

Neutrino Physics

(Neutrino Masses, Mixing, Oscillations, Leptonic CP Violation and Beyond)

S. T. Petcov

SISSA/INFN, Trieste, Italy, and
Kavli IPMU, University of Tokyo, Japan

CERN-JINR European HEP School
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Plan of the Lectures

1. Introduction.
 2. Massive Neutrinos, Neutrino Mixing and Oscillations: Overview.
 3. Three Neutrino Mixing. Massive Majorana versus Massive Dirac Neutrinos I. Dirac and Majorana CP Violation.
 - 4. Neutrino Oscillations in Vacuum: Theory and Experimental Evidences.
 - 5. Matter Effects in Neutrino Oscillations: Theory.
 - Neutrino Oscillations in the Earth.
 - CP Violation in Neutrino Oscillations.
 - Flavour Conversions of Solar Neutrinos.
 6. Three Neutrino Mixing: the Angle θ_{13} and Indications for Dirac CP Violation.
 7. Open Questions in the Physics of Massive Neutrinos.
 8. Understanding the Pattern of Neutrino Mixing.
 9. The Absolute Scale of Neutrino Masses.
 10. The Nature of Massive Neutrinos.
 - Massive Majorana versus Massive Dirac Neutrinos.
 - II. Origins of Dirac and Majorana Massive Neutrinos.
 - The Seesaw Mechanisms of Neutrino Mass Generation.
 11. Determining the Nature of Massive Neutrinos.
 12. Future LBL Neutrino Oscillation Experiments on $\text{sgn}(\Delta m_{31}^2)$ and CP Violation (?).
 13. Conclusions.
- Time permitting, will cover also: Leptogenesis Scenario of Generation of the Baryon Asymmetry of the Universe. Dirac and Majorana Leptonic CP-Violation and Leptogenesis.

3 Families of Fundamental Particles

$$\begin{pmatrix} \nu_e & u \\ e & d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & c \\ \mu & s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & t \\ \tau & b \end{pmatrix} \quad + \text{ their antiparticles}$$

- 3 types (flavours) of active ν 's and $\tilde{\nu}$'s
- The notion of "type" ("flavour") - dynamical;
 ν_e : $\nu_e + n \rightarrow e^- + p$; ν_μ : $\pi^+ \rightarrow \mu^+ + \nu_\mu$; etc.
- The flavour of a given neutrino is Lorentz invariant.
- $\nu_l \neq \nu_{l'}, \tilde{\nu}_l \neq \tilde{\nu}_{l'}, l \neq l' = e, \mu, \tau; \nu_l \neq \tilde{\nu}_{l'}, l, l' = e, \mu, \tau$.

The states must be orthogonal (within the precision of the corresponding data): $\langle \nu'_l | \nu_l \rangle = \delta_{ll}, \langle \tilde{\nu}'_l | \tilde{\nu}_l \rangle = \delta_{ll}, \langle \tilde{\nu}'_l | \nu_l \rangle = 0$.

- Data (relativistic ν 's): ν_l ($\tilde{\nu}_l$) - predominantly LH (RH).
Standard Theory: ν_l , $\tilde{\nu}_l$ - $\nu_{lL}(x)$;
 $\nu_{lL}(x)$ form doublets with $l_L(x)$, $l = e\mu, \tau$:

$$\begin{pmatrix} \nu_{lL}(x) \\ l_L(x) \end{pmatrix} \quad l = e, \mu, \tau.$$

- No (compelling) evidence for existence of (relativistic) ν 's ($\tilde{\nu}$'s) which are predominantly RH (LH): ν_R ($\tilde{\nu}_L$).
If ν_R , $\tilde{\nu}_L$ exist, must have much weaker interaction than ν_l , $\tilde{\nu}_l$: ν_R , $\tilde{\nu}_L$ - “sterile”, “inert”.

In the formalism of the ST, ν_R and $\tilde{\nu}_L$ - RH ν fields $\nu_R(x)$; can be introduced in the ST as $SU(2)_L$ singlets.

B. Pontecorvo, 1967

No experimental indications exist at present whether the SM should be minimally extended to include $\nu_R(x)$, and if it should, how many $\nu_R(x)$ should be introduced.

$\nu_R(x)$ appear in many extensions of the ST, notably in $SO(10)$ GUT's.

The RH ν 's can play crucial role

- i) in the generation of $m(\nu) \neq 0$,
- ii) in understanding why $m(\nu) \ll m_l, m_q$,
- iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each $\nu_{lL}(x)$ there corresponds a $\nu_{lR}(x)$, $l = e, \mu, \tau$.

ST + $m(\nu) = 0$: $L_l = \text{const.}$, $l = e, \mu, \tau$;
 $L \equiv L_e + L_\mu + L_\tau = \text{const.}$

There have been remarkable discoveries in neutrino physics in the last ~ 15 years.

Compellings Evidence for ν -Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z -, L/E - dependences of μ -like events

Dominant $\nu_{\mu} \rightarrow \nu_{\tau}$ K2K, MINOS, T2K; CNGS (OPERA)

$-\nu_{\odot}$: Homestake, Kamiokande, SAGE, GALLEX/GNO

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ BOREXINO

$-\bar{\nu}_e$ (from reactors): Daya Bay, RENO, Double Chooz

Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS (ν_{μ} from accelerators): $\nu_{\mu} \rightarrow \nu_e$

Compelling Evidences for ν -Oscillations: ν mixing

$$|\nu_l\rangle = \sum_{j=1}^n U_{lj}^* |\nu_j\rangle, \quad \nu_j : m_j \neq 0; \quad l = e, \mu, \tau; \quad n \geq 3;$$

$$\nu_{l\text{L}}(x) = \sum_{j=1}^n U_{lj} \nu_{j\text{L}}(x), \quad \nu_{j\text{L}}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

$\nu_j, m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 ν s are light: $\nu_{1,2,3}, m_{1,2,3} \lesssim 1$ eV.

The Charged Current Weak Interaction Lagrangian:

$$\mathcal{L}^{CC}(x) = -\frac{g}{2\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}(x) \gamma_\alpha (1 - \gamma_5) \nu_{lL}(x) W^\alpha(x) + \text{h.c.},$$

$$\nu_{lL}(x) = \sum_{j=1}^n U_{lj} \nu_{jL}(x), \quad \nu_{jL}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, \dots$) if, e.g., **sterile** ν_R , $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,\dots} \sim 1$ eV;

in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analyses of short baseline (SBL) reactor neutrino oscillation data (“reactor neutrino anomaly”, data of radioactive source calibration of the solar neutrino SAGE and GALLEX experiments (“Gallium anomaly”));

ii) $M_{4,5,\dots} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models;
 $M_{4,5,\dots} \sim (10^9 - 10^{13})$ GeV, “classical” seesaw models.

We can also have, in principle:

$m_4 \sim 1$ eV ($\nu_{e(\mu)} \rightarrow \nu_S$), $m_5 \sim 5$ keV (DM), $M_6 \sim (10 - 10^3)$ GeV (seesaw).

All compelling data compatible with 3- ν mixing:

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (at least: $|U_{l,n}| \lesssim (0.1)$, $l = e, \mu, n = 4, 5, \dots$).

$\nu_j, m_j \neq 0$: Dirac or Majorana particles.

3- ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E ; at distance L : $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$$

Three Neutrino Mixing

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL}$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

n	2	3	4	
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6

CP-violating phases:

- ν_j - Dirac: $\frac{1}{2}(n-1)(n-2)$ 0 1 3
- ν_j - Majorana: $\frac{1}{2}n(n-1)$ 1 3 6

$n = 3$: 1 Dirac and

2 additional CP-violating phases, Majorana phases

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C (\bar{\chi}_k(x))^T = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

- Invariant under proper Lorentz transformations.
- Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1) : \chi_k(x) \rightarrow e^{i\alpha} \chi_k(x) - \text{impossible}$$

- $\chi_k(x)$ cannot absorb phases.
- $Q_{U(1)} = 0 : Q_{\text{el}} = 0, L_l = 0, L = 0, \dots$
- $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
- $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$ -Dirac, $\chi(x)$ -Majorana

$$\langle 0 | T(\Psi_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\Psi_\alpha(x) \Psi_\beta(y)) | 0 \rangle = 0 , \quad \langle 0 | T(\bar{\Psi}_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = 0 .$$

$$\langle 0 | T(\chi_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\chi_\alpha(x) \chi_\beta(y)) | 0 \rangle = -\xi^* S_{\alpha\kappa}^F(x - y) C_{\kappa\beta} ,$$

$$\langle 0 | T(\bar{\chi}_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = \xi C_{\alpha\kappa}^{-1} S_{\kappa\beta}^F(x - y)$$

$$U_{CP} \ \chi(x) \ U_{CP}^{-1} = \eta_{CP} \ \gamma_0 \ \chi(x') , \quad \eta_{CP} = \pm i .$$

PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2\dots$
S.M. Bilenky, J. Hosek, S.T.P., 1980
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ)
- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), Capozzi et al. NO (IO).
F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

- Fogli et al., Phys. Rev. D86 (2012) 013012, global analysis, b.f.v.: $\sin^2 \theta_{13} = 0.0241$ (0.0244), NO (IO).
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5}$ eV $^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3}$ eV 2 , $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), NH (IH).
- $1\sigma(\Delta m_{21}^2) = 2.6\%$, $1\sigma(\sin^2 \theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m_{31(23)}^2|) = 2.6\%$, $1\sigma(\sin^2 \theta_{23}) = 9.6\%$;
- $1\sigma(\sin^2 \theta_{13}) = 8.5\%$;
- $3\sigma(\Delta m_{21}^2) : (6.99 - 8.18) \times 10^{-5}$ eV 2 ; $3\sigma(\sin^2 \theta_{12}) : (0.259 - 0.359)$;
- $3\sigma(|\Delta m_{31(23)}^2|) : 2.27(2.23) - 2.65(2.60) \times 10^{-3}$ eV 2 ;
 $3\sigma(\sin^2 \theta_{23}) : 0.374(0.380) - 0.628(0.641)$;
- $3\sigma(\sin^2 \theta_{13}) : 0.0176(0.0178) - 0.0295(0.0298)$.

- Dirac phase δ : $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{CP}^{(ll')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data: $|J_{CP}| \lesssim 0.040 |\sin \delta|$ (can be relatively large!).

- Majorana phases α_{21}, α_{31} :

– $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

- $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay depends on α_{21}, α_{31} ;
- $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
- BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

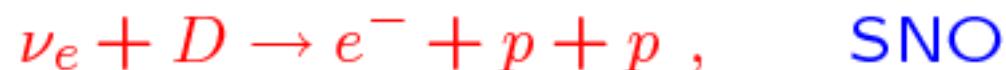
Solar Neutrinos ν_e , $E \sim 1$ MeV: B. Pontecorvo 1946



R. Davis et al., 1967 - 1996: 615 t C_2Cl_4 ; 0.5 Ar atoms/day, exposure 60 days.

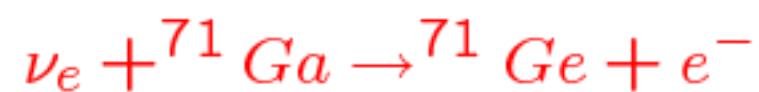


Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);



Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water (D_2O)



SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003

Atmospheric Neutrinos $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$, $E \sim 1$ GeV (0.20 - 100 GeV)

$$\nu_\mu + N \rightarrow \mu^- + X, \quad \bar{\nu}_\mu + N \rightarrow \mu^+ + X$$

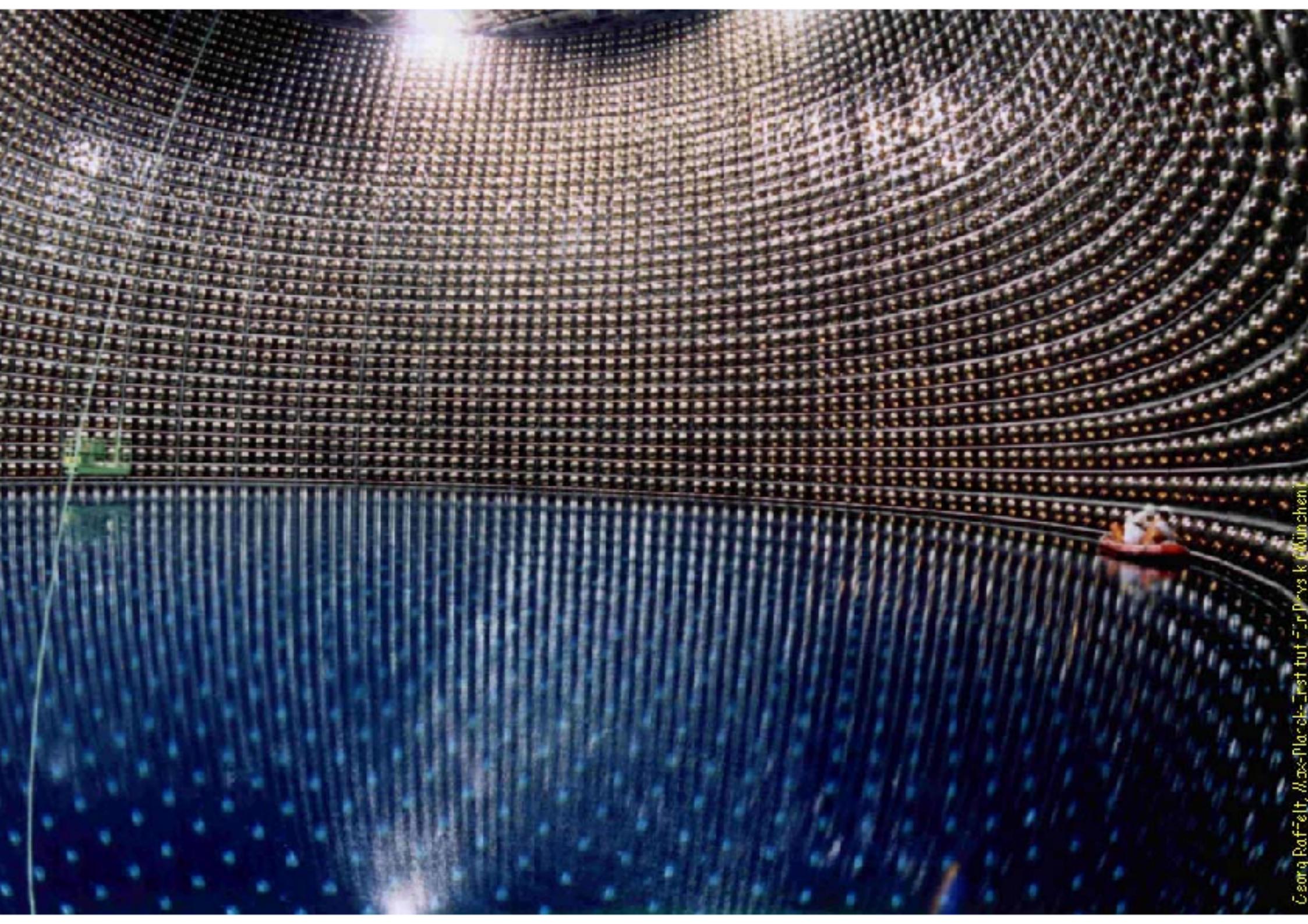
$$\nu_e + N \rightarrow e^- + X, \quad \bar{\nu}_e + N \rightarrow e^+ + X$$

