

## Dirac CP-Nonconservation: $\delta$ in $U_{\text{PMNS}}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP  $\alpha_{21}, \alpha_{31}$

CP-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' = e, \mu, \tau$$

N. Cabibbo, 1978  
S.M. Bilenky, J. Hosek, S.T.P., 1980;  
V. Barger, S. Pakvasa et al., 1980.

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l': \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-Invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3 $\nu$ -mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' = e, \mu, \tau$$

$$A_T^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

$$A_{T(\text{CP})}^{(e,\mu)} = A_{T(\text{CP})}^{(\mu,\tau)} = -A_{T(\text{CP})}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa et al., 1980

In vacuum:

$$A_{CP(T)}^{(e,\mu)} = J_{CP} F_{osc}^{vac}$$

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{osc}^{vac} = \sin\left(\frac{\Delta m_{21}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{32}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{13}^2}{2E}L\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

$$\text{CP : } P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$\text{CPT : } P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance (**Earth**)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

In matter with constant density:  $A_T^{(e,\mu)} = J_{CP}^{\text{mat}} F_{osc}^{\text{mat}}$

$$J_{CP}^{\text{mat}} = J_{CP}^{\text{vac}} R_{CP}$$

$R_{CP}$  does not depend on  $\theta_{23}$  and  $\delta$ ;  $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Up to 2nd order in the two small parameters  $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$  and  $\sin^2 \theta_{13} \ll 1$ :

$$P_m^{3\nu \text{ man}}(\nu_e \rightarrow \nu_\mu) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

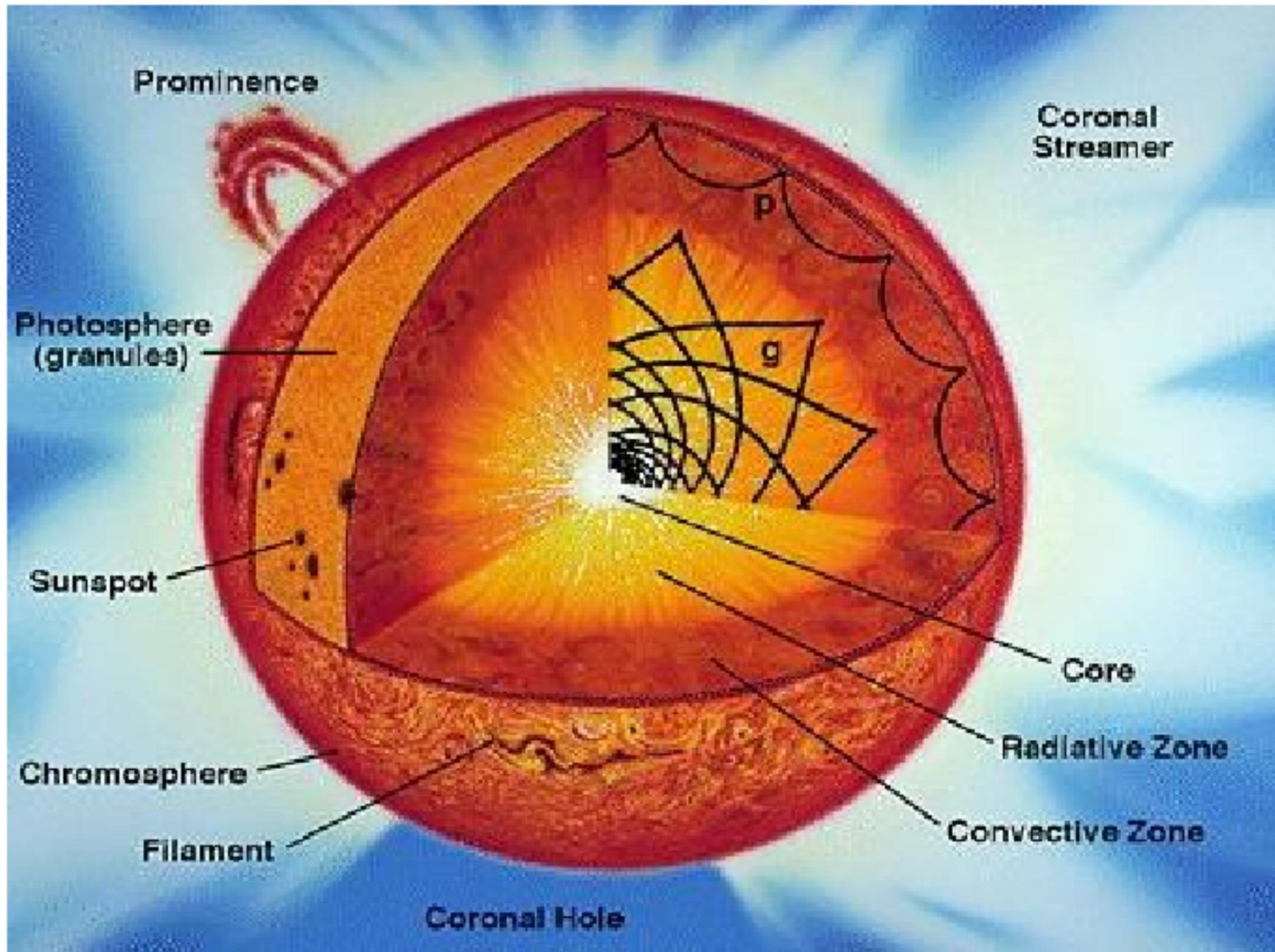
$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = \alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{\text{man}} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_e \rightarrow \bar{\nu}_\mu: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

