Study of magnetic monopole condensation using surface operators

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Surface operators

The magnetic field flow (Abelian case):

$$B = \int_S H \cdot dS = \oint_C A \cdot dl$$

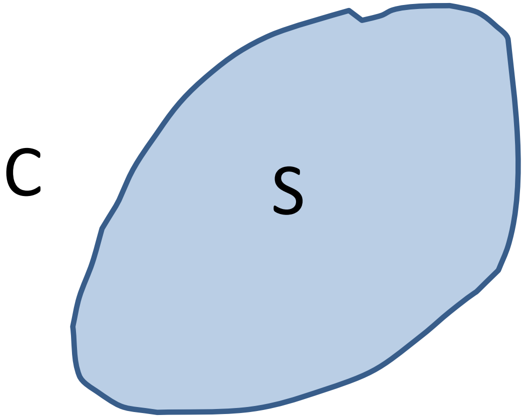
For closed surface (C=0), in the absence of monopoles: B=0

Thus, we study:

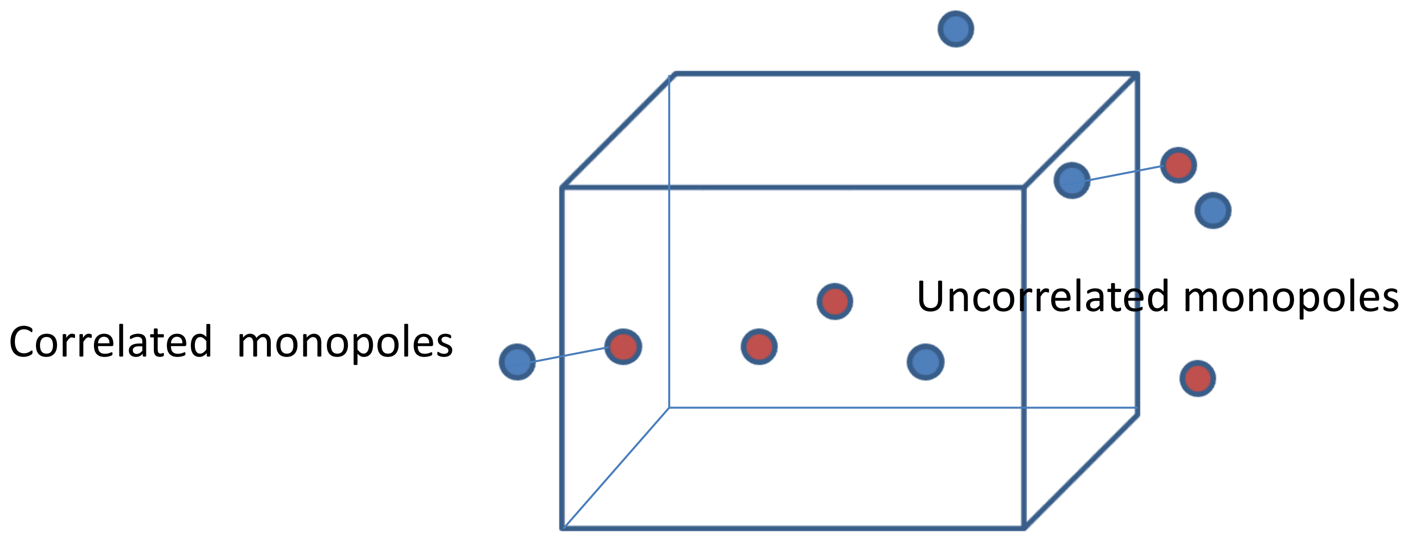
$$e^{i\kappa \int_S H \cdot dS} \neq 1$$

- For a general gauge group

$$W_S = Tr\left(P e^{ig \int_S F d\sigma}\right)$$



Monopoles distribution



Random walk argument:

- Spatial volume V with surface S

1) Uncorrelated monopoles and antimonopoles distributed in the volume:

$$n_M \propto V \qquad \frac{1}{2\pi}\delta B = n_M - n_{\overline{M}} \propto \sqrt{n_M} \propto \sqrt{V}$$

Volume law

2) Monopole is bound to antimonopole. The pairs broken by the surface contribute only:

$$n_D \propto S \qquad \frac{1}{2\pi}\delta B = n_D - n_{\overline{D}} \propto \sqrt{n_D} \propto \sqrt{S}$$