K2K, MINOS, T2K, ν_μ ($\bar{\nu}_\mu$), $E \sim 1$ GeV

$$\nu_\mu + N \rightarrow \mu^- + X \quad (\nu_e + N \rightarrow e^- + X)$$

Reactor $\bar{\nu}_e$: CHOOZ, KamLAND, Double Chooz, RENO, Daya Bay ($E \cong 2 - 8$ MeV)

$$\bar{\nu}_e + p \rightarrow e^+ + n$$



Georg Raafelt, Max-Planck-Institut für Physik (Würzburg)



Neutrino Oscillations in Vacuum

Suppose at $t = 0$ in vacuum

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta,$$

$$|\nu_{\mu(\tau)}\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta; \quad \nu_{1,2} : m_{1,2} \neq 0$$

After time t in vacuum

$$|\nu_e\rangle_t = e^{-iE_1 t} |\nu_1\rangle \cos\theta + e^{-iE_2 t} |\nu_2\rangle \sin\theta, \quad E_{1,2} = \sqrt{p^2 + m_{1,2}^2}$$

$$A(\nu_e \rightarrow \nu_{\mu}; t) = \langle \nu_{\mu} | \nu_e \rangle_t = \frac{1}{2} \sin 2\theta (e^{-iE_2 t} - e^{-iE_1 t})$$

$$P(\nu_e \rightarrow \nu_{\mu}; t) = \frac{1}{2} \sin^2 2\theta (1 - \cos(E_2 - E_1)t)$$

$$P(\nu_e \rightarrow \nu_e; t) \equiv P_{ee} = 1 - P(\nu_e \rightarrow \nu_{\mu}; t)$$

V. Gribov, B. Pontecorvo, 1969

Neutrinos are relativistic: $t \cong L$, $E_2 - E_1 \cong (m_2^2 - m_1^2)/(2p)$

$$(E_2 - E_1)t \cong (m_2^2 - m_1^2)L/(2p) = 2\pi \frac{L}{L_{osc}^{vac}}, \quad L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$

$$P(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta (1 - \cos 2\pi \frac{L}{L_{osc}^{vac}}), \quad L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$

$$L_{osc}^{vac} \cong 2.5 \text{ m } \frac{E[\text{MeV}]}{\Delta m^2[\text{eV}^2]}$$

$$E \cong 3 \text{ MeV}, \quad \Delta m^2[\text{eV}^2] \cong 8 \times 10^{-5} : \quad L_{osc}^{vac} \cong 100 \text{ km}$$

$$E \cong 1 \text{ GeV}, \quad \Delta m^2[\text{eV}^2] \cong 2.5 \times 10^{-3} : \quad L_{osc}^{vac} \cong 1000 \text{ km}$$

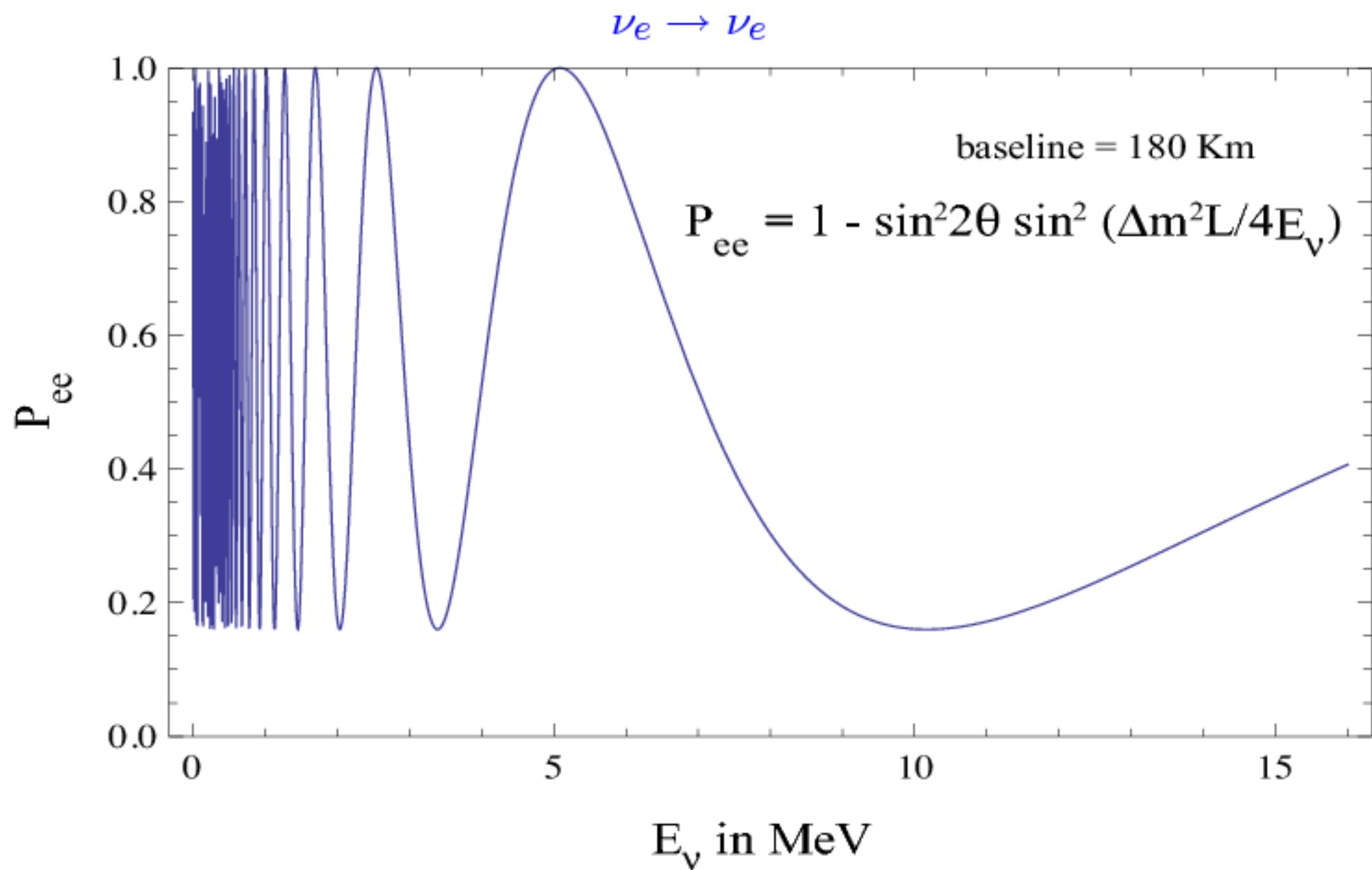
Effects of oscillations observable if

$$\sin^2 2\theta - sufficiently \ large, \quad L \gtrsim L_{osc}^{vac}$$

Two basic parameters: $\sin^2 2\theta, \Delta m^2$

SK, K2K, MINOS; CNGS (OPERA): dominant $\nu_\mu \rightarrow \nu_\tau$

KamLAND: $\bar{\nu}_e \rightarrow \bar{\nu}_e; \bar{\nu}_e \rightarrow (\bar{\nu}_\mu + \bar{\nu}_\tau)/\sqrt{2}$



Source	Type of ν	$\bar{E}[\text{MeV}]$	$L[\text{km}]$	$\min(\Delta m^2)[\text{eV}^2]$
Reactor	$\tilde{\nu}_e$	~ 1	1	$\sim 10^{-3}$
Reactor	$\tilde{\nu}_e$	~ 1	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	1	~ 1
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric ν's	$\nu_{\mu,e}, \tilde{\nu}_{\mu,e}$	$\sim 10^3$	10^4	$\sim 10^{-4}$
Sun	ν_e	~ 1	1.5×10^8	$\sim 10^{-11}$

Correspond to: CHOOZ, Double Chooz, RENO, Daya Bay ($L \sim 1$ km), KamLAND ($L \sim 100$ km);

$\tilde{\nu}_e$ disappearance; $E = (1.8 \div 8.0)$ MeV;

to accelerator experiments - past ($L \sim 1$ km);

past, current: K2K ($L \sim 250$ km), MINOS ($L \sim 730$ km), ν_μ disappearance; OPERA ($L \sim 730$ km), $\nu_\mu \rightarrow \nu_\tau$;

T2K ($L \sim 295$ km), future NO ν A ($L \sim 800$ km), ν_μ disappearance, $\nu_\mu \rightarrow \nu_e$; $E \sim 1$ GeV;

SK experiment studying atmospheric $\nu_\mu, \tilde{\nu}_\mu, \nu_e, \tilde{\nu}_e$ ($E \cong 0.1 \div 100$ GeV), and solar ν_e ($E \cong 5 \div 14$ MeV) oscillations, and to the solar ν experiments ($E \cong 0.29 \div 14$ MeV).

$$|\nu_l\rangle = \sum_j U_{lj}^* |\nu_j; \tilde{p}_j\rangle, \quad l = e, \mu, \tau$$

$\pi^+ \rightarrow \mu^+ + \nu_\mu$ decay at rest:

$$E_j = E + m_j^2/(2m_\pi), \quad p_j = E - \xi m_j^2/(2E), \quad E = (m_\pi/2)(1 - m_\mu^2/m_\pi^2) \cong 30 \text{ MeV}, \quad \xi = (1 + m_\mu^2/m_\pi^2)/2 \cong 0.8.$$

Taking $m_j = 1$ eV: $E_j \cong E(1 + 1.2 \times 10^{-16})$,
 $p_j \cong E(1 - 4.4 \times 10^{-16})$.

Problem avoided if one uses the fact that the ν_j state is entangled with the μ^+ state.

$$A(\nu_l \rightarrow \nu_{l'}) = \sum_j U_{l'j} D_j U_{jl}^\dagger, \quad l, l' = e, \mu, \tau,$$

$$D_j = e^{-i\tilde{p}_j(x_f - x_0)} = e^{-i(E_j T - p_j L)}, \quad p_j \equiv |\mathbf{p}_j|.$$

$$\begin{aligned}\delta\varphi_{jk} &= (E_j - E_K)T - (p_j - p_k)L \\ &= (E_j - E_K) \left[T - \frac{E_j + E_K}{p_j + p_k} L \right] + \frac{m_j^2 - m_k^2}{p_j + p_k} L;\end{aligned}$$

First term - negligible:

- L and T related: $T = (E_j + E_k) L / (p_j + p_k) = L/\bar{v}$,
 $\bar{v} = (E_j/(E_j + E_k))v_j + (E_k/(E_j + E_k))v_k$ - the “average” velocity of ν_j and ν_k ,
 $v_{j,k} = p_{j,k}/E_{j,k}$;
- $E_j = E_k = E_0$;
- $p_j = p_k = p$
 (additionally suppressed by $(m_j^2 + m_k^2)/p^2$: $L = T$ up to $\sim m_{j,k}^2/p^2$);
- $E_j \neq E_k, p_j \neq p_k, j \neq k$: **the same conclusion**
 (neutrinos are relativistic, $L \cong T$ up to corrections $\sim m_{j,k}^2/E_{j,k}^2$).

$$\delta\varphi_{jk} \cong \frac{m_j^2 - m_k^2}{2p} L = 2\pi \frac{L}{L_{jk}^v} \text{sgn}(m_j^2 - m_k^2), \quad p = (p_j + p_k)/2,$$

$$L_{jk}^v = 4\pi \frac{p}{|\Delta m_{jk}^2|} \cong 2.5 \text{ m} \frac{p[\text{MeV}]}{|\Delta m_{jk}^2|[\text{eV}^2]}$$

is the neutrino oscillation length associated with Δm_{jk}^2 .

- One can safely neglect the dependence of p_j and p_k on the masses m_j and m_k and consider p to be the zero neutrino mass momentum, $p = E$.
- The phase $\delta\varphi_{jk}$ is Lorentz invariant.

$$\sigma_{m^2} = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2}$$

Condition for producing coherently ν_1, ν_2, \dots :

$$\sigma_{m^2} > |\Delta m_{jk}^2|$$

The equation used above corresponds to a plane wave description of the propagation of neutrinos ν_j . It accounts only for the movement of the center of the wave packet describing ν_j . In the wave packet treatment of the problem, the interference between the states of ν_j and ν_k is subject to a number of conditions, the localisation condition (in space and time) and the condition of overlapping of the wave packets of ν_j and ν_k at the detection point being the most important. For relativistic neutrinos, the localisation condition in space reads: $\sigma_{xP}, \sigma_{xD} < L_{jk}^v/(2\pi)$, $\sigma_{xP(D)}$ being the spatial width of the production (detection) wave packet. Thus, the interference will not be suppressed if the spatial width of the neutrino wave packets determined by the neutrino production and detection processes is smaller than the corresponding oscillation length in vacuum. In order for the interference to be nonzero, the wave packets describing ν_j and ν_k should also overlap in the point of neutrino detection. This requires that the spatial separation between the two wave packets at the point of neutrinos detection, caused by the two wave packets having different group velocities $v_j \neq v_k$, satisfies $|(v_j - v_k)T| \ll \max(\sigma_{xP}, \sigma_{xD})$. If the interval of time T is not measured, T in the preceding condition must be replaced by the distance L between the neutrino source and the detector.

Examples

- Spatial localisation condition

ΔL - dimensions of the ν - source (and/or detector):

$$2\pi \Delta L / L_{jk}^v \lesssim 1.$$

- Time localisation condition

ΔE - detector's energy resolution:

$$2\pi (L/L_{jk}^v) (\Delta E/E) \lesssim 1.$$

If $2\pi \Delta L / L_{jk}^v \gg 1$, and/or $2\pi (L/L_{jk}^v) (\Delta E/E) \gg 1$,

$$\bar{P}(\nu_l \rightarrow \nu_{l'}) = \bar{P}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \cong \sum_j |U_{l'j}|^2 |U_{lj}|^2$$

Two-Neutrino Oscillations in Vacuum

SK ((100-12742) km), K2K (250 km); CNGS (OPERA),
MINOS (730 km); T2K (295 km); dominant $\nu_\mu \rightarrow \nu_\tau$;

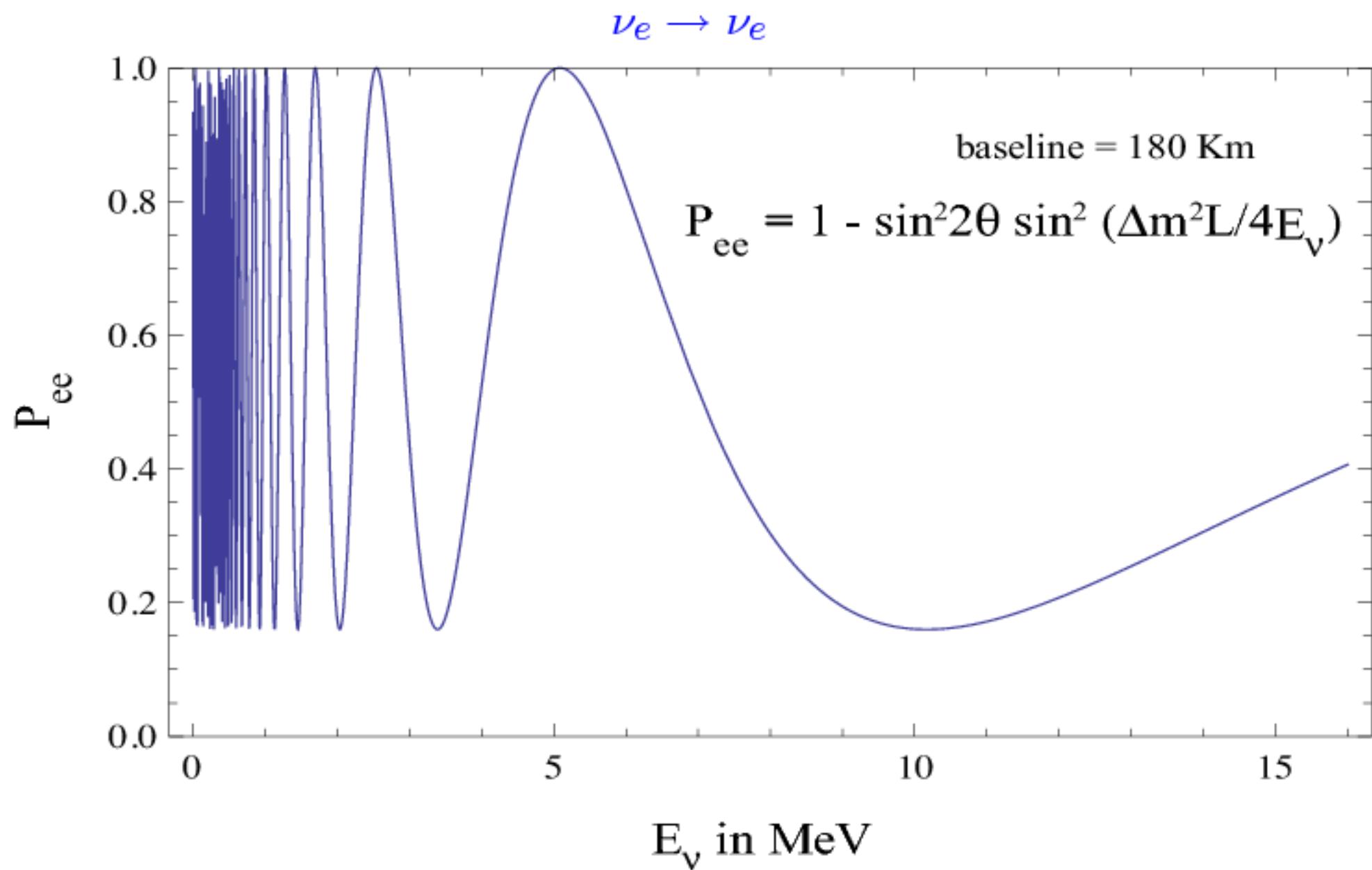
$$P(\nu_\mu \rightarrow \nu_\tau; L) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau; L) \cong \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2 L}{4E};$$
$$P(\nu_\mu \rightarrow \nu_\mu; L) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu; L) = 1 - P(\nu_\mu \rightarrow \nu_\tau; L).$$

KamLAND (~ 180 km): $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2} \sin^2 2\theta_{12} (1 - \cos \frac{\Delta m_{21}^2 L}{2E}).$$

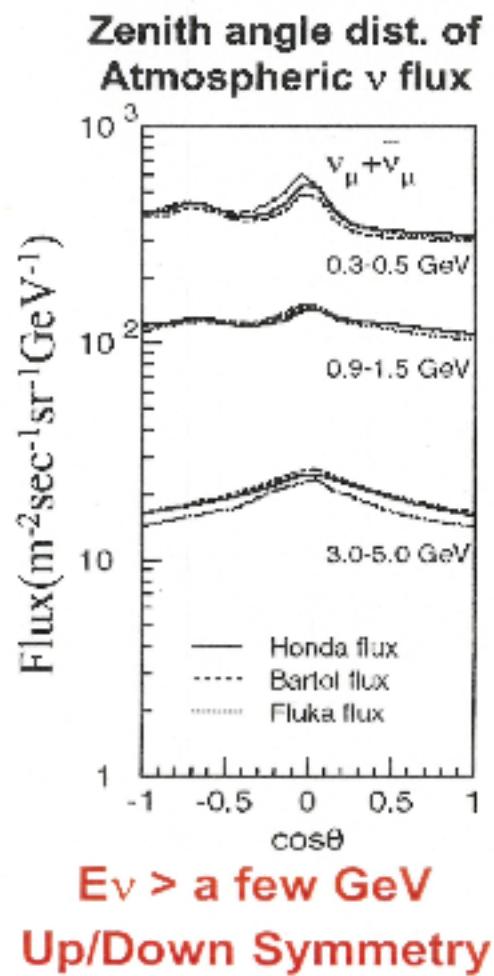
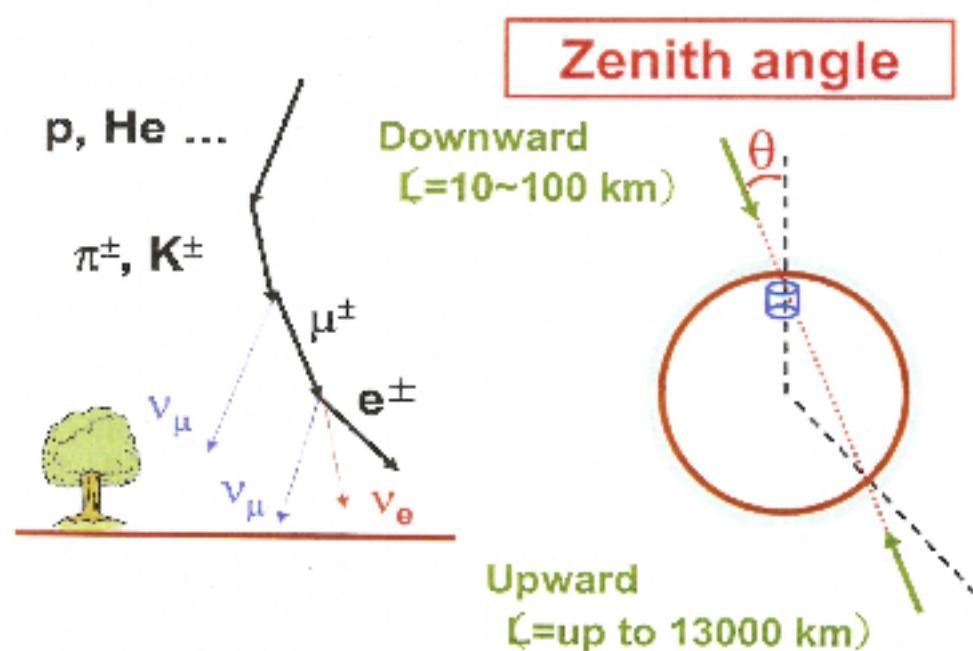
CHOOZ, Double Chooz, Daya Bay, RENO (~ 1 km):
 $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2} \sin^2 2\theta_{13} (1 - \cos \frac{\Delta m_{31}^2 L}{2E}).$$

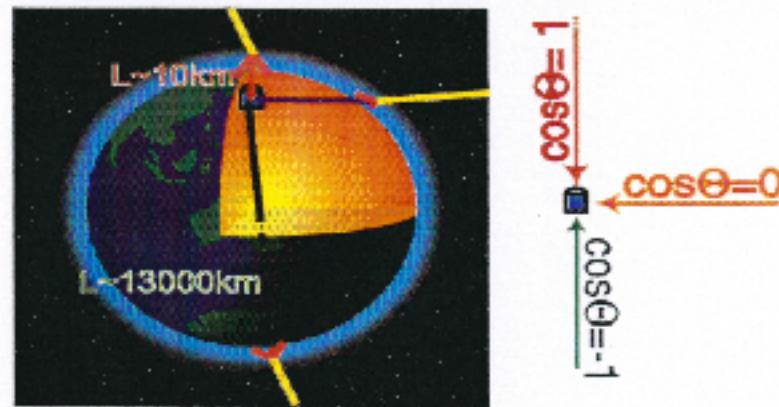


Observing the Oscillations of Neutrinos

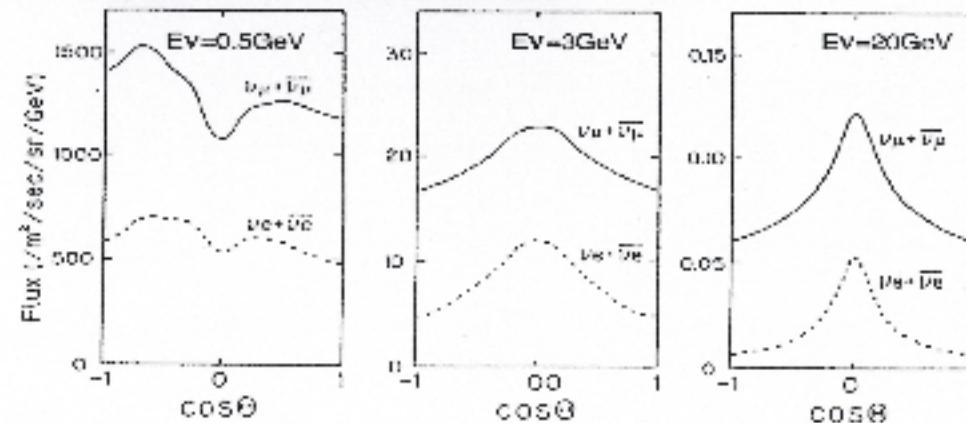
Atmospheric neutrinos



Zenith angle distribution(1D)



Calculated zenith angle distribution

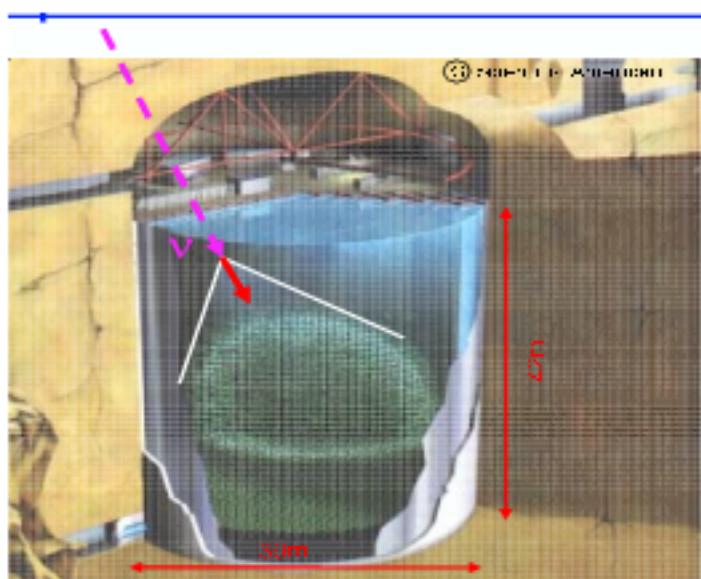


For $E_\nu >$ a few GeV,

Upward / downward = 1 (within a few %)

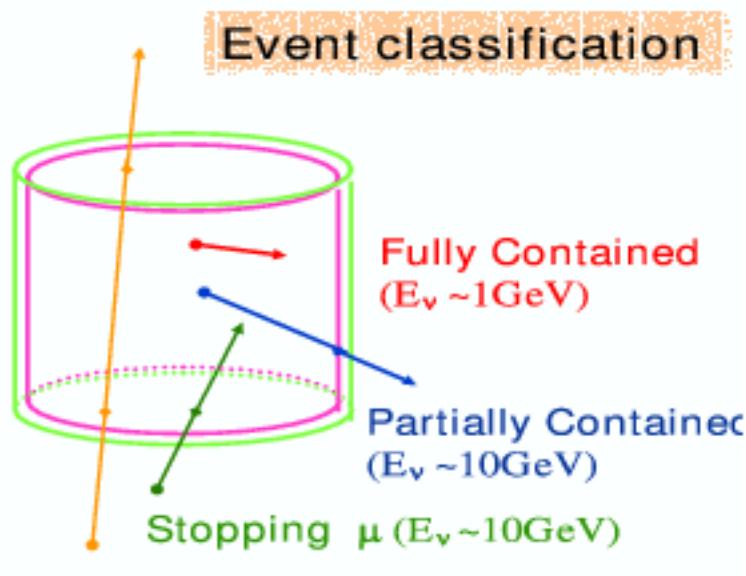


Up/Down asymmetry for neutrino oscillations



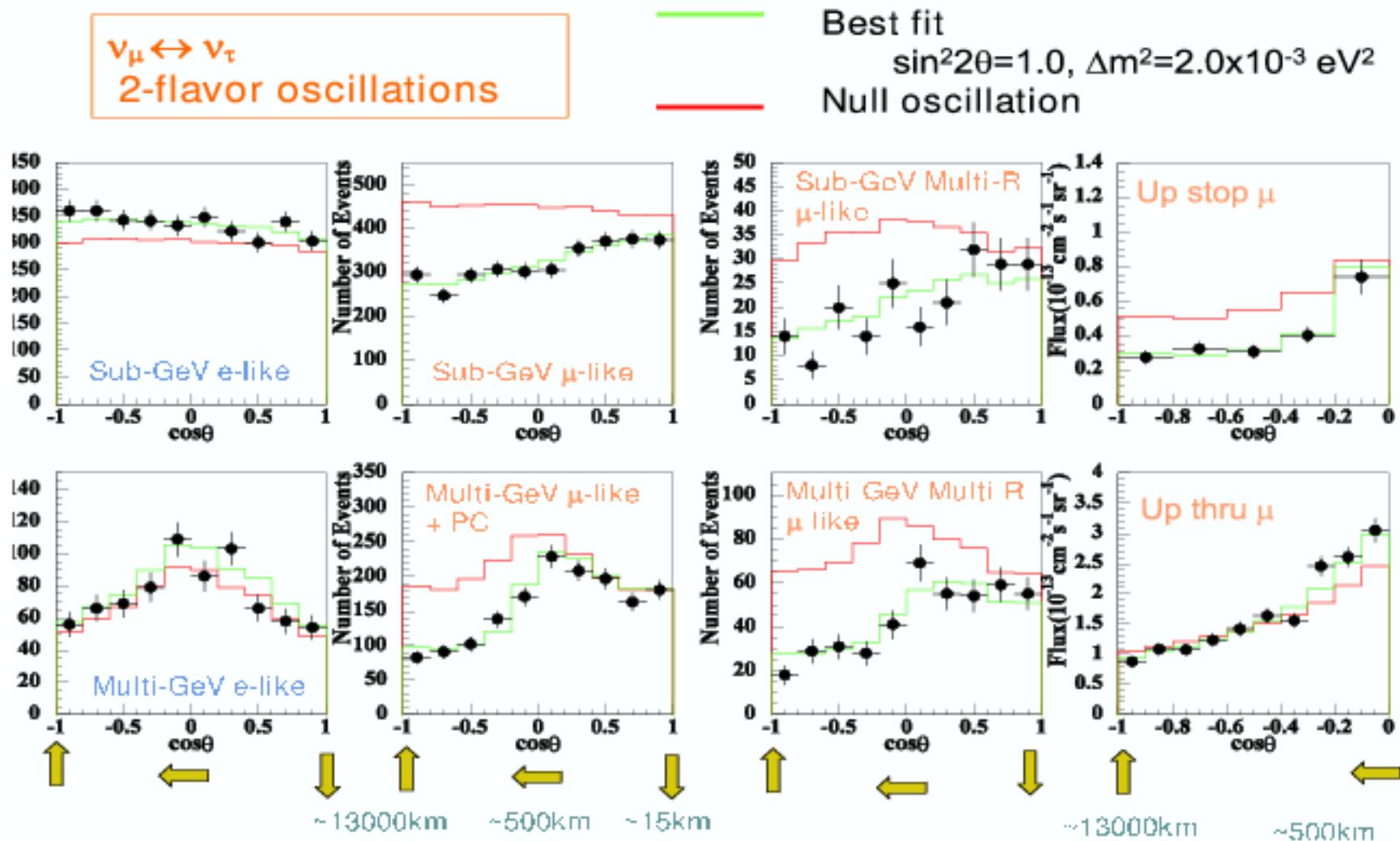
Water Cherenkov detector

1000 m underground
50,000 ton (22,500 ton fid.)
inner-detector(ID): 11,146
outer-detector(OD): 1,885

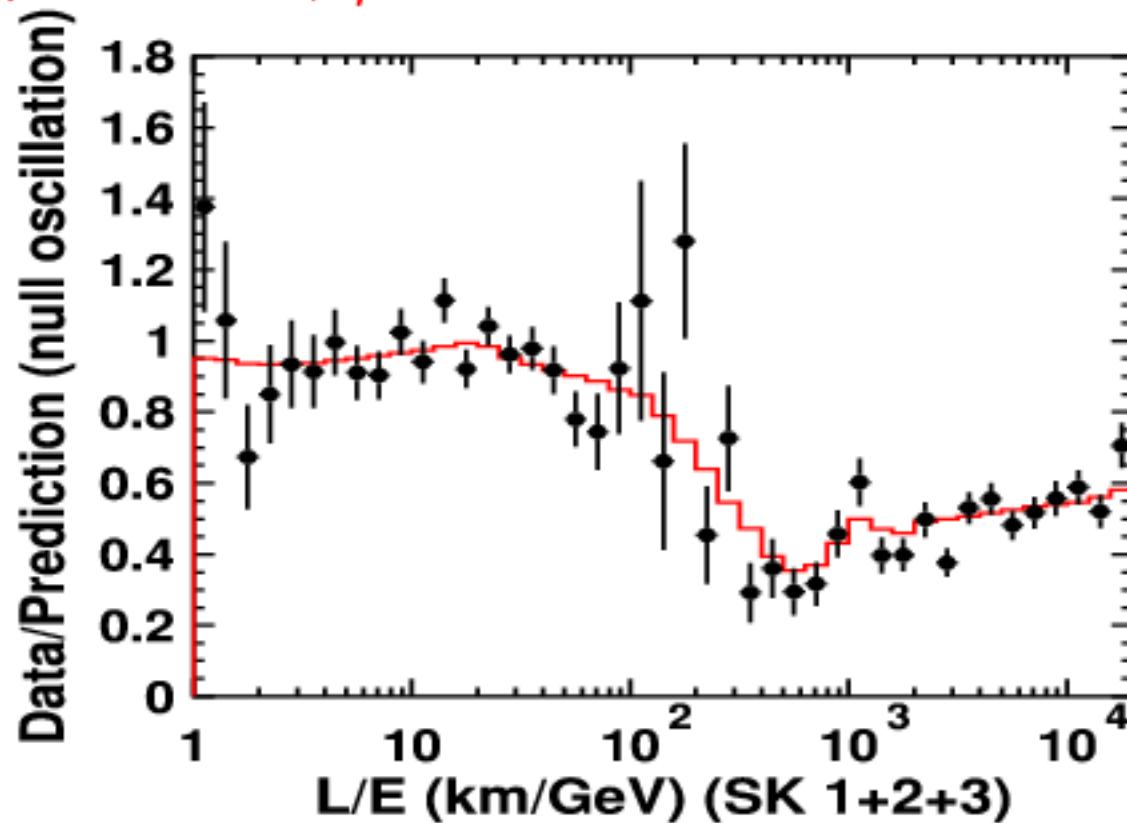


20 inch PMTs(SK-I)
8 inch PMTs

Zenith angle distributions



SK: L/E Dependence, μ -Like Events

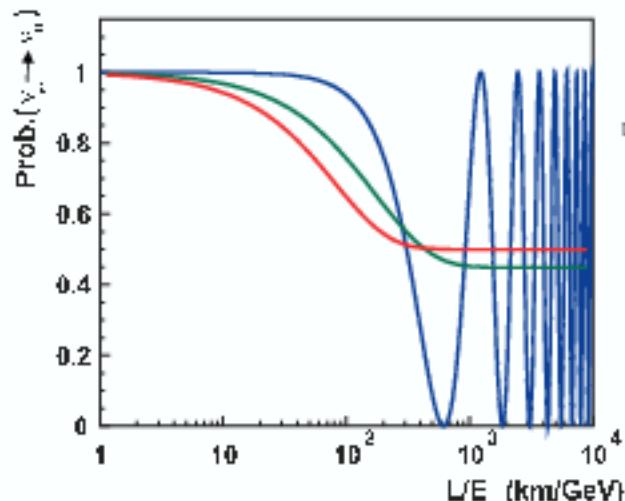


L/E analysis

Neutrino oscillation : $P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(1.27 \frac{\Delta m^2 L}{E})$

Neutrino decay : $P_{\mu\mu} = (\cos^2 \theta + \sin^2 \theta \times \exp(-\frac{m}{2\tau} \frac{L}{E}))^2$

Neutrino decoherence : $P_{\mu\mu} = 1 - \frac{1}{2} \sin^2 2\theta \times (1 - \exp(-\gamma_0 \frac{L}{E}))$



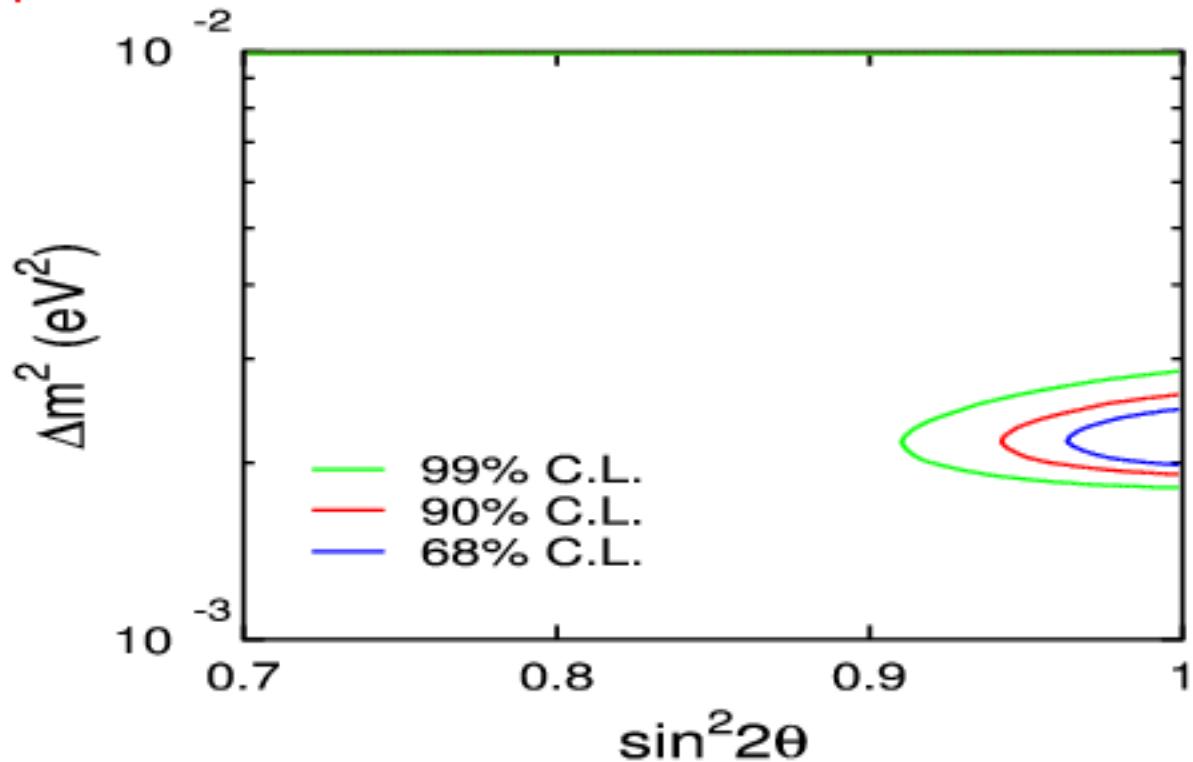
Use events with high resolution in L/E

→ The first dip can be observed

→ Direct evidence for oscillations

→ Strong constraint to oscillation parameters, especially Δm^2 value

SK: Atmospheric ν Data



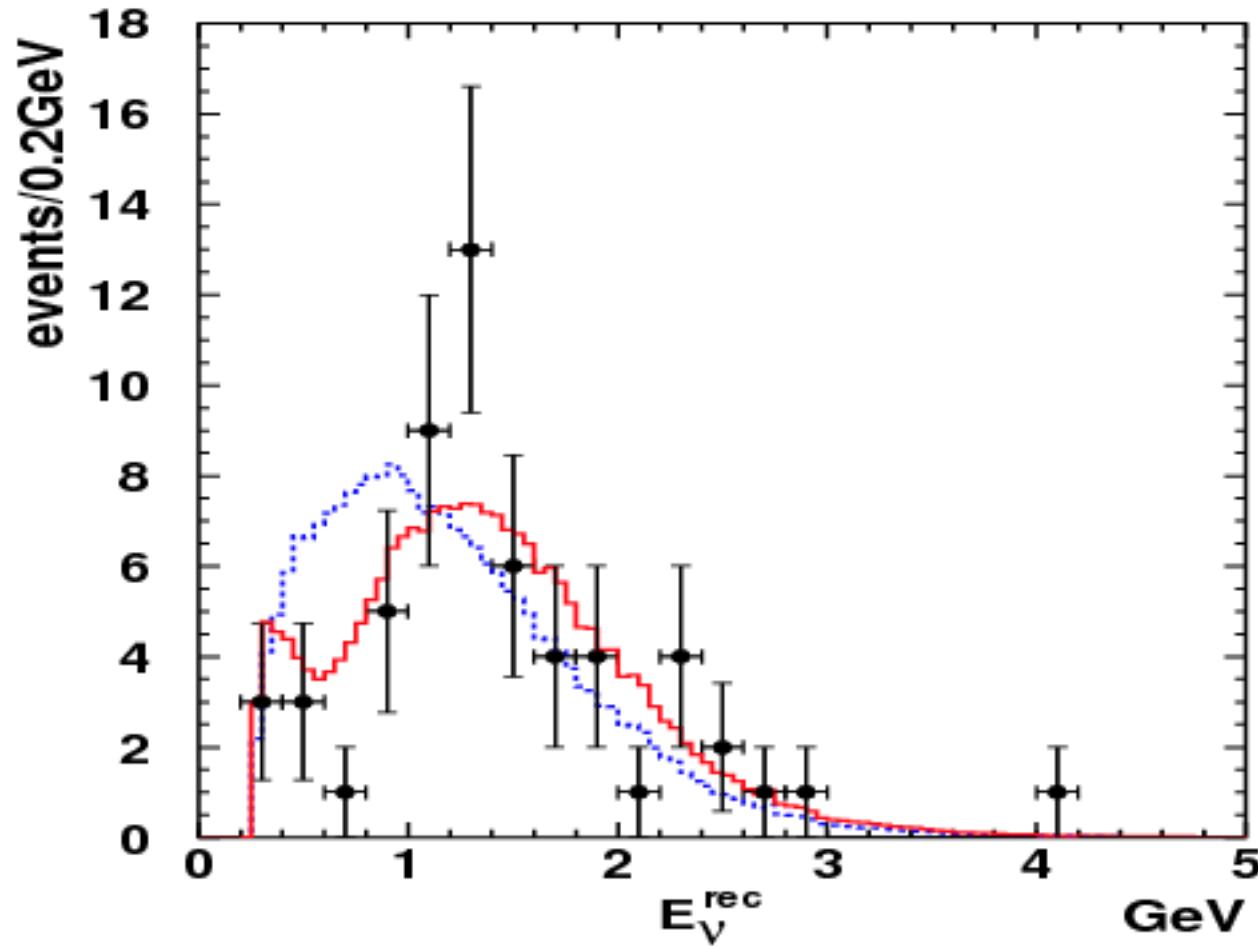
$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 = 2.4 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{atm}} \equiv \sin^2 2\theta_{23} = 1.0 ;$$

$$\Delta m_{31}^2 = (1.9 - 2.9) \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} \geq 0.92, \quad 99\% \text{ C.L.}$$

- sign of Δm_{atm}^2 not determined. If $\theta_{23} \neq \frac{\pi}{4}$: $\theta_{23}, (\frac{\pi}{4} - \theta_{23})$ ambiguity.

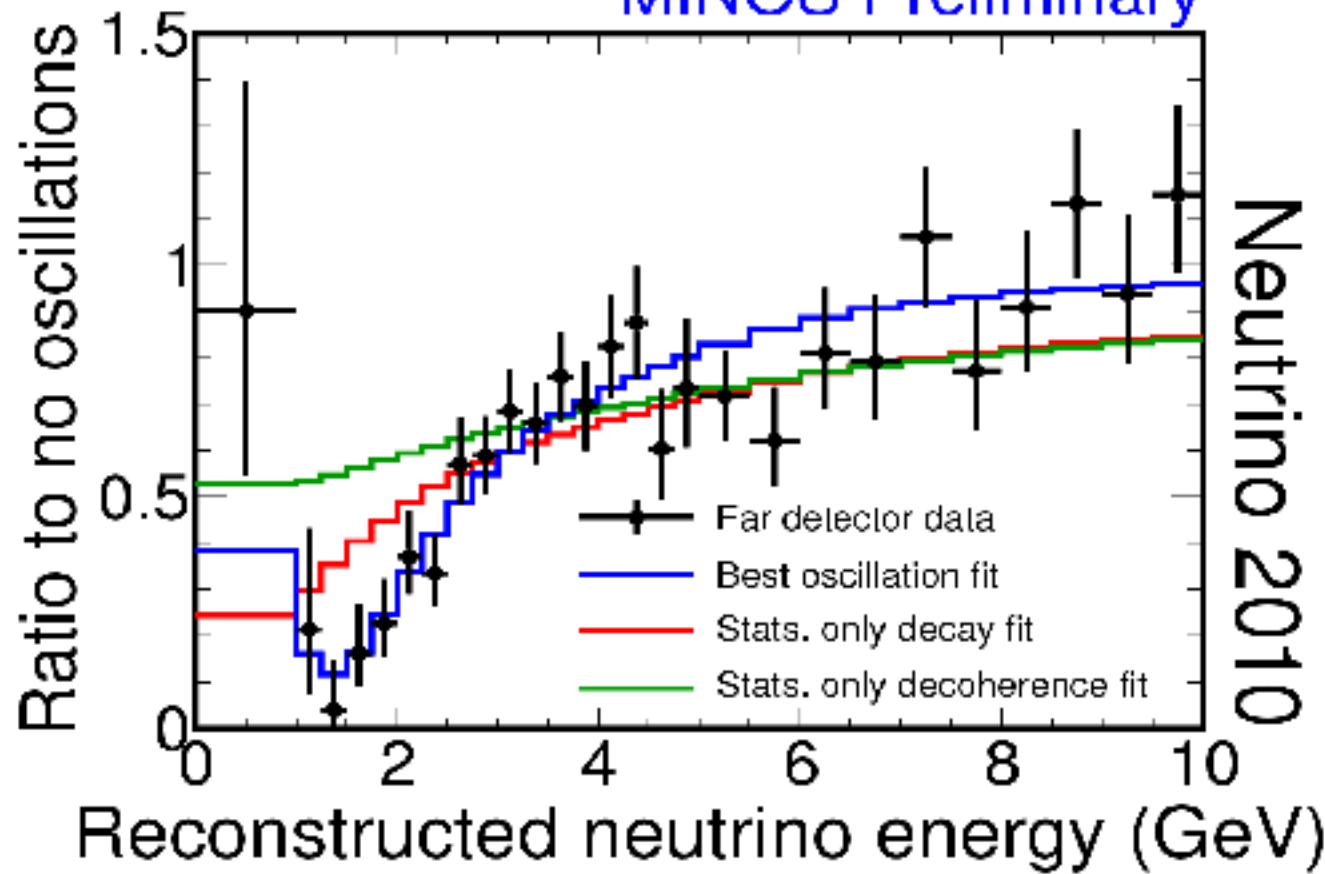
3- ν mixing: $\Delta m_{31}^2 > 0, m_1 < m_2 < m_3$ (NH); $\Delta m_{31}^2 < 0, m_3 < m_1 < m_2$ (IH).

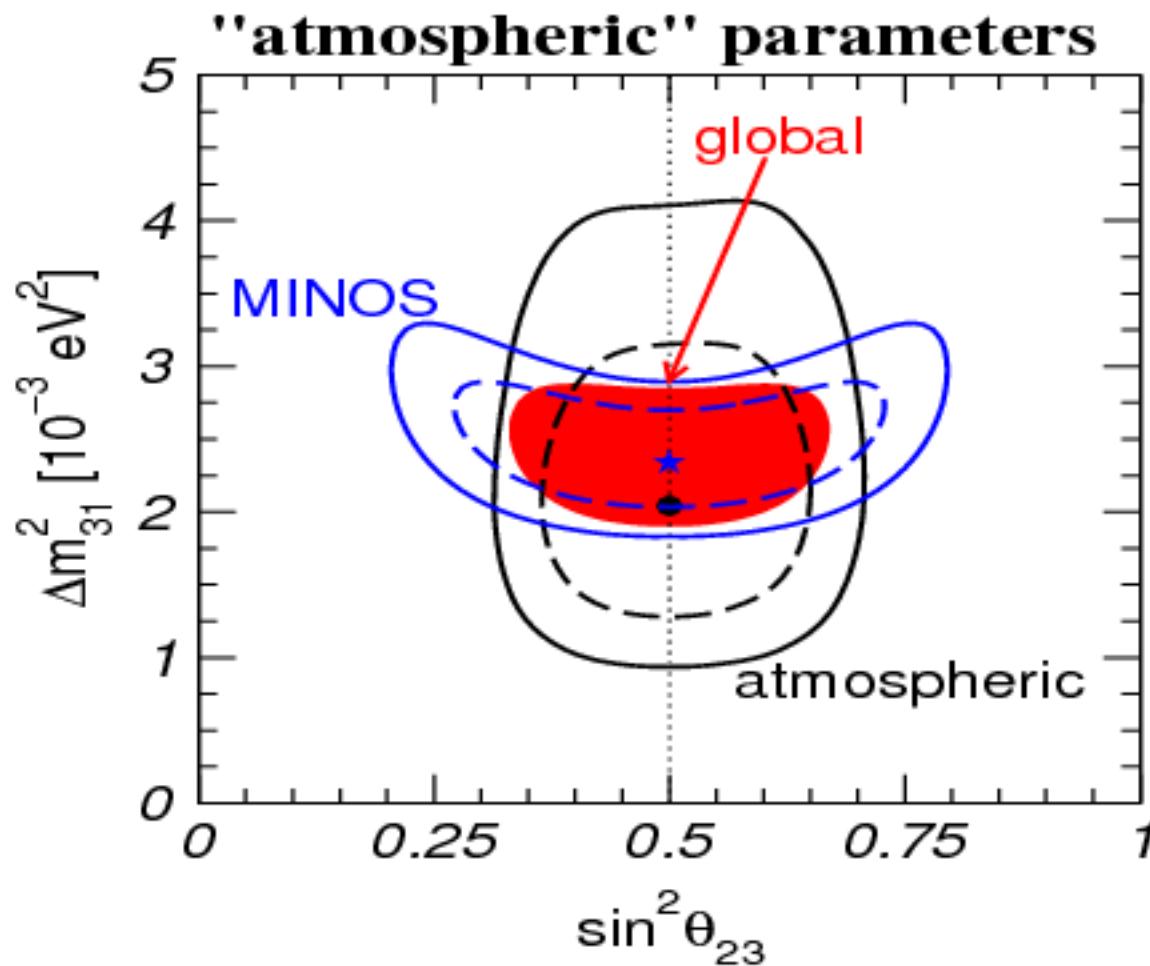
K2K: ν_μ Spectrum (ν_μ "disappearance")



MINOS: ν_μ Spectrum (ν_μ "disappearance")

MINOS Preliminary





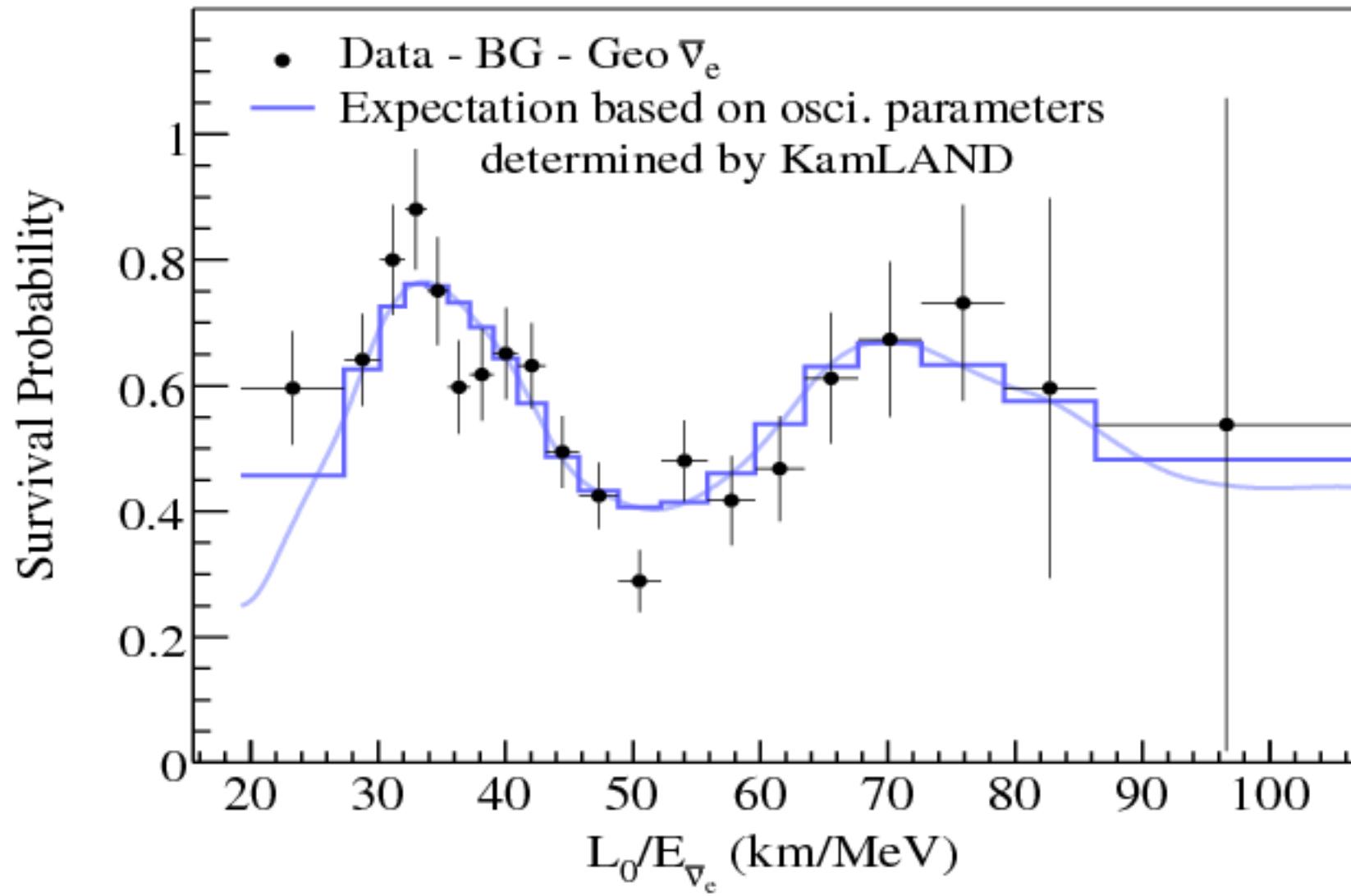
- sign of Δm_{31}^2 not determined;

T. Schwetz, arXiv:0710.5027[hep-ph]

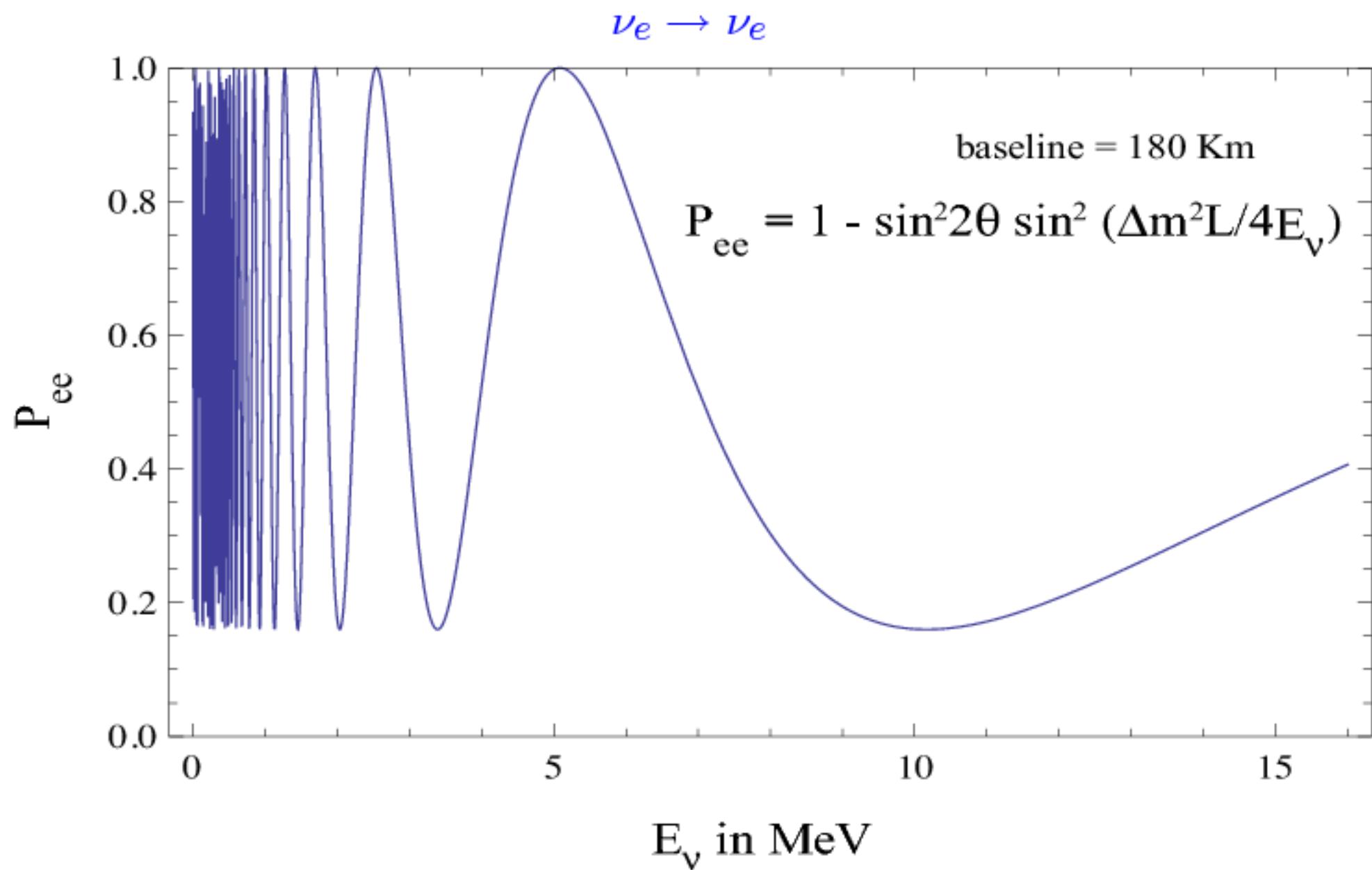
3-ν mixing: $\Delta m_{31}^2 > 0$, $m_1 < m_2 < m_3$ (normal ordering (NO));

$\Delta m_{31}^2 < 0$, $m_3 < m_1 < m_2$ (inverted ordering (IO)).

- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23} , $(\frac{\pi}{4} - \theta_{23})$ ambiguity.



KamLAND: L/E -Dependence (reactor $\bar{\nu}_e$, $\bar{L} = 180$ km, $E = (1.8 - 10)$ MeV)



Solar Neutrinos: ν_e , $E \sim (0.26 - 14.4)$ MeV

Super-Kamiokande, $E \cong (5.0 - 14.4)$ MeV

$$\begin{aligned} R(SK) &\propto \Phi_E^0(\nu_e) \sum_{l=e,\mu,\tau} P(\nu_e \rightarrow \nu_l) \sigma(\nu_l e^- \rightarrow \nu_l e^-) \\ &= \sigma(\nu_e e^- \rightarrow \nu_e e^-) [\Phi_E^0(\nu_e) P(\nu_e \rightarrow \nu_e) \\ &\quad + \Phi_E^0(\nu_e) (1 - P(\nu_e \rightarrow \nu_e)) \frac{\sigma(\nu_{\mu(\tau)} e^- \rightarrow \nu_{\mu(\tau)} e^-)}{\sigma(\nu_e e^- \rightarrow \nu_e e^-)}] \\ &= \sigma(\nu_e e^- \rightarrow \nu_e e^-) [\Phi_E(\nu_e) + 0.16(\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau))] \end{aligned}$$

$$\begin{aligned} P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) + P(\nu_e \rightarrow \nu_\tau) &= 1, \\ \sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-) &= \sigma(\nu_\tau e^- \rightarrow \nu_\tau e^-). \end{aligned}$$

SNO, CC: $E \cong (5.0 - 14.4)$ MeV

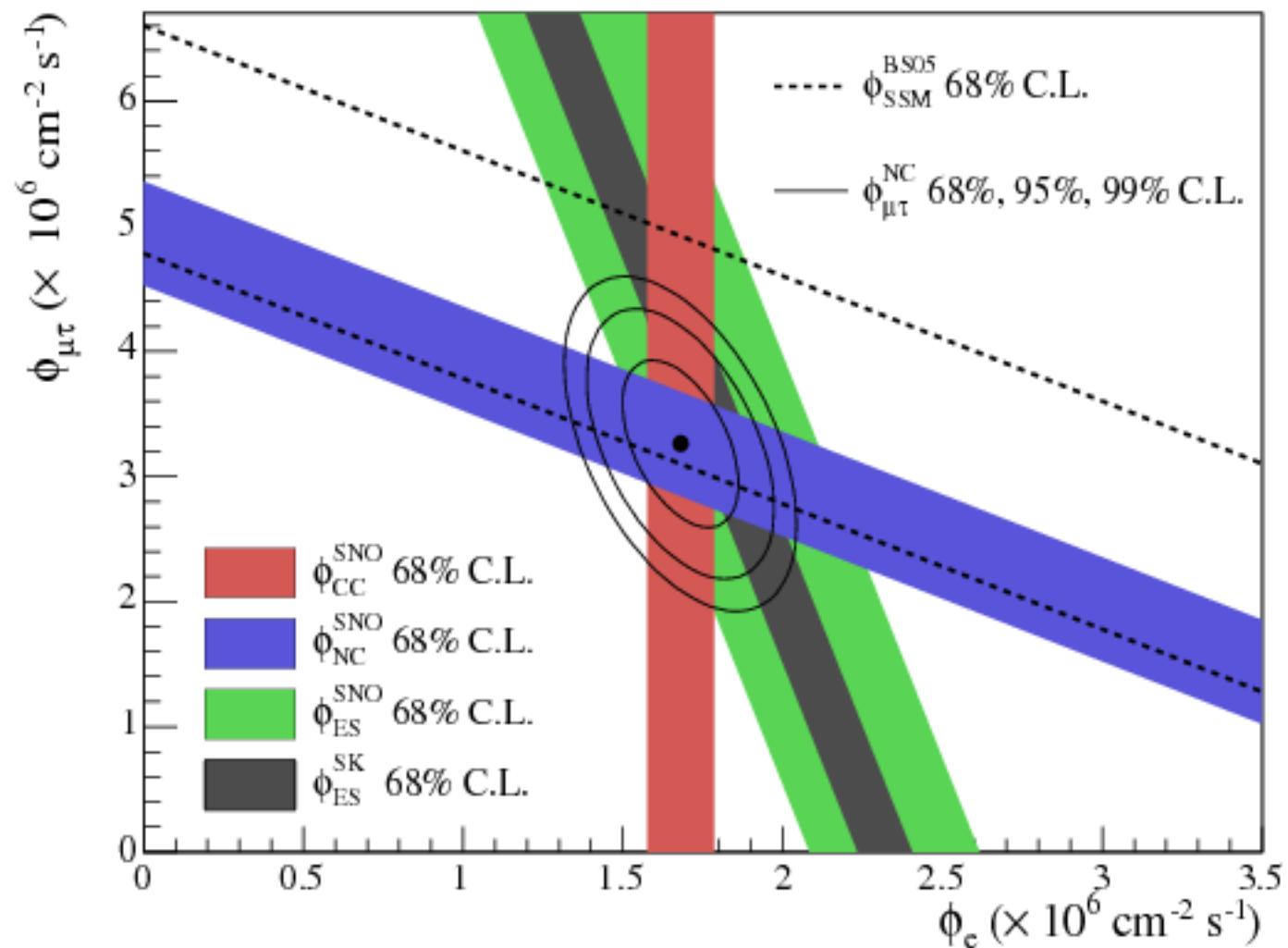


$$R(SNO) \propto \sigma(\nu_e + D \rightarrow e^- + p + p) \Phi_E^0(\nu_e) P(\nu_e \rightarrow \nu_e) \\ = \sigma(\nu_e + D \rightarrow e^- + p + p) \Phi_E(\nu_e)$$

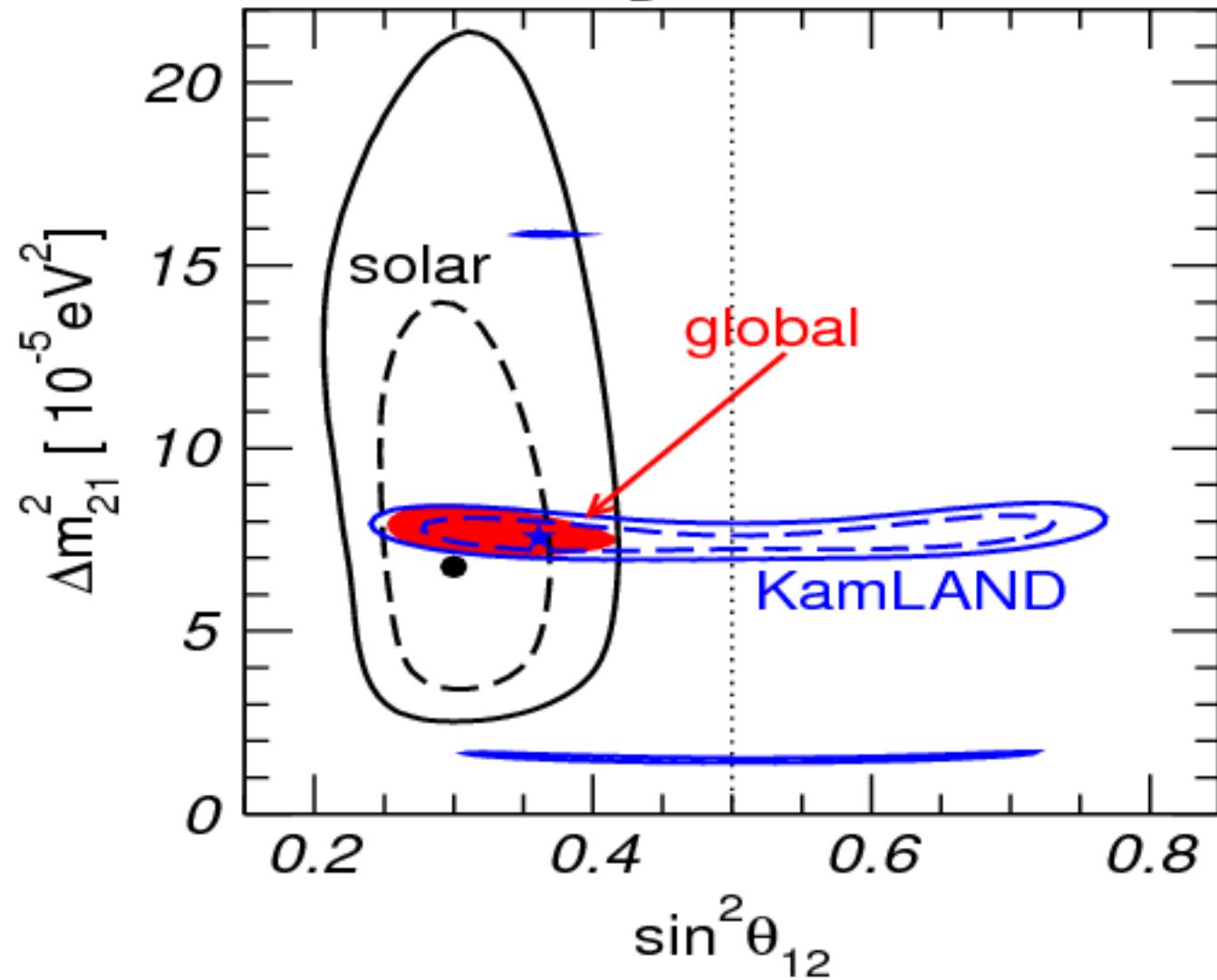
$$SK: \Phi^{SK}(\nu_\odot) = \Phi_E(\nu_e) + 0.16(\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau))$$

$$SNO \text{ CC: } \Phi^{SNO}(\nu_\odot) = \Phi_E(\nu_e)$$

$$\text{No oscillations: } \Phi_E(\nu_\mu) + \Phi_E(\nu_\tau) = 0, \quad \Phi^{SK}(\nu_\odot) = \Phi^{SNO}(\nu_\odot)$$



"solar" parameters



Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

CP-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' = e, \mu, \tau$$

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P., 1980;
V. Barger, S. Pakvasa et al., 1980.

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l': \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-Invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3 ν -mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' = e, \mu, \tau$$

$$A_T^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

$$A_{T(\text{CP})}^{(e,\mu)} = A_{T(\text{CP})}^{(\mu,\tau)} = -A_{T(\text{CP})}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa et al., 1980

In vacuum:

$$A_{CP(T)}^{(e,\mu)} = J_{CP} F_{osc}^{vac}$$

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{osc}^{vac} = \sin\left(\frac{\Delta m_{21}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{32}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{13}^2}{2E}L\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

$$\text{CP : } P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$\text{CPT : } P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance (**Earth**)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

In matter with constant density: $A_T^{(e,\mu)} = J_{CP}^{\text{mat}} F_{osc}^{\text{mat}}$

$$J_{CP}^{\text{mat}} = J_{CP}^{\text{vac}} R_{CP}$$

R_{CP} does not depend on θ_{23} and δ ; $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Rephasing Invariants Associated with CPVP

Dirac phase δ :

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21} , α_{31} :

$$S_1 = \text{Im} \{ U_{e1} U_{e3}^* \}, \quad S_2 = \text{Im} \{ U_{e2} U_{e3}^* \} \quad (\text{not unique}); \quad \text{or}$$

$$S'_1 = \text{Im} \{ U_{\tau 1} U_{\tau 2}^* \}, \quad S'_2 = \text{Im} \{ U_{\tau 2} U_{\tau 3}^* \}$$

J.F. Nieves and P. Pal, 1987, 2001

G.C. Branco et al., 1986

J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im} \{ U_{e1} U_{e3}^* \} \neq 0$ and $\text{Re} \{ U_{e1} U_{e3}^* \} \neq 0$.

S_1 , S_2 appear in $|<m>|$ in $(\beta\beta)_{0\nu}$ -decay.

In general, J_{CP} , S_1 and S_2 are independent.

LECTURE II

Matter Effects in Neutrino Oscillations

Matter can affect strongly ν -oscillations:

Mean free path in matter with $\bar{\rho} = \bar{\rho}(\text{Earth})$: $N \cong 4N_A cm^{-3}$,

$E \sim 1 \text{ MeV}$, $L_f \sim 2.5 \times 10^{14} \text{ km}$; $R_E = 6371 \text{ km}$

$E \sim 1 \text{ GeV}$, $L_f \sim 2.5 \times 10^8 \text{ km}$

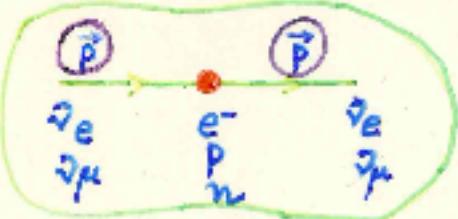
For $\bar{\rho} = \rho(\text{center of the Sun})$: $N \cong 100 N_A cm^{-3}$,

$E \sim 1 \text{ MeV}$, $L_f \sim 10^{13} \text{ km}$; $R_{\text{Sun}} = 6.96 \times 10^5 \text{ km}$

ν coherent scattering on e^- , p , n - effective potential
(index of refraction)

The presence of matter can change
drastically the pattern of ν -oscillations

$$H_{\text{mat}} = H_{\text{vac}} + H_{\text{int}}$$



$$n(\nu_e) \neq 1, n(\nu_\mu) \neq 1$$

$$n(\nu_e) - n(\nu_\mu) = \frac{2\pi}{p^2} [F_{\nu_e e^-}^V(0) - F_{\nu_\mu e^-}^V(0)] = \\ = + \frac{2\pi}{p^2} \left\{ \begin{array}{c} \nu_e \rightarrow e^- \\ \nu_\mu \rightarrow e^- \\ \text{W}^- \end{array} \right. + \begin{array}{c} \nu_e \rightarrow e^- \\ \nu_\mu \rightarrow e^- \\ Z^0 \end{array} - \begin{array}{c} \nu_\mu \rightarrow e^- \\ \nu_\mu \rightarrow e^- \\ Z^0 \end{array} \right\}$$

$$= -\frac{1}{p} \sqrt{2} G_F N_e$$

ν coherent scattering on e^- , p , n - effective potential
(index of refraction)

$$V_{e\mu} = V(\nu_e) - V(\nu_\mu) = \sqrt{2}G_F N_e$$

$$\bar{V}_{e\mu} = V(\bar{\nu}_e) - V(\bar{\nu}_\mu) = -\sqrt{2}G_F N_e$$

$$V_{\mu\tau} = V(\nu_\mu) - V(\nu_\tau) = 0 \text{ (leading order)}$$

L. Wolfenstein, 1978; V. Barger et al., 1980; P. Langacker et al., 1983

$$V_{es} = V(\nu_e) - V(\nu_s) = \sqrt{2}G_F(N_e - \frac{1}{2}N_n)$$

$$\bar{V}_{es} = V(\bar{\nu}_e) - V(\bar{\nu}_s) = -\sqrt{2}G_F(N_e - \frac{1}{2}N_n) = -V_{es}$$

$$V_{\mu s} = V(\nu_\mu) - V(\nu_s) = \sqrt{2}G_F(-\frac{1}{2}N_n)$$

$$\bar{V}_{\mu s} = V(\bar{\nu}_\mu) - V(\bar{\nu}_s) = -\sqrt{2}G_F(-\frac{1}{2}N_n) = -V_{\mu s}$$

$V_{e\mu} \neq \bar{V}_{e\mu}$: CP, CPT violated

P. Langacker, S.T.P. et al., 1987

Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the background of electrons, protons and neutrinos, which generates an effective potential in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$.

This modifies the neutrino mixing since the eigenstates and the eigenvalues of H_{vac} and of $H = H_{vac} + V_{eff}$ are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$$i\frac{d}{dt} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} \quad (1)$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$.

$$\epsilon(t) = \frac{1}{2} \left[\frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t) \right],$$

$$\epsilon'(t) = \frac{\Delta m^2}{4E} \sin 2\theta, \text{ with } \Delta m^2 = m_2^2 - m_1^2.$$

In matter, $H_m = H_0 + H_{int}$.

$H_0|\nu_{1,2}\rangle = E_{1,2}|\nu_{1,2}\rangle$, not eigenstates of H_m .

Consider first $N_e = \text{const.}$

$$H_m |\nu_{1,2}^m\rangle = E_{1,2}^m |\nu_{1,2}^m\rangle.$$

Then at $t = 0$ in matter

$$|\nu_e\rangle = |\nu_1^m\rangle \cos\theta_m + |\nu_2^m\rangle \sin\theta_m,$$

$$|\nu_{\mu(\tau)}\rangle = -|\nu_1^m\rangle \sin\theta_m + |\nu_2^m\rangle \cos\theta_m;$$

$$\sin 2\theta_m = \frac{\epsilon'}{\sqrt{\epsilon^2 + \epsilon'^2}} = \frac{\tan 2\theta}{\sqrt{(1 - \frac{N_e}{N_e^{res}})^2 + \tan^2 2\theta}},$$

$$\cos 2\theta_m = \frac{1 - N_e/N_e^{res}}{\sqrt{(1 - \frac{N_e}{N_e^{res}})^2 + \tan^2 2\theta}},$$

$$N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F} \cong 6.56 \times 10^6 \frac{\Delta m^2 [\text{eV}^2]}{E [\text{MeV}]} \cos 2\theta \text{ cm}^{-3} \text{ N}_A,$$

$$E_2^m - E_1^m = \frac{\Delta m^2}{2E} \left((1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta \right)^{\frac{1}{2}}$$

$$P_m^{2\nu}(\nu_e \rightarrow \nu_\mu) = |A_\mu(t)|^2 = \frac{1}{2} \sin^2 2\theta_m [1 - \cos 2\pi \frac{L}{L_m}],$$

$$L_m = \frac{E_2^m - E_1^m}{2\pi} = L^v \left((1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta \right)^{-\frac{1}{2}}.$$

The resonance condition: $N_e = N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$

At the resonance:

$$\sin^2 2\theta_m = 1, \min(E_2^m - E_1^m), L_m^{res} = L^v / \sin 2\theta.$$

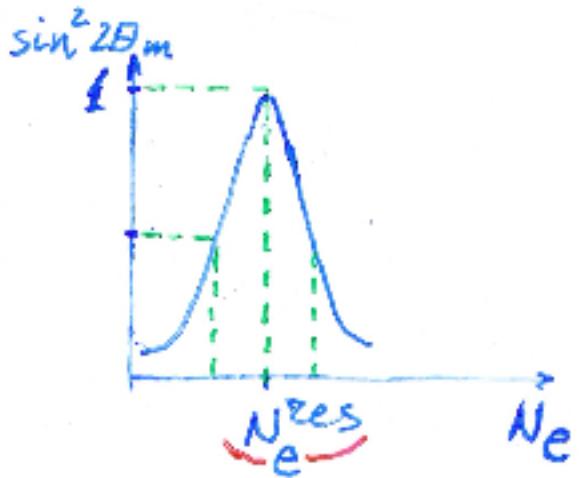
Limiting cases:

$$N_e \ll N_e^{res}: \theta_m \cong \theta, E_{1,2}^m \cong E_{1,2}, L_m \cong L^v.$$

$$N_e \gg N_e^{res}: \theta_m \cong \frac{\pi}{2}, \nu_e \rightarrow \nu_\mu \text{ suppressed.}$$

$$\text{In this case: } |\nu_e\rangle \cong |\nu_2^m\rangle, |\nu_\mu\rangle = -|\nu_1^m\rangle.$$

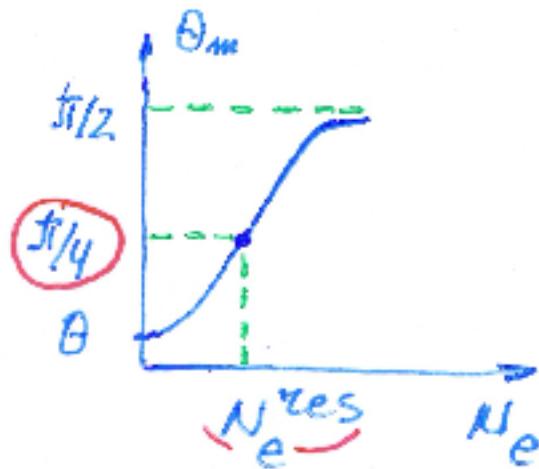
$$N_e^{\text{res}} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$$



$$N_e \gg N_e^{\text{res}}, \quad \theta_m \approx \frac{\pi}{2}$$

$$N_e \ll N_e^{\text{res}}, \quad \theta_m \approx \theta$$

$$N_e = N_e^{\text{res}}, \quad \theta_m \approx \frac{\pi}{4}$$



$$\Delta N_e^{\text{res}} = 2N_e^{\text{res}} \tan 2\theta$$

$$E_2^m - E_1^m \Big|_{\text{res}} = \min (E_2^m - E_1^m)$$

Antineutrinos: $N_e \rightarrow (-N_e)$

$\Delta m^2 \cos 2\theta > 0$: $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ suppressed by matter; $\nu_e \rightarrow \nu_\mu$ can be enhanced.

$\Delta m^2 \cos 2\theta < 0$: $\nu_e \rightarrow \nu_\mu$ suppressed by matter; $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ can be enhanced.

V. Barger et al., 1980; S.P. Mikheyev, A.Yu. Smirnov, 1985

Oscillations in matter (Earth, Sun) are neither CP- nor CPT- invariant.

P. Langacker, S.T.P., S. Toshev, G. Steigman, 1987

Earth: $\bar{N}_e^{mant} \sim 2.3 N_A \text{ cm}^{-3}$, $\bar{N}_e^{core} \sim 6.0 N_A \text{ cm}^{-3}$

$$P^m(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta_m (1 - \cos 2\pi \frac{L}{L_{osc}^m}), \quad L_{osc}^m \sim L_{osc}^{vac}$$

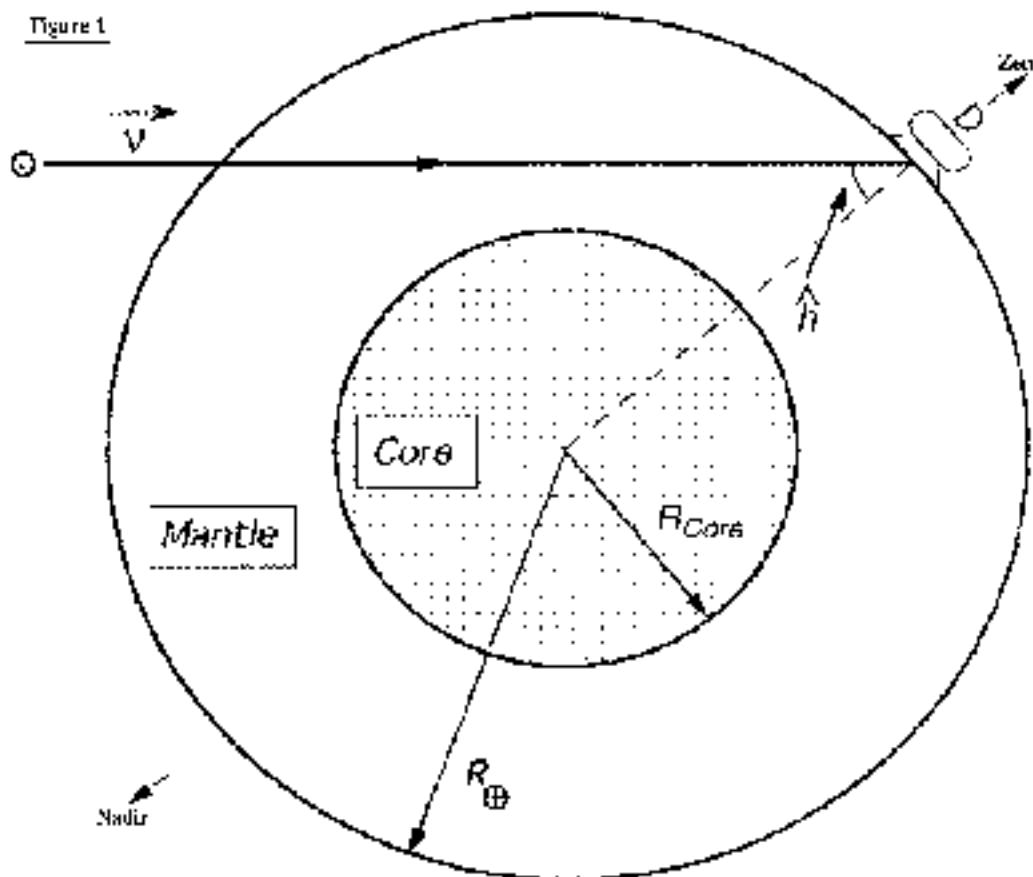
$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta + \sin^2 2\theta}, \quad N_e^{res} \equiv \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$$

$N_e = N_e^{res}$: MSW (Mikheyev, Smirnov, Wolfenstein) resonance

$\Delta m^2 \cos 2\theta > 0$: $\nu_e \rightarrow \nu_\mu$

$\Delta m^2 \cos 2\theta < 0$: $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$

The Earth



Earth: $R_{core} = 3446 \text{ km}$, $R_{mant} = 2885 \text{ km}$

Earth: $\bar{N}_e^{mant} \sim 2.3 \text{ } N_A \text{ cm}^{-3}$, $\bar{N}_e^{core} \sim 5.7 \text{ } N_A \text{ cm}^{-3}$

The Earth

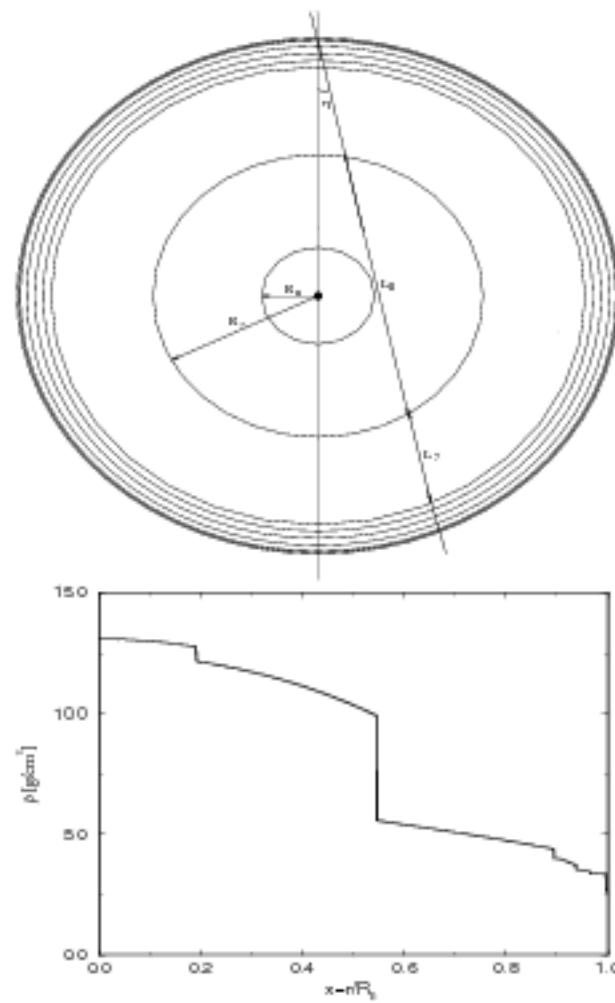
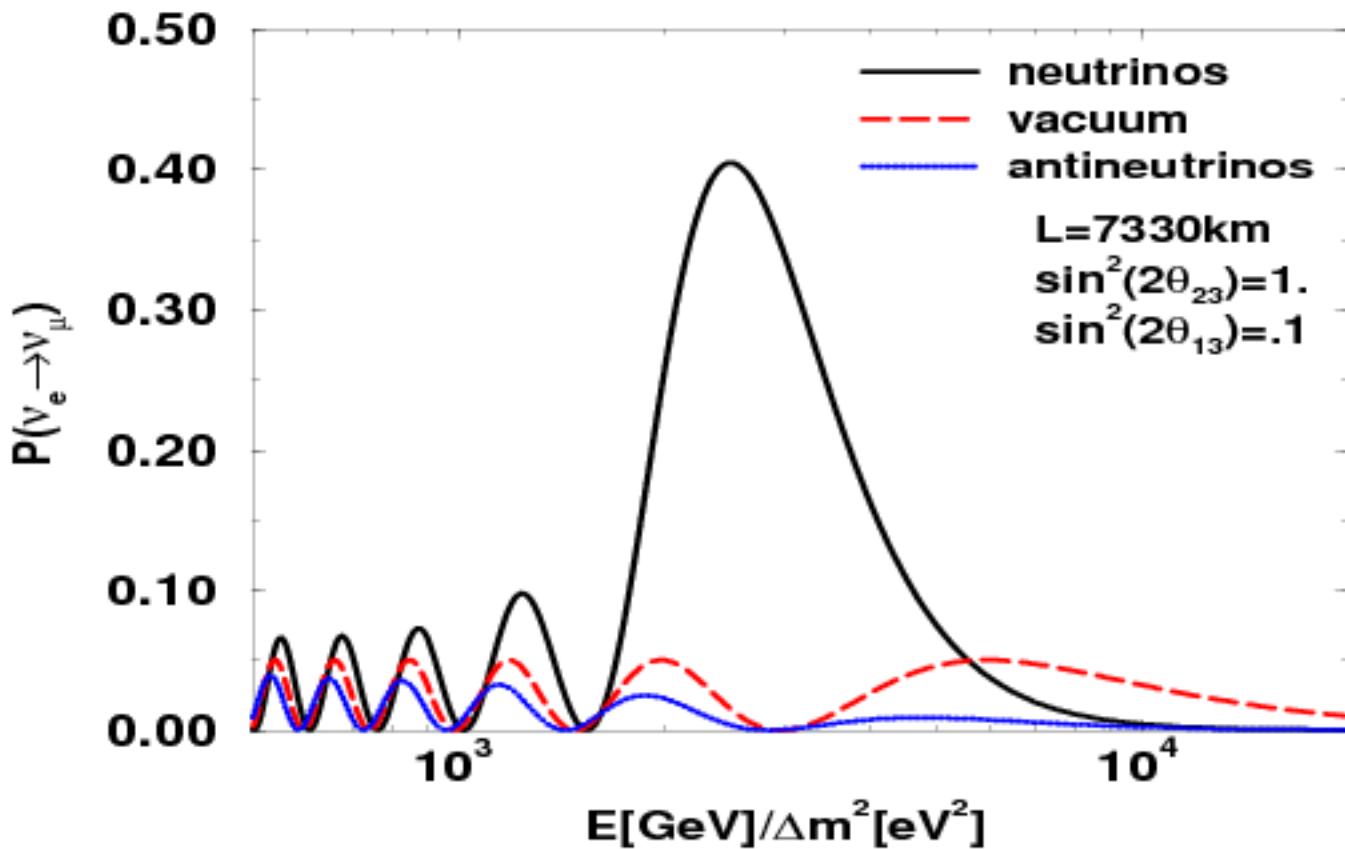


FIG. 1. Density profile of the Earth.

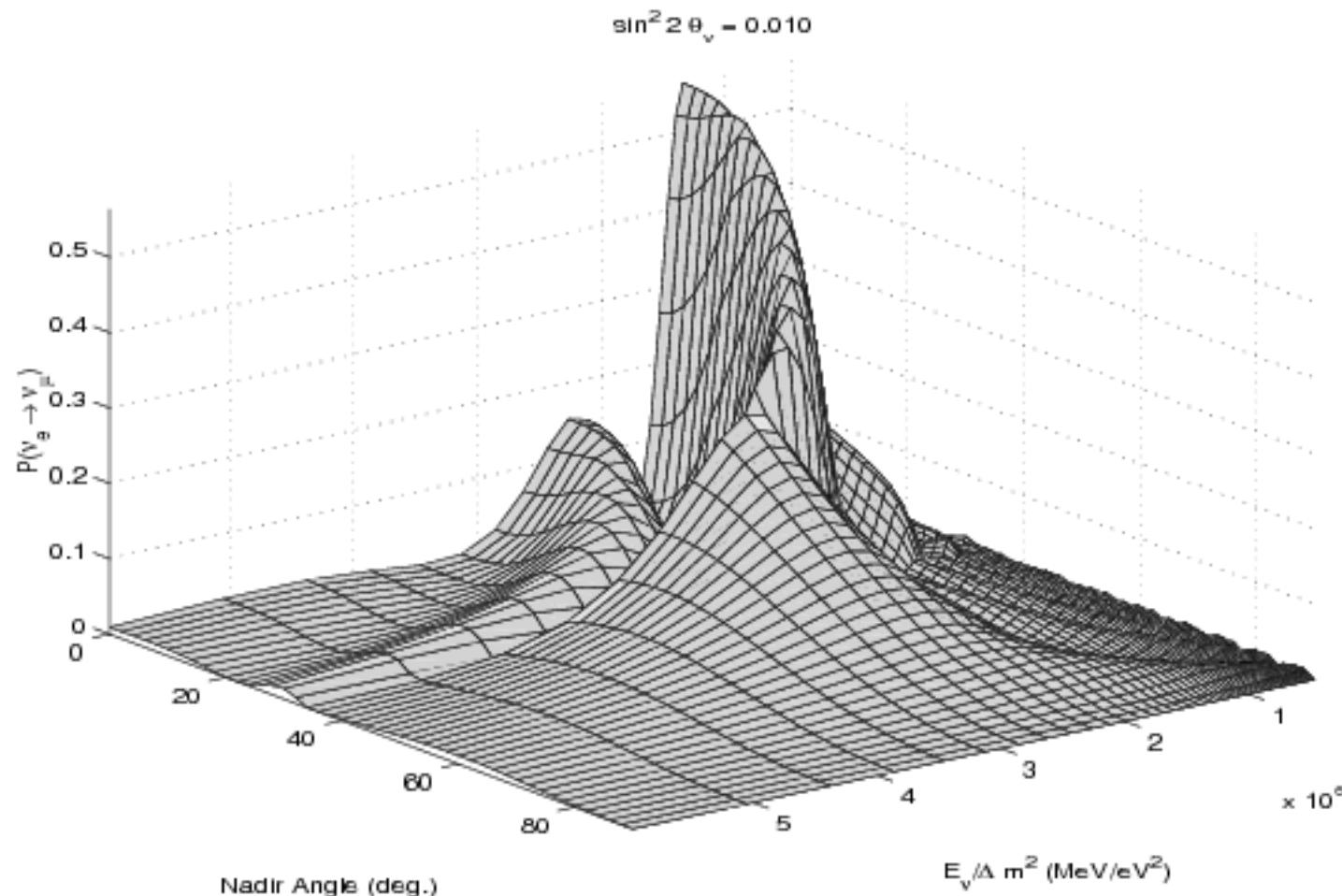
$R_c = 3446$ km, $R_m = 2885$ km; $\bar{N}_e^{mant} \sim 2.3 N_A cm^{-3}$, $\bar{N}_e^{core} \sim 5.7 N_A cm^{-3}$

Earth matter effect in $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (MSW)



$$\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2, E^{\text{res}} = 6.25 \text{ GeV}; P^{3\nu} = \sin^2 \theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}; N_e^{\text{res}} \cong 2.3 \text{ cm}^{-3} N_A; L_m^{\text{res}} = L^{\nu} / \sin 2\theta_{13} \cong 6250 / 0.32 \text{ km}; 2\pi L / L_m \cong 0.75\pi (\neq \pi).$$

Earth matter effects in $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (NOLR)



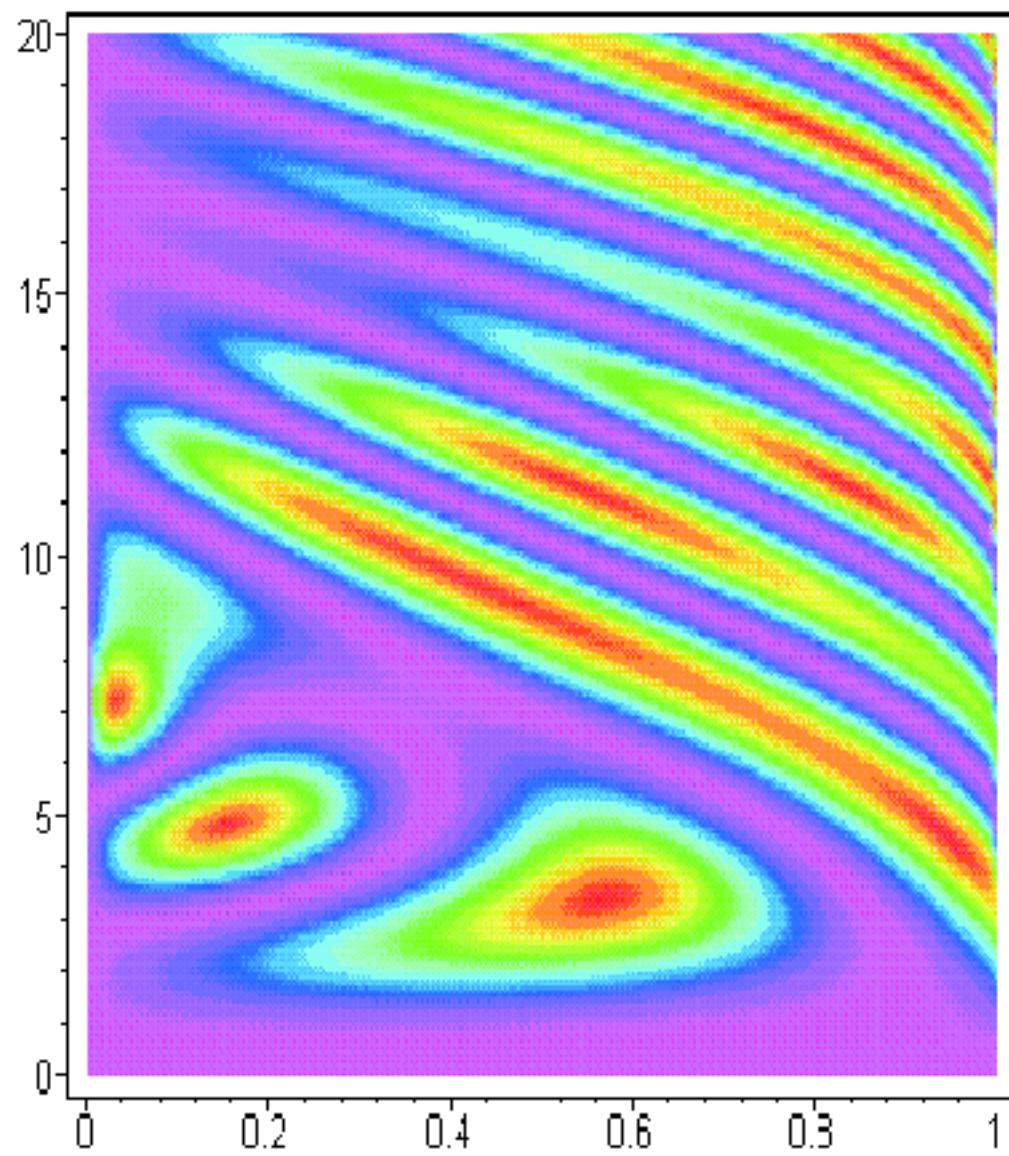
S.T.P., 1998;

M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$P(\nu_e \rightarrow \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)})$, $\theta_\nu \equiv \theta_{13}$, $\Delta m^2 \equiv \Delta m_{\text{atm}}^2$;

Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);

Local maxima: MSW effect in the Earth mantle or core.



$(s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: "Dark Red Spots"**, $P_{2\nu} = 1$;
Vertical axis: $\Delta m^2/E$ [$10^{-7} \text{eV}^2/\text{MeV}$]; **horizontal axis:** $\sin^2 2\theta_{13}$; $\theta_n = 0$

M. Chizhov, S.T.P., 1999 (hep-ph/9903399, 9903424)

- For Earth center crossing ν 's ($\theta_n = 0$) and, e.g. $\sin^2 2\theta_{13} = 0.01$, NOLR occurs at $E \cong 4$ GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV 2).

S.T.P., hep-ph/9805262

- For the Earth core crossing ν 's: $P_{2\nu} = 1$ due to NOLR when

$$\tan \Phi^{\text{man}}/2 \equiv \tan \phi' = \pm \sqrt{\frac{-\cos 2\theta_m''}{\cos(2\theta_m'' - 4\theta_m')}} ,$$

$$\tan \Phi^{\text{core}}/2 \equiv \tan \phi'' = \pm \sqrt{\frac{\cos 2\theta_m'}{-\cos(2\theta_m'') \cos(2\theta_m'' - 4\theta_m')}} ,$$

Φ^{man} (Φ^{core}) - phase accumulated in the Earth mantle (core),
 θ_m' (θ_m'') - the mixing angle in the Earth mantle (core).

$P_{2\nu} = 1$ due to NOLR for $\theta_n = 0$ (Earth center crossing ν 's) at,
e.g. $\sin^2 2\theta_{13} = 0.034; 0.154$, $E \cong 3.5; 5.2$ GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV 2).

M. Chizhov, S.T.P., Phys. Rev. Lett. 83 (1999) 1096 (hep-ph/9903399); Phys. Rev. Lett. 85 (2000) 3979 (hep-ph/0504247); Phys. Rev. D63 (2001) 073003 (hep-ph/9903424).