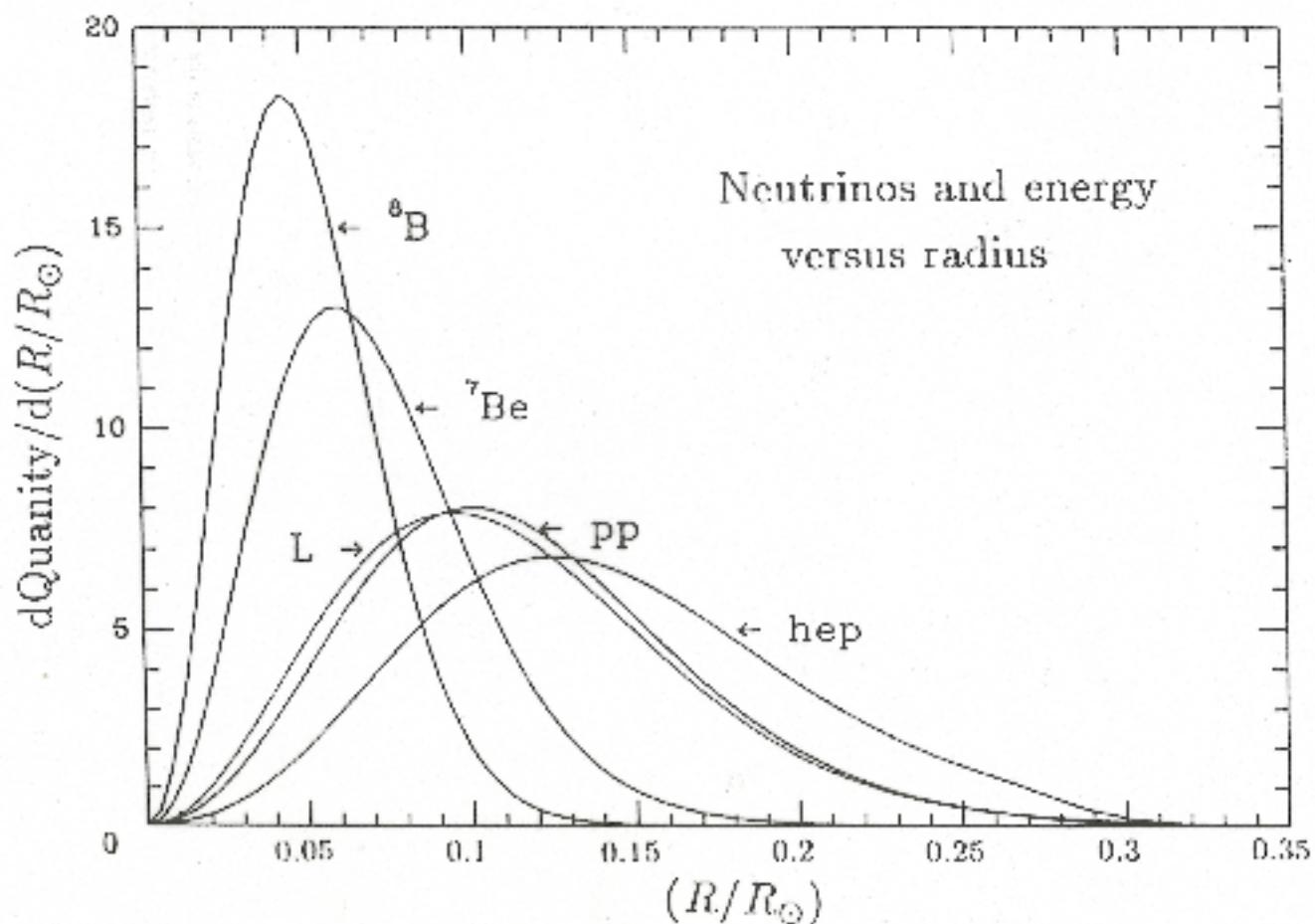


## Solar Neutrino Production: pp Chain

REACTION	TERM. (%)	$\nu$ ENERGY (MeV)
$p + p \rightarrow ^2H + e^+ + \nu_e$	(99.96)	$\leq 0.420$
or		
$p + e^- + p \rightarrow ^2H + \nu_e$	(0.14)	1.442
$^2H + p \rightarrow ^3He + \gamma$	(100)	
$^3He + ^3He \rightarrow \alpha + 2p$	(85)	
or		
$^3He + ^4He \rightarrow ^7Be + \gamma$	(15)	
$^7Be + e^- \rightarrow ^7Li + \nu_e$	(15)	$\begin{cases} 0.861 & 90\% \\ 0.383 & 10\% \end{cases}$
$^7Li + p \rightarrow 2\alpha$		
or		
$^7Be + p \rightarrow ^8B + \gamma$	(0.02)	
$^8B \rightarrow ^8Be^* + e^- + \nu_e$		$< 1.5$
$^8Be^* \rightarrow 2\alpha$		
or		
$^3He + p \rightarrow ^4He + e^+ + \nu_e$	(0.000004)	18.8



- *pp* neutrinos,  $E \leq 0.420$  MeV,  $\bar{E} = 0.265$  MeV,
- ${}^7\text{Be}$  neutrinos,  $E=0.862$  MeV (89.7% of the flux),  $0.384$  MeV (10.3%) ,
- ${}^8\text{B}$  neutrinos,  $E \leq 14.40$  MeV,  $\bar{E} = 6.71$  MeV,
- *pep* neutrinos,  $E=1.442$  MeV,
- of  ${}^{13}\text{N}$ ,  $E \leq 1.199$  MeV,  $\bar{E} = 0.707$  MeV,
- of  ${}^{15}\text{O}$ ,  $E \leq 1.732$  MeV,  $\bar{E} = 0.997$  MeV.



Flux	BP'00	Cl-Ar	Ga-Ge
$\Phi_{\text{pp}} \times 10^{-10}$	5.95(1 $^{+0.01}_{-0.01}$ )	0.00	69.7
$\Phi_{\text{pep}} \times 10^{-8}$	1.40(1 $^{+0.01}_{-0.01}$ )	0.22	2.8
$\Phi_{\text{Be}} \times 10^{-9}$	4.77(1 $^{+0.09}_{-0.09}$ )	1.15	34.2
$\Phi_{\text{B}} \times 10^{-6}$	5.93(1 $^{+0.14}_{-0.15}$ )	6.76	14.2
$\Phi_{\text{N}} \times 10^{-8}$	5.48(1 $^{+0.19}_{-0.13}$ )	0.09	3.4
$\Phi_{\text{O}} \times 10^{-8}$	4.80(1 $^{+0.22}_{-0.15}$ )	0.33	5.5
Total		8.55 $^{+1.1}_{-1.2}$	129.8 $^{+9}_{-7}$

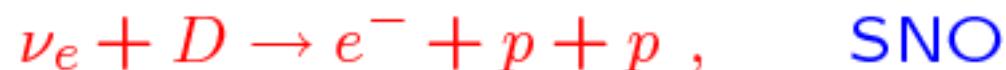
## Solar Neutrinos $\nu_e$ , $E \sim 1$ MeV: B. Pontecorvo 1946



R. Davis et al., 1967 - 1996: 615 t  $C_2Cl_4$ ; 0.5 Ar atoms/day, exposure 60 days.

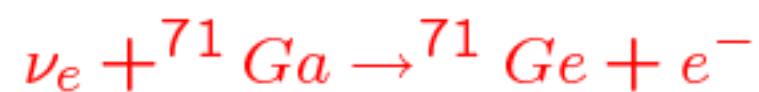


Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 - );



Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water ( $D_2O$ )



SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003

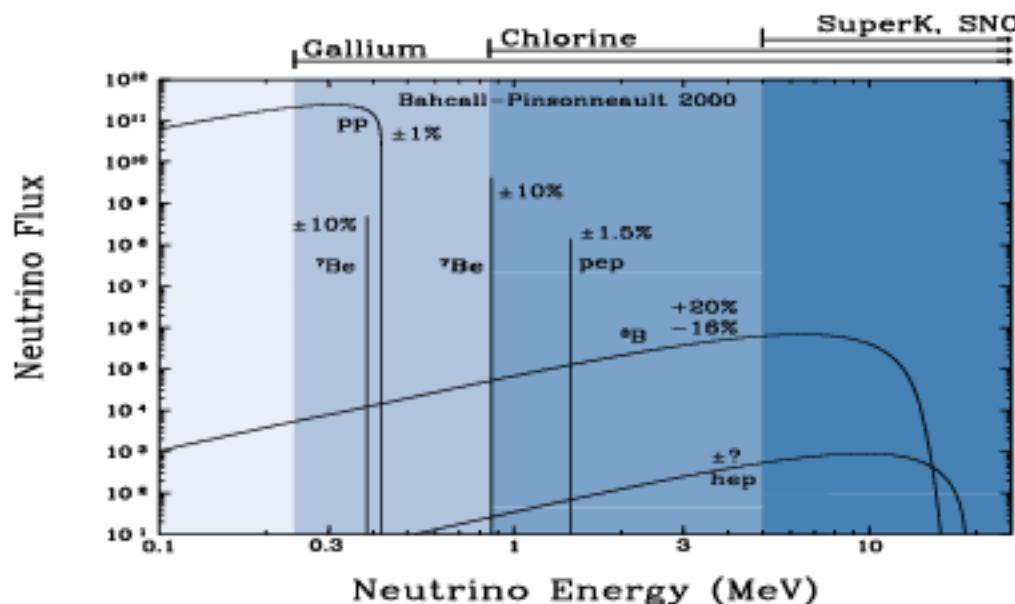


Figure 2: Differential Standard Solar Model neutrino fluxes [14].

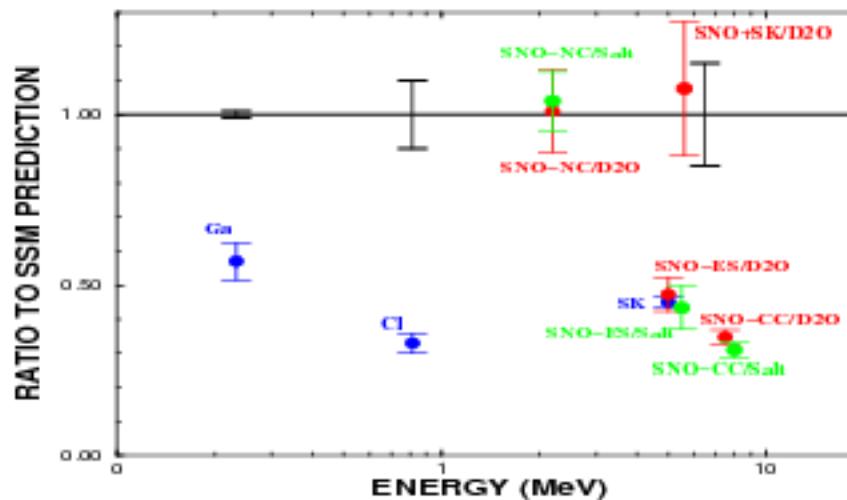
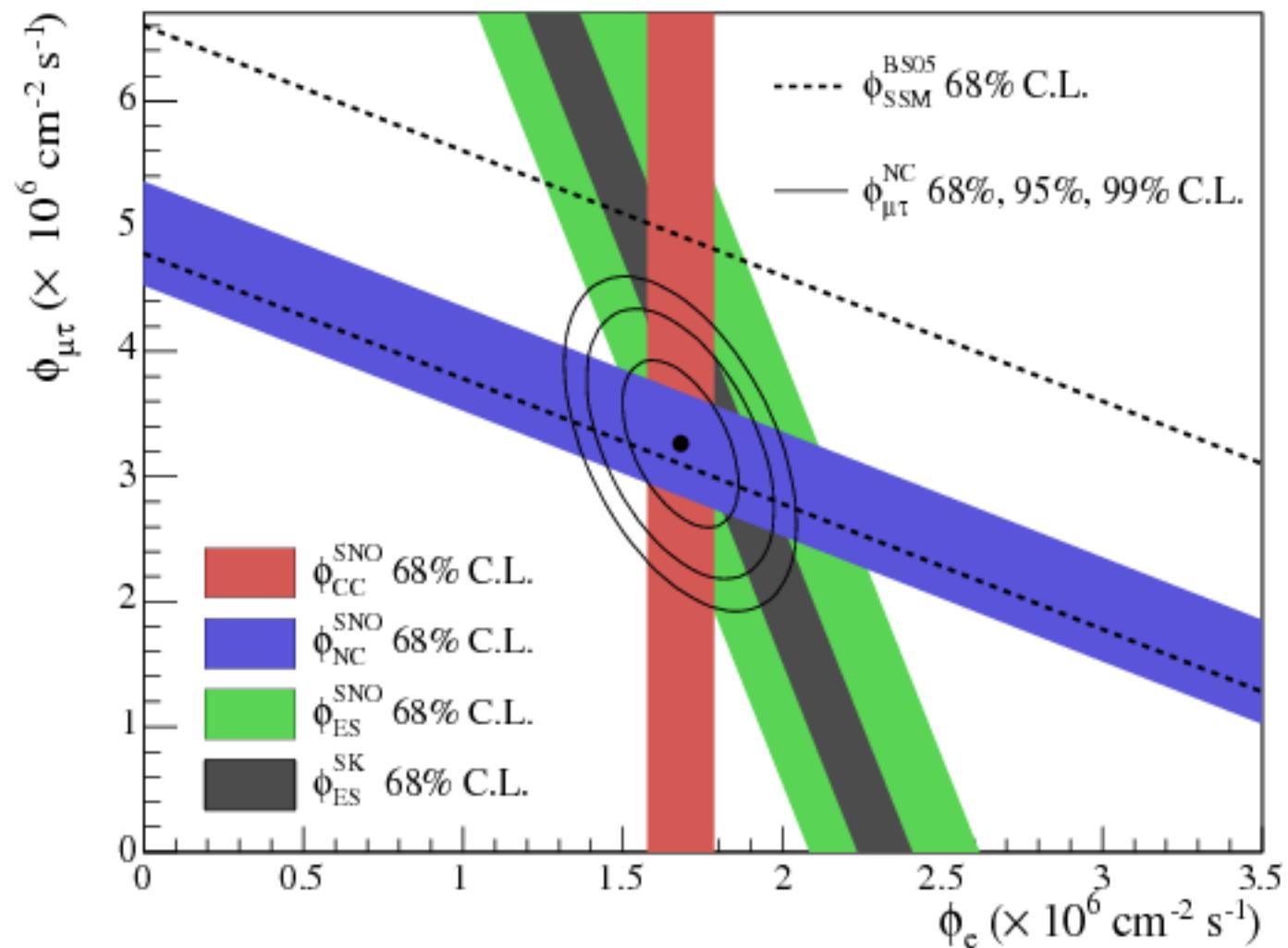