M.Teper, 1986

Area law

Geometry argument: from surface to lines

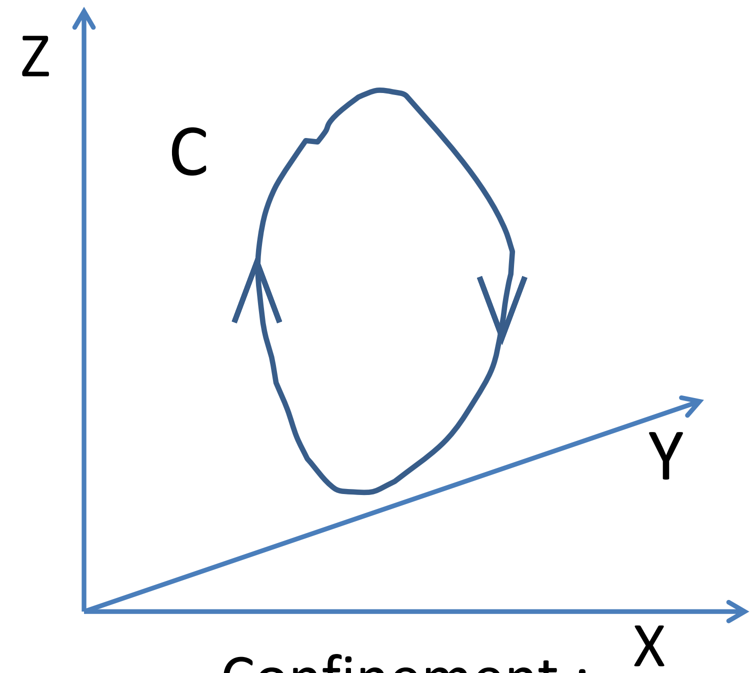
- In the limit β **-> 0** the 4D SYM theory reduces to a pure (non- supersymmetric) three-dimensional Yang-Mills theory on **S.**
- In this limit, a temporal surface operator turns into a line operator (supported on γ) in the 3D theory.
- Therefore, surface operators in the four-dimensional gauge theory exhibit volume (resp. area) law whenever the corresponding line operators in the 3D theory exhibit area (resp. circumference) law.

S. Gukov, E. Witten

Line operator

- Wilson loop operator:

$$W_C = Tr\left(Pe^{ig\oint_C A dc}\right)$$



Deconfinement:

Area law:

$$W_C \propto e^{-\sigma S(C)}$$

Confinement :

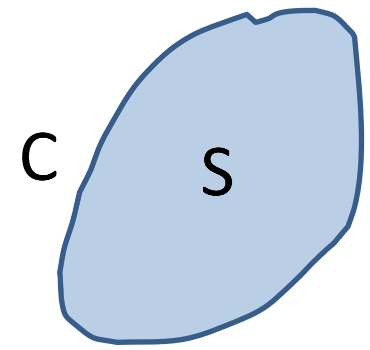
Perimeter law:

$$W_C \propto e^{-kp(C)}$$

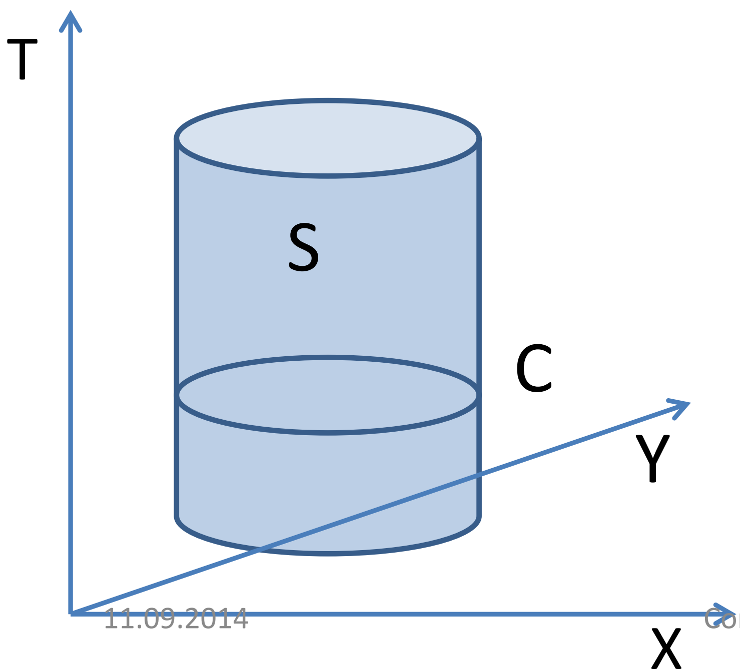
Line – Surface operators correspondence

- Stokes theorem:

$$Tr\left(Pe^{ig\oint_{C} A\,dc}\right) = Tr\left(Pe^{ig\int_{S} F\,d\sigma}\right)$$



- In general case:



Spatial Wilson loop: perimeter law

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Temporal surface operator: area law

Motivations

Spatial surface:

- Can be sensitive to monopole condensate
- Can distinguish correlated and uncorrelated monopoles and antimonopoles
- Another order parameter

Temporal surface

- Additional probe for the theory phase states and nonlocal objects dynamics

Calculation within SU(2) LQCD

- Surface operator on lattice:

$$W_S = Re\, S \prod e^{\imath\theta_p} \qquad \theta_p = g \int_S F d\sigma$$

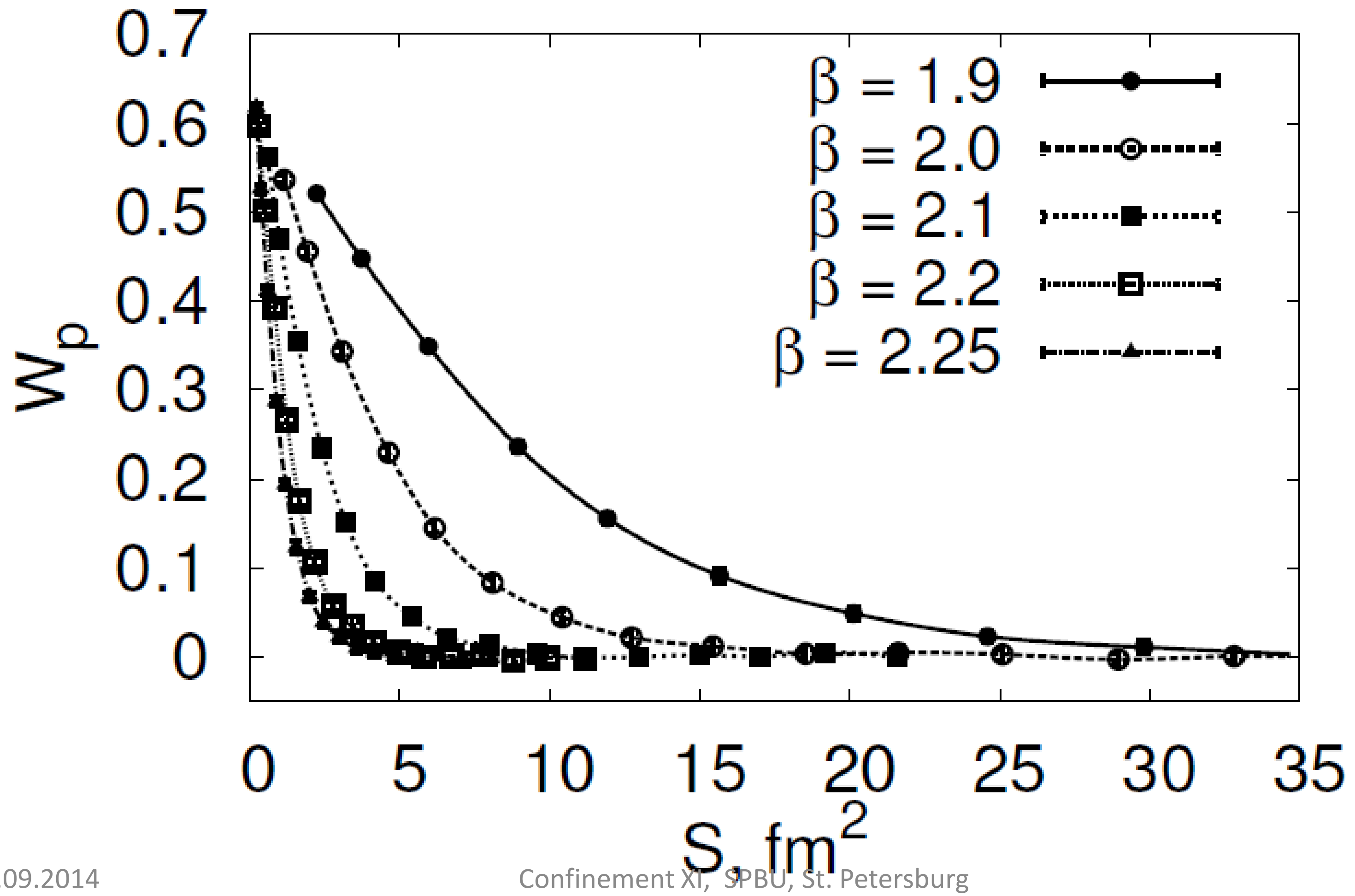
- SU(2) pure gauge:

$$F_p = \widehat{1} \cos\theta_p + \imath\, n_i \sigma_i \sin\theta_p$$

$$\theta_p = \arccos\left(\frac{1}{2} Tr\, F_p\right)$$

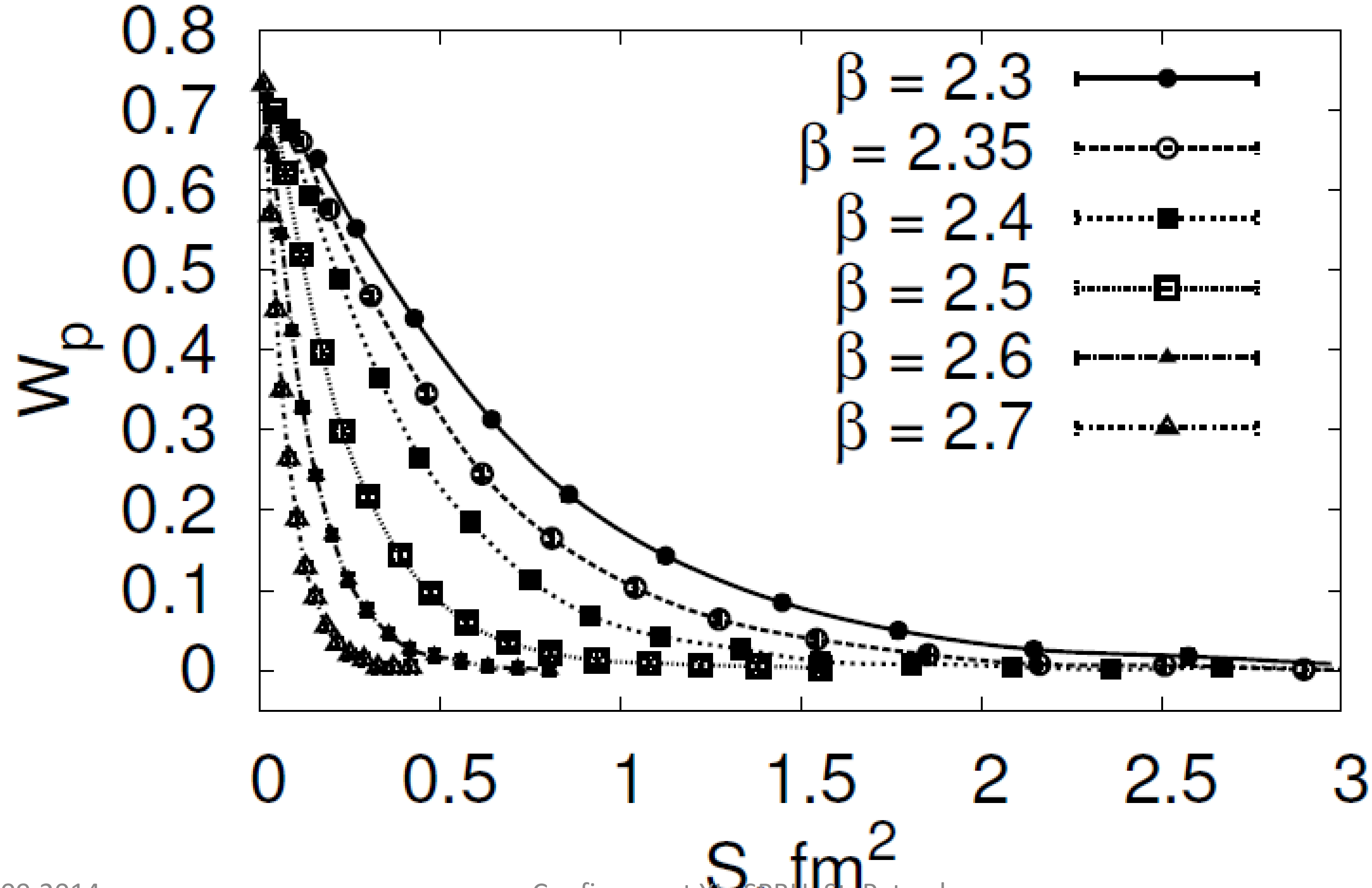
Surface operator fit (confinement)

Lattice size 4x40^3



Surface operator fit (deconfinement)

Lattice size 4x40^3



Surface operator calculation

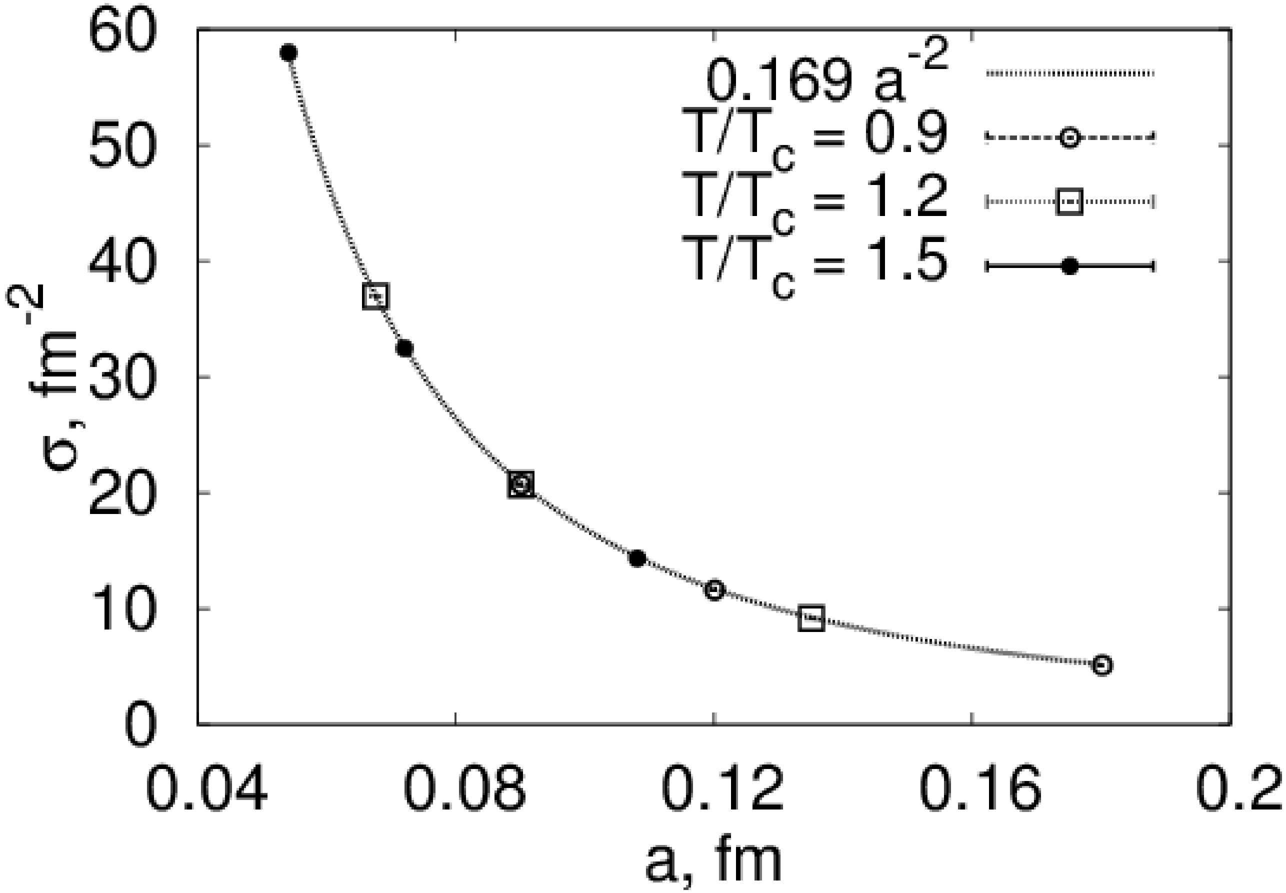
Surface operator parameterization:

$$W_S = Ce^{-\sigma S - \gamma V}$$

$$\sigma(a,T) = \sigma_{ph}(T) + \sigma_{div}(a,T)$$

$$\gamma(a,T) = \gamma_{ph}(T) + \gamma_{div}(a,T)$$

Area law: divergent part

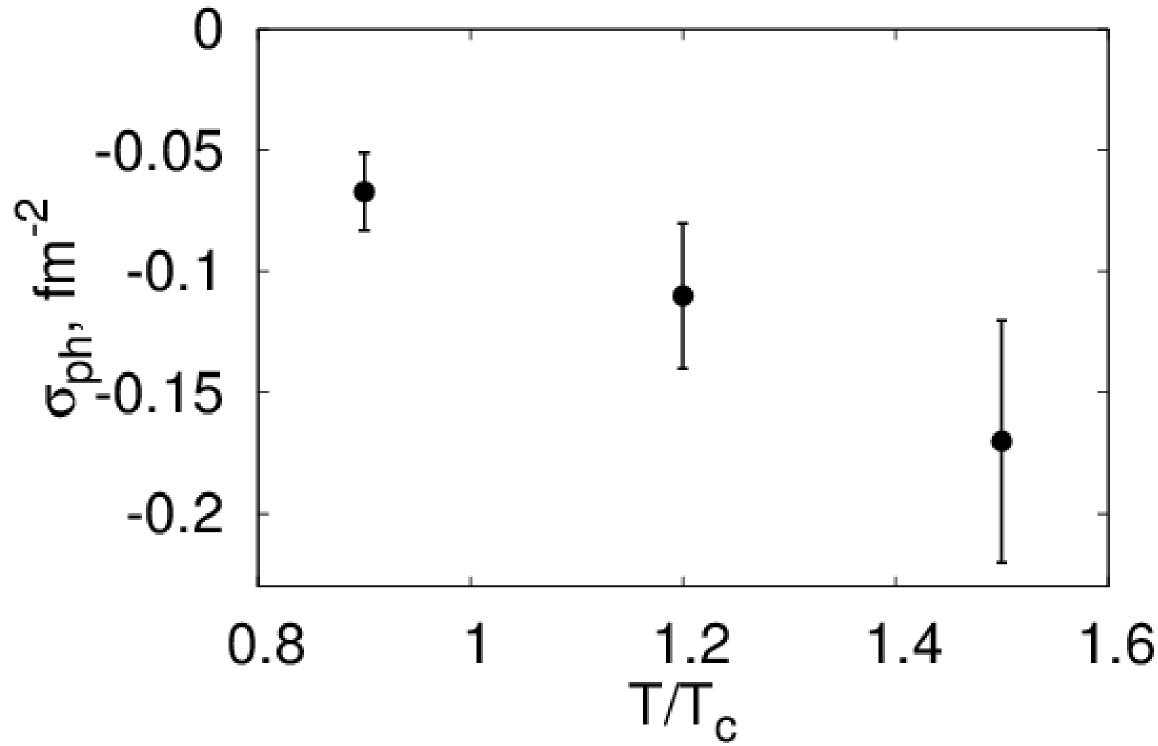


$$\sigma_{div} = \frac{\sigma_1}{a^2}$$

$$\sigma_1 = 0.1685 \pm 0.0005$$

- No temperature dependence of the divergent part

Area law: physical part

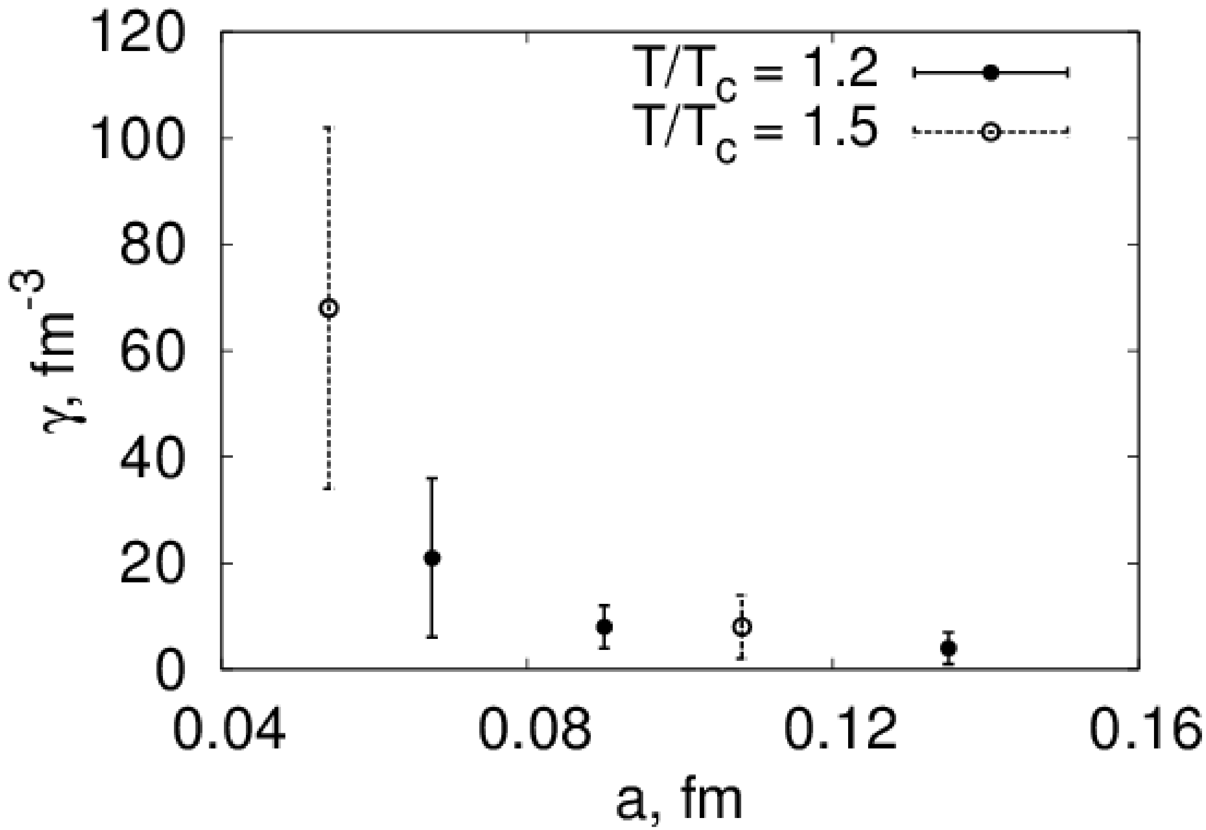


$$\sigma_{ph}(T) \approx -\sigma_0 \frac{T^2}{T_C^2}$$

$$\sigma_0 \approx 0.077\, fm^{-2}$$

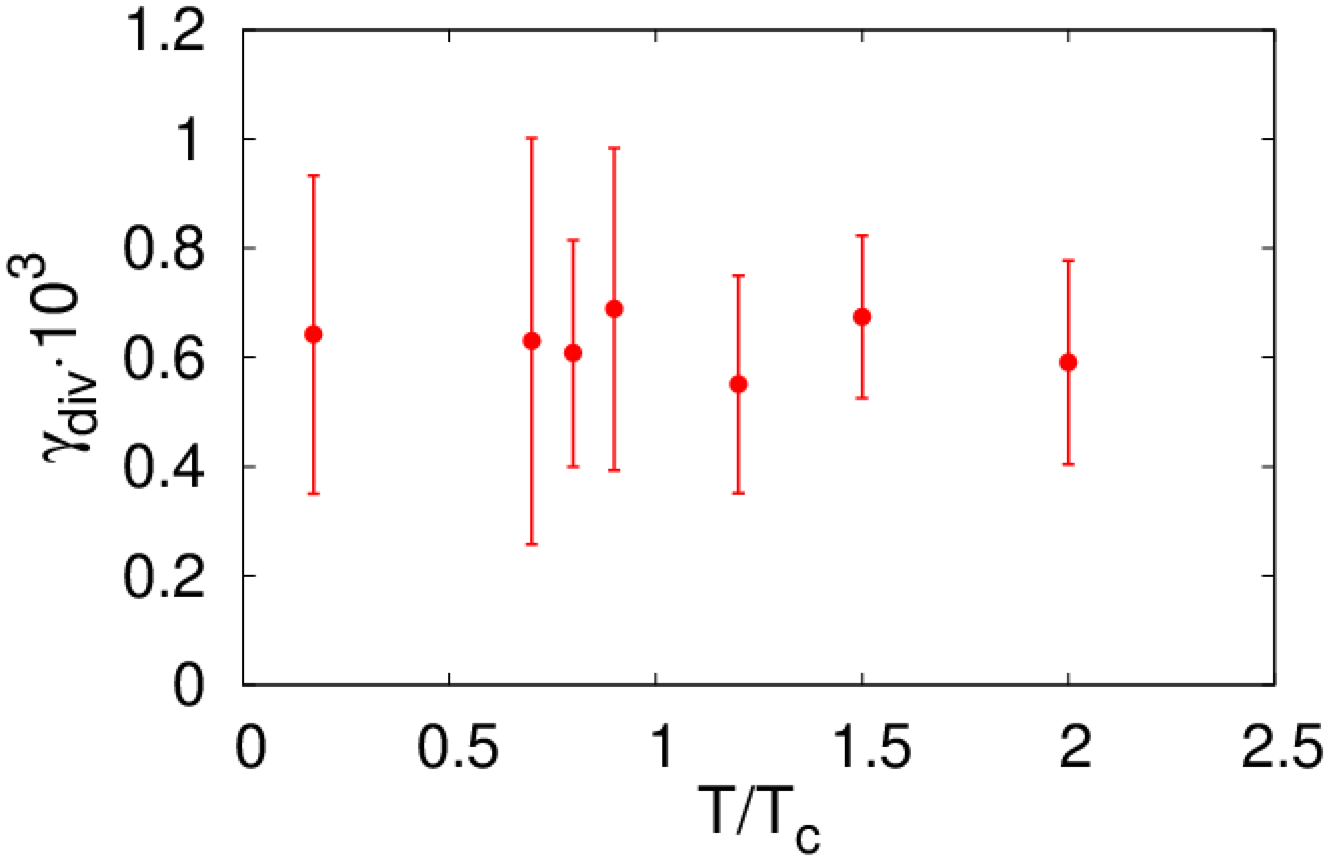
Smooth behavior at the phase transition point:
There is the color dipole condensate in both phases

Volume law: divergent part



$$\gamma_{div} = \frac{\gamma_1}{a^3}$$

Volume law: divergent part



$$\gamma_{div} = \frac{\gamma_1}{a^3}$$

$$\gamma_1 \approx 0.0006$$

Volume law: physical part

$$\frac{T}{T_c} = 0.9$$

$$\gamma_{ph} \approx 7.47\ fm^{-3}$$

$$\frac{T}{T_c} = 1.2, \ldots 2$$

$$\gamma_{ph} \approx 0$$

- An indication that there is uncorrelated monopole condensation in the confinement phase

Surface operator calculation

Surface operator parameterization:

$$W_S = Ce^{-\sigma S - \gamma V}$$

$$\sigma(a,T) = \sigma_{ph}(T) + \sigma_{div}(a,T)$$

Confinement and deconfinement:

$$\sigma(a,T) = -\sigma_0 \frac{T^2}{T_C^2} + \frac{\sigma_1}{a^2}$$

$$\gamma(a,T) = \gamma_{ph}(T) + \gamma_{div}(a,T)$$

Confinement:

$$\gamma(a,T) = \gamma_0 + \frac{\gamma_1}{a^3}$$

Deconfinement:

$$\gamma(a,T) = \frac{\gamma_1}{a^3}$$

Conclusion

- The spatial surface operator exhibits area law in confinement and deconfinement phases.
- The magnetic field flow grows with surface area grow, what corresponds the random walk argument.
- The magnetic field flow grows with temperature. (Dipole density grows with T)
- There is an indication that the spatial surface operator exhibits volume law in the confinement phase (uncorrelated monopoles condensation?). Needs more statistics for a definite conclusion.