Lattice QCD study of strangeness S=-2 two-baryon system

Kenji Sasaki (*CCS, University of Tsukuba*)

for HAL QCD collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

S. Aoki (*YITP*)	**B. Charron** (*Univ. of Tokyo*)	**T. Doi** (*RIKEN*)	**F. Etminan** (*Univ. of Birjand*)	**T. Hatsuda** (*RIKEN*)
Y. Ikeda (*RIKEN*)	**T. Inoue** (*Nihon Univ.*)	**N. Ishii** (*RCNP*)	**K. Murano** (*RCNP*)	**H. Nemura** (*Univ. of Tsukuba*)
M. Yamada (*Univ. of Tsukuba*)	**V. Krejcirik** (*RIKEN*)			

Contents

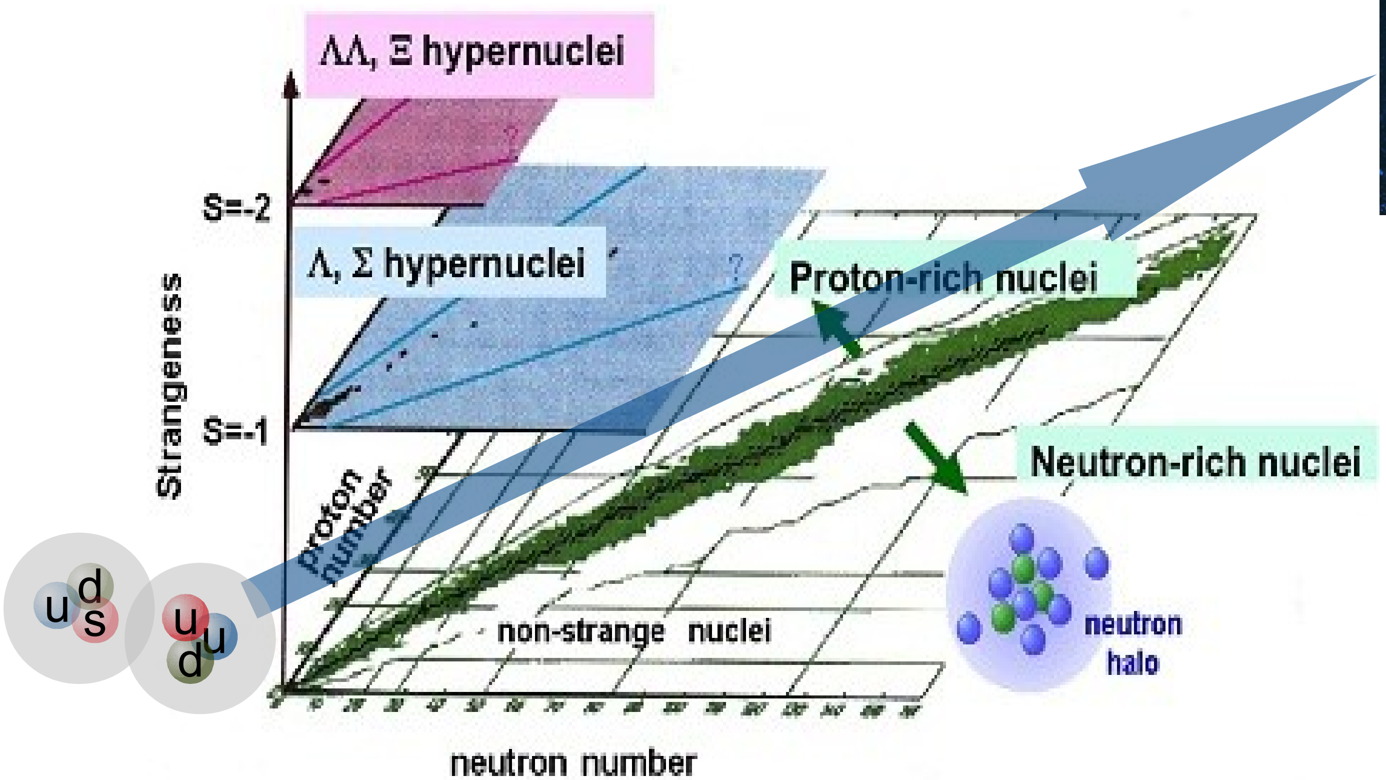
- Introduction
- HAL QCD strategy
- S=-2 Baryon-Baryon potential
  - in SU(3) limit
  - with SU(3) breaking
- Summary and Outlook