Figure 3: Comparison of measurements to Standard Solar Model predictions.

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Experiment	Observed rate/BP04 prediction	Predicted Rate at global best-fit	Predicted Rate at solar best-fit
Ga	$0.52 \pm 0.029$	0.555	0.540
Cl	$0.301 \pm 0.027$	0.356	0.345
SK(ES)	$0.406 \pm 0.014$	0.394	0.395
SNO(CC)	$0.274 \pm 0.019$	0.289	0.289
SNO(ES)	$0.38 \pm 0.052$	0.386	0.386
SNO(NC)	$0.895 \pm 0.08$	0.889	0.908

The observed rates w.r.t predictions from the latest Standard Solar Model BP04. Shown are also the predicted rates for the best fit values of  $\Delta m_{21}^2$  and  $\sin^2 \theta_{12}$ , obtained in the analysis of the i) global solar neutrino data, and ii) global solar neutrino +KamLAND data.



## MSW Transitions of Solar Neutrinos in the Sun and the Hydrogen Atom

$$i \frac{d}{dt} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} \quad (2)$$

where  $\alpha = \nu_e$ ,  $\beta = \nu_{\mu(\tau)}$ .

$$\epsilon(t) = \frac{1}{2} \left[ \frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t) \right],$$

$$\epsilon'(t) = \frac{\Delta m^2}{4E} \sin 2\theta, \text{ with } \Delta m^2 = m_2^2 - m_1^2.$$

- Standard Solar Models

$$N_e(t) = N_e(t_0) \exp \left\{ -\frac{t-t_0}{r_0} \right\}, \quad r_0 \sim 0.1 R_\odot, \quad R_\odot = 6.96 \times 10^5 \text{ km}$$

The region of  $\nu_{\odot}$  production:  $r \lesssim 0.2R_{\odot}$

$$20 N_A \text{ cm}^{-3} \lesssim N_e(x_0) \lesssim 100 N_A \text{ cm}^{-3}$$

Suppose  $N_e(x_0) \gg N_e^{res}$ :  $|\nu_e\rangle \cong |\nu_2^m\rangle$ .

Possible evolution:

The system stays at this level; at the surface:  $|\nu_2^m\rangle = |\nu_2\rangle$

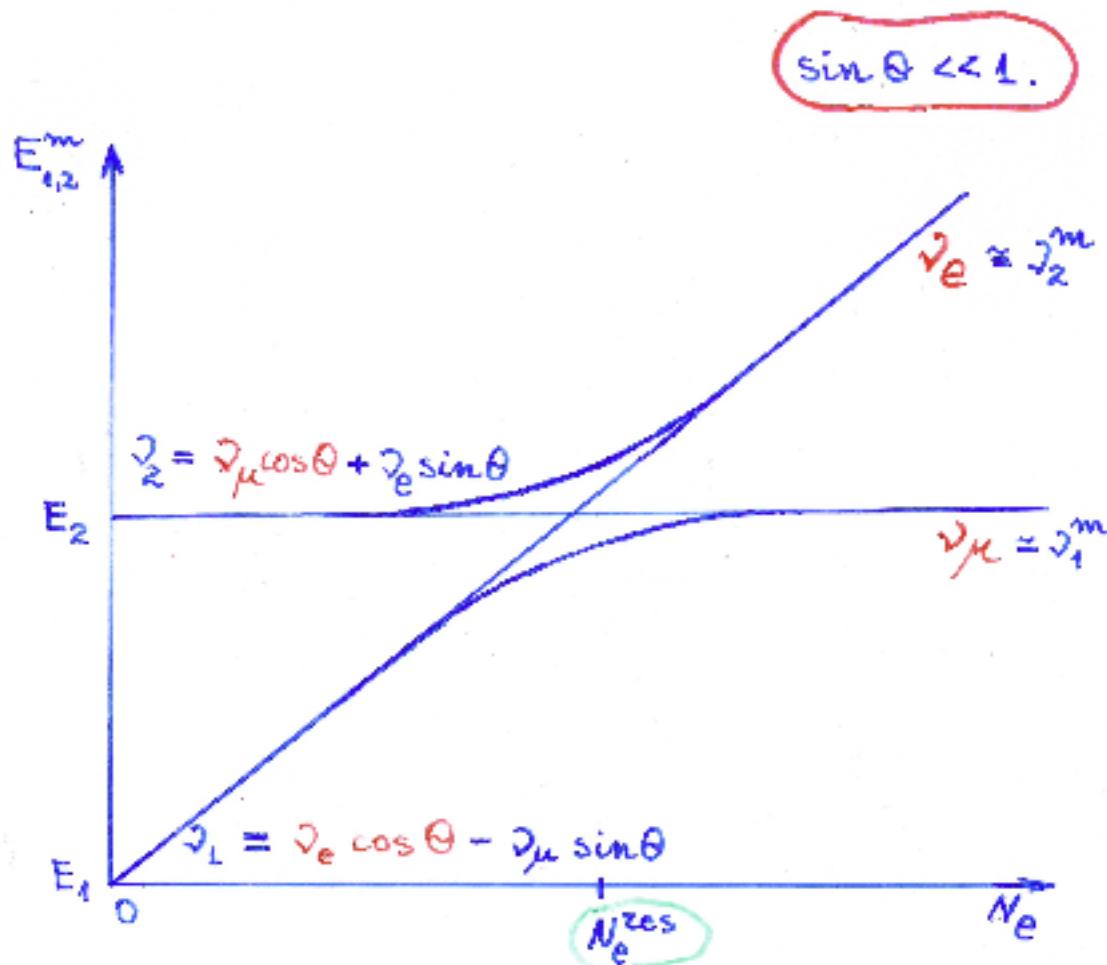
$$P(\nu_e \rightarrow \nu_e) \cong |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2 \theta, \quad \text{Adiabatic}$$

At  $N_e = N_e^{res}$ , where  $E_2^m - E_1^m$  is minimal, the system jumps to lower level  $|\nu_1^m\rangle$ ; at the surface:  $|\nu_1^m\rangle = |\nu_1\rangle$

$$P(\nu_e \rightarrow \nu_e) \cong |\langle \nu_e | \nu_1 \rangle|^2 = \cos^2 \theta, \quad \text{Nonadiabatic}$$

Type of transition:  $P' \equiv P(\nu_2^m(t_0) \rightarrow \nu_1)$ , jump probability

4.



1.  $P(\tilde{v}_2 \rightarrow \tilde{v}_1; t_0, t_0) \equiv P$  - negligible : adiabatic transition
2.  $P'$  - nonnegligible : nonadiabatic

Introducing the dimensionless variable

$$Z = ir_0\sqrt{2}G_F N_e(t_0)e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t = t_0),$$

and making the substitution

$$A_e(t, t_0) = (Z/Z_0)^{c-a} e^{-(Z-Z_0)+i \int_{t_0}^t \epsilon(t') dt'} A'_e(t, t_0),$$

$A'_e(t, t_0)$  satisfies the confluent hypergeometric equation (CHE):

$$\left\{ Z \frac{d^2}{dZ^2} + (c - Z) \frac{d}{dZ} - a \right\} A'_e(t, t_0) = 0,$$

where

$$a = 1 + ir_0 \frac{\Delta m^2}{2E} \sin^2 \theta, \quad c = 1 + ir_0 \frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the  $\nu_e$  oscillations in the Sun, coincides in form with the Schroedinger (energy eigenvalue) equation obeyed by the radial part,  $\psi_{kl}(r)$ , of the non-relativistic wave function of the hydrogen atom,

$$\Psi(\vec{r}) = \frac{1}{r} \psi_{kl}(r) Y_{lm}(\theta', \phi'),$$

$r$ ,  $\theta'$  and  $\phi'$  are the spherical coordinates of the electron in the proton's rest frame,  $l$  and  $m$  are the orbital momentum quantum numbers ( $m = -l, \dots, l$ ),  $k$  is the quantum number labeling (together with  $l$ ) the electron energy (the principal quantum number is equal to  $(k+l)$ ),  $E_{kl}$  ( $E_{kl} < 0$ ), and  $Y_{lm}(\theta', \phi')$  are the spherical harmonics. The function

$$\psi'_{kl}(Z) = Z^{-c/2} e^{Z/2} \psi_{kl}(r)$$

satisfies the confluent hypergeometric equation in which the variable  $Z$  and the parameters  $a$  and  $c$  are in this case related to the physical quantities characterizing the hydrogen atom:

$$Z = 2 \frac{r}{a_0} \sqrt{-E_{kl}/E_I}, \quad a \equiv a_{kl} = l + 1 - \sqrt{-E_I/E_{kl}}, \quad c \equiv c_l = 2(l+1),$$

$a_0 = \hbar/(m_e e^2)$  is the Bohr radius and  $E_I = m_e e^4/(2\hbar^2) \cong 13.6$  eV is the ionization energy of the hydrogen atom.

Quite remarkably, the behavior of such different physical systems as solar neutrinos undergoing MSW transitions in the Sun and the non-relativistic hydrogen atom are governed by one and the same differential equation.

Any solution - linear combination of two linearly independent solutions:

$$\Phi(a, c; Z), \quad Z^{1-c} \Phi(a - c + 1, 2 - c; Z); \quad \Phi(a', c'; Z = 0) = 1, \quad a', c' \neq 0, -1, -2, \dots$$

$$A(\nu_e \rightarrow \nu_{\mu(\tau)}) = \frac{1}{2} \sin 2\theta \left\{ \Phi(a - c, 2 - c; Z_0) - e^{i(t-t_0)\frac{\Delta m^2}{2E}} \Phi(a - 1, c; Z_0) \right\}.$$

Sun:  $N_e(x) \cong N_e(x_0)e^{-\frac{x}{r_0}}$ ,  $r_0 \cong 0.1R_\odot$ ,  $R_\odot \cong 7 \times 10^5$  km

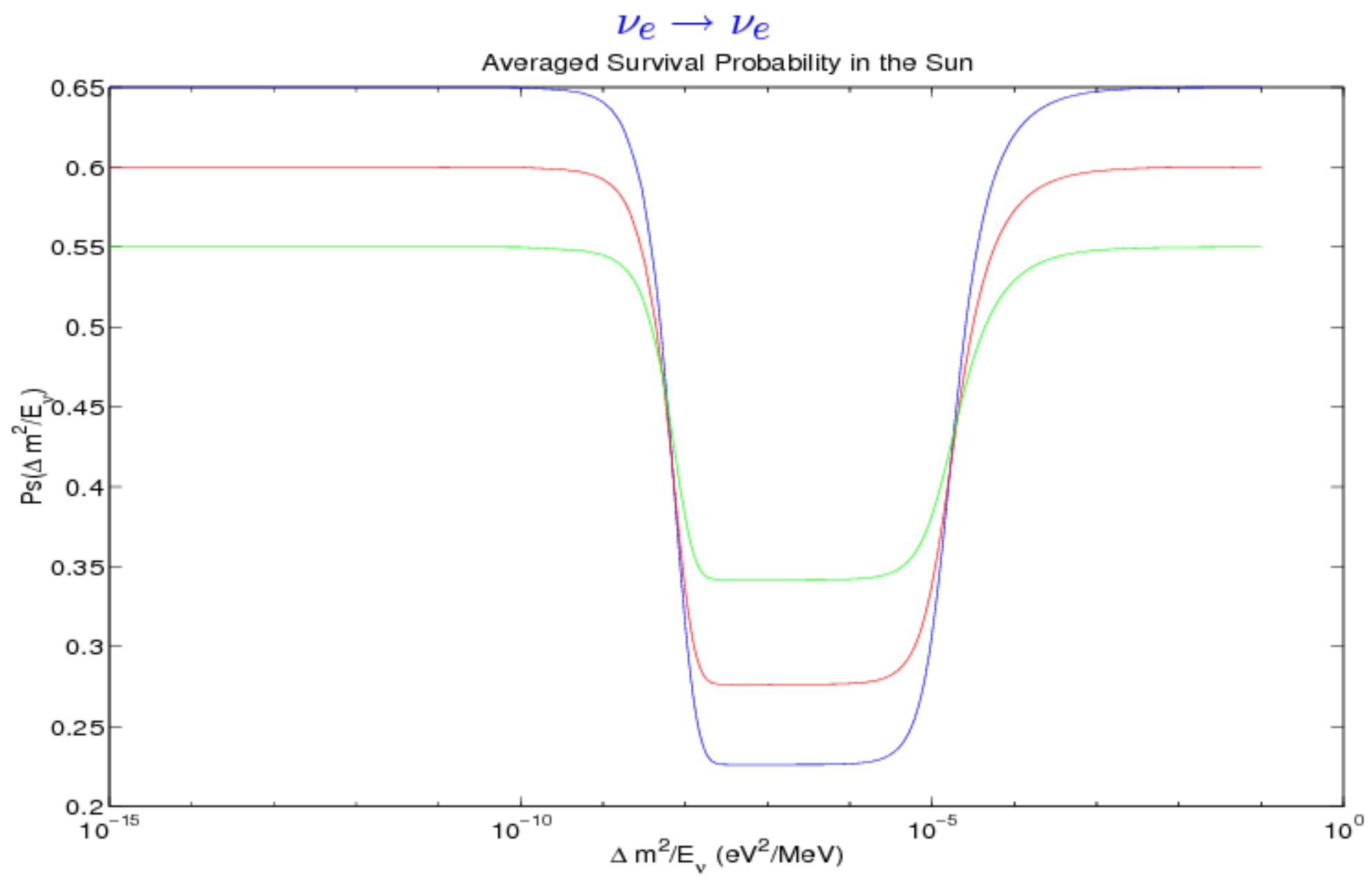
The region of  $\nu_\odot$  production:

$$20 \text{ } N_A \text{ cm}^{-3} \lesssim N_e(x_0) \lesssim 100 \text{ } N_A \text{ cm}^{-3}; \quad |Z_0| > 500 \text{ (!)}$$

The solar  $\nu_e$  survival probability:

$$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E} \sin^2 \theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$



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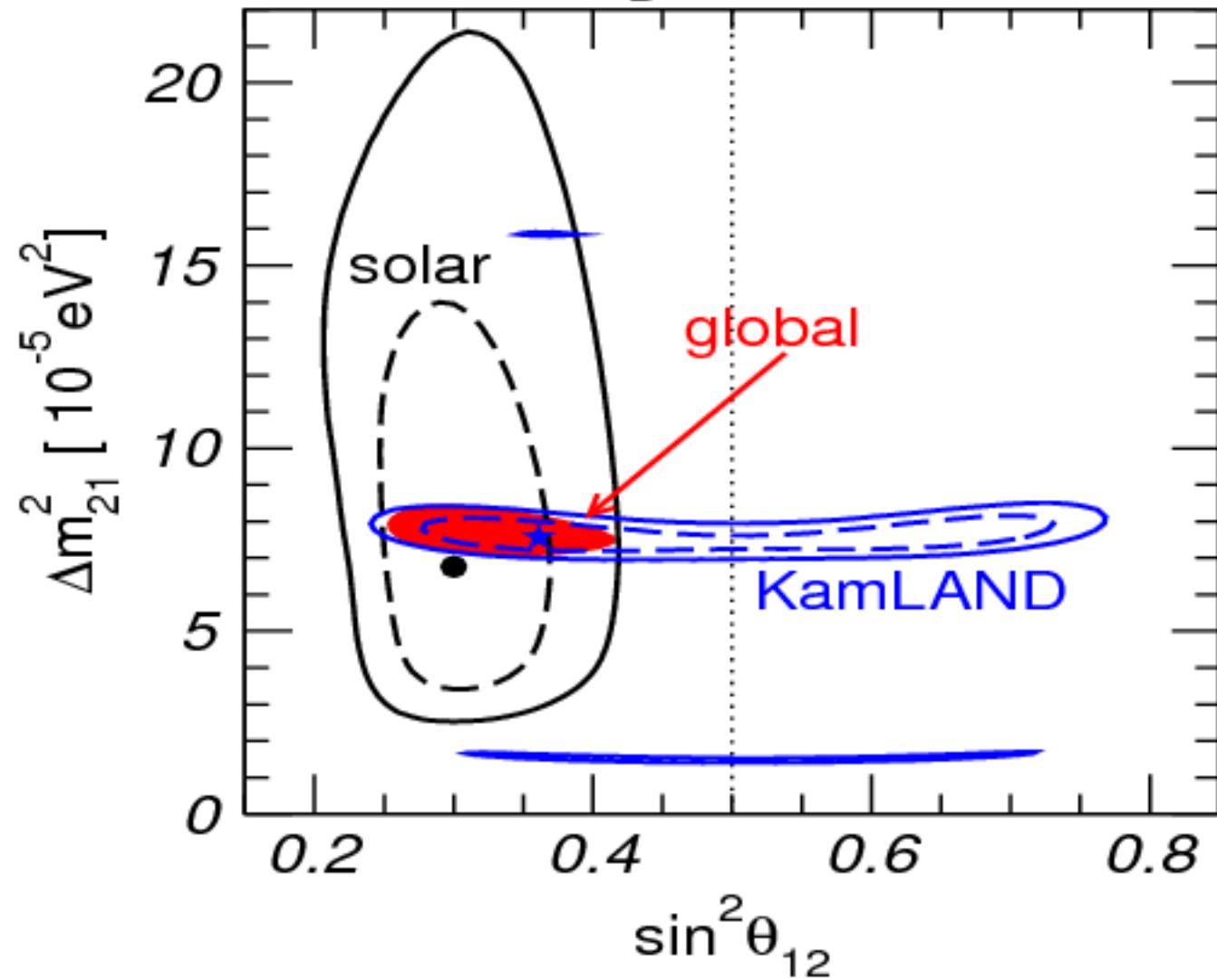
Case 1:  $\cos 2\theta_m^0 = -1$ ,  $P' = 0$ ,  $\bar{P} = \frac{1}{2}(1 - \cos 2\theta)$ .

Case 2:  $\theta_m^0 = \theta$ ,  $P' = 0$ ,  $\bar{P}(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta$

Case 1: SNO, Super Kamiokande;  $\bar{P} \cong 0.3$ :  $\cos 2\theta > 0$ !

Case 2:  $pp$  neutrinos.

## "solar" parameters



J.N. BAHCALL, H.C. GONZALEZ-GARCIA, C. PEÑA GARAY '01

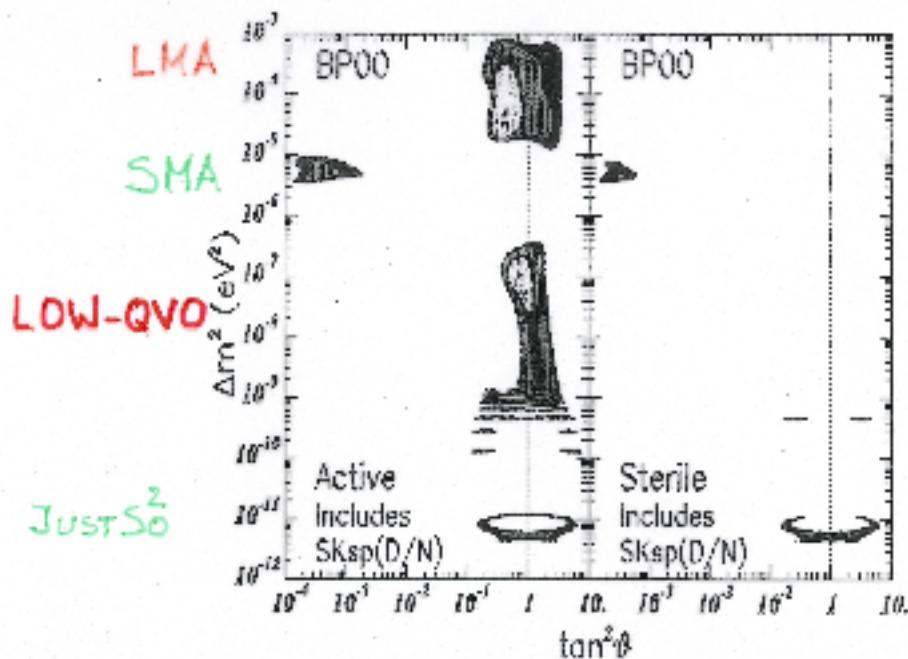


Figure 1: Global solutions including all available solar neutrino data. The input data include the total rates from the Chlorine [2], Gallium (averaged) [3, 5, 4], Super-Kamiokande [6], and SNO [1] experiments, as well as the recoil electron energy spectrum measured by Super-Kamiokande during the day and separately the energy spectrum measured at night. The C.L. contours shown in the figure are 90%, 95%, 99%, and 99.73% ( $3\sigma$ ). The allowed regions are constrained below  $10^{-9} \text{ eV}^2$  by the Chooz reactor measurements [22]. The local best-fit points are marked by dark circles. The theoretical errors for the BP2000 neutrino fluxes are included in the analysis.

# LECTURE III

The reference scheme: 3- $\nu$  mixing

$$\nu_{l\text{L}} = \sum_{j=1}^3 U_{lj} \nu_{j\text{L}} \quad l = e, \mu, \tau.$$

3-flavour neutrino oscillation probabilities

$$P(\nu_l \rightarrow \nu_{l'}) : m_2^2 - m_1^2 \equiv \Delta m_{21}^2, \quad m_3^2 - m_1^2 \equiv \Delta m_{31}^2$$

## PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$ ,  $c_{ij} \equiv \cos \theta_{ij}$ ,  $\theta_{ij} = [0, \frac{\pi}{2}]$ ,
- $\delta$  - Dirac CPV phase,  $\delta = [0, 2\pi]$ ; CP inv.:  $\delta = 0, \pi, 2\pi$ ;
- $\alpha_{21}$ ,  $\alpha_{31}$  - Majorana CPV phases; CP inv.:  $\alpha_{21(31)} = k(k')\pi$ ,  $k(k') = 0, 1, 2, \dots$   
S.M. Bilenky, J. Hosek, S.T.P., 1980
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$ ,  $\sin^2 \theta_{12} \cong 0.308$ ,  $\cos 2\theta_{12} \gtrsim 0.28$  ( $3\sigma$ ),
- $|\Delta m_{31(32)}^2| \cong 2.47 \text{ (2.42)} \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 \theta_{23} \cong 0.437$  (0.455), NO (IO),
- $\theta_{13}$  - the CHOOZ angle:  $\sin^2 \theta_{13} = 0.0234$  (0.0240), Capozzi et al. NO (IO).  
F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

## 3- $\nu$ Mixing Analysis: $\Delta m_{\odot}^2 \ll |\Delta m_{\text{atm}}^2|$

$$P_{\odot}^{3\nu} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} P_{\odot}^{2\nu},$$

$$P_{\odot}^{2\nu} = \bar{P}_{\odot}^{2\nu} + P_{\odot \text{ osc}}^{2\nu},$$

$$\bar{P}_{\odot}^{2\nu} = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_{12}^m(t_0) \cos 2\theta_{12} \quad (\theta_{12} \equiv \theta_{\odot}),$$

$P' = 0$ : L. Wolfenstein, 1978; S. Mikheyev, A. Smirnov, 1985;

$P' \neq 0$  (general or LZ): S. Parke, W. Haxton, 1986;

$P'$ -double exponential,  $P_{\odot \text{ osc}}^{2\nu}$ : S.T.P., 1988

$$N_e \rightarrow N_e \cos^2 \theta_{13},$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}} \sin^2 \theta_{12} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}, \quad r_0 \sim 0.1 R_{\odot}$$

