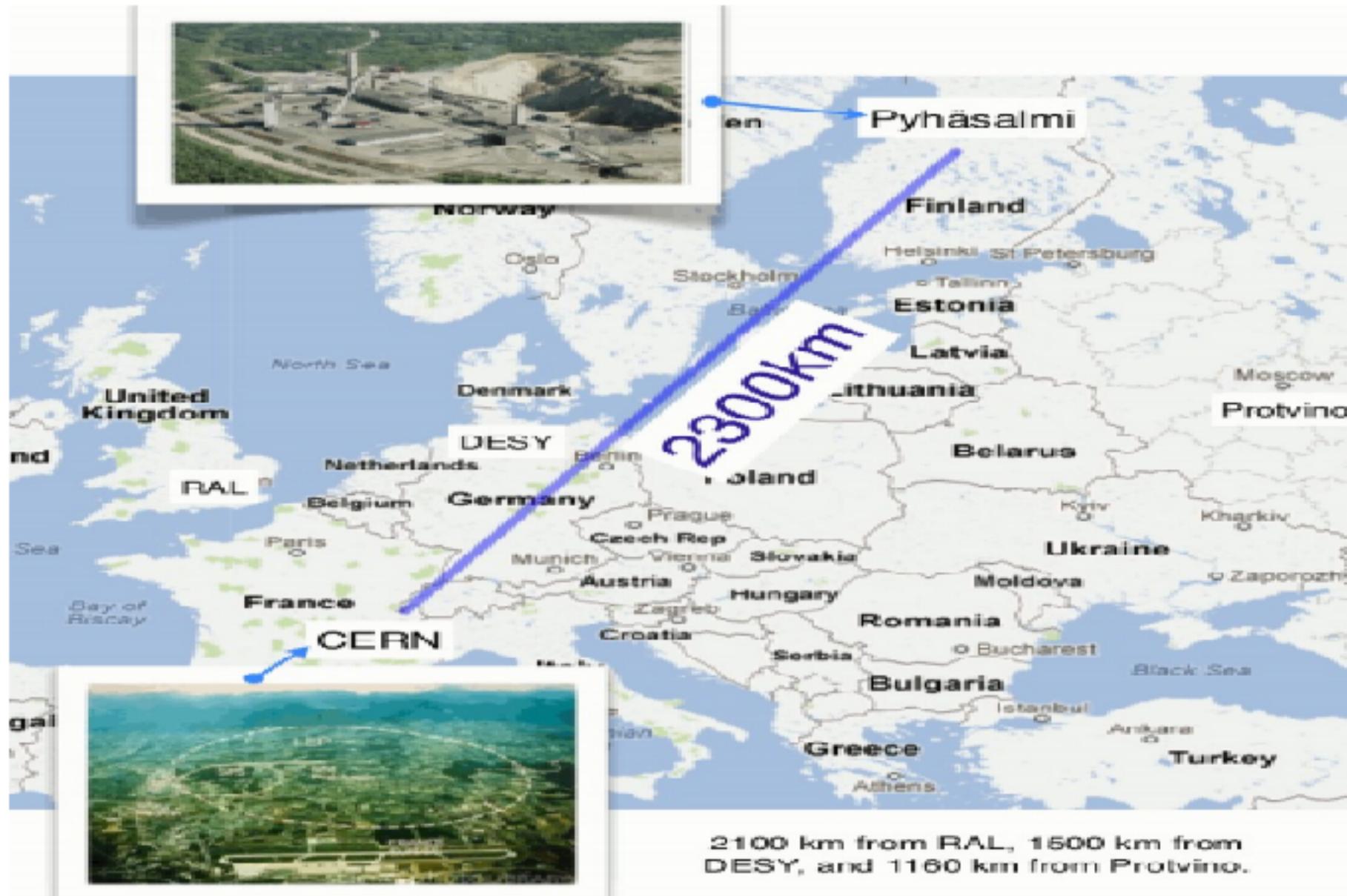
**LBNE:** Fermilab-DUSEL,  $L = 1290$  km, 700 KW wide band  $\nu$  beam (first and second osc. maxima at  $E = 2.4$  GeV and 0.8 GeV); 2 or 3 100 kt Water Cherenkov with 15% to 30% PMT coverage, or multiple 17 kt fiducial volume LAr detectors; plans to run 5 years with  $\nu_\mu$  and 5 years with  $\bar{\nu}_\mu$ ; 2023 (?)

**LAGUNA-LBNO:** CERN-Pyhasalmi,  $L = 2290$  km, wide band  $\nu_\mu$  1.6 MW super beam (first and second osc. maxima at  $E \cong 4$  GeV and 1.5 GeV); 440 kt Water Cherenkov, or 100 kt LAr, or 50 kt liquid scintillator detector; 2023 (?)



## Long-Baseline Neutrino Experiment

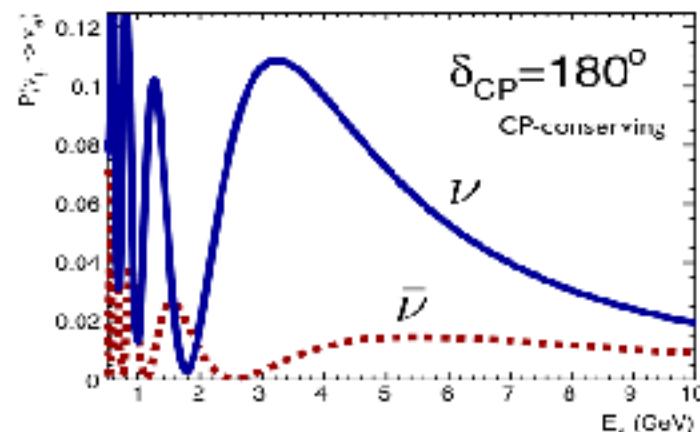
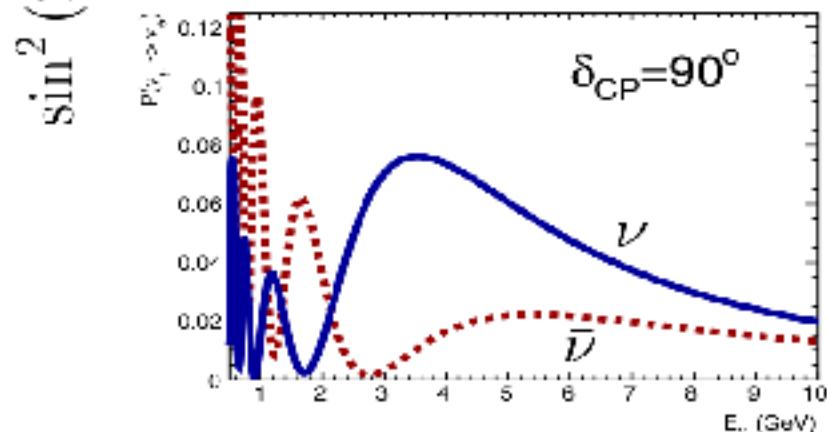
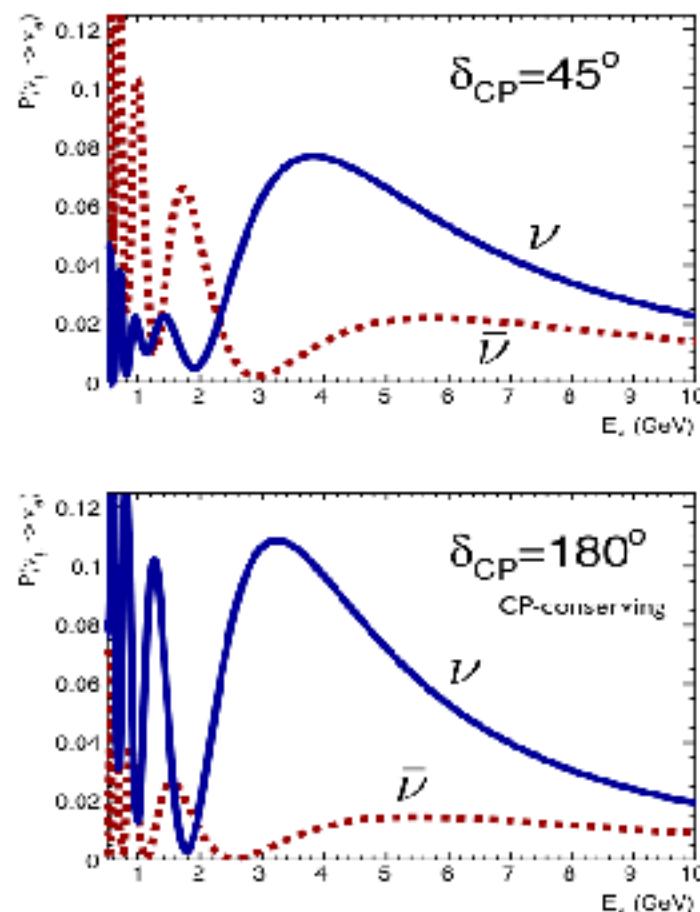
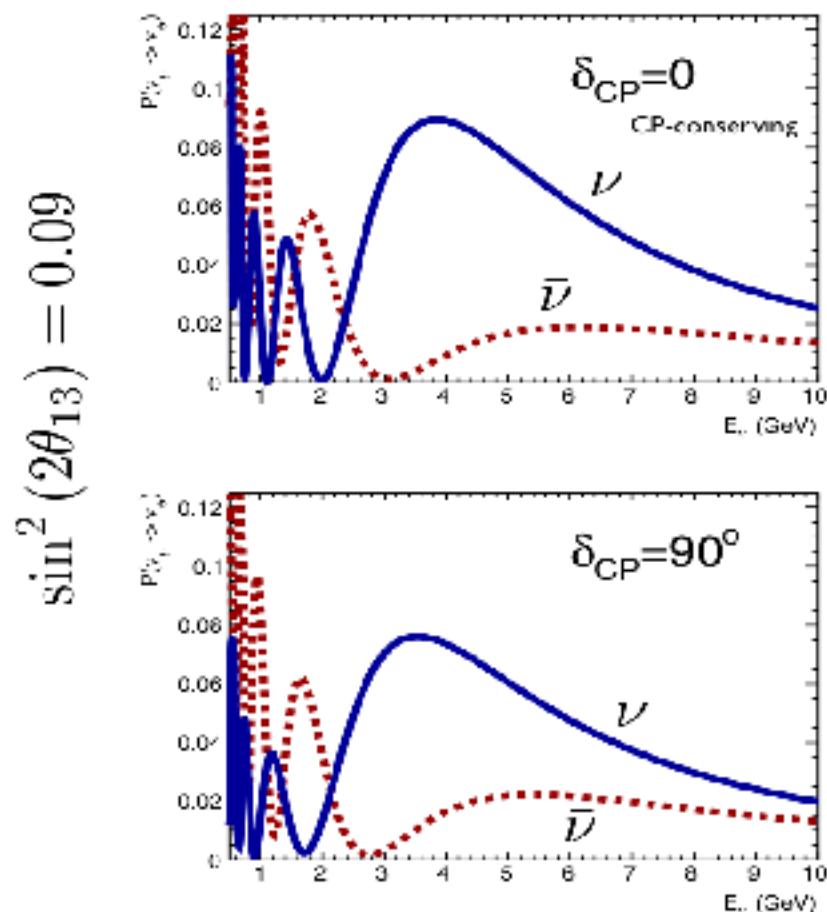




# CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★Normal mass hierarchy

L=2300 km

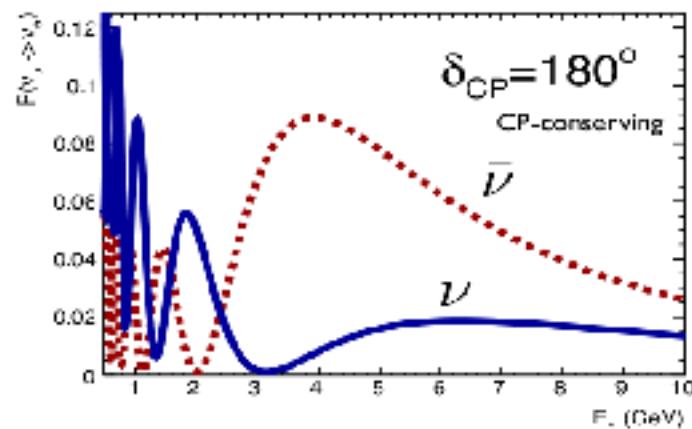
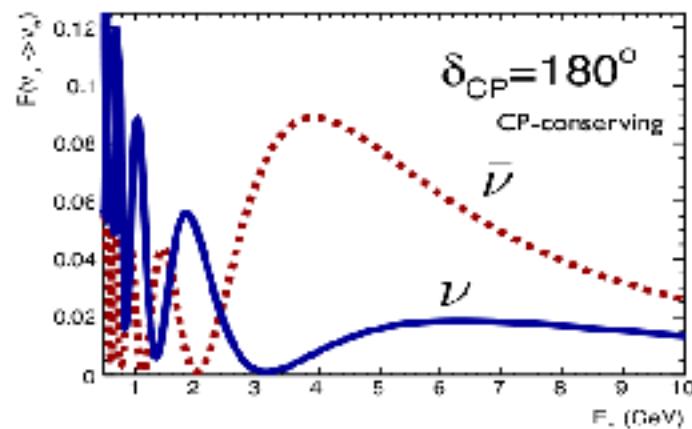
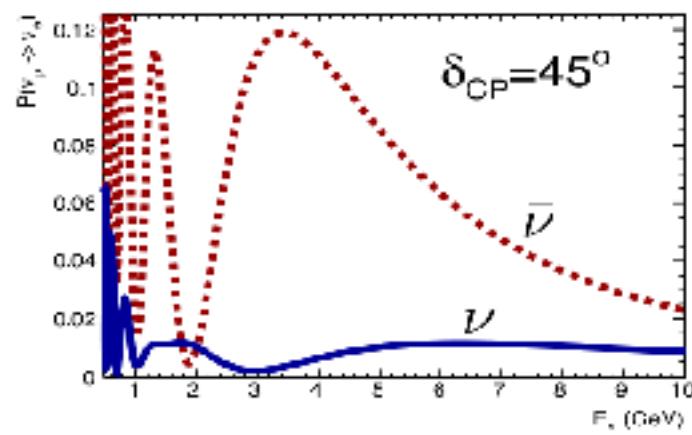
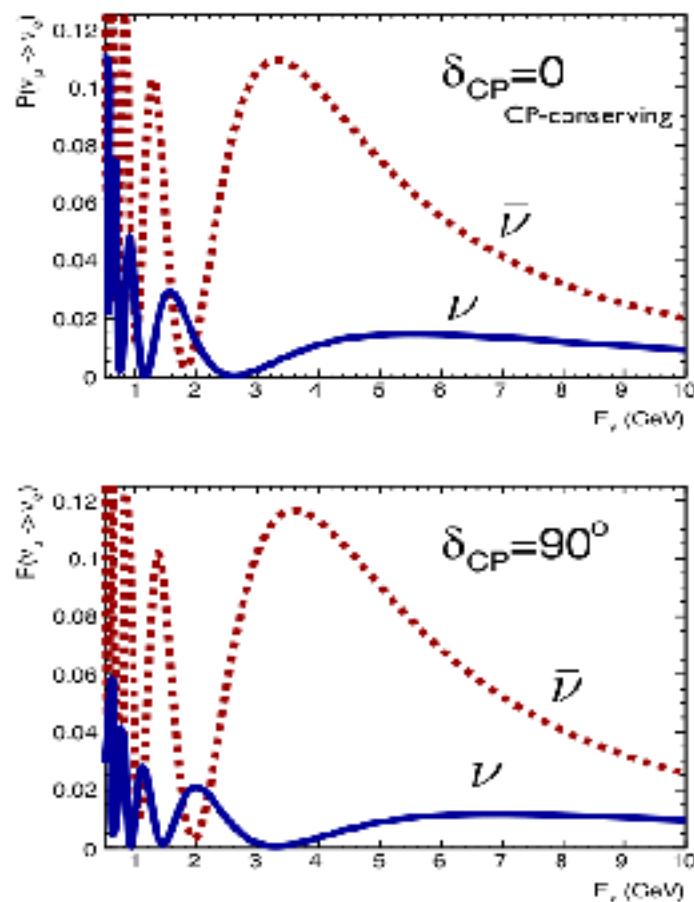


# CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★ Inverted mass hierarchy

L=2300 km

$$\sin^2(2\theta_{13}) = 0.09$$



The Nature of Massive Neutrinos II:  
Origins of Dirac and Majorana Massive Neutrinos

- Massive Dirac Neutrinos:  $U(1)$ , Conserved (Additive) Charge, e.g.,  $L$ .
- Massive Majorana Neutrinos: No Conserved (Additive) Charge(s).

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term  $\mathcal{L}_m^\nu(x)$  neutrinos have, more precisely, by the symmetries  $\mathcal{L}_m^\nu(x)$  and the total Lagrangian  $\mathcal{L}(x)$  of the theory have.

Mass Term: any by-linear in fermion (neutrino) fields invariant under the proper Lorentz transformations.

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term  $\mathcal{L}_m^\nu(x)$  neutrinos have, more precisely, by the symmetries  $\mathcal{L}_m^\nu(x)$  and the total Lagrangian  $\mathcal{L}(x)$  of the theory have.

- Dirac Neutrinos: Dirac Mass Term, requires  $\nu_R(x)$  -  $SU(2)_L$  singlet RH  $\nu$  fields

$$\mathcal{L}_D^\nu(x) = -\overline{\nu_{lR}}(x) M_{Dl} \nu_{lL}(x) + h.c. , \quad M_D - \text{complex}$$

- $\mathcal{L}_D^\nu(x)$  conserves  $L$ :  $L = \text{const.}$

$$M_D = V M_D^{\text{diag}} W^\dagger, \quad V, U - \text{unitary (bi-unitary transformation)}, \quad W \equiv U_{\text{PMNS}}$$

- ST + 3  $\nu_R(x)$  - RH  $\nu$  fields:  $n = 3$

$$\begin{aligned} \mathcal{L}_Y(x) &= Y_{l'l}^\nu \overline{\nu_{lR}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ M_D &= \frac{v}{\sqrt{2}} Y^\nu, \quad v = 246 \text{ GeV}. \end{aligned}$$

No explanation why  $m(\nu_j) \ll m_l, m_q$ .

No DM candidate.

No mechanism for generation of the observed BAU.

The LFV processes  $\mu^+ \rightarrow e^+ + \gamma$  decay,  $\mu^- \rightarrow e^- + e^+ + e^-$  decay,  $\tau^- \rightarrow e^- + \gamma$  decay, etc. are allowed.

However, they are predicted to proceed with unobservable rates:

$$BR(\mu \rightarrow e + \gamma) = \frac{3\alpha}{32\pi} \left| U_{ej} U_{\mu j}^* \frac{m_j^2}{M_W^2} \right|^2 \cong (2.5 - 3.9) \times 10^{-55},$$
$$M_W \cong 80 \text{ GeV, the } W^\pm \text{ - mass}$$

S.T.P., 1976

“New Physics”:  $\nu_l \rightarrow \nu_{l'}, \bar{\nu}_l \rightarrow \bar{\nu}_{l'}$ ,  $l, l' = e, \mu, \tau$  oscillations.

- Majorana  $\nu_j$ : Majorana Mass Term of  $\nu_{lL}(x)$ ,  $l = e, \mu, \tau$

$$\mathcal{L}_M^\nu(x) = \frac{1}{2} \nu_{lL}^\top(x) C^{-1} M_{ll} \nu_{lL}(x) + h.c. , \quad C^{-1} \gamma_\alpha C = -\gamma_\alpha^\top$$

- If  $M_{ll} \neq 0$ ,  $L_l \neq \text{const.}$ ,  $L \neq \text{const.}$ ,  $n = 3$

- $\nu_{lL}(x)$ -fermions:  $M = M^\top$ , complex.

$M^{\text{diag}} = U^\top M U$ ,  $U$  – unitary (congruent transformation);  $U \equiv U_{\text{PMNS}}$

$$\nu_j \equiv \chi_j(x) = U_{jl}^\dagger \nu_{lL}(x) + U_{jl}^* \nu_{lR}^c = C (\bar{\chi}_j(x))^\top, \quad m_j \neq 0, \quad j = 1, 2, 3$$

CP-invariance:  $M^* = M$ ,  $M$  - real, symmetric.

$$M^{\text{diag}} = (m'_1, m'_2, m'_3): \quad m'_j = \rho_j m_j, \quad m_j \geq 0, \quad \rho_j = \pm 1$$

$$\chi_j: \quad m_j \geq 0: \quad \eta_{CP}(\chi_j) = i\rho_j$$

$\mathcal{L}_M^\nu(x)$  not possible in the ST: requires New Physics Beyond the ST

$(\beta\beta)_{0\nu}$ -decay is allowed; typically also  $BR(\mu \rightarrow e + \gamma)$ ,  $BR(\mu \rightarrow 3e)$ ,  $CR(\mu^- + N \rightarrow e^- + N)$  can be "large", i.e., in the range of sensitivity of ongoing (MEG) and future planned experiments.

- Majorana  $\nu_j$ : Dirac+Majorana Mass Term; requires both  $\nu_{lL}(x)$  and  $\nu_{lR}(x)$ :

$$\mathcal{L}_{D+M}^\nu(x) = -\overline{\nu_{lR}}(x) M_{DlR} \nu_{lL}(x) + \frac{1}{2} \nu_{lL}^\top(x) C^{-1} M_{ll}^{LL} \nu_{lL}(x) + \frac{1}{2} \nu_{lR}^\top(x) C^{-1} (M^{RR})_{ll}^\dagger \nu_{lR}(x) + h.c. ,$$

$$M = \begin{pmatrix} M^{LL} & M_D^{RR} \\ M_D^T & M^{RR} \end{pmatrix} = M^T \quad \left( (M^{LL})^T = M^{LL}, \quad (M^{RR})^T = M^{RR} \right)$$

- If  $M_{DlR} \neq 0$  and  $M_{ll}^{LL} \neq 0$  and/or  $M_{ll}^{RR} \neq 0$ :  $L_l \neq const.$ ,  $L \neq const.$ ;  $n = 6$  ( $> 3$ )
- $M = M^T$ , complex.

$$M^{diag} = W^T M W, \quad W - \text{unitary}, \quad 6 \times 6; \quad W^T \equiv (U^T \quad V^T); \quad U \equiv U_{\text{PMNS}} : \quad 3 \times 6.$$

$$\nu_{lL}(x) = \sum_{j=1}^6 U_{lj} \chi_j(x), \quad \chi_j(x) - \text{Majorana}, \quad m_j \neq 0, \quad l = e, \mu, \tau;$$

$$\nu_{lL}^C(x) \equiv C (\overline{\nu_{lR}}(x))^\top = \sum_{j=1}^6 V_{lj} \chi_j(x), \quad \nu_{lL}^C(x) : \text{sterile antineutrino}$$

$\mathcal{L}_{D+M}^\nu(x)$  possible in the ST +  $\nu_{lR}$ :  $M^{LL} = 0$

$(\beta\beta)_{0\nu}$ -decay is allowed;  
phenomenology depends on the relative magnitude of  $M_D$  and  $M^{RR}$ .

## Dirac - Majorana Relation (if any...)

Majorana Mass Term of  $\nu_{lL}(x)$ ,  $l = e, \mu, \tau$ , can lead to Dirac neutrinos with definite mass if it conserves some lepton charge:

$$\mathcal{L}_M^\nu(x) = -\frac{1}{2} \overline{\nu_{l'R}^c}(x) M_{ll} \nu_{lL}(x) + h.c. , \quad \nu_{l'R}^c \equiv C (\overline{\nu_{l'L}}(x))^T$$

$\mathcal{L}_M^\nu(x)$  conserves, e.g.  $L' = L_e - L_\mu - L_\tau$  if only  $M_{e\mu} = M_{\mu e}, M_{e\tau} = M_{\tau e} \neq 0$   
S.T.P., 1982

- Dirac  $\nu$ ,  $\Psi$ , is equivalent to two Majorana  $\nu$ 's,  $\chi_{1,2}$ , having the same (positive) mass, opposite CP-parities, and which are "maximally mixed":

$$\Psi(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}, \quad m_1 = m_2 = m_D > 0, \quad \eta_{jCP} = i\rho_j, \quad \rho_1 = -\rho_2 \quad (C (\overline{\chi_j})^T = \rho_j \chi_j)$$

$$\text{Example ZKM } \nu : \quad \nu_{eL}(x) = \Psi_L = \frac{\chi_{1L} + \chi_{2L}}{\sqrt{2}}, \quad \nu_{\mu L}(x) = \Psi_L^C = \frac{\chi_{1L} - \chi_{2L}}{\sqrt{2}}$$

- Pseudo-Dirac Neutrino: the symmetry of  $\mathcal{L}_M^\nu(x)$  is not a symmetry of  $\mathcal{L}_{tot}(x)$

Suppose:  $\nu_{eL}(x) = \Psi_L = (\chi_{1L} + \chi_{2L})/\sqrt{2}$ , and to "leading order"  $m_1 = m_2$ , but due to "higher order" corrections  $m_1 \neq m_2$ ,  $|m_2 - m_1| \equiv |\Delta m| \ll m_{1,2}$

All Majorana effects  $\sim \Delta m$

- Suppose:  $m_1 = m_2$ ,  $\rho_1 = -\rho_2$ , but  $\chi_{1,2}$  are not maximally mixed:

$$\nu_{eL}(x) = \chi_{1L} \cos \phi + \chi_{2L} \sin \phi = \Psi_L \cos \phi' + \Psi_L^C \sin \phi'$$

All Majorana effects are  $\sim m_D \cos \phi' \sin \phi'$

In the case of conserved  $L' = L_e - L_\mu - L_\tau$ :

$$M = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix}$$

$\theta_{12} = \pi/4$ ,  $\theta_{13} = 0$ ,  $\tan \theta_{23} = M_{e\tau}/M_{e\mu}$ ,

$m_3 = 0$  - spectrum with IH,  $m_1 = m_2$ ,  $\chi_{1,2}$  - equivalent to one Dirac  $\nu, \Psi$ .

Adding  $L'$ -breaking term, e.g.  $M_{ee}$ ,  $|M_{ee}|/\sqrt{M_{e\mu}^2 + M_{e\tau}^2} \sim 0.01$ , leads to  $m_1 \neq m_2$  compatible with  $\Delta m^2_{\odot}$ .

## The Nature of Massive Neutrinos III: The Seesaw Mechanisms of Neutrino Mass Generation

- Explain the smallness of  $\nu$ -masses.
- Through leptogenesis theory link the  $\nu$ -mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

## Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the ST

Type I seesaw mechanism:  $\nu_{lR}$  - RH  $\nu$ s' (heavy).

Type II seesaw mechanism:  $H(x)$  - a triplet of  $H^0, H^-, H^{--}$  Higgs fields (HTM).

Type III seesaw mechanism:  $T(x)$  - a triplet of fermion fields.

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos  $\nu_j$  - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ( $(\beta\beta)_{0\nu}$ -decay, LFV processes, etc.) and New Physics at LHC.

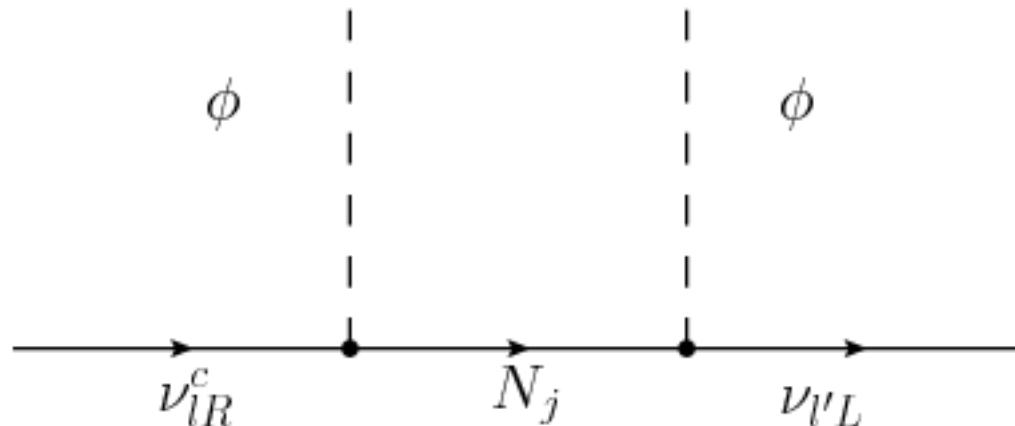
## Type I Seesaw Mechanism

- Requires both  $\nu_{lL}(x)$  and  $\nu_{l'R}(x)$ .
- Dirac+Majorana Mass Term:  $M^{LL} = 0$ ,  $|M_D| = v Y^\nu / \sqrt{2} | << |M^{RR}|$ .
- Diagonalising  $M^{RR}$ :  $N_j$  - heavy Majorana neutrinos,  $M_j \sim \text{TeV}$ ; or  $(10^9 - 10^{13}) \text{ GeV}$  in GUTs.

For sufficiently large  $M_j$ , Majorana mass term for  $\nu_{lL}(x)$ :

$$M_\nu \cong v_u^2 (Y^\nu)^T M_j^{-1} Y^\nu = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger .$$

$v_u Y^\nu = M_D$ ,  $M_D \sim 1 \text{ GeV}$ ,  $M_j = 10^{10} \text{ GeV}$ :  $M_\nu \sim 0.1 \text{ eV}$ .



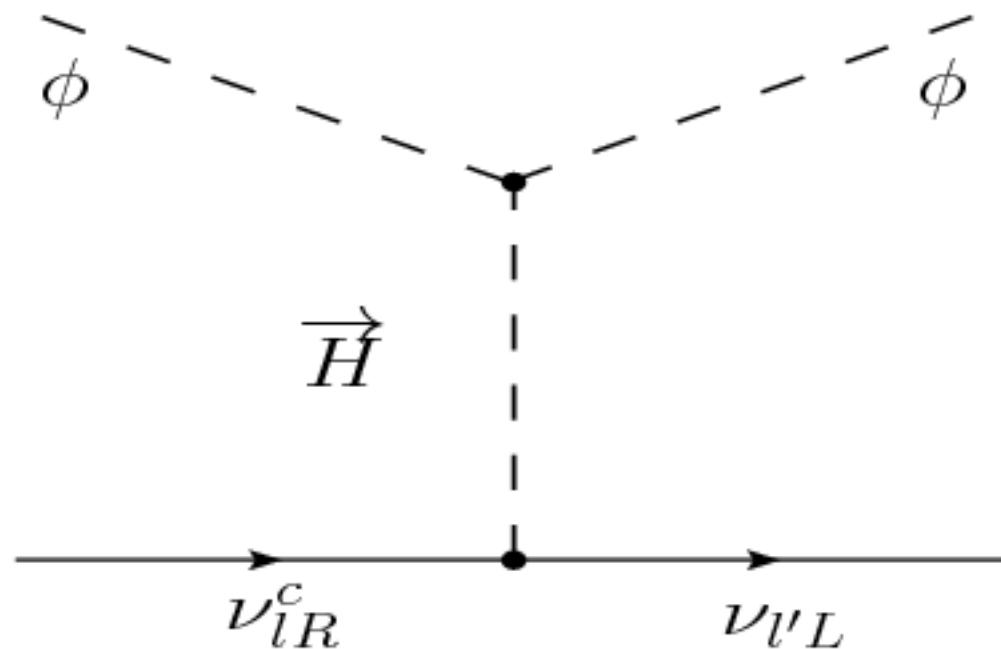
- $\nu_{l'R}(x)$ : Majorana mass term at "high scale" ( $\sim$ TeV; or  $(10^9 - 10^{13})$  GeV in  $SO(10)$  GUT)

$$\mathcal{L}_M^\nu(x) = + \frac{1}{2} \nu_{l'R}^\top(x) C^{-1} (M^{RR})_{ll'}^\dagger \nu_{lR}(x) + h.c. = - \frac{1}{2} \sum_j \bar{N}_j M_j N_j ,$$

- Yukawa type coupling of  $\nu_{lL}(x)$  and  $\nu_{l'R}(x)$  involving  $\Phi(x)$ :

$$\begin{aligned} \mathcal{L}_Y(x) &= \bar{Y}_{ll'}^\nu \overline{\nu_{l'R}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ &= Y_{jl}^\nu \overline{N_{jR}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ M_D &= \frac{v}{\sqrt{2}} Y^\nu , \quad v = 246 \text{ GeV} . \end{aligned}$$

## Type II Seesaw Mechanism

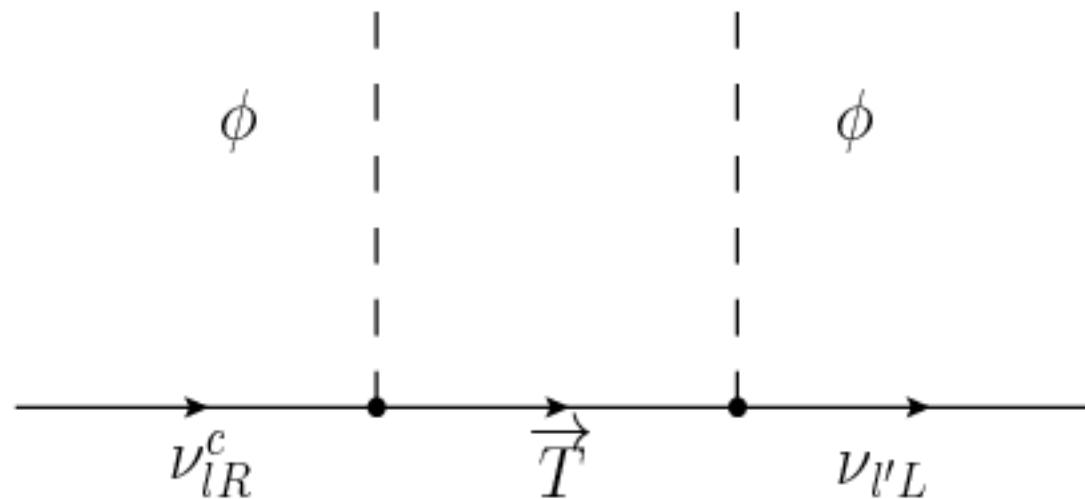


Due to I. Girardi

$$M_\nu \cong h v^2 M_H^{-1} = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger .$$

$h \sim 10^{-2}$ ,  $v = 246$  GeV,  $M_H \sim 10^{12}$  GeV:  $M_\nu \sim 0.6$  eV.

### Type III Seesaw Mechanism



$$M_\nu \cong v^2 \ (Y_T)^T M_T^{-1} Y_T = U_{\text{PMNS}}^* \ m_\nu^{\text{diag}} \ U_{\text{PMNS}}^\dagger .$$

$v \ Y_T \sim 1 \text{ GeV}, \ M_T \sim 10^{10} \text{ GeV}: \ M_\nu \sim 0.1 \text{ eV}.$

## TeV Scale Type I See-Saw Mechanism

Type I see-saw mechanism, heavy Majorana neutrinos  $N_j$  at the TeV scale:

$$m_\nu \simeq - M_D \hat{M}_N^{-1} M_D^T, \quad \hat{M} = \text{diag}(M_1, M_2, M_3), \quad M_j \sim (100 - 1000) \text{ GeV}.$$

$$\mathcal{L}_{CC}^N = -\frac{g}{2\sqrt{2}} \bar{\ell} \gamma_\alpha (RV)_{\ell k} (1 - \gamma_5) N_k W^\alpha + \text{h.c.}, \quad (RV)_{\ell k} \equiv U_{\ell 3+k},$$

$$\mathcal{L}_{NC}^N = -\frac{g}{2c_w} \overline{\nu_{\ell L}} \gamma_\alpha (RV)_{\ell k} N_{kL} Z^\alpha + \text{h.c.}$$

- All low-energy constraints can be satisfied in a scheme with two heavy Majorana neutrinos  $N_{1,2}$ , which form a pseudo-Dirac pair:

$$M_2 = M_1(1+z), \quad 0 < z \ll 1.$$

- Only NH and IH  $\nu$  mass spectra possible:  $\min(m_j) = 0$ .

- Requirements:  $|(\mathcal{R}V)_{\ell k}|$  “sizable”  
+ reproducing correctly the neutrino oscillation data:

$$|(\mathcal{R}V)_{\ell 1}|^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_3}{m_2 + m_3} \left| U_{\ell 3} + i\sqrt{m_2/m_3} U_{\ell 2} \right|^2, \quad \text{NH},$$

$$|(\mathcal{R}V)_{\ell 1}|^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_2}{m_1 + m_2} \left| U_{\ell 2} + i\sqrt{m_1/m_2} U_{\ell 1} \right|^2 \cong \frac{1}{4} \frac{y^2 v^2}{M_1^2} |U_{\ell 2} + iU_{\ell 1}|^2, \quad \text{IH},$$

$$(\mathcal{R}V)_{\ell 2} = \pm i (\mathcal{R}V)_{\ell 1} \sqrt{\frac{M_1}{M_2}}, \quad \ell = e, \mu, \tau,$$

$y$ - the maximum eigenvalue of  $Y^\nu$ ,  $v_u \simeq 174$  GeV.

4 parameters:  $M$ ,  $z$ ,  $y$  and a phase  $\omega$ . A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

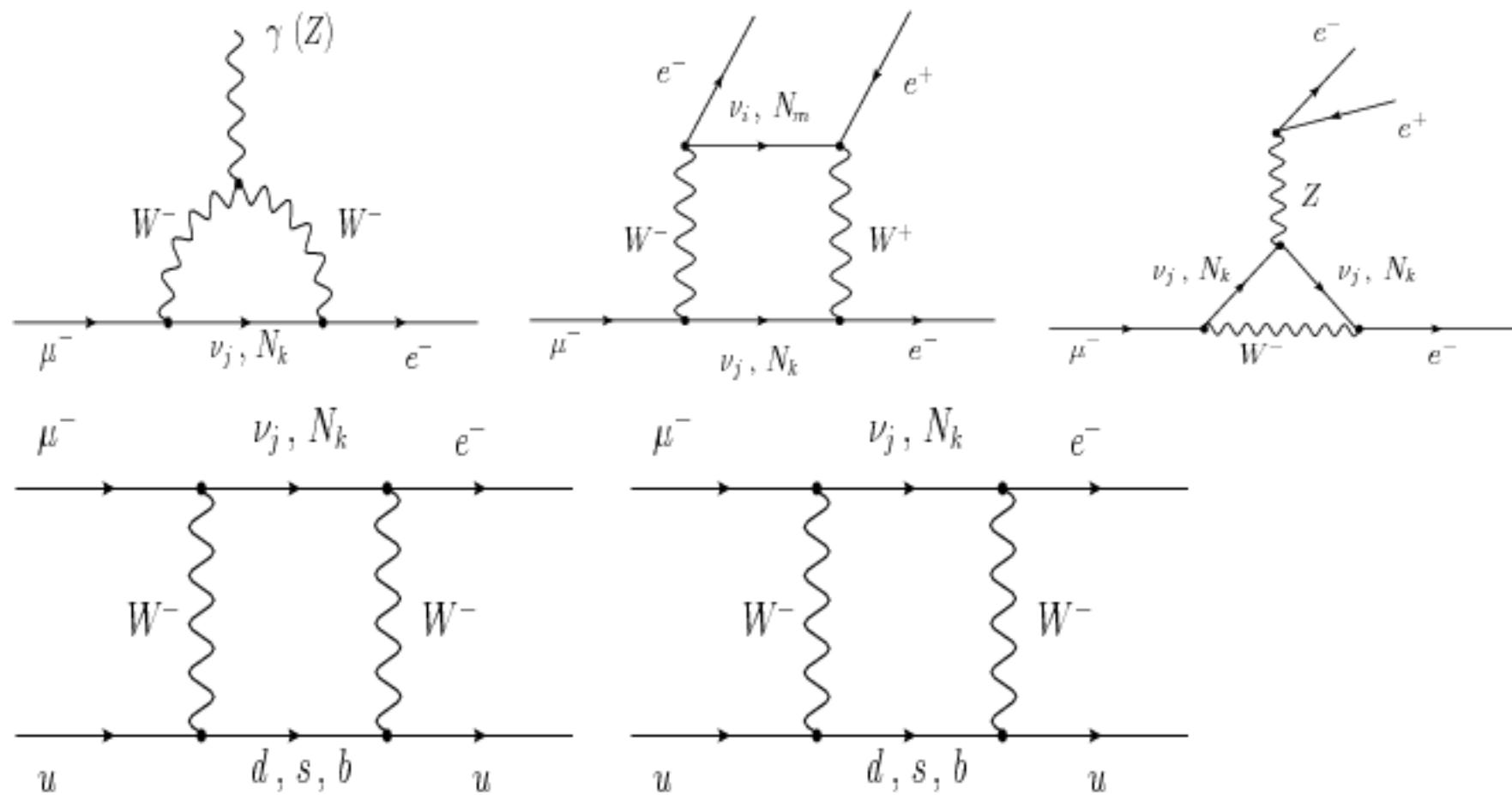
Low energy data:

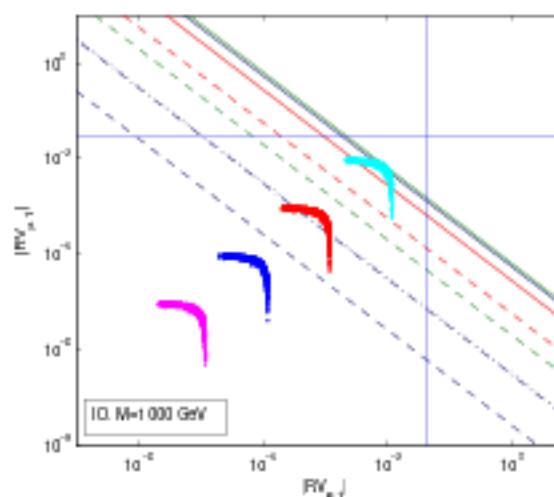
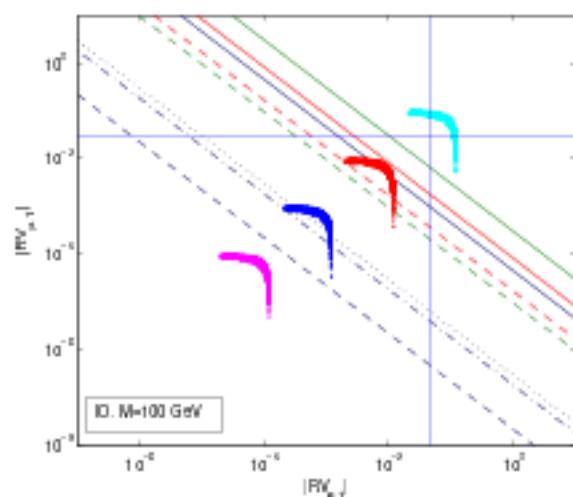
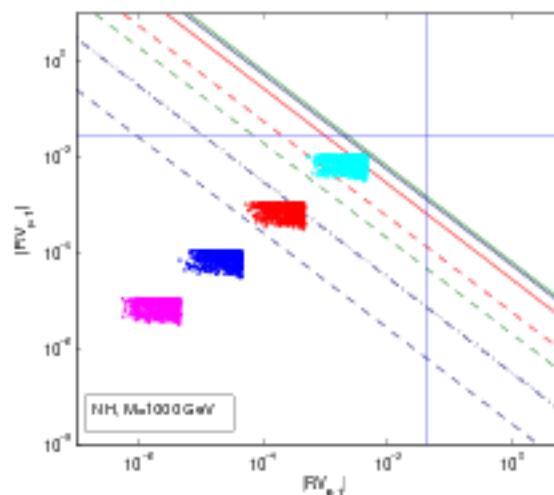
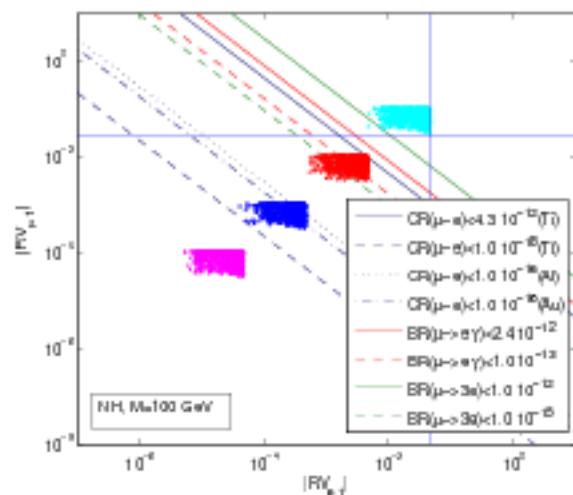
$$\begin{aligned} |(\mathcal{R}V)_{e1}|^2 &\lesssim 2 \times 10^{-3}, \\ |(\mathcal{R}V)_{\mu 1}|^2 &\lesssim 0.8 \times 10^{-3}, \\ |(\mathcal{R}V)_{\tau 1}|^2 &\lesssim 2.6 \times 10^{-3}. \end{aligned}$$

S. Antusch et al., 2008

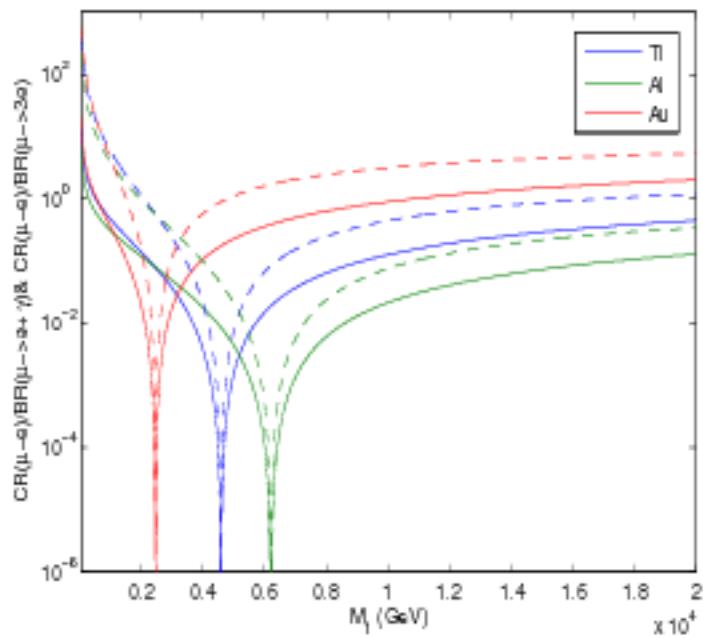
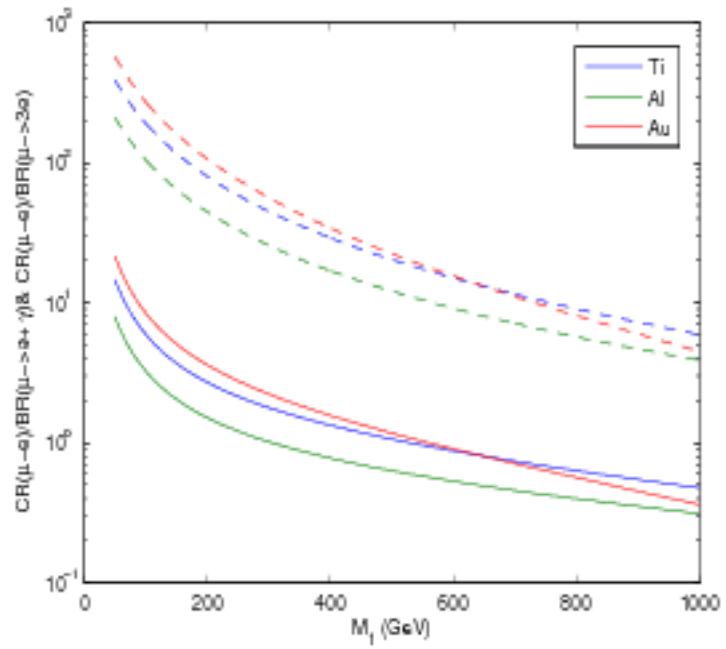
Observation of  $N_{1,2}$  at LHC - problematic.

**LFV processes:**  $\mu^- \rightarrow e^- + \gamma$ ,  $\mu^- \rightarrow 3e^-$ ,  $\mu^- + N \rightarrow e^- + N$ : can proceed with exchange of virtual  $N_j$ :





Current limits and potential sensitivity to  $|RV_{e1}|$  and  $|RV_{\mu 1}|$  from data on LFV processes for NH (upper panels) and IH (lower panels) spectra, for  $M_1 = 100$  ( $1000$ ) GeV and, *i*)  $y = 0.0001$  (magenta pts), *ii*)  $y = 0.001$  (blue pts), *iii*)  $y = 0.01$  (red pts) and *iv*)  $y = 0.1$  (cyan pts).



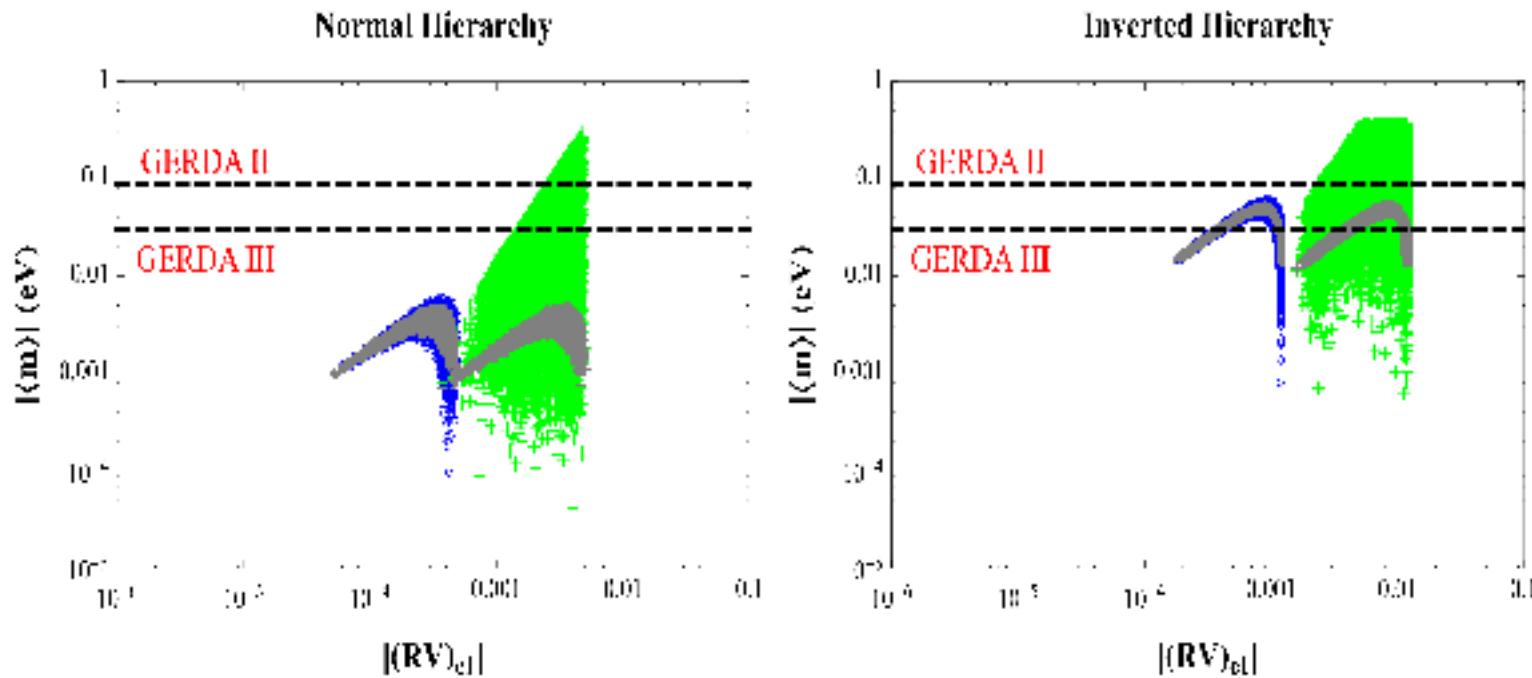
The ratio of the  $\mu - e$  relative conversion rate and the branching ratio of the I)  $\mu \rightarrow e\gamma$  decay (solid lines), II)  $\mu \rightarrow 3e$  decay (dashed lines), versus the type I see-saw mass scale  $M_1$ , for three different nuclei:  $^{48}\text{Ti}$  (blue lines),  $^{27}\text{Al}$  (green lines) and  $^{197}\text{Au}$  (red lines).

The exchange of virtual  $N_j$  gives a contribution to  $|\langle m \rangle|$  :

$$|\langle m \rangle| \cong \left| \sum_i (U_{PMNS})_{ei}^2 m_i - \sum_k f(A, M_k) (RV)_{ek}^2 \frac{(0.9 \text{ GeV})^2}{M_k} \right|,$$
$$f(A, M_k) \cong f(A).$$

For, e.g.,  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{130}\text{Te}$  and  $^{136}\text{Xe}$ , the function  $f(A)$  takes the values  $f(A) \cong 0.033, 0.079, 0.073, 0.085$  and  $0.068$ , respectively.

- The Predictions for  $|\langle m \rangle|$  can be modified considerably.



$|\langle m \rangle|$  vs  $|(RV)_{e1}|$  for  $^{76}\text{Ge}$  in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for  $M_1 = 100$  GeV and *i*)  $y = 0.001$  (blue), *ii*)  $y = 0.01$  (green). The gray markers correspond to  $|\langle m \rangle^{\text{std}}| = |\sum_i (U_{PMNS})_{ei}^2 m_i|$ .

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

## Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{5.4}$ ,  $\theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4} (?)$ ,  $\theta_{13} \cong \frac{\pi}{20}$

$$U_{\text{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.20$ ,  $\theta_{13} \cong 0 + \pi/20$ ,  $\theta_{23} \cong \pi/4 - 0.11$ .
- $U_{\text{PMNS}}$  due to new approximate symmetry?

### A Natural Possibility (vast literature):

$$U = U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \psi) Q(\phi, \varphi) U_{\text{tri,bim,LC}} P(\alpha_{21}, \alpha_{31}),$$

with

$$U_{\text{tri}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{bim}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \psi)$  - from diagonalization of the  $l^-$  mass matrix;
- $U_{\text{bim,tri,LC}} P(\alpha_{21}, \alpha_{31})$  - from diagonalization of the  $\nu$  mass matrix;
- $Q(\phi, \varphi)$ , - from diagonalization of both the  $l^-$  and  $\nu$  mass matrices.

$U_{\text{tri(bim)}}$ : Groups  $A_4$ ,  $S_4$ ,  $T'$ , ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552;  
S. King and Ch. Luhn, arXiv:1301.1340)

- $U_{\text{bim}}$ : alternatively  $U(1)$ ,  $L' = L_e - L_\mu - L_\tau$

S.T.P., 1982

**None of the symmetries leading to  $U_{\text{tri}}$ ,  $U_{\text{bim}}$  or other approximate forms of  $U_{\text{PMNS}}$  can be exact.**

**Which is the correct approximate symmetry, i.e., approximate form of  $U_{\text{PMNS}}$  (if any)?**

**In the two cases of  $U_\nu$  given by  $U_{\text{tri}}$ , or  $U_{\text{bim}}$ , the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.**

**Depending on the symmetry leading to  $U_{\text{tri,bim}}$  and on the form of  $U_{\text{lep}}$ , one obtains different experimentally testable predictions for the sum of the neutrino masses, the neutrino mass spectrum, the nature (Dirac or Majorana) of  $\nu_j$  and the CP violating phases in the neutrino mixing matrix. Future data will help us understand whether there is some new fundamental symmetry behind the observed patterns of neutrino mixing and  $\Delta m_{ij}^2$ .**

## Predictions for $\delta$

Assume:

- $U_{PMNS} = U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \psi) Q(\phi, \varphi) U_{\text{TBM}, \text{BM}} P(\alpha_{21}, \alpha_{31})$ ,
- $U_{\text{lep}}^\dagger$  - minimal, such that
  - i)  $\sin \theta_{13} \cong 0.16$ ; BM:  $\sin^2 \theta_{12} \cong 0.31$ ;
  - ii)  $\sin^2 \theta_{23}$  can deviate significantly (by more than  $\sin^2 \theta_{13}$ ) from 0.5 (b.f.v. = 0.42-0.43).

From i), ii) +  $m_e \ll m_\mu \ll m_\tau$ :

$$U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \psi) = R_{12}(\theta_{12}^\ell) R_{23}(\theta_{23}^\ell), \quad Q(\phi, \varphi) = \text{diag}(1, e^{i\phi}, 1)$$

Leads to  $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})$  - new sum rules for  $\delta$ !

For  $U_{\text{TBM}}$ :

$$\cos \delta = \frac{\tan \theta_{23}}{3 \sin 2\theta_{12} \sin \theta_{13}} [1 + (3 \sin^2 \theta_{12} - 2)(1 - \cot^2 \theta_{23} \sin^2 \theta_{13})]$$

For  $U_{\text{TBM}} + \text{b.f.v.}$  of  $\theta_{12}, \theta_{23}, \theta_{13}$ :

$$\delta \cong 3\pi/2 \text{ or } \pi/2 \quad (\delta = 266^\circ \text{ or } \delta = 94^\circ)$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

$T'$  model of lepton flavour:  $U_{\text{TBM}}$ ,  $\delta \cong 3\pi/2$  or  $\pi/2$ .

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

For  $U_{\text{BM}}$ :

$$\cos \delta = -\frac{1}{2 \sin \theta_{13}} \cot 2\theta_{12} \tan \theta_{23} (1 - \cot^2 \theta_{23} \sin^2 \theta_{13}).$$

For  $U_{\text{BM}} + \text{b.f.v.}$  of  $\theta_{12}, \theta_{23}, \theta_{13}$ :

$$\delta \cong \pi$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevilla, arXiv:1302.

$T'$  model of lepton flavour:  $U_{\text{TBM}}$ ,  $\delta \cong 3\pi/2$  or  $\pi/2$ .

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

- Light neutrino masses: type I seesaw mechanism.
- $\nu_j$  - Majorana particles.
- Diagonalisation of  $M_\nu$ :  $U_{\text{TBM}}\Phi$ ,  $\Phi = \text{diag}(1, 1, 1(i))$
- $U_{\text{TBM}}$  “corrected” by  
 $U_{\text{lep}}^\dagger Q = R_{12}(\theta_{12}^\ell) R_{23}(\theta_{23}^\ell) Q$ ,  $Q = \text{diag}(1, e^{i\phi}, 1)$

$T'$  model of lepton flavour:  $U_{\text{TBM}}$ ,  $\delta \cong 3\pi/2$  or  $\pi/2$ .

- $T'$ : double covering of  $A_4$  (tetrahedral symmetry group).
- $T'$ : **1**, **1'**, **1''**; **2**, **2'**, **2''**; **3**.
- $T'$  model:  $\psi_{eL}(x), \psi_{\mu L}(x), \psi_{\tau L}(x)$  - triplet of  $T'$ ;  
 $e_R(x), \mu_R(x)$  - a doublet,  $\tau_R(x)$  - a singlet, of  $T'$ ;  
 $\nu_{eR}(x), \nu_{\mu R}(x), \nu_{\tau R}(x)$  - a triplet of  $T'$ ;  
the Higgs doublets  $H_u(x), H_d(x)$  - singlets of  $T'$ .
- The discrete symmetries of the model are  $T' \times H_{CP} \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2$ , the  $Z_n$  factors being the shaping symmetries of the superpotential required to forbid unwanted operators.

## Predictions of the $T'$ Model

- $m_{1,2,3}$  determined by 2 real parameters +  $\Phi^2$ :

$$\text{NO spectrum A : } (m_1, m_2, m_3) = (4.43, 9.75, 48.73) \cdot 10^{-3} \text{ eV}$$

$$\text{NO spectrum B : } (m_1, m_2, m_3) = (5.87, 10.48, 48.88) \cdot 10^{-3} \text{ eV}$$

$$\text{IO spectrum : } (m_1, m_2, m_3) = (51.53, 52.26, 17.34) \cdot 10^{-3} \text{ eV}$$

$$\text{NO A : } \sum_{j=1}^3 m_j = 6.29 \times 10^{-2} \text{ eV},$$

$$\text{NO B : } \sum_{j=1}^3 m_j = 6.52 \times 10^{-2} \text{ eV},$$

$$\text{IO : } \sum_{j=1}^3 m_j = 12.11 \times 10^{-2} \text{ eV},$$

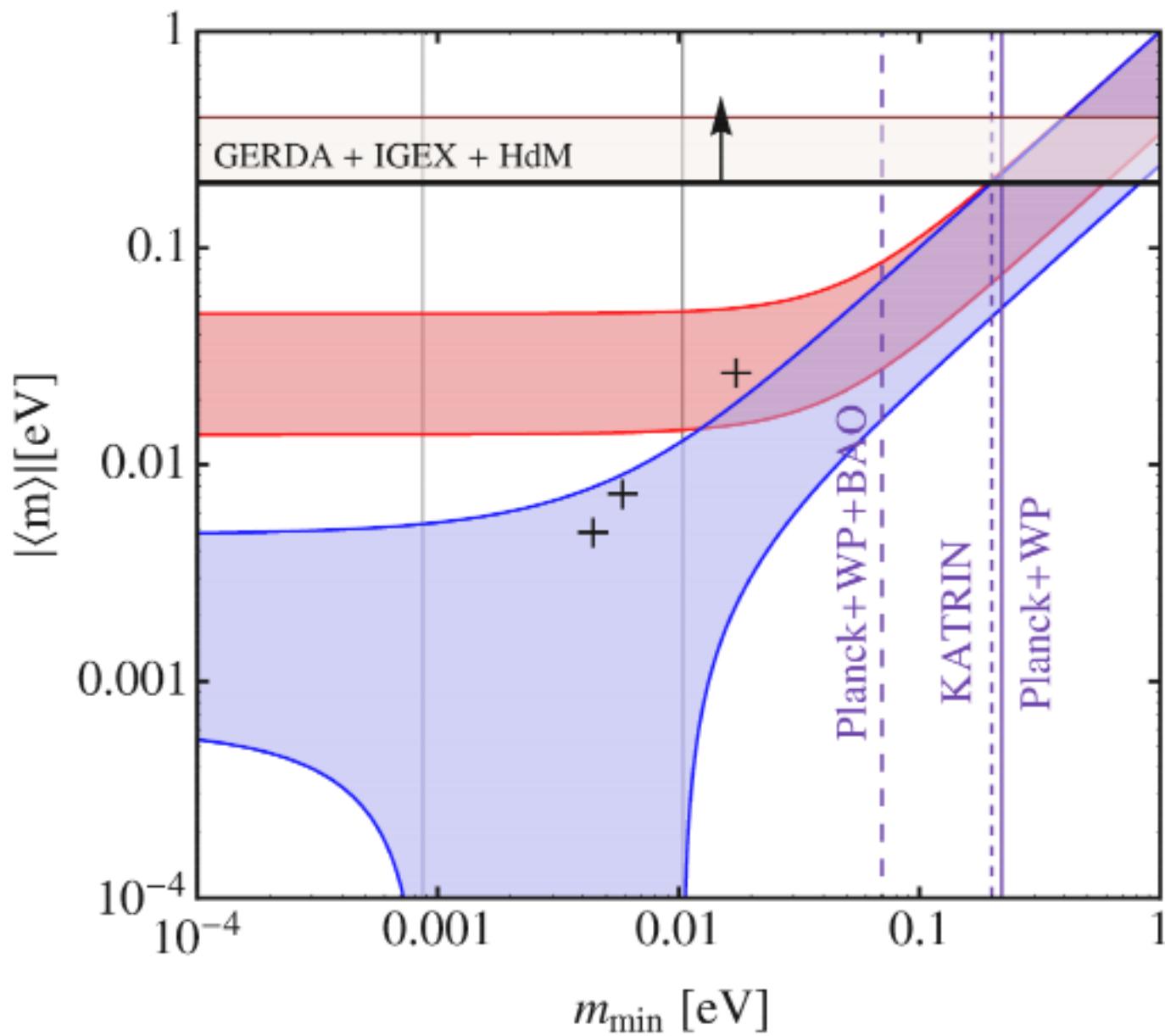
- $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$  determined by 3 real parameters.

Given the values of  $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$  are predicted:

$\delta \cong 3\pi/2$  ( $266^\circ$ ) (or  $\pi/2$  ( $94^\circ$ ));

NO A:  $\alpha_{21} \cong +47.0^\circ$  (or  $-47.0^\circ$ ) ( $+2\pi$ ),

$\alpha_{31} \cong -23.8^\circ$  (or  $+23.8^\circ$ ) ( $+2\pi$ ).



**Instead of Conclusions**

**The future of neutrino physics is bright.**