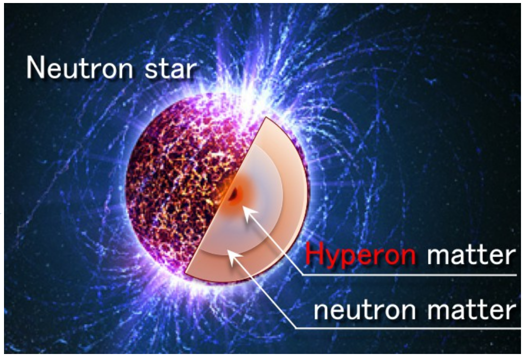
Introduction

Introduction

BB interactions are inputs for nuclear structure, astrophysical phenomena

Once we obtain a “proper” nuclear potential,
we apply them to the structure of (hyper-) nucleus
and neutron star calculation.





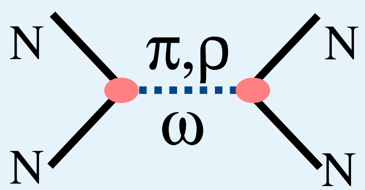
How do we obtain the nuclear force?

Introduction

Approaches to baryon-baryon interactions

Meson exchange model

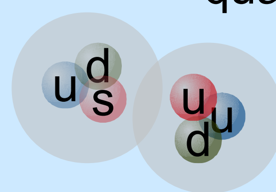
Described by hadron dof



Repulsion is generated
by ω meson exchange and
phenomenological repul. core

Quark cluster model

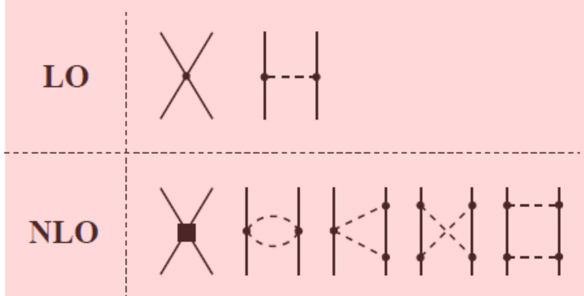
Effective meson ex
+ quark anti-symmetrization



Quark Pauli effects
Color magnetic int.
are taking into account

Effective Field theory

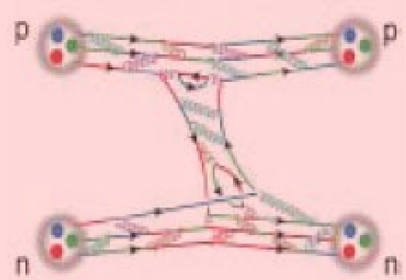
Systematic calc. respecting with
symmetry of QCD



Short range int.
parametrized by
contact term

First principle calculation

Direct derivation of nuclear force
from QCD



Particularly
difficult problem...

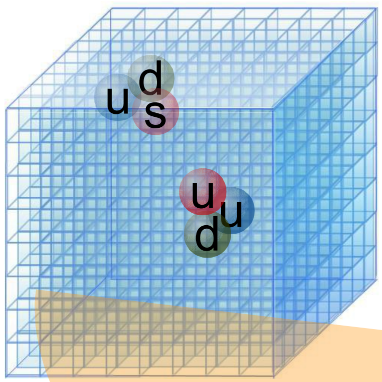
HAL QCD strategy

QCD to hadronic interactions

Start with the fundamental theory,QCD, to obtain a “proper” interaction

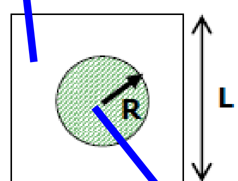
$$L_{QCD}=\bar{q}(i\gamma_\mu D^\mu - m)q + \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Lattice QCD simulation