S.T.P., 1988

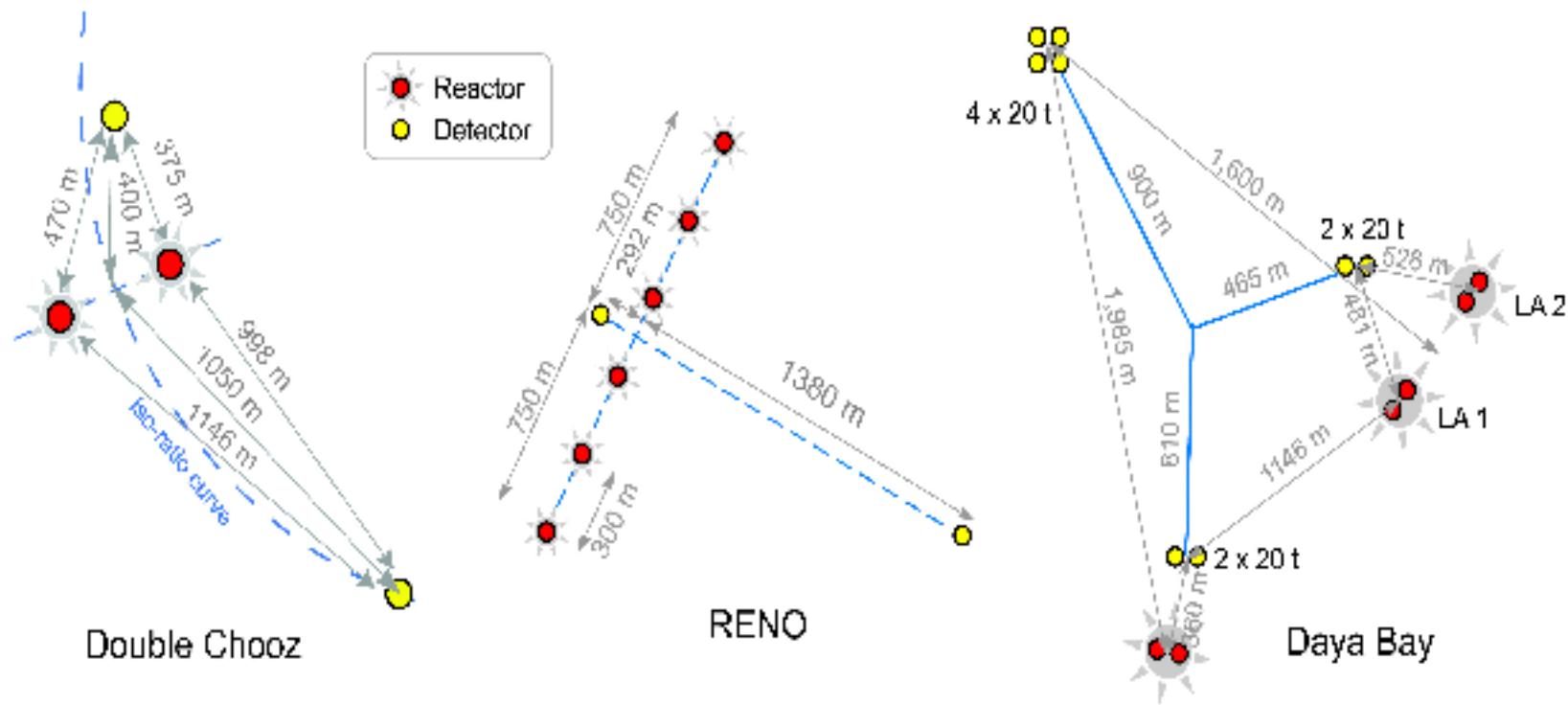
LMA:  $P' \ll 1, \quad \langle P_{\odot \text{ osc}}^{2\nu} \rangle \cong 0$

J. Rich, S.T.P., 1988

$$P_{\text{KL}}^{3\nu} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} \left[ 1 - \sin^2 2\theta_{12} \sin^2 \left( \frac{\Delta m_{\odot}^2}{4E} L \right) \right]$$

$$P_{\text{CHOOZ}}^{3\nu} \cong 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{\text{atm}}^2}{4E} L \right)$$

- March 8, 2012, Daya Bay:  $5.2\sigma$  evidence for  $\theta_{13} \neq 0$ ,  
 $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$ .
- April 4, 2012, RENO:  $4.9\sigma$  evidence for  $\theta_{13} \neq 0$ ,  
 $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$ .
- Nu'2012 (June 4-9, 2012), T2K, Double Chooz:  $3.2\sigma$  and  $2.9\sigma$  evidence for  $\theta_{13} \neq 0$ .
- Daya Bay, 23/08/2013:  
 $\sin^2 2\theta_{13} = 0.090 \pm 0.009$ .
- RENO, 12/09/2013 (TAUP 2013):  
 $\sin^2 2\theta_{13} = 0.100 \pm 0.010$  (*stat.*)  $\pm 0.012$ .



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]

## T2K: Search for $\nu_\mu \rightarrow \nu_e$ oscillations

T2K: first results March 2011 (2 events);  
June 14, 2011 (6 events): evidence for  $\theta_{13} \neq 0$  at  $2.5\sigma$ ;  
**July, 2013 (28 events).**

For  $|\Delta m_{23}^2| = 2.4 \times 10^{-3}$  eV $^2$ ,  $\sin^2 2\theta_{23} = 1$ ,  $\delta = 0$ , NO  
(IO) spectrum:

$$\sin^2 2\theta_{13} = 0.18, \text{ best fit.}$$

This value is by a factor of  $\sim 2$  bigger than the value obtained in the Daya Bay and RENO experiments.



Up to 2nd order in the two small parameters  $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$  and  $\sin^2 \theta_{13} \ll 1$ :

$$P_m^{3\nu \text{ man}}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{\text{man}} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31(32)}^2)$  not determined

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0$ , normal mass ordering (NO)

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0$ , inverted mass ordering (IO)

Convention:  $m_1 < m_2 < m_3$  - NO,  $m_3 < m_1 < m_2$  - IO

$$\Delta m_{31}^2(\text{NO}) = -\Delta m_{32}^2(\text{IO})$$

$$m_1 \ll m_2 < m_3, \quad \text{NH},$$

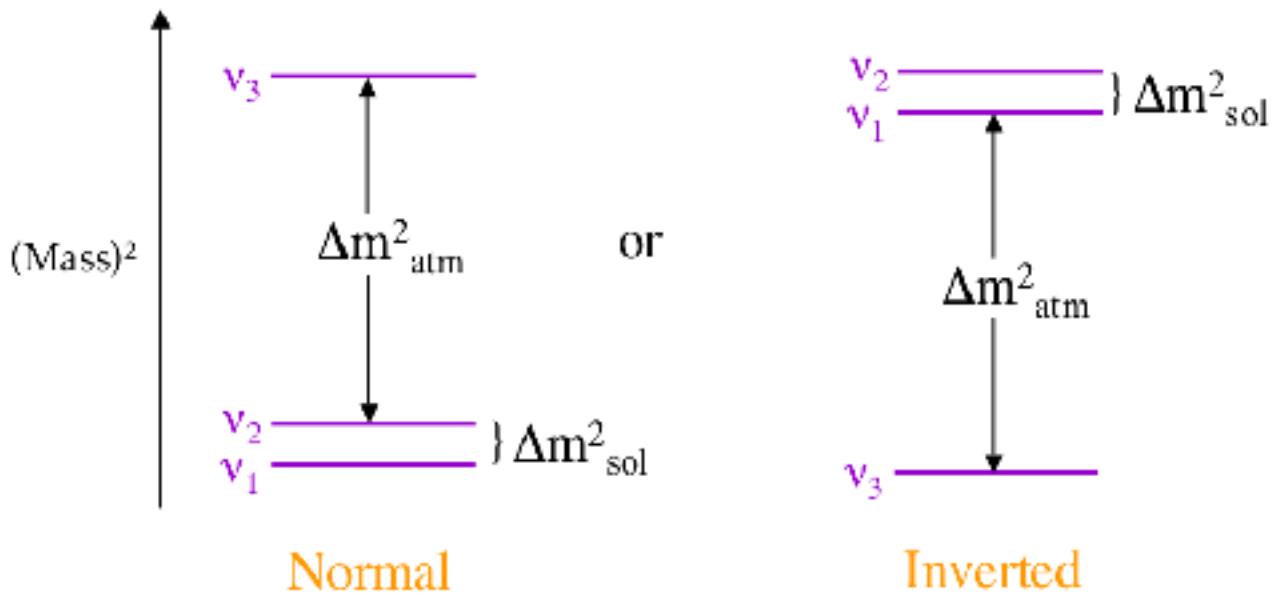
$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD}; \quad m_j \gtrsim 0.10 \text{ eV}.$$

- $m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}$  - NO;

- $m_1 = \sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}, \quad m_2 = \sqrt{m_3^2 + \Delta m_{23}^2}$  - IO;

## The (Mass)<sup>2</sup> Spectrum



$$\Delta m^2_{\text{sol}} \approx 7.6 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{\text{atm}} \approx 2.4 \times 10^{-3} \text{ eV}^2$$

Are there *more* mass eigenstates, as LSND suggests,  
and MiniBooNE recently hints?

- Dirac phase  $\delta$ :  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ ,  $l \neq l'$ ;  $A_{CP}^{(ll')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$ :

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data:  $|J_{CP}| \lesssim 0.035$  (can be relatively large!); b.f.v. with  $\delta = 3\pi/2$ :  $J_{CP} \cong -0.035$ .

- Majorana phases  $\alpha_{21}, \alpha_{31}$ :

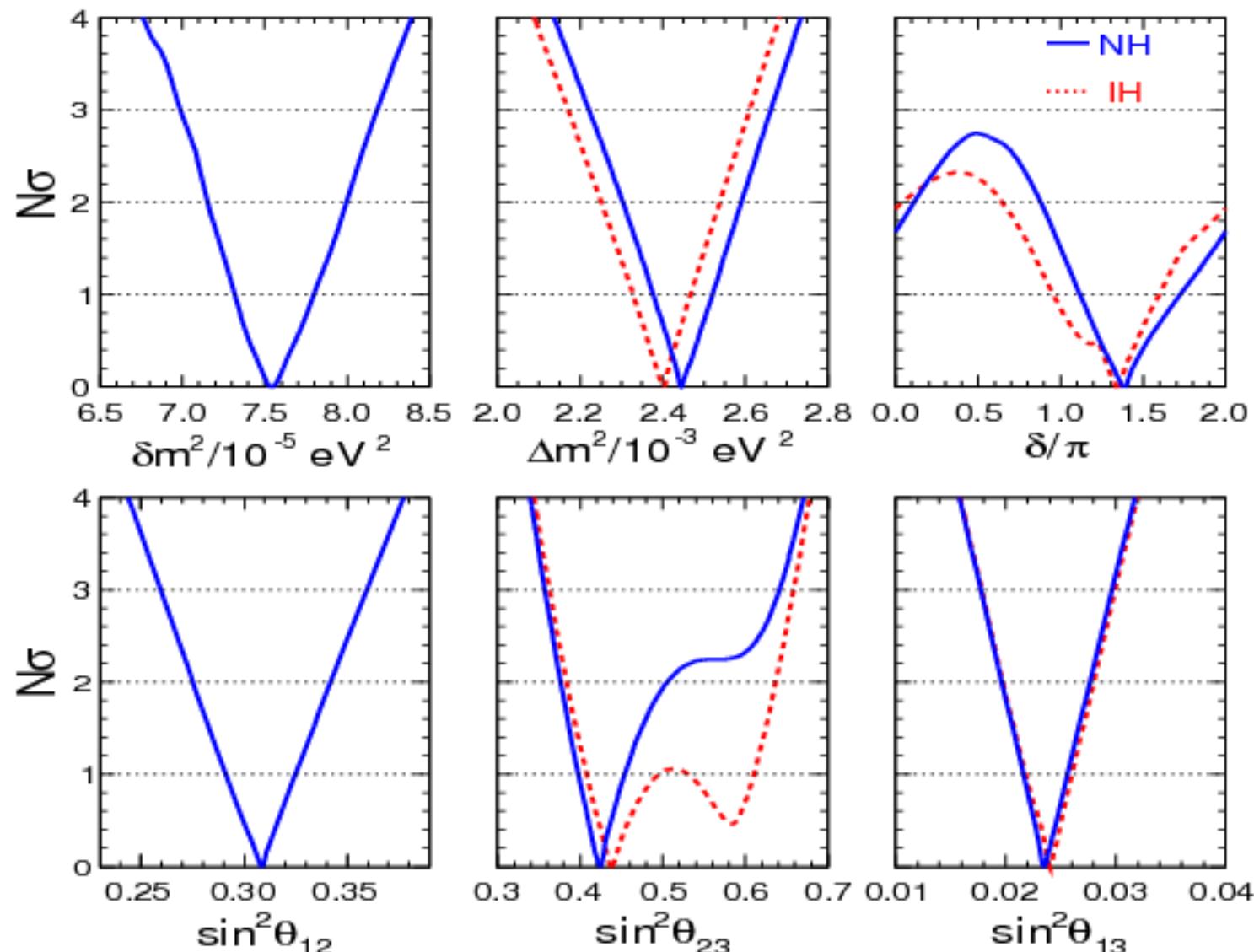
–  $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$  not sensitive;

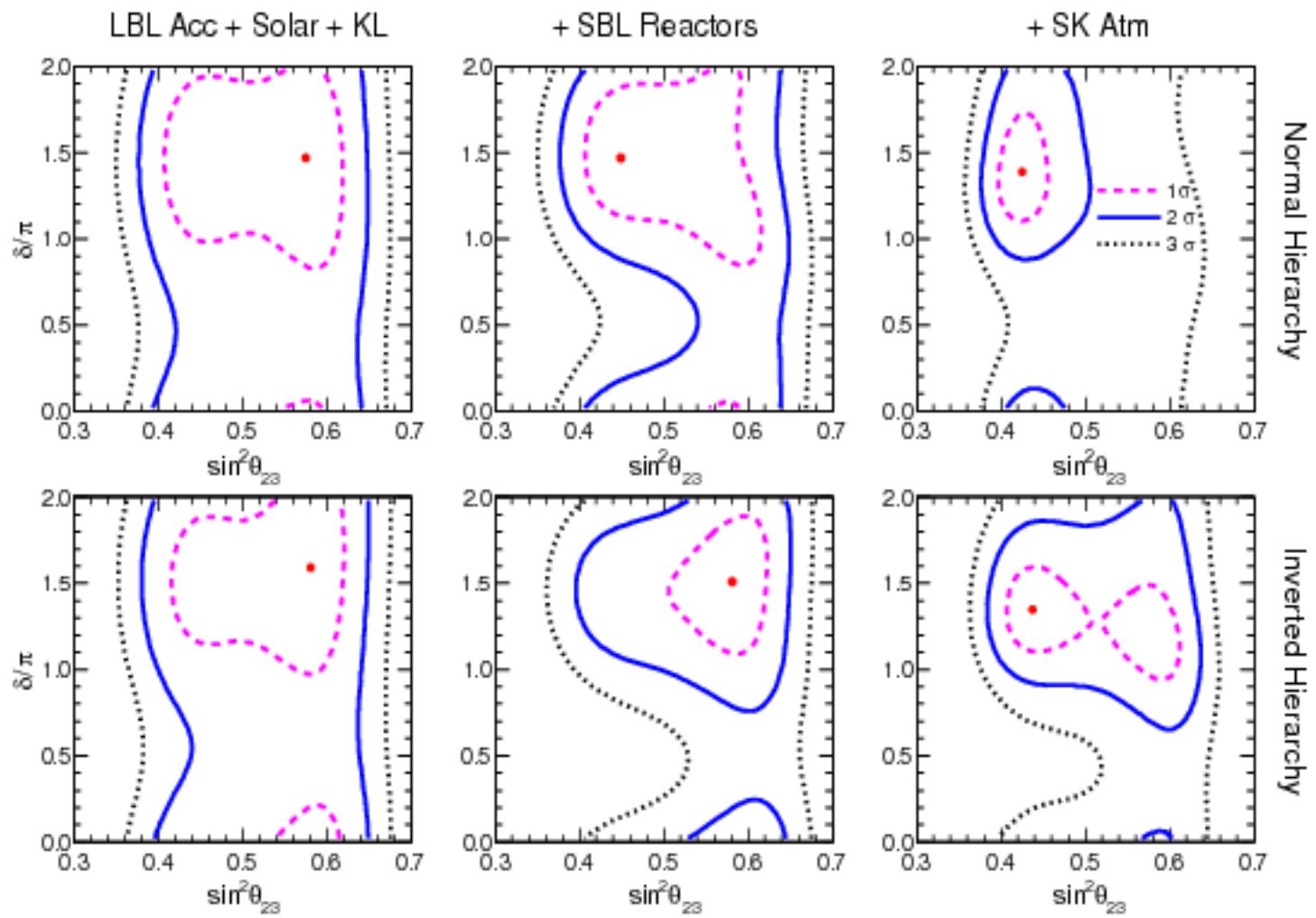
S.M. Bilenky, J. Hosek, S.T.P., 1980;  
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

- $|\langle m \rangle|$  in  $(\beta\beta)_{0\nu}$ -decay depends on  $\alpha_{21}, \alpha_{31}$ ;
- $\Gamma(\mu \rightarrow e + \gamma)$  etc. in SUSY theories depend on  $\alpha_{21,31}$ ;
- BAU, leptogenesis scenario:  $\delta, \alpha_{21,31}$  !

$$\begin{aligned}
J_{CP} &= \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} \\
&= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta
\end{aligned}$$

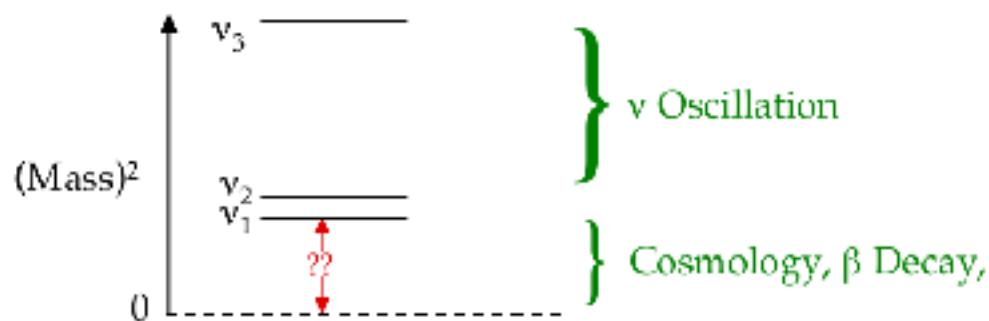
# LBL Acc + Solar + KL + SBL Reactors + SK Atm





## Absolute Neutrino Mass Scale

### The Absolute Scale of Neutrino Mass



How far above zero  
is the whole pattern?

Oscillation Data  $\Rightarrow \sqrt{\Delta m_{\text{atm}}^2} < \text{Mass[Heaviest } v_i]$

## Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on  ${}^3\text{H} \rightarrow {}^3\text{He} + \text{e}^- + \bar{\nu}_e$ :  
 $m_{\nu_e} < 2.2 \text{ eV}$  (95% C.L.)

We have  $m_{\nu_e} \cong m_{1,2,3}$  in the case of QD spectrum. The upcoming KATRIN experiment is planned to reach sensitivity

KATRIN:  $m_{\nu_e} \sim 0.2 \text{ eV}$

i.e., it will probe the region of the QD spectrum.

Improved  $\beta$  energy resolution requires a **BIG**  $\beta$  spectrometer.





## Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on  $\sum_j m_j$ : the Planck + WMAP (low  $l \leq 25$ ) + ACT (large  $l \geq 2500$ ) CMB data +  $\Lambda$ CDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

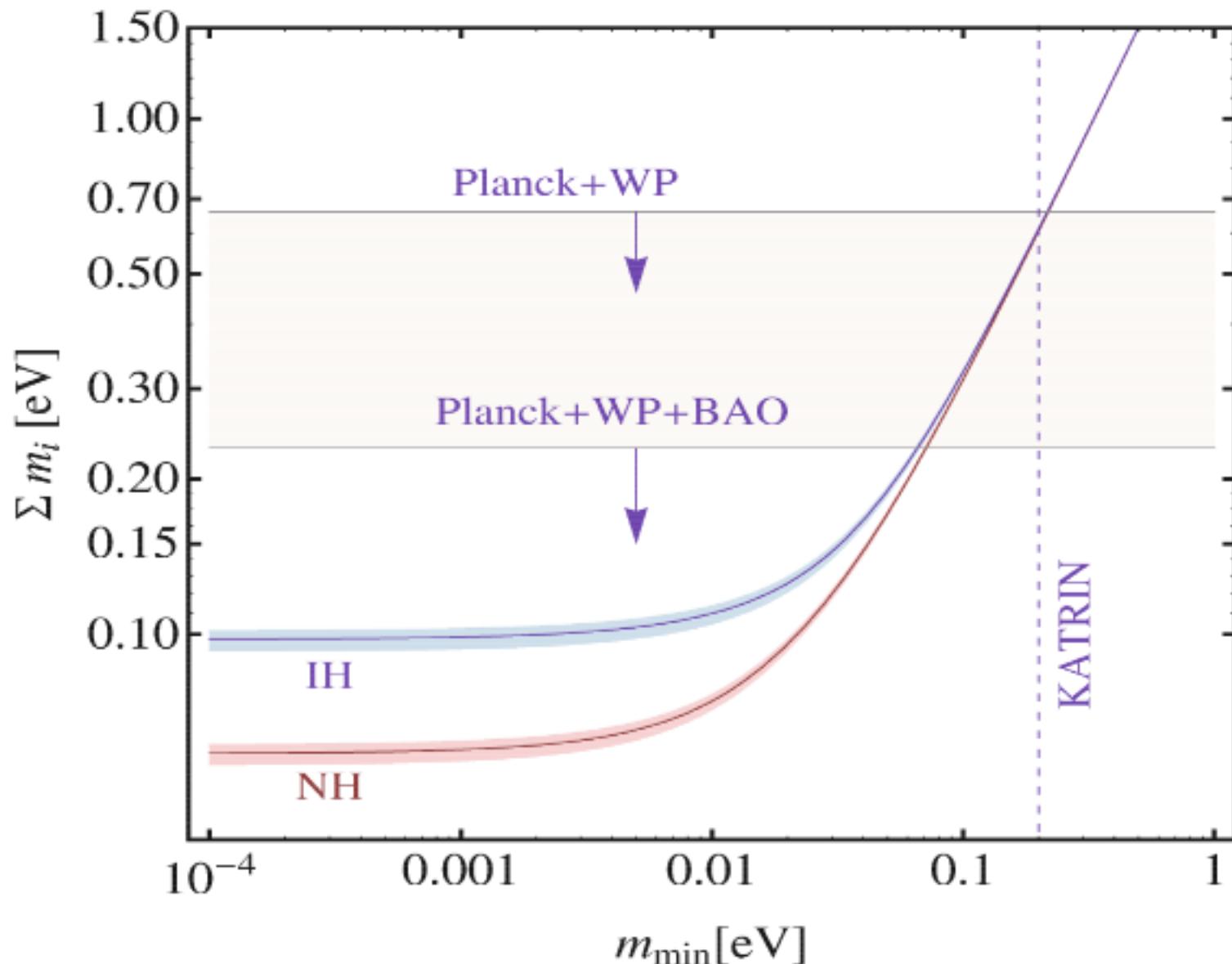
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and Planck experiments might allow to determine

$$\sum_j m_j : \quad \delta \cong (0.01 - 0.04) \text{ eV.}$$

NH:  $\sum_j m_j \leq 0.05 \text{ eV } (3\sigma)$ ;

IH:  $\sum_j m_j \geq 0.10 \text{ eV } (3\sigma)$ .

## Mass and Hierarchy from Cosmology



These data imply that

$$m_{\nu_j} \ll m_{e,\mu,\tau}, m_q, \quad q = u, c, t, d, s, b$$

For  $m_{\nu_j} \lesssim 1$  eV:  $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family:  $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^2$

## Future Progress

- Determination of the nature - Dirac or Majorana, of  $\nu_j$  .
- Determination of  $\text{sgn}(\Delta m_{\text{atm}}^2)$ , type of  $\nu$ - mass spectrum

$$m_1 \ll m_2 \ll m_3, \quad \text{NH},$$

$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD}; \quad m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of  $\nu_j$ -masses, or  $\min(m_j)$ .
- Status of the CP-symmetry in the lepton sector: violated due to  $\delta$  (Dirac), and/or due to  $\alpha_{21}$ ,  $\alpha_{31}$  (Majorana)?
- High precision determination of  $\Delta m_{\odot}^2$ ,  $\theta_{12}$ ,  $\Delta m_{\text{atm}}^2$ ,  $\theta_{23}$ ,  $\theta_{13}$
- Searching for possible manifestations, other than  $\nu_l$ -oscillations, of the non-conservation of  $L_l$ ,  $l = e, \mu, \tau$ , such as  $\mu \rightarrow e + \gamma$ ,  $\tau \rightarrow \mu + \gamma$ , etc. decays.

- Understanding at fundamental level the mechanism giving rise to the  $\nu$ - masses and mixing and to the  $L_l$ -non-conservation. Includes understanding
  - the origin of the observed patterns of  $\nu$ -mixing and  $\nu$ -masses ;
  - the physical origin of *CPV* phases in  $U_{\text{PMNS}}$  ;
  - Are the observed patterns of  $\nu$ -mixing and of  $\Delta m^2_{21,31}$  related to the existence of a new symmetry?
  - Is there any relations between  $q$ -mixing and  $\nu$ - mixing? Is  $\theta_{12} + \theta_c = \pi/4$  ?
  - Is  $\theta_{23} = \pi/4$ , or  $\theta_{23} > \pi/4$  or else  $\theta_{23} < \pi/4$ ?
  - Is there any correlation between the values of *CPV* phases and of mixing angles in  $U_{\text{PMNS}}$ ?
- Progress in the theory of  $\nu$ -mixing might lead to a better understanding of the origin of the BAU.
  - Can the Majorana and/or Dirac CPVP in  $U_{\text{PMNS}}$  be the leptogenesis CPV parameters at the origin of BAU?

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos.
- determination of the neutrino mass hierarchy;
- determination of the absolute neutrino mass scale (or  $\min(m_j)$ );
- determination of the status of the CP symmetry in the lepton sector.

Large  $\sin \theta_{13} \cong 0.16$  (Daya Bay, RENO) - far-reaching implications for the program of research in neutrino physics:

- For the determination of the type of  $\nu$ - mass spectrum (or of  $\text{sgn}(\Delta m_{\text{atm}}^2)$ ) in neutrino oscillation experiments.
- For understanding the pattern of the neutrino mixing and its origins (symmetry, etc.?).
- For the predictions for the  $(\beta\beta)_{0\nu}$ -decay effective Majorana mass in the case of NH light  $\nu$  mass spectrum (possibility of a strong suppression).

Large  $\sin \theta_{13} \cong 0.16$  (Daya Bay, RENO) +  $\delta = 3\pi/2$  - far-reaching implications:

- For the searches for CP violation in  $\nu$ -oscillations; for the b.f.v. one has  $J_{CP} \cong -0.030$ ;
- Important implications also for the “flavoured” leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to  $\delta$ , a necessary condition for reproducing the observed BAU is

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

The Nature of Massive Neutrinos I:  
Majorana versus Dirac Massive Neutrinos