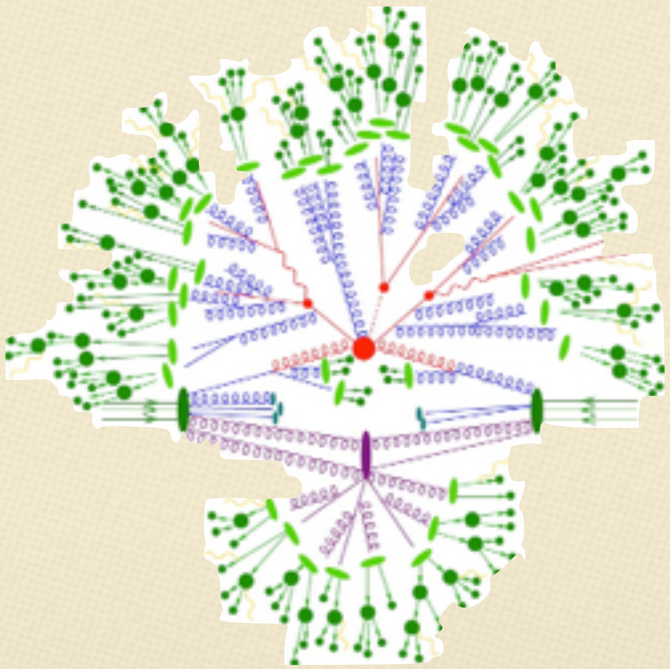
$$\frac{1}{4}\mathrm{Tr}[G^2] - \bar{\psi}(\not{D} - m)\psi$$ ☞



QCD

Eric Laenen
CERN school 2014

Lecture 4:

All orders in QCD: resummation

So far

- We have discussed the structure of fixed order calculations in perturbative QCD
  - PDF's: what they mean and how they are made
  - LO: methods for computation, e.g. spinor helicity methods, MHV amplitudes, recursion relations
  - NLO: divergences, how they are consistently removed
- We also discussed the basics of the parton shower, and reviewed (superficially) how it is used in Monte Carlo's.
  - Monte Carlo's can produce an arbitrary number of partons, but that does not make them NNN…NLO accurate
  - Yet they should get some of the all-order soft and collinear physics right.
- This lecture: how can we say something systematic about all-order predictions, even though we cannot compute arbitrarily higher orders exactly?
  - Aka: "Resummation"

Perturbative series in QFT

- Typical perturbative behavior of observable

$$\hat{O}_2 = 1 + \alpha(L^2 + L + 1) + \alpha^2(L^4 + L^3 + L^2 + L + 1) + \ldots$$

  - α is the coupling of the theory (QCD, QED, ..)
  - L is some numerically large logarithm
  - "1" = π^2, ln2, anything no
  - Notice: *effective* expansion parameter is αL^2. Problem occurs if is this >1!!
  - Possible fix: reorganize/resum terms such that

$$\hat{O} = 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \ldots$$

$$= \exp\left(\underbrace{\underbrace{L g_1(\alpha_s L)}_{LL} + g_2(\alpha_s L)}_{NLL} + \alpha_s g_3(\alpha_s L) + \ldots \right) \underbrace{C(\alpha_s)}_{\text{constants}}$$

$$+ \text{ suppressed terms}$$

- Notice the definition of LL, NLL, etc

LL, NLL,.. and matching to fixed order

- This is nomenclature you see very often: leading-log, next-to-leading log, etc
  - Here is the schematic overview of accuracy in resummation

$$O = \alpha_s^p \left(\underbrace{\underbrace{C_0}_{\text{LL,NLL}} + C_1\alpha_s + \ldots}_{\text{NNLL}} \right) \exp\left[\underbrace{\underbrace{\underbrace{(\sum_{n=1} \alpha_s^n L^{n+1} c_n)}_{\text{LL}} + (\sum_{n=1} \alpha_s^n L^n d_n)}_{\text{NLL}} + (\sum_{n=1} \alpha_s^n L^{n-1} e_n)}_{\text{NNLL}} + \ldots \right]$$

  - This is a systematic expansion in α_s in the exponent
    - ✓ If we can find the coefficients c_n, d_n, e_n, C_0, C_1 etc
  - It is directly clear how to combine this with an exact NLO or NNLO calculation
    - ✓ Expand the resummed version to the next order in α_s . Add the NLO and resummed, but subtract the order α_s -expanded resummed result, to avoid double counting.

$$O_{\text{NLO matched}} = O_{\text{NLO}} + O_{\text{resummed}} - (O_{\text{resummed}})\Big|_{\text{expanded to } \mathcal{O}(\alpha_s)}$$

      - generalization to NNLO is obvious

- But what can L be the logarithm of?

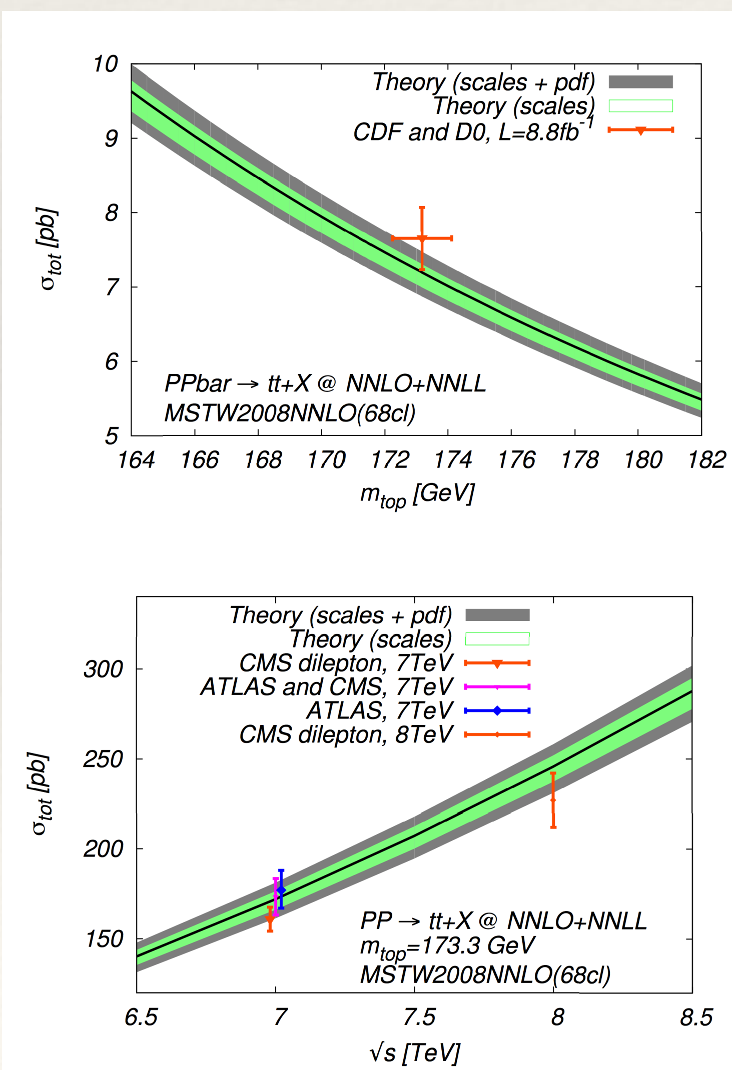
Benefits of resummation

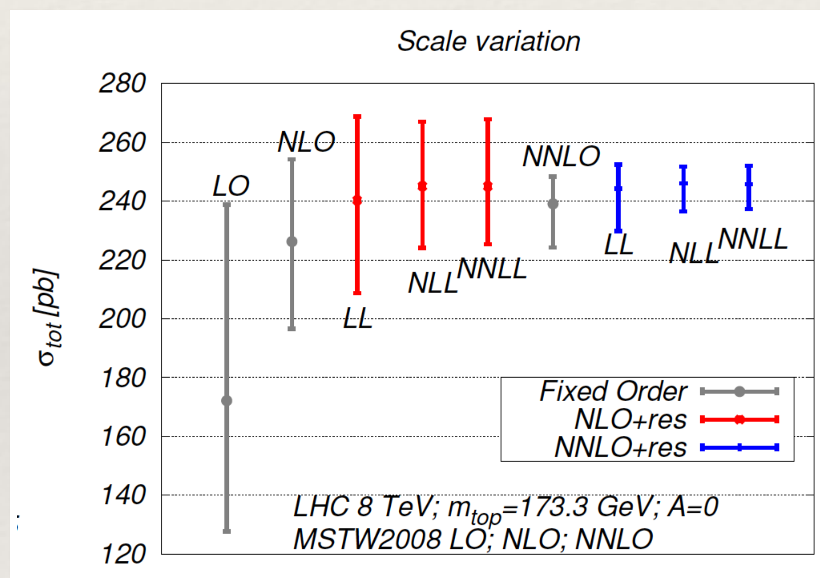
- It can rescue predictive power
  - when perturbative series converges poorly
  - and can predict terms in next order when they are not known exactly yet ("approximate NNLO")
    - ✓ by expanding the resummed cross section to that order
- Better physics description (small p_T e.g., more later)
- Lessens the renormalization/factorization scale uncertainty,
  - the inclusive top quark cross section
  - the Higgs cross section

NNLO-NNLL inclusive cross section

Baernreuther, Fiedler, Mitov, Czakon

- A milestone in QCD, with clear benefits. Logarithm is “threshold logarithm”
  - precision top physics is here
  - new calculational methods developed
  - use for gluon density at large x, and α_s Czakon, Mitov, Mangano, Rojo





Concurrent uncertainties:

Scales	~ 3%
pdf (at 68%cl)	~ 2-3%
α_S (parametric)	~ 1.5%
m_{top} (parametric)	~ 3%

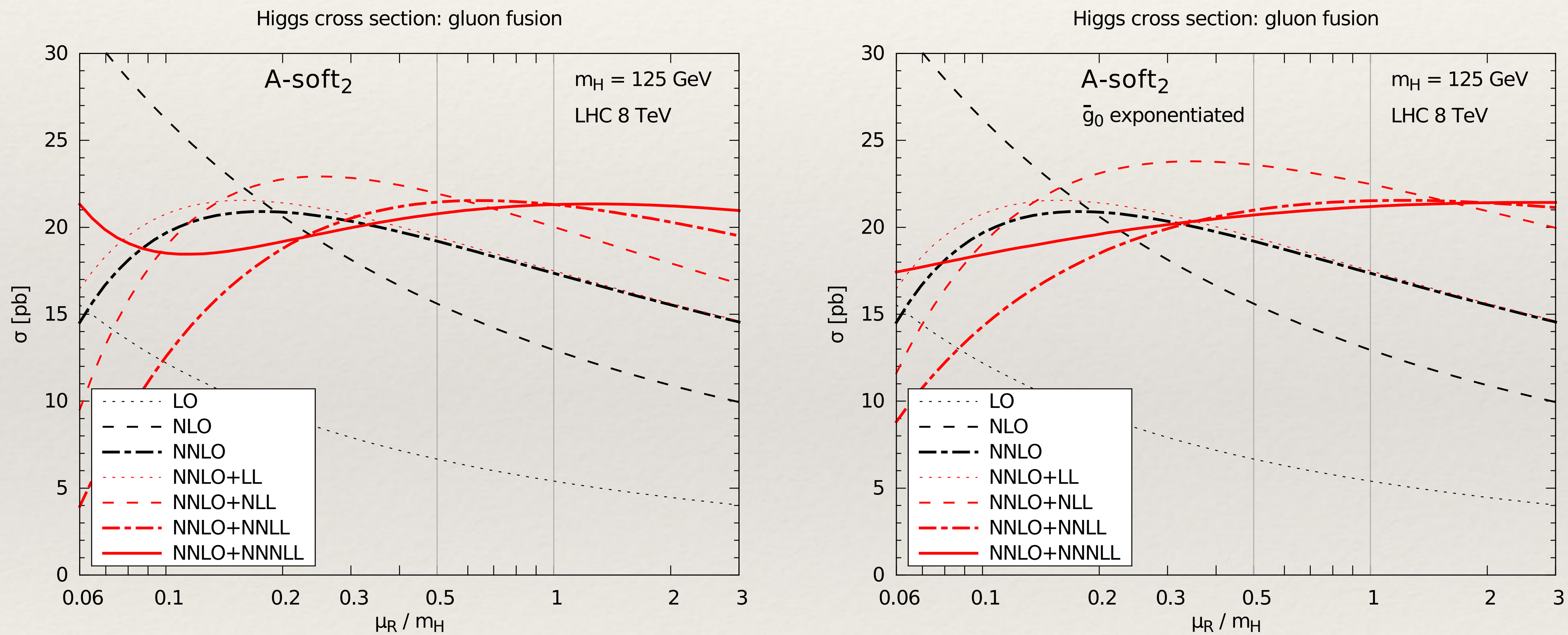
Soft gluon resummation makes a difference

5% -> 3%

N^{3}LL resummation for Higgs production

Bonvini, Marzani

- Logarithm is threshold logarithm
  - Nice progression, especially with exponentiated constants

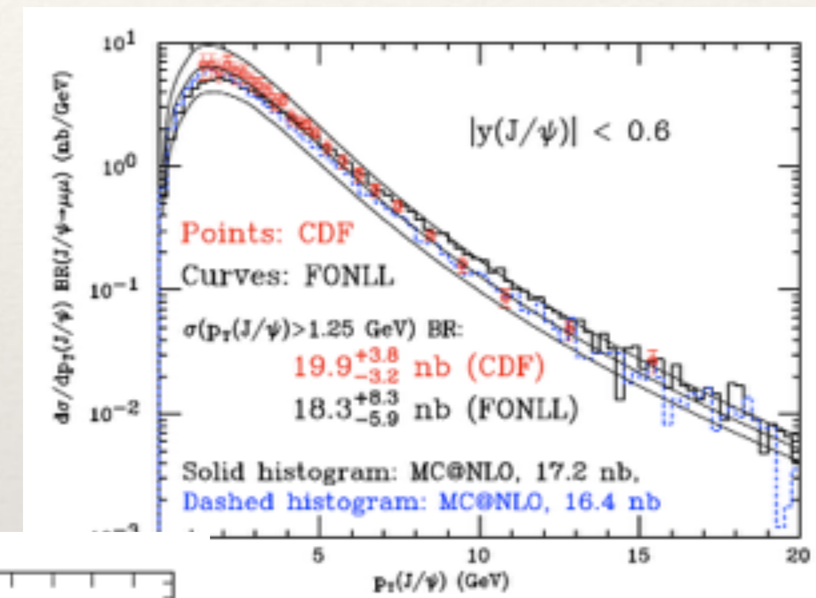


Resummation of what logarithm?

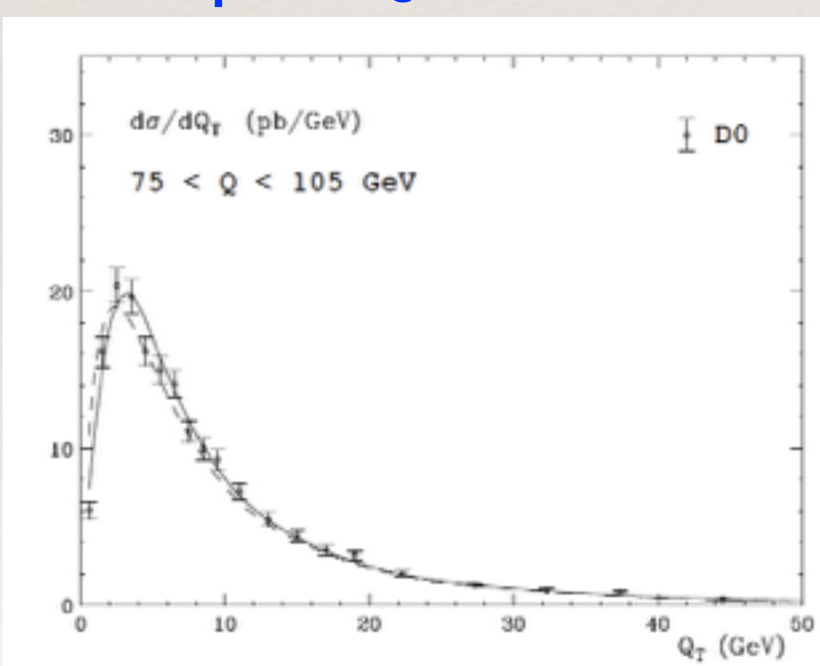
- So many variables, so many logs,...

$\ln(1-T)$ $\ln(p_T/m_Z)$ $\ln(k_T)$ $\ln(1/x)$ $\ln(1-x)$ $\ln(N)$ $\ln(\mu)$ $\ln(b)$

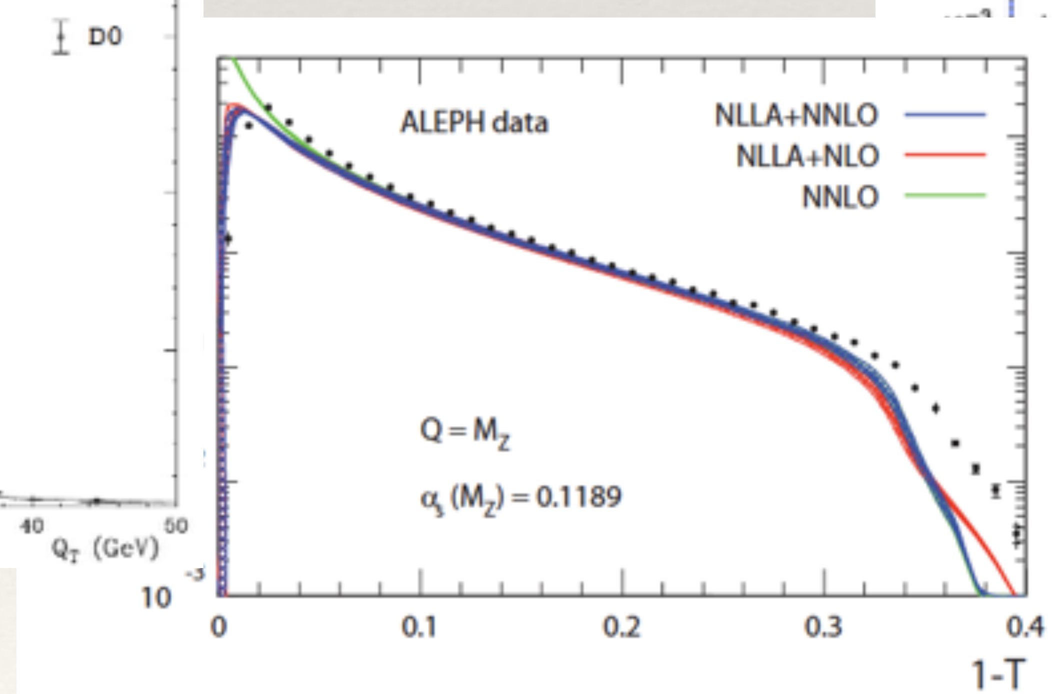
pT of b @ Tevatron



pT of Z @ Tevatron



Thrust T @ LEP



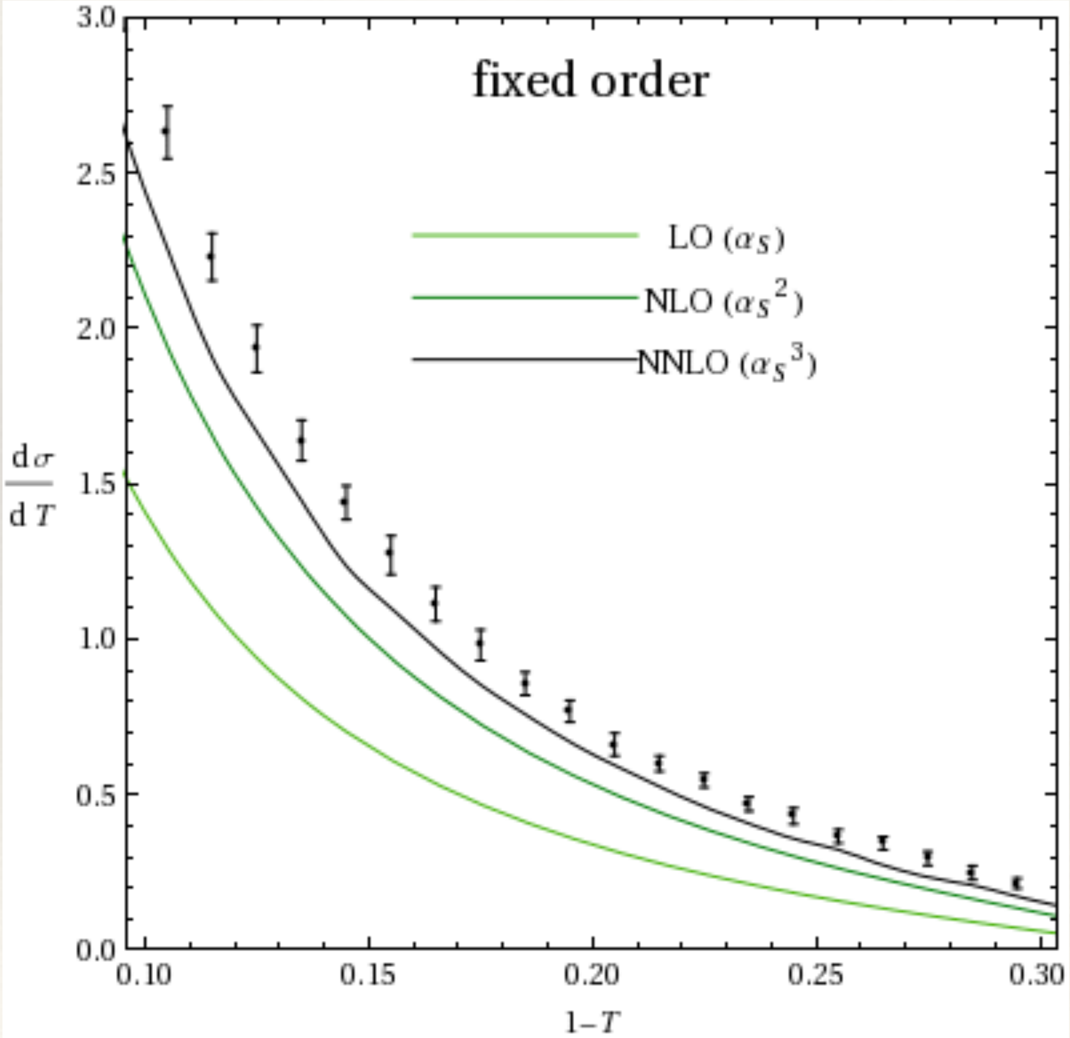
1st example of double logs: thrust

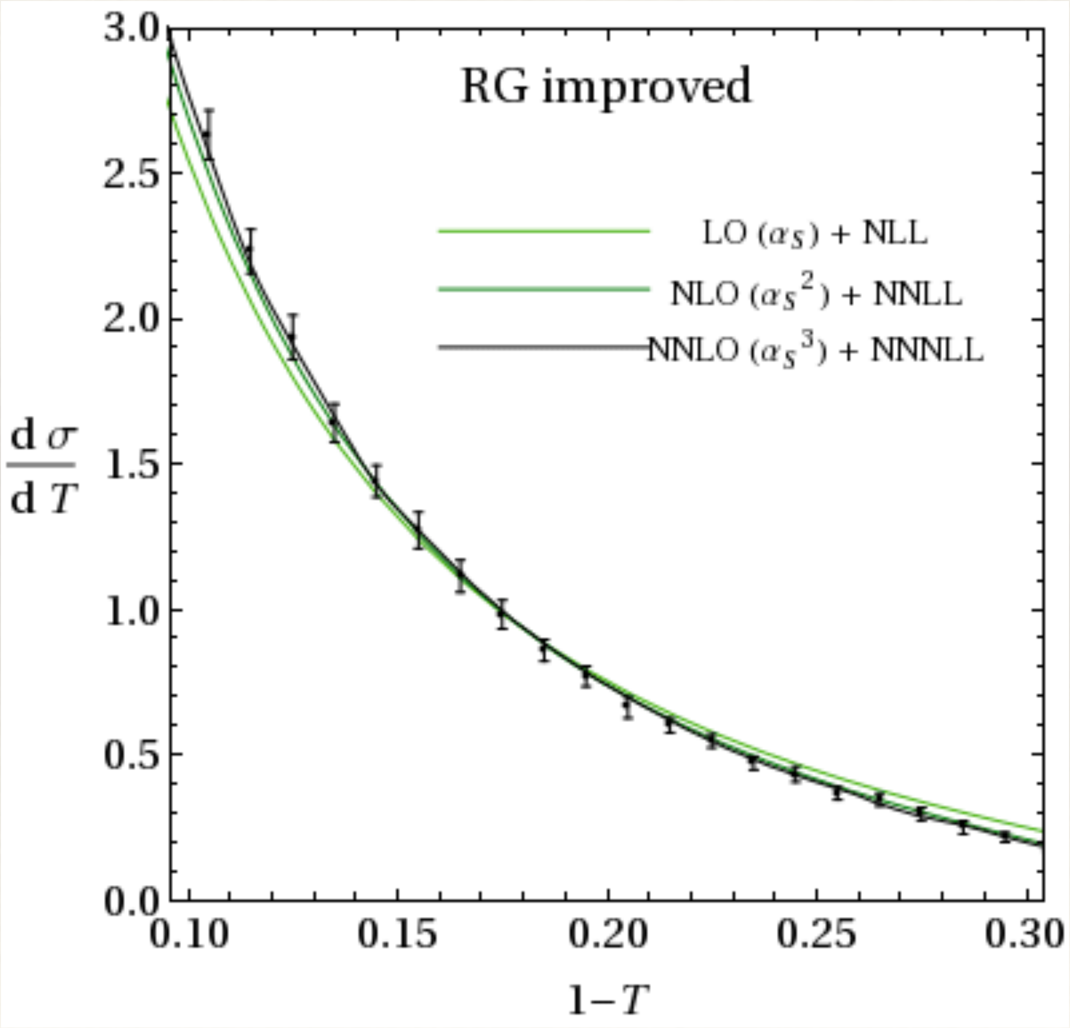
- Near T=1 the final state looks like two very narrow jets
  - emission must then be either very soft, and/or very collinear. Large logs:

$$\ln^2(1-T)$$

  - Data (ALEPH) vs fixed order and vs resummation

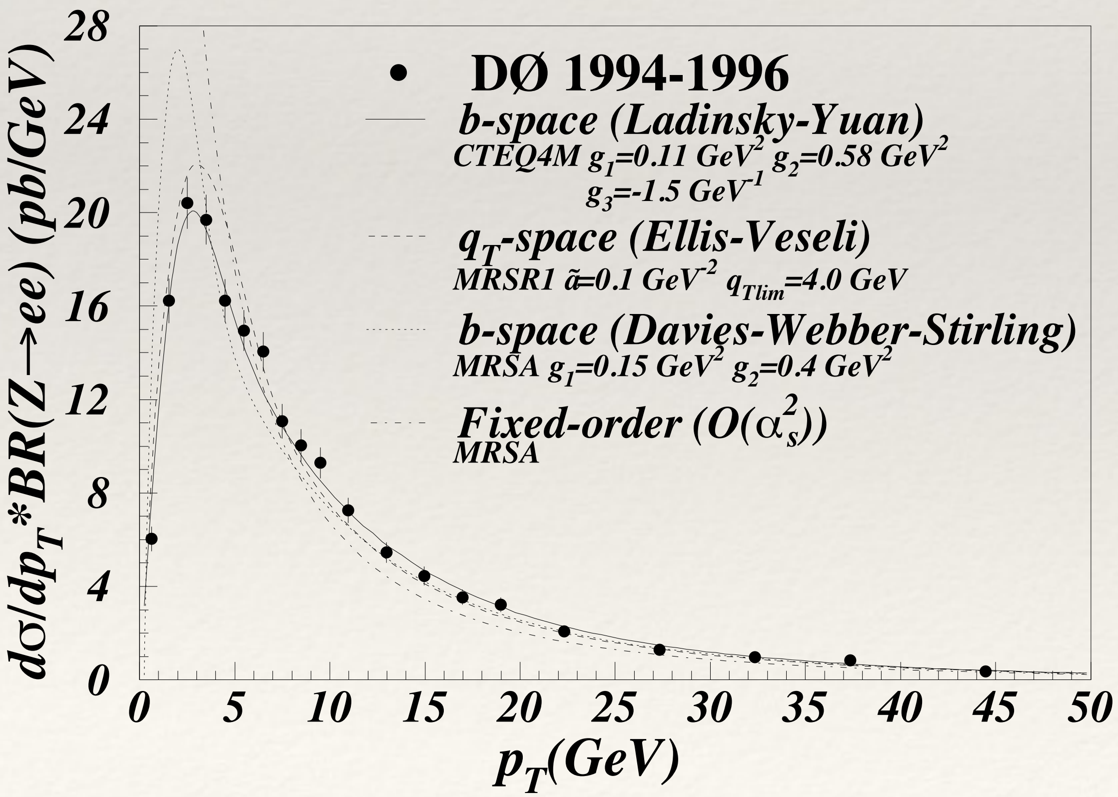
Becher, Schwartz

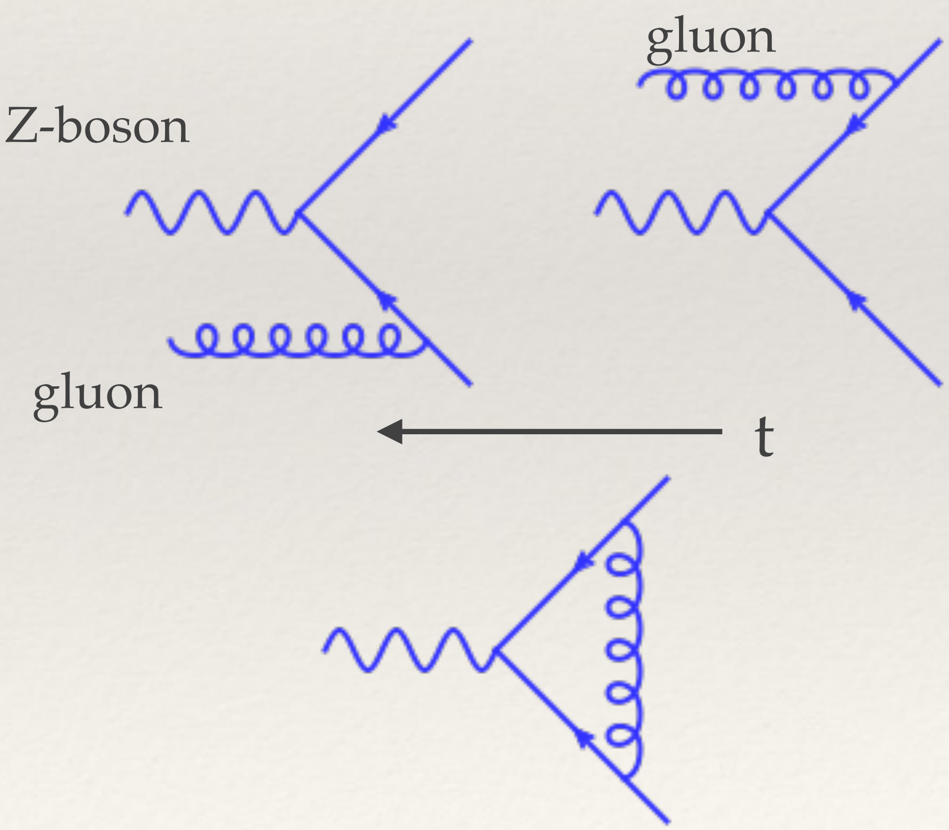




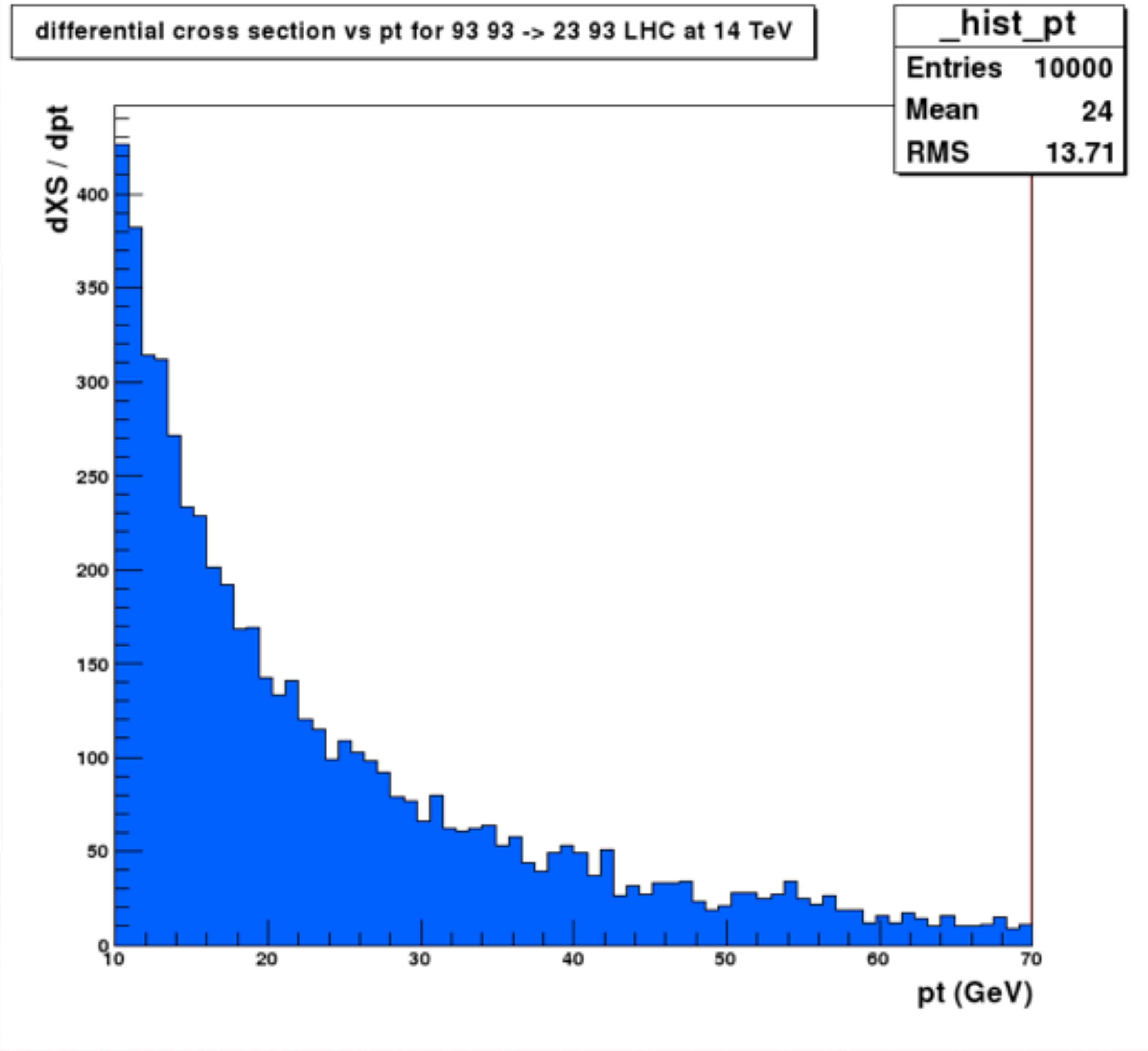
2nd example of double log: recoil logs

- Eg. pT of Z-bosons produced at Tevatron
  - Z-boson gets p_T from recoil agains (soft) gluons
  - Visible logs (argument made of measured quantities)
    - 1 emission: with gluon very soft: divergent
      - virtual: large negative bin at pT=0
  - The turn-over at pT around 5 GeV is only explained by resummation, not by finite order calculations





Divergence near p_T=0



⇔

Physics near small p_T

- At finite order

$$\frac{d\sigma}{dp_T} = c_0\delta(p_T) + \alpha_s \left(c_2^1 \frac{\ln p_T}{p_T} + c_1^1 \frac{1}{p_T} + c_0^1 \delta(p_T) \right) + \ldots$$

  - hence the real divergence toward p_T near zero

- Resummed

$$\frac{d\sigma}{dp_T} = c_0 \exp\left[-c_2^1 \alpha_s \ln^2(p_T) + \ldots \right]$$

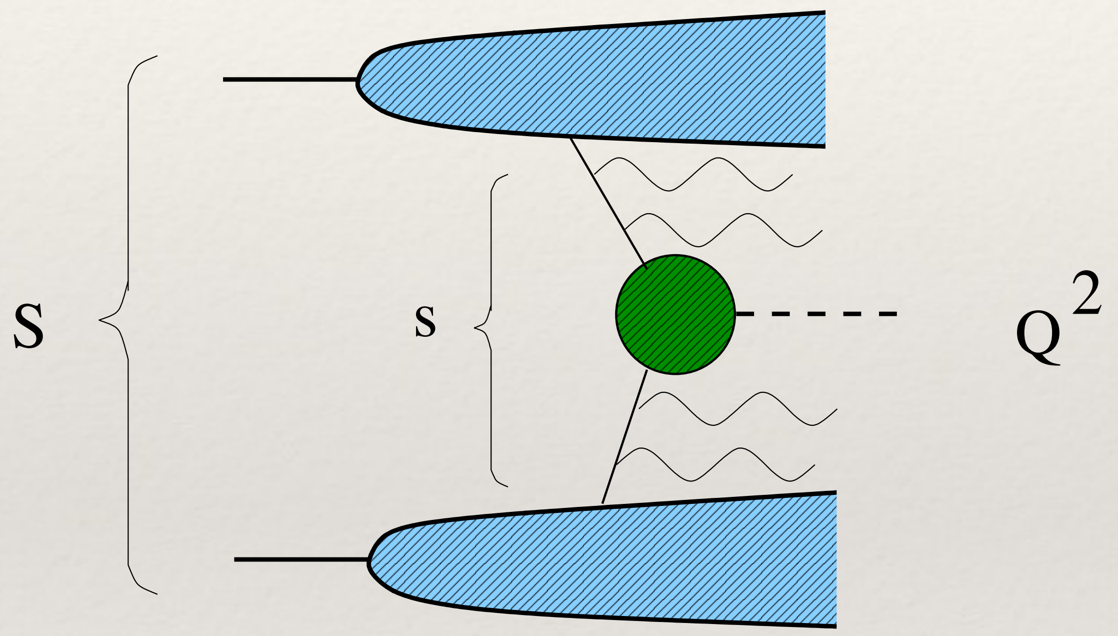
  - ✓ this is also the effective behaviour of the parton shower there

- Notice:
  - finite order oscillates wildly near small p_T, and may be negative
  - resummed is positive, and it tracks the data well

- Physics of resummed answer:
  - probability of the process **not** to emit at small p_T is vanishingly small
    - ✓ There is violent acceleration of color charges after all..

3rd example of double log: threshold logs

- Logarithm of "energy above threshold Q^2" $\ln^2(1 - Q^2/s)$
  - "Invisible" logs": argument made up of integration variables
  - Typical effect: enhancement of cross section



$$S \geq s \geq Q^2$$

Threshold log rule of thumb

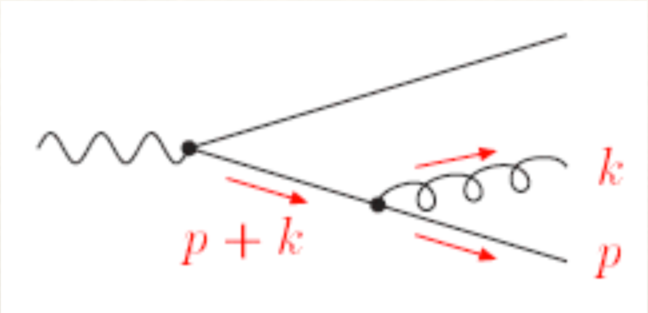
- Why do they increase the cross section? (N large = near threshold)

$$\sigma_{partonic,resum}(N) = \frac{\sigma_{hadronic}(N)}{\phi^2(N)} = \frac{\exp(-\ln^2 N)}{\left(\exp(-\ln^2 N)\right)^2} = \exp(+\ln^2 N)$$

- In words:
  - The hadronic cross section is a product/convolution of PDF's and the partonic cross section
  - In both factors emissions may, and should occur.
    - ✓ The contribution from the PDF's is too stingy
    - ✓ The partonic cross section has to overcompensate in order to get the right amount for the hadronic cross section

Reminder of origin of double ("Sudakov") logs

- Double logarithms in cross sections are related to IR divergences

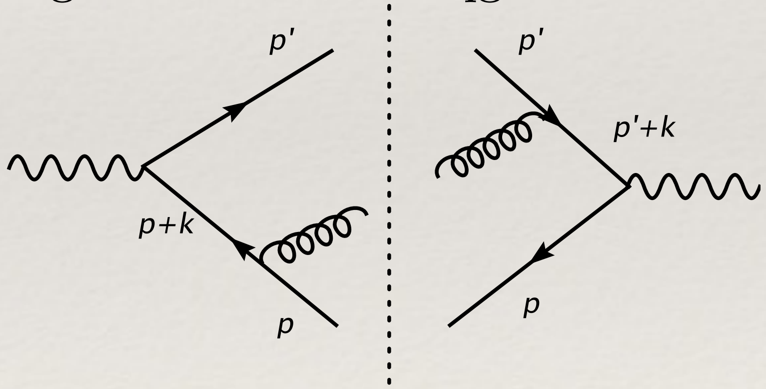


$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_{\rm g}E_{\rm q}(1-\cos\theta_{\rm qg})}$$

Phase space integration

$$\alpha_s \int \frac{d^{4-2\epsilon}k}{(2\pi)^4}\,\frac{p\cdot p'}{p\cdot k\, p'\cdot k} \sim \alpha_s \int^K \frac{dE_{\rm g}E_{\rm g}^{-\epsilon}}{E_{\rm g}} \int \frac{d\theta_{\rm qg}\sin^{-\epsilon}\theta_{\rm qg}}{\theta_{\rm qg}}$$

$$\sim \alpha_s\,(\frac{1}{\epsilon^2} + \ln^2(K))\,.$$



Decoupling of IR effects

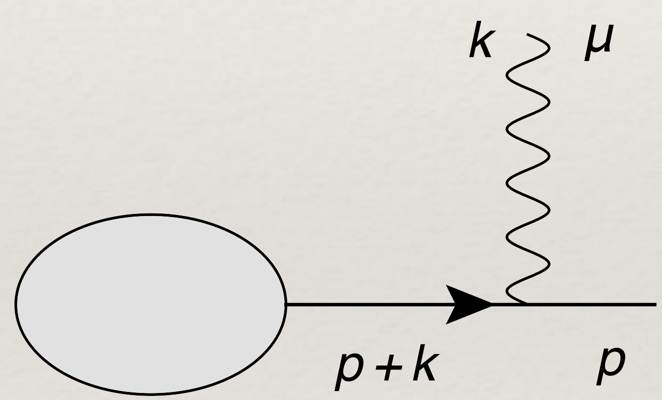
- We saw already for the spinor methods that the part with the soft (k5) gluon decouples from the rest

$$\mathcal{M}(1^+,2^-,3^+,4^-,5^+) = 2\sqrt{2}e^2 g T_a \frac{\langle 24\rangle^2}{\langle 12\rangle\langle 35\rangle\langle 45\rangle} = \frac{\langle 24\rangle^2}{\langle 12\rangle\langle 34\rangle} \times \frac{\langle 34\rangle}{\langle 35\rangle\langle 45\rangle}$$

- This is in fact quite general, and it is essence of why we can resum double logarithms to all orders.
- The soft approximation is also known as the “eikonal” approximation
  - Simplification, gives nice and very insightful results

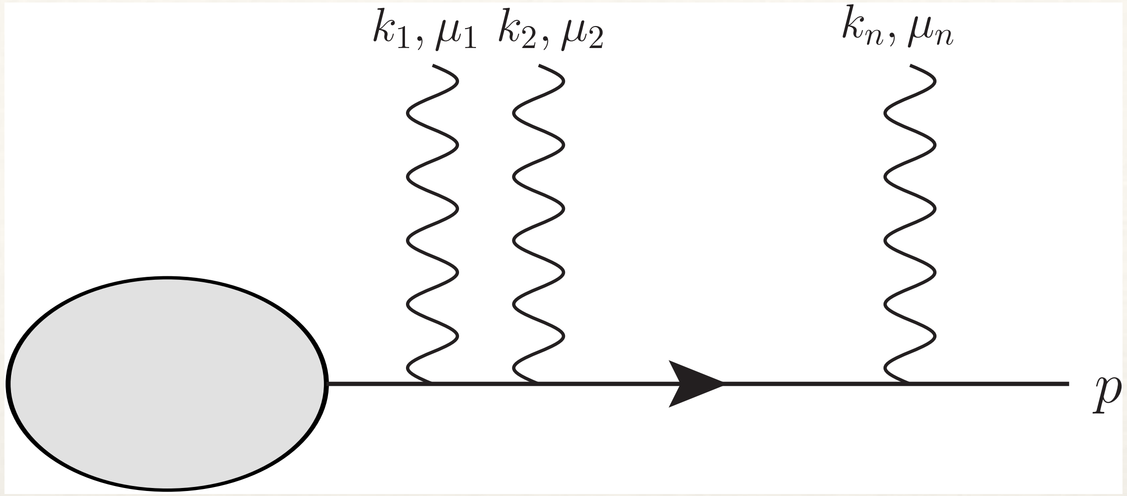
Basics of eikonal approximation: QED

- Charged particle emits softly
  - Propagator: expand numerator & denominator in soft momentum, keep lowest order
  - Vertex: expand in soft momentum, keep lowest order



$$\frac{(p+k)^\mu + p^\mu}{2p\cdot k + k^2} \longrightarrow \frac{2p^\mu}{2p\cdot k}$$

Basics of eikonal approximation in QED



Exact: $$\frac{1}{(p+K_1)^2}(2p+K_2+K_1)^{\mu_1}\dots\frac{1}{(p+K_n)^2}(2p+K_n)^{\mu_n}, \quad K_i=\sum_{m=i}^{n}k_m.$$

Approx: $$\frac{1}{2pK_1}2p^{\mu_1}\dots\frac{1}{2pK_n}2p^{\mu_n}$$

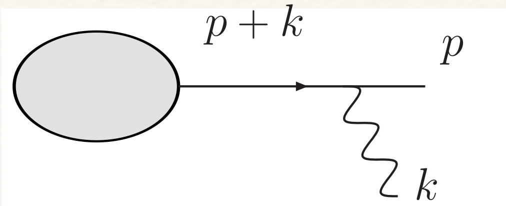
Eikonal identity: $$\frac{1}{p\cdot(k_1+k_2)\,p\cdot k_2}+\frac{1}{p\cdot(k_1+k_2)\,p\cdot k_1}=\frac{1}{p\cdot k_1\,p\cdot k_2}$$

Sum over all perm's: $$\prod_i \frac{p^{\mu_i}}{p\cdot k_i}.$$

Independent, uncorrelated emissions, Poisson process

Eikonal approximation: no dependence on emitter spin

- Emitter spin becomes irrelevant in eikonal approximation
  - Fermion



$$M\left(\frac{i(\not{p}+\not{k})}{(p+k)^2}\left(-ig_s\gamma^\mu\right)u(p)\right.$$

  - Approximate, and use Dirac equation $\not{p}u(p) = 0$
  - Result:

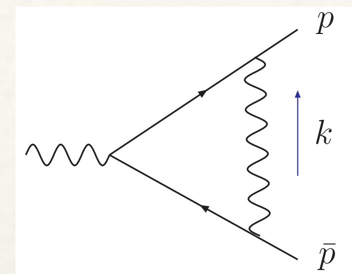
$$g\,(M\,u(p)) \times \frac{p^\mu}{p\cdot k}$$

  - Two things have happened
    - ✓ No sign of emitter spin anymore
    - ✓ Coupling of photon proportional to p^μ !
- Decoupling again of emission and emitter

Eikonal exponentiation

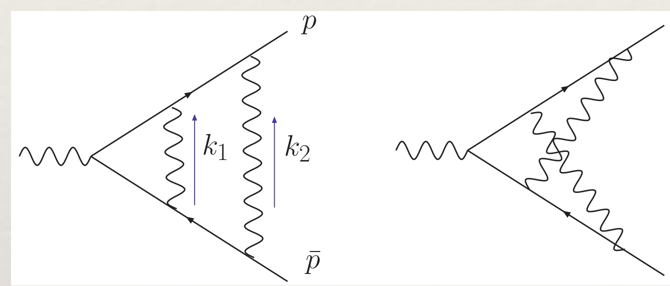
- In the eikonal approximation, suddenly we see very interesting patterns.

One loop vertex correction, in eikonal approximation



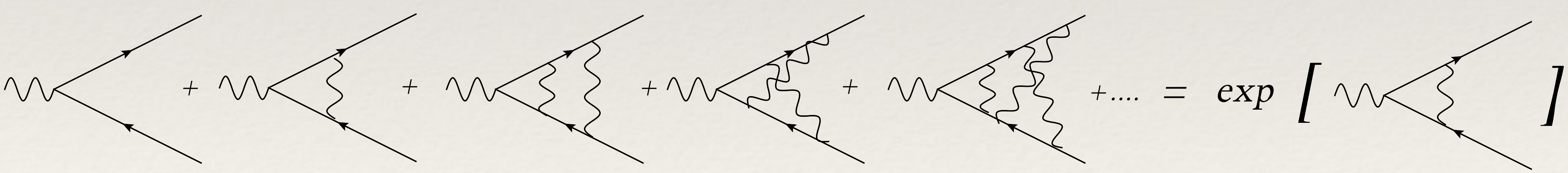
$$\mathcal{A}_0 \int d^n k \frac{1}{k^2} \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)}$$

Two loop vertex correction, in eikonal approximation



$$\mathcal{A}_0 \frac{1}{2} \left(\int d^n k \frac{1}{k^2} \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)} \right)^2$$

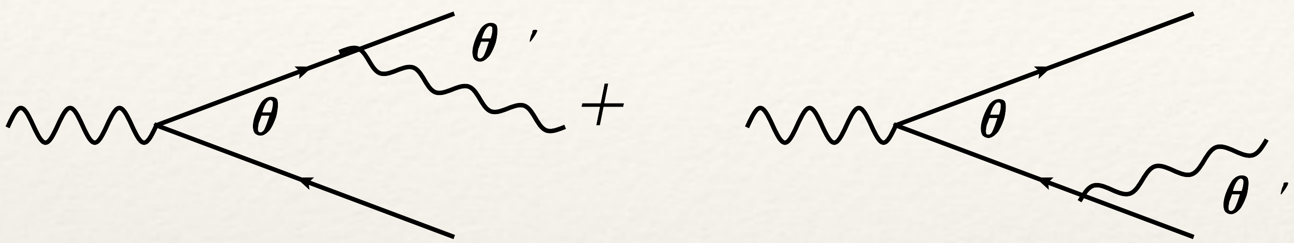
Exponential series! A really beautiful result



Yennie, Frautschi, Suura

Another eikonal effect: coherence in emission

- Eikonal approximation in amplitude, coherence possible
  - First in QED



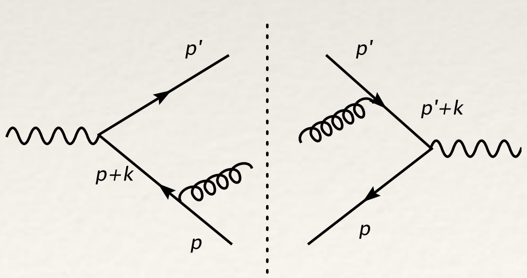
  - Square the amplitude, take the eikonal approximation, and combine with phase. Result

$$d\sigma_R = d\sigma \frac{\alpha_s}{2\pi} \frac{dE}{E} d\cos\theta d\phi \ E^2 \frac{p \cdot \bar{p}}{p \cdot k \, \bar{p} \cdot k}$$

  - Only non-zero when θ'<θ : angular ordering after azimuthal integral
    - ✓ photon that is too soft only see the sum of the charges, which is zero here.
  - In QCD very similar result (after being a little bit more careful with color charges). Radiation function

$$W_{ij} = \frac{\omega^2 p_i \cdot p_j}{p_i \cdot q \, p_j \cdot q} = \frac{1 - v_i v_j \cos\theta_{ij}}{(1 - v_i \cos\theta_{iq})(1 - v_j \cos\theta_{jq})}$$

    - ✓ clearly has eikonal form. Notice, it is an interference effect:



Color coherence

- Decompose into a part for emitter i and one for emitter j

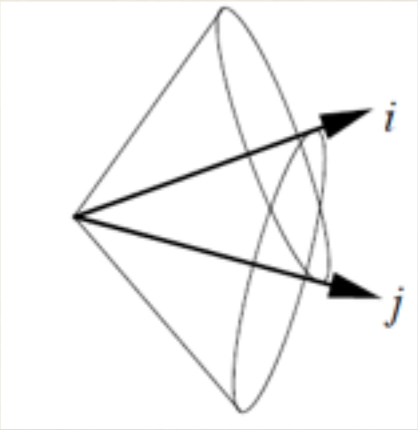
$$W_{ij} = W_{ij}^{[i]} + W_{ij}^{[j]}$$

- where

$$W_{ij}^{[i]} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{jq}}\right)$$

  - can be checked. Just substitute..

- Then one can show (as an exact result)

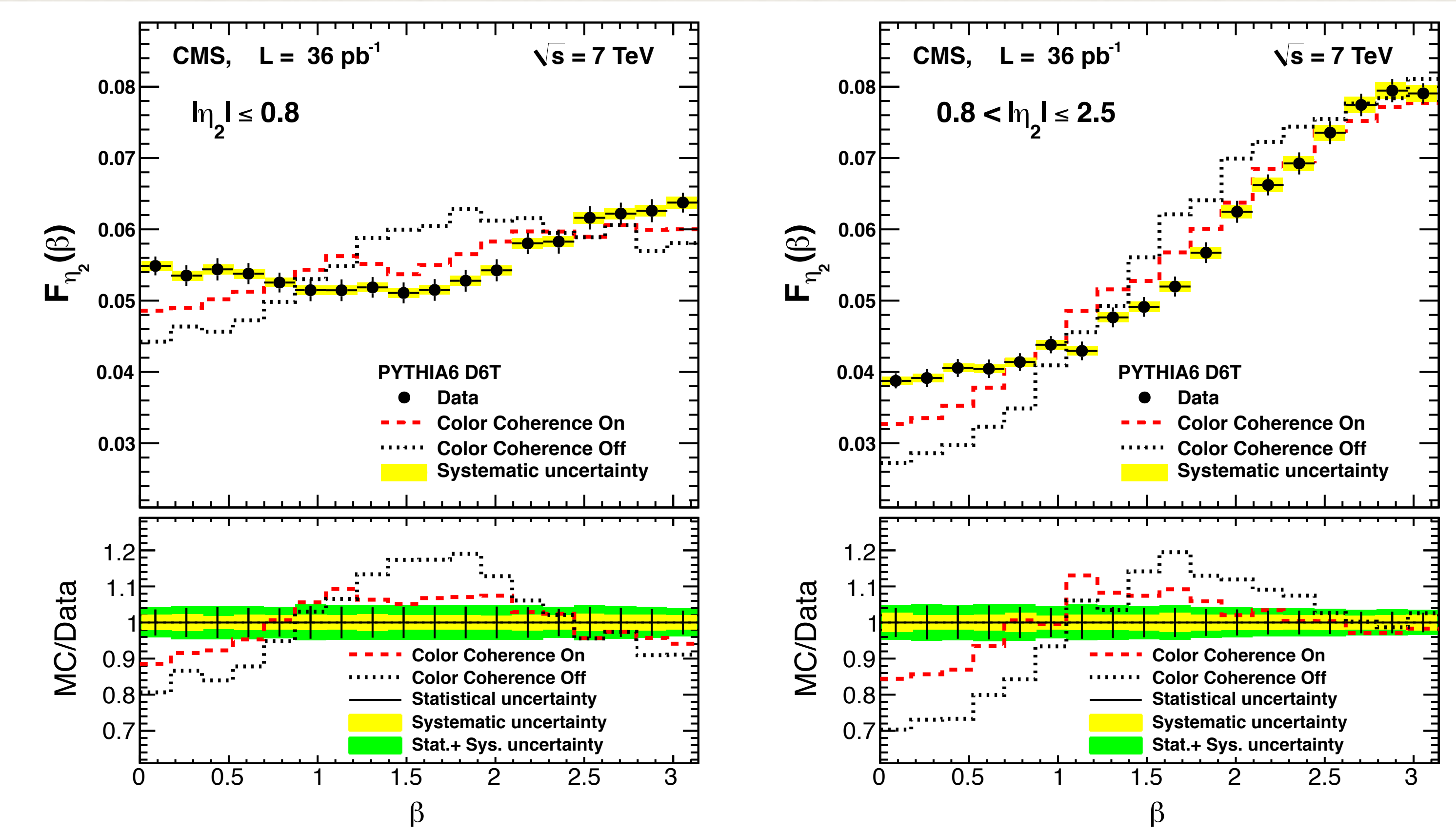


$$\int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} W_{ij}^{[i]} = \begin{cases} \frac{1}{1-\cos\theta_{iq}} & \text{if } \theta_{iq} < \theta_{ij} \\ 0 & \text{otherwise.} \end{cases}$$

  - color coherence/angular ordering in QCD!
    - ✓ but after azimuthal integration
  - Built-in to the HERWIG parton shower.
  - There is evidence for this in data

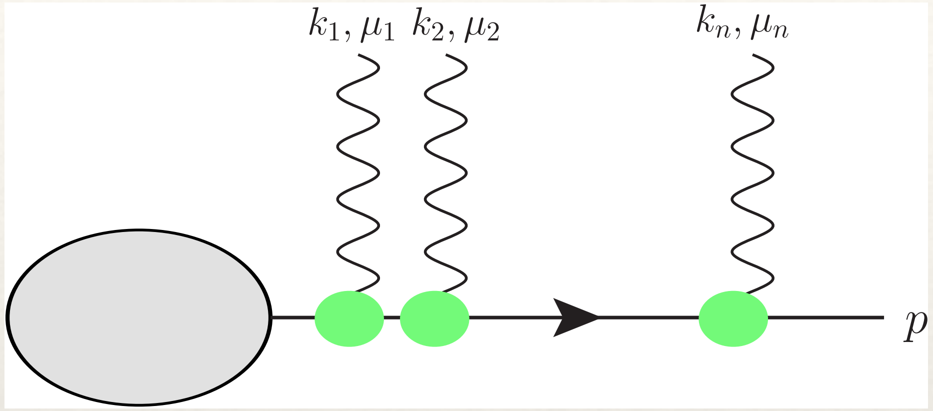
Color coherence in 3-jet events

- Recent CMS study (based on earlier CDF study): order the 3 jets in pT.
- Then study angular correlations between 2nd and 3rd jet. Expressed through parameter β.
  - ✓ β = 0 will be enhanced, when second jet is central
- Study through swithing color coherence on and off in Pythia
  - ✓ Switched on works better, but no real satisfactory description of data



Non-abelian eikonal approximation

- Same methods as for QED, but organization harder: SU(3) generator at every vertex



  - now no obvious decorrelation

Order the T_a according to λ

- Key "object": Wilson line

$$\Phi_n(\lambda_2, \lambda_1) = P \exp\left[ig \int_{\lambda_1}^{\lambda_2} d\lambda\, n \cdot A^a(\lambda n)\, T_a\right]$$

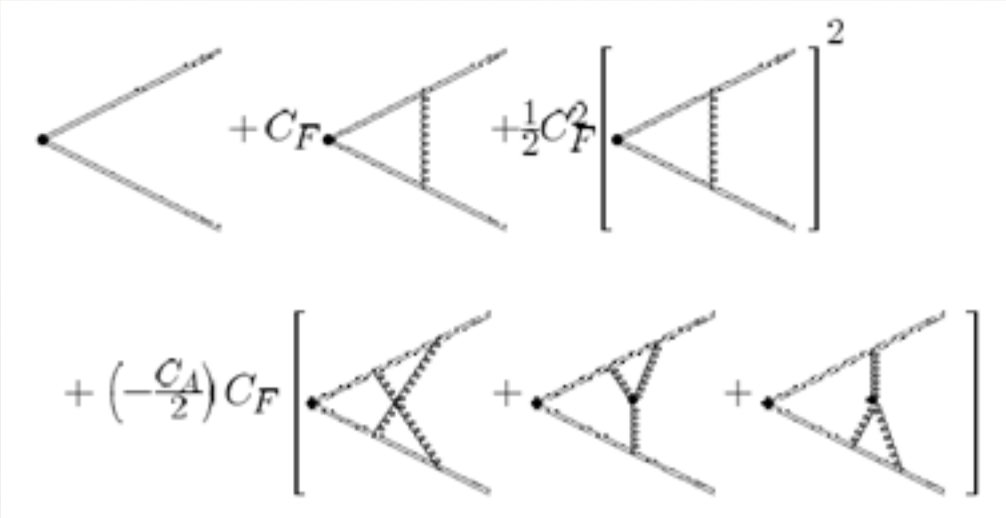
  - Order by order in "g", it generates QCD eikonal Feynman rules, including the SU(3) generators

Non-abelian exponentiation: webs

Gatheral; Frenkel, Taylor; Sterman

- Take quark - antiquark line, connect with soft gluons in all possible ways, and use eikonal approximation
- Exponentiation still occurs! Sum of all eikonal diagrams D with color factor C and momentum space part F

$$\sum C(D)\mathcal{F}(D) = \exp\left[\bar{C}(D)W(D)\right]$$



- A selection of diagrams in exponent, but with modified color weights: “webs”
  - ✓ Easy to select webs: they must be two-eikonal line irreducible
  - ✓ More difficult to compute the modified color factors, but can be done also

Multiple colored lines

- Structure

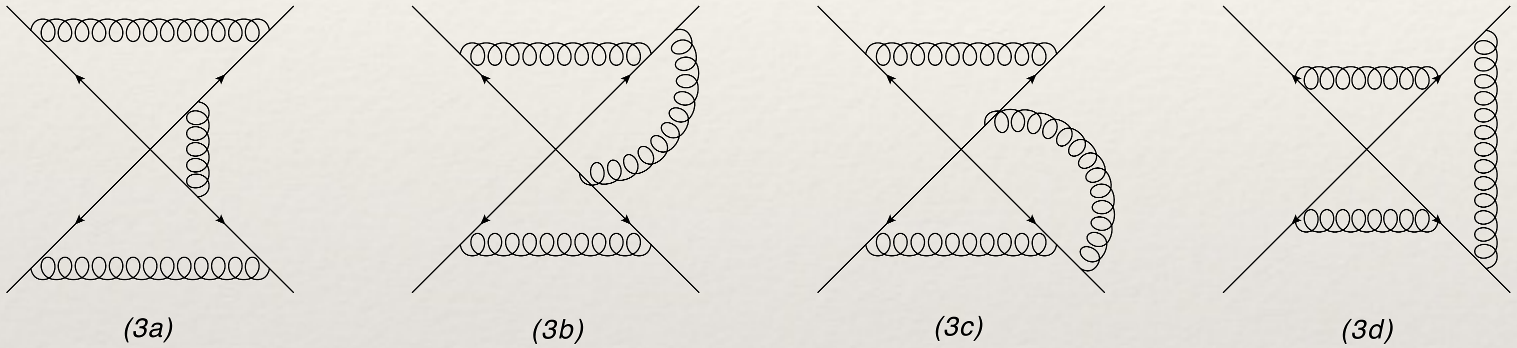
Projector matrix

$$\sum \mathcal{F}(D)C(D) = \exp[\sum_{d,d'} \mathcal{F}(d) R_{dd'} C(d')]$$

$$\sum_{d'} R_{dd'} = 0$$

Eigenvalues 0 or 1

  - multi-parton webs are “closed sets” of diagrams, with modified color factors



= Multiparton Web

- Closed form solution for modified color factor

$$\frac{1}{6}\Big[C(3a) - C(3b) - C(3c) + C(3d)\Big] \times \Big[M(3a) - 2M(3b) - 2M(3c) + M(3d)\Big]$$

  - Interesting properties of projector matrix (reduces degree of divergence)

Projector matrix

$$\sum \mathcal{F}(D)C(D) = \exp[\sum_{d,d'} \mathcal{F}(d) R_{dd'} C(d')]$$

Gardi, White

- Projects out contributions that come from exponentiation of lower order diagrams
  - ✓ Interesting combinatorial aspects (Stirling numbers)
  - ✓ Proof of idempotency and zero sum row property
- Combinatorics involves quite interesting for mathematicians

How to resum?

- There are many ways, depending on
  - the observable
  - the logarithm
  - the resummer
- Here we take as key notions
  - factorization
  - approximations for kinematic limit (eikonal approximation e.g.)

Resummation 101

- Cross section for n extra gluons

Phase space measure **Squared matrix element**

$$\sigma(n) = \frac{1}{2s}\int d\Phi_{n+1}(P, k_1, \ldots, k_n) \times |\mathcal{M}(P, k_1, \ldots, k_n)|^2$$

- When emissions are soft, can factorize phase space measure and matrix element [eikonal approximation]

$$d\Phi_{n+1}(P, k_1, \ldots, k_n) \longrightarrow d\Phi(P) \times \Big(d\Phi_1(k)\Big)^n \frac{1}{n!}$$

- Sum over all orders

$$|\mathcal{M}(P, k_1, \ldots, k_n)|^2 \longrightarrow |\mathcal{M}(P)|^2 \times \Big(|\mathcal{M}_{1\ \text{emission}}(k)|^2\Big)^n$$

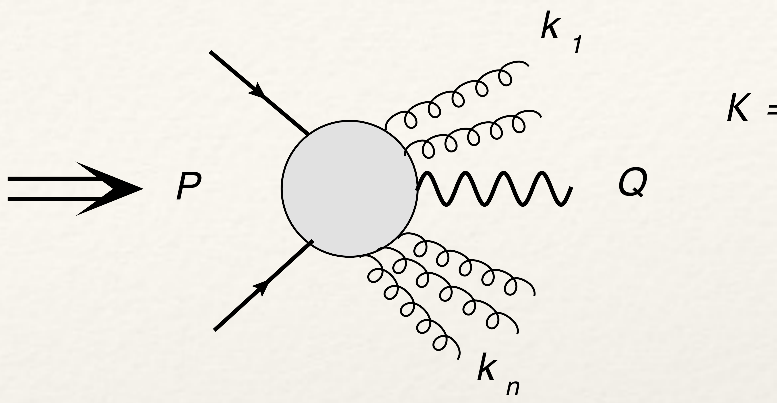
$$\sum_n \sigma(n) = \sigma(0) \times \exp\left[\int d\Phi_1(k)|\mathcal{M}_{1\ \text{emission}}(k)|^2\right]$$

- Incorporate Theta or Delta functions in space space
  - but these must factorize similarly, or they cannot go into exponent

Phase space in resummation

- Kinematic condition expresses "z" in terms of gluon energies

$$s = Q^2 - 2P\cdot K - K^2 \qquad \delta\Big(1 - \frac{Q^2}{s} - \sum_i \frac{2k_i^0}{\sqrt{s}}\Big)$$



  - or conservation of transverse momentum

$$\delta^2(Q_T - \sum_i p_T^i)$$

- Transform (e.g. Laplace/Mellin or Fourier) factorizes the phase space

$$\int_0^\infty dw\, e^{-w\,N}\delta\Big(w - \sum_i w_i\Big) = \prod_i \exp(-w_i N) \qquad \int d^2Q_T\, e^{ib\cdot Q_T}\, \delta^2(Q_T - \sum_i p_T^i) = \prod_i e^{ib\cdot p_T^i}$$

- So can go into exponent

$$\sum_n \sigma(n) = \sigma(0) \times \exp\Big[\int d\Phi_1(k)|\mathcal{M}_{1\ \mathrm{emission}}(k)|^2(\exp(-w\,N) - 1)\Big]$$

  - Large logs: ln(N) or ln(bQ)

Resummation and factorization

- Very generically, if a quantity factorizes, one can resum it. Let us consider UV renormalization as an example.
  - Renormalization; factorizes UV modes into Z-factor (Λ is an ultraviolet cut-off)

$$G_B(g_B, \Lambda, p) = Z\Big(\frac{\Lambda}{\mu}, g_R(\mu)\Big) \times G_R\Big(g_R(\mu), \frac{p}{\mu}\Big)$$

  - Evolution equation (here RG equation)

$$\mu \frac{d}{d\mu} \ln G_R\Big(g_R(\mu), \frac{p}{\mu}\Big) = -\mu \frac{d}{d\mu} \ln Z\Big(\frac{\Lambda}{\mu}, g_R(\mu)\Big) = \gamma(g_R(\mu))$$

    - ✓ Notice that γ can only depend on, the only common variable
  - Solving the differential equation = resumming

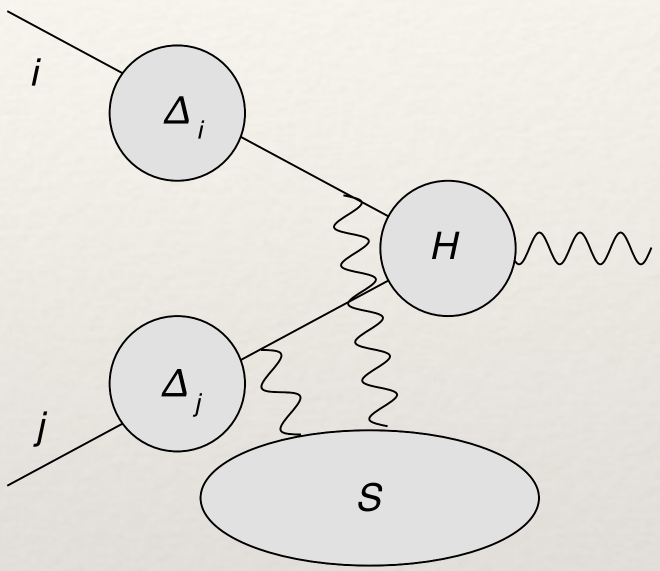
$$G_R\left(\frac{p}{\mu}, g_R(\mu)\right) = G_R\left(1, g_R(p)\right) \underbrace{\exp\left[\int_p^\mu \frac{d\lambda}{\lambda} \gamma(g_R(\lambda))\right]}_{\text{resummed}}$$

    - ✓ The exponent will be a series in $g_R(\mu)$.
      - Exercise: compute the first term in the exponent. Answer: $\frac{g_R^2(p)}{4\pi} \gamma^{(1)} \ln \frac{\mu}{p}$
- This is a very general notion, and can be seen as the basis of resummation.

Factorization and resummation for Drell-Yan

$$\sigma(N) = \Delta(N,\mu,\xi_1)\Delta(N,\mu,\xi_2)S(N,\mu,\xi_1,\xi_2)H(\mu)$$

- Near threshold, cross section is equivalent to product of 4 well-defined functions
- Demand independence of
  - renormalization scale μ
  - gauge dependence parameter ξ
    - ✓ find exponent of double logarithm



$$0 = \mu\frac{d}{d\mu}\sigma(N) = \xi_1\frac{d}{d\xi_1}\sigma(N) = \xi_2\frac{d}{d\xi_2}\sigma(N)$$

Contopanagos, EL, Sterman

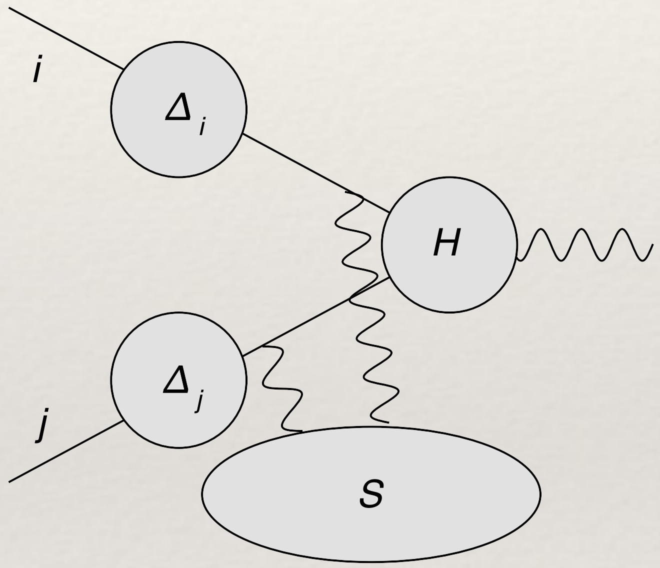
$$\Delta = \exp[\int\frac{d\mu}{\mu}\int\frac{d\xi}{\xi}..]$$

Factorization for threshold resummation

- $\Delta_i(N)$: initial state soft+collinear radiation effects
  - real+virtual
  - $\alpha_s^n \ln^{2n} N$

$$\sigma(N) = \sum_{ij} \phi_i(N)\phi_j(N) \times \underbrace{\left[\Delta_i(N)\Delta_j(N)S_{ij}(N)\, H_{ij}\right]}_{\hat{\sigma}_{ij}(N)}$$

- $S_{ij}(N)$: soft, non-collinear radiation effects
  - $\alpha_s^n \ln^n N$
- H: hard function, no soft and collinear effects



$$\Delta_i(N) = \exp\left[\ln N \frac{C_F}{2\pi b_0 \lambda}\{2\lambda + (1-2\lambda)\ln(1-2\lambda)\} + ..\right]$$
$$= \exp\left[\frac{2\alpha_s C_F}{\pi}\ln^2 N + ..\right]$$

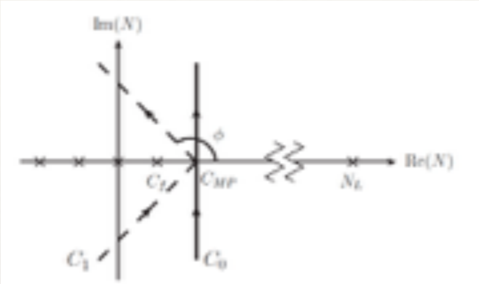
From N space back to momentum-space

- Parton cross section derived in N space

$$\sigma^{(\mathrm{res})}_{h_1 h_2 \to kl}(\rho^2, \{m^2\}, \mu_R^2, \mu_F^2) = \frac{1}{\pi} \int_0^\infty dy\, \mathrm{Im}\,[e^{i\phi}\, \rho^{-C_{\mathrm{MP}} - y e^{i\phi}} \times \sigma^{(\mathrm{res})}_{h_1 h_2 \to kl}(N = C_{\mathrm{MP}} + y e^{i\phi}, \{m^2\}, \mu_R^2, \mu_F^2)\,]$$

- PDF's in N space
  - Use initial conditions in N-space, then QCD-PEGASUS evolution (A. Vogt)
- Use inverse Mellin transform
  - Avoid Landau pole singularity with Minimal Prescription (go left..)
    - ✓ gives Good numerical stability

Catani, Mangano
Nason, Trentadue



- Exercise:
  - function $f(x) = x^p$
  - Melling transform $f(N) = \int_0^1 dx\, x^{N-1} x^p = \frac{1}{N+p}$
  - Inverse Mellin transform $f(x) = \frac{1}{2\pi i} \int dN x^{-N} \frac{1}{N+p} = x^p$
    - ✓ Correct!

Resummed Drell-Yan/Higgs cross section

Sterman; Catani, Trentadue

Threshold-resummed Drell-Yan cross section

Functions in exponent depend only on coupling

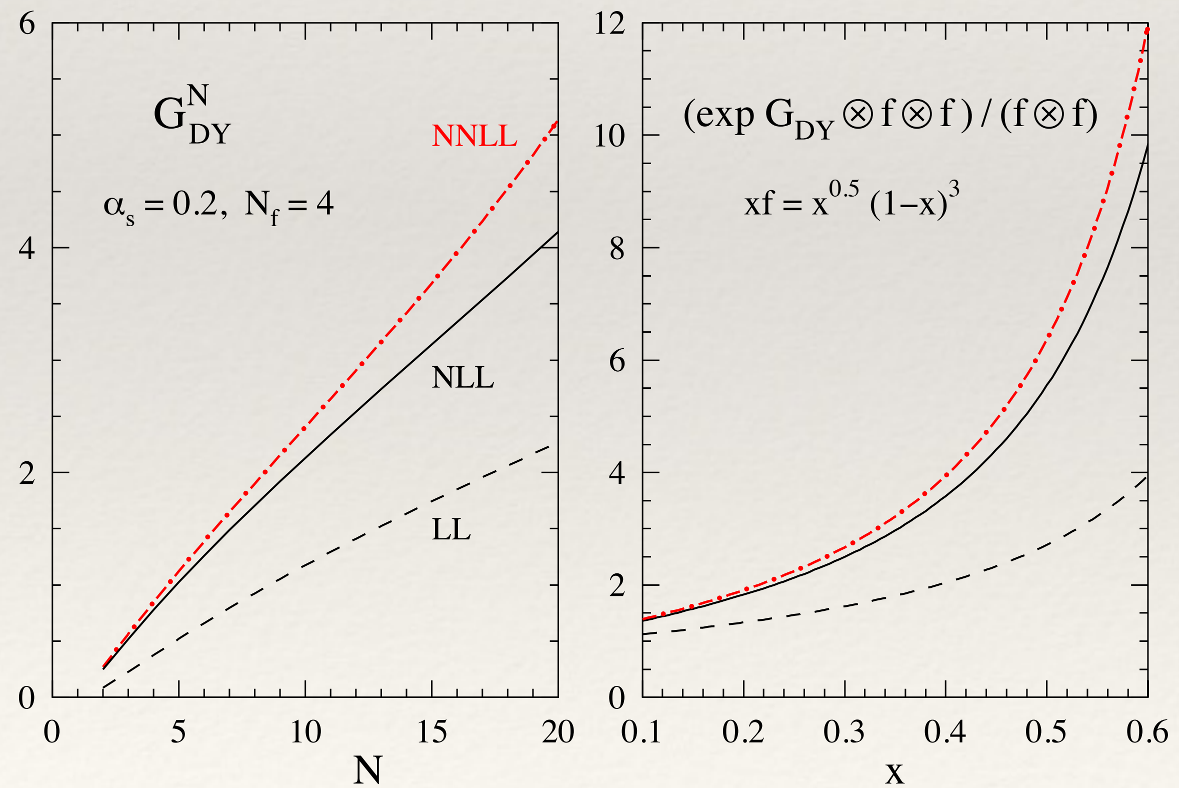
$$\frac{d\sigma^{\rm resum}}{dQ^2}(z) = \int_C \frac{dN}{2\pi i}\, z^{-N}\, \hat\sigma(N)$$

$$\sigma(N) = \exp\left[-\int_0^1 dx\, \frac{x^{N-1}-1}{1-x} \left\{ \int_{Q^2}^{Q^2(1-x)^2} \frac{d\mu}{\mu} A(\alpha_s(\mu)) + D(\alpha_s((1-x)Q)) \right\} \right] \times (1+\alpha_s(Q^2)\frac{C_F}{\pi} + \ldots)$$

$$\hat\sigma_{DY}(N,Q^2) = g_0(Q^2)\, \exp\left[G^N_{DY}(Q^2)\right]$$

$$G^N_{DY} = \ln N g_1(\lambda) + g_2(\lambda) + \alpha_s g_3(\lambda) + \ldots, \qquad \lambda = \beta_0 \alpha_s \ln N$$

A. Vogt



Good convergence in exponent

Resummation vs parton shower

- Both account for emission to all orders in perturbative QCD. It's accuracy vs flexibility
  - Resummation: a formula
    - ✓ accuracy to LL, NLL, NNLL depending on what the theorists did. For specific observables
  - Parton shower: generate events
    - ✓ very flexible, can use for any observables
    - ✓ but, on the downside, in essence only LL accuracte (it never has all the NLL information in it, because that is to some extent observable dependent).
      - Progress is being made here however

Final summary

- Many concepts in perturbative QCD were discussed, in both their essence and some technical aspects
  - Formal: symmetries, renormalization, asymptotic freedom
  - Finite orders, IR and COL divergence-handling
  - Parton showers
  - Modern methods: spinor helicity methods, and a glimpse of the NLO revolution
  - All-orders: resummation, why and how
    - ✓ here there is quite a bit of physics insight possible
- My hope: that when you see such concepts in workshops or talks, you now have a sense about what this is about.
    - ✓ Don't be blinded by the technicalities, there is room for a lot of physics intuition in QCD
- Especially I hope that you will feel free to ask, and discuss with QCD theorists when you have questions and/or ideas. Just as how you have done here. I think the success of the LHC and its research program depend on this!