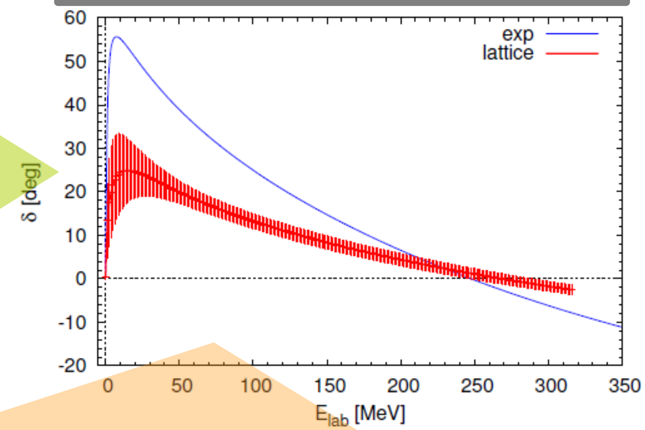


Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

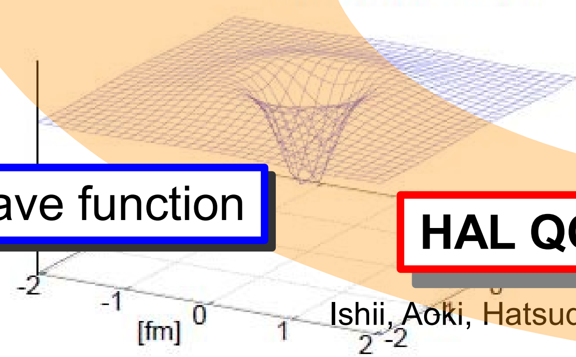


BB scattering phase shift



Guaranteed to be the same

Kurth et al JHEP 1312 (2013) 015

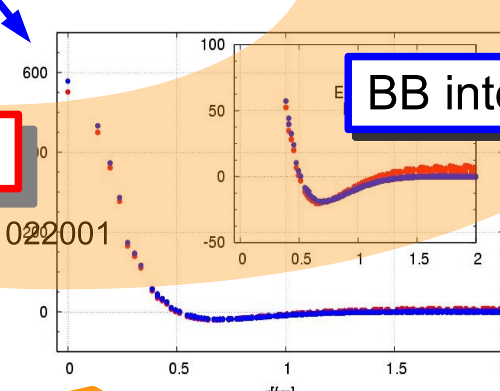


NBS wave function

HAL QCD method

Ishii, Aoki, Hatsuda, PRL**99** (2007) 022001

BB interaction (potential)



The potential is proper for the phase shift by QCD

Coupled channel Schrödinger equation

Preparation for the NBS wave function

Two-channel coupling case

$$\Psi^{\alpha}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0|(B_1 B_2)^{\alpha}(t,\vec{r})|E\rangle$$

$$\Psi^{\beta}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0|(B_1 B_2)^{\beta}(t,\vec{x})|E\rangle$$

- in asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- In interaction region : $(p^2+\nabla^2)\Psi(E,\vec{r})=K(E,\vec{r})$

Inside the interaction range

In the *leading order of velocity expansion* of non-local potential,

Coupled channel Schrödinger equation.

$$\left(\frac{p_\alpha^2}{2\mu_\alpha}+\frac{\nabla^2}{2\mu_\alpha}\right)\psi^{\alpha}(\vec{x},E)=V^{\alpha}{}_{\alpha}(\vec{x})\psi^{\alpha}(\vec{x},E)+V^{\alpha}{}_{\beta}(\vec{x})\psi^{\beta}(\vec{x},E)$$

μ_α : *reduced mass*

p_α : asymptotic momentum.

Asymptotic momentum are replaced by the time-derivative of *R*.

$$R_I^{B_1 B_2}(t,\vec{r})=\sum_{\vec{x}}\langle 0|B_1(t,\vec{x}+\vec{r})B_2(t,\vec{x})\bar{I}(0)|0\rangle e^{(m_1+m_2)t}$$

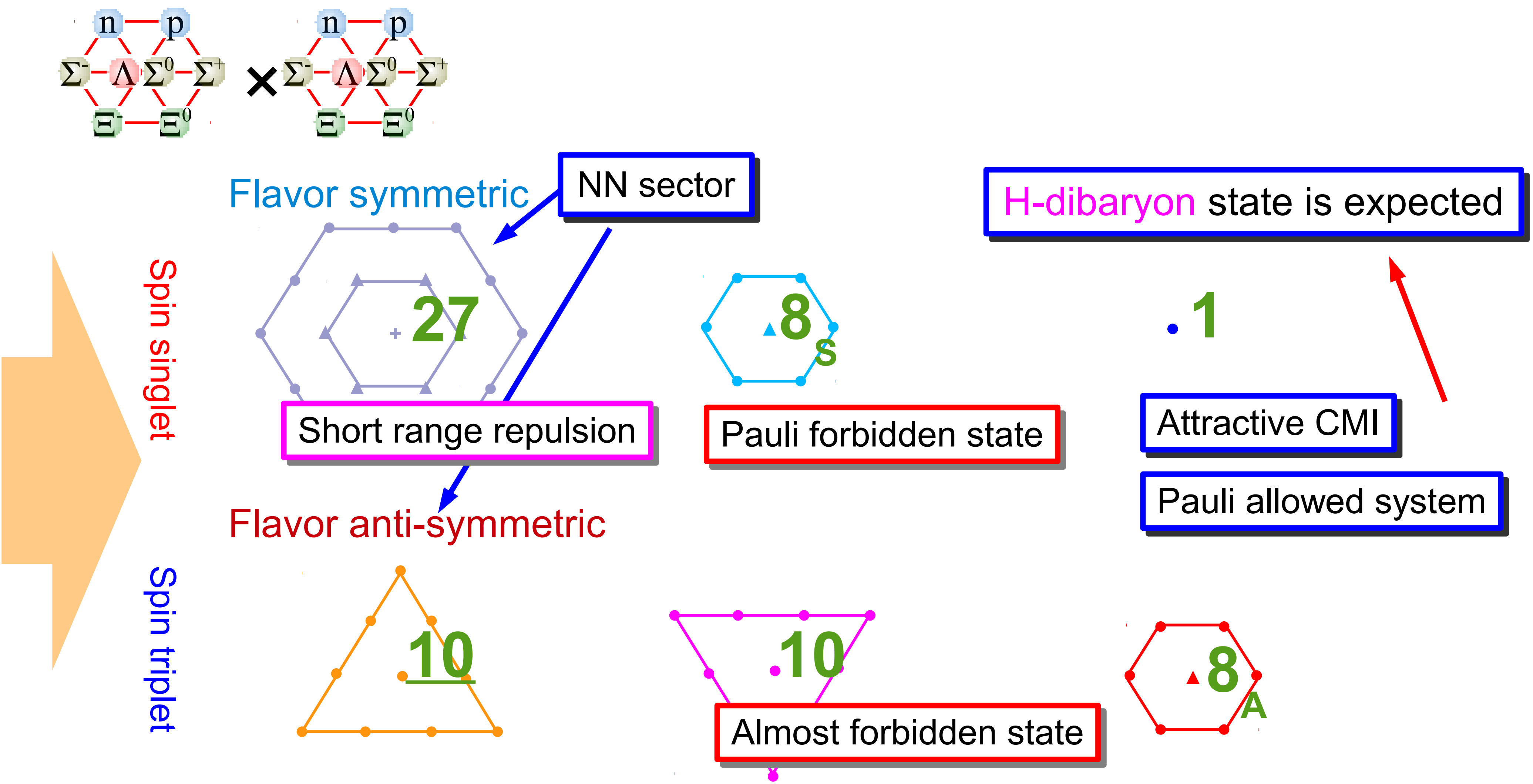
$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})x \\ V^{\beta}{}_{\alpha}(\vec{r})x^{-1} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} (\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t})R^{\alpha}_{I1}(\vec{r},E) & (\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t})R^{\beta}_{I2}(\vec{r},E) \\ (\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t})R^{\alpha}_{I1}(\vec{r},E) & (\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t})R^{\beta}_{I2}(\vec{r},E) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{I1}(\vec{r},E) & R^{\beta}_{I1}(\vec{r},E) \\ R^{\alpha}_{I2}(\vec{r},E) & R^{\beta}_{I2}(\vec{r},E) \end{pmatrix}^{-1}$$

$$x=\frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

Strangeness S=-2 BB potential
in SU(3) limit

SU(3) classification for BB interaction

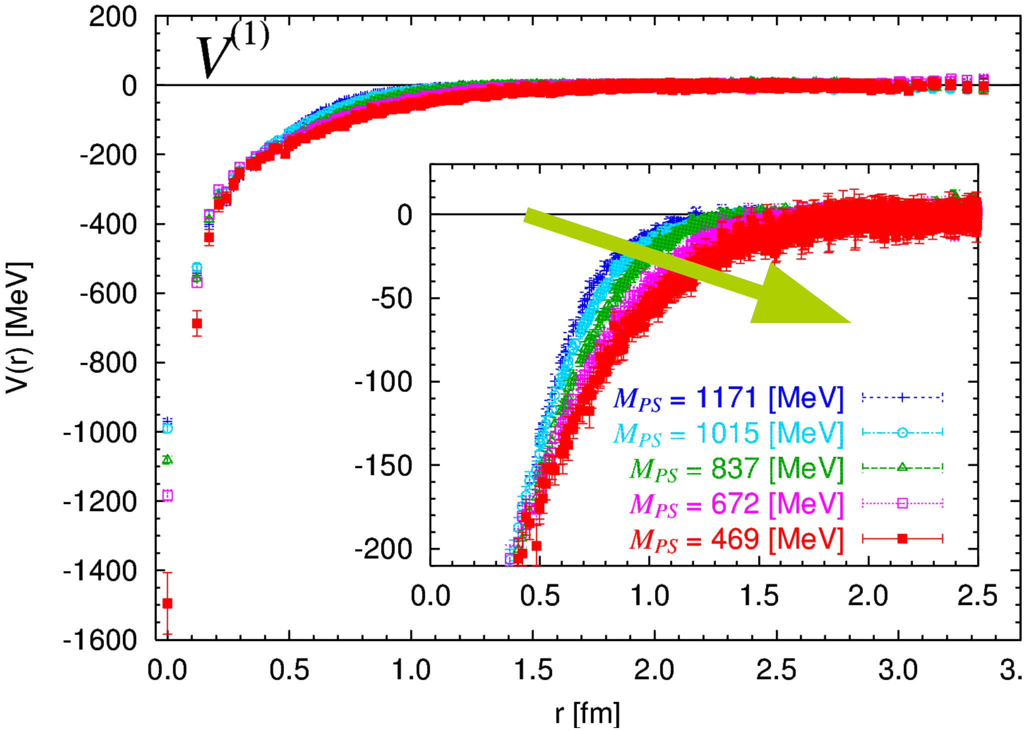
Features of two-octet baryon system

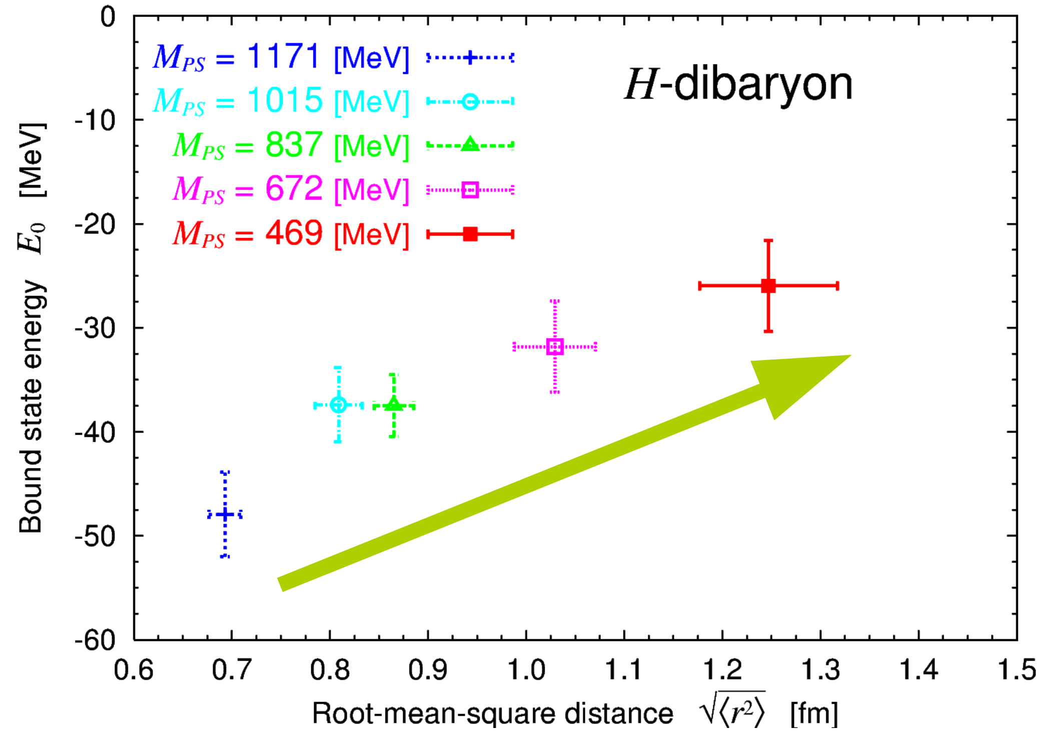


Strangeness brought us deeper understanding of BB interaction.

Looking for H-dibaryon in SU(3) limit

- 3 flavor gauge configurations with L = 3.872 [fm]..
- Flat wall source is considered to produce S-wave B-B state.





- There is a 6q bound state in this mass range with SU(3) symmetry.

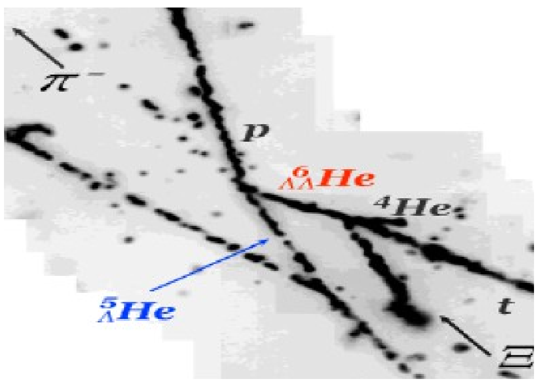
Growth of kinetic energy of baryon pair could be quicker than enhancement of attraction.

Recent results for H-particle

Conclusions of the "NAGARA Event"

K.Nakazawa and KEK-E176 & E373 collaborators

PRL87(2001)212502



Λ–N attraction

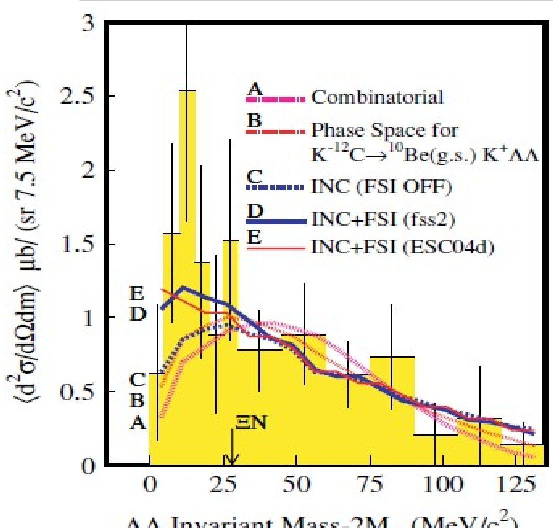
Λ–Λ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV

Conclusions of $^{12}C(K^-,K^+\Lambda\Lambda)$ reaction

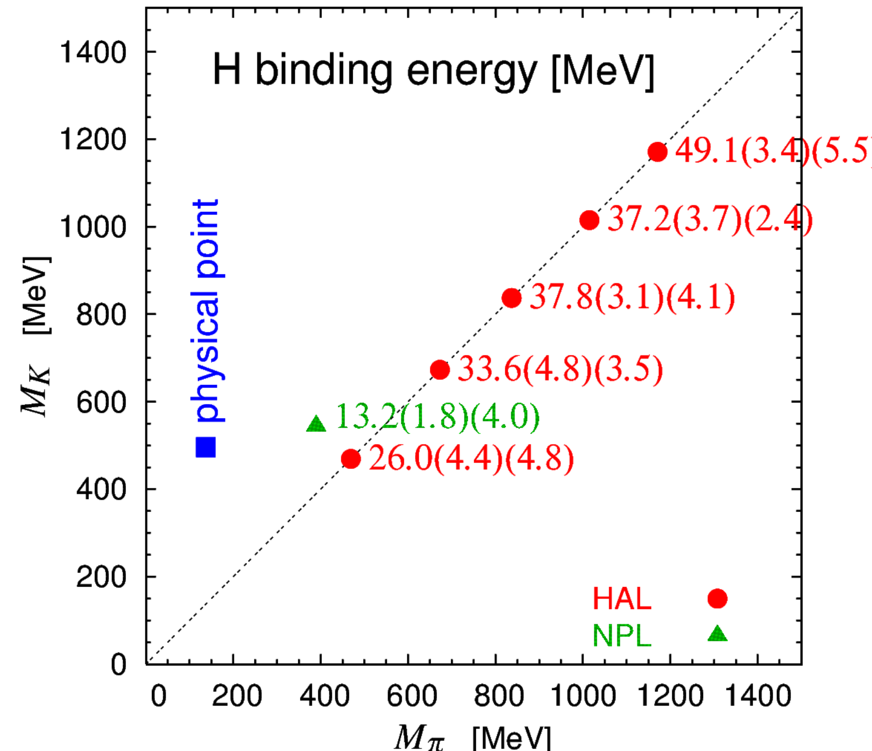
C.J.Yoon and KEK-PS E522 collaborators

PRC75(2007)022201(R)



the enhancement
was found below 30 MeV

Lattice QCD simulations



**At the physical point
H-dibaryon resonance??**

Beyond SU(3) simulation is necessary
toward the physical point !!

Strangeness S=-2 BB potential

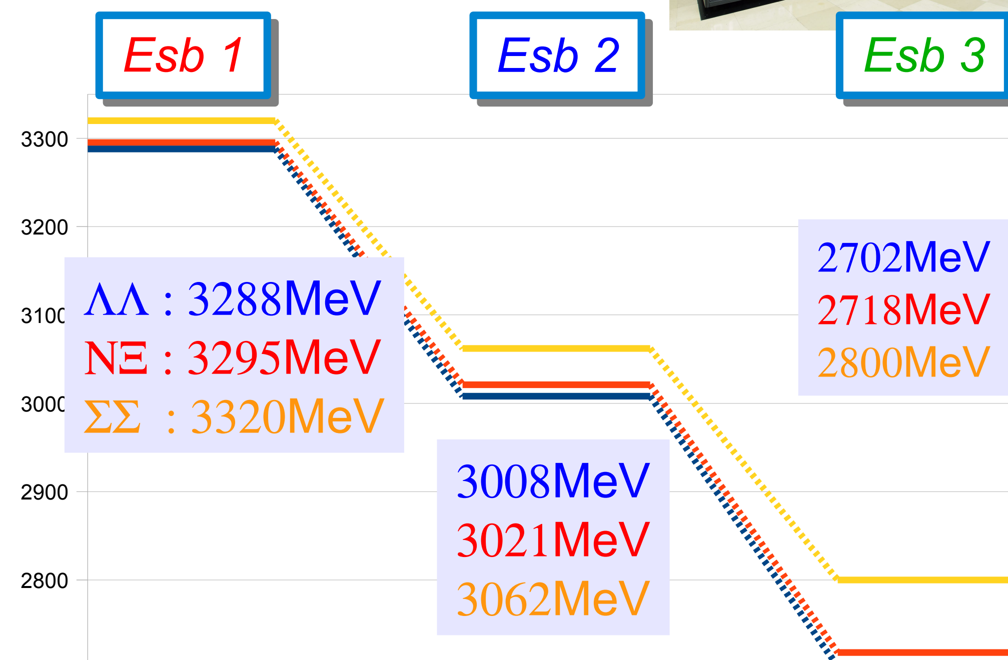
beyond SU(3) limit

Numerical setup

- 2+1 flavor gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved Wilson quark action
  - β = 1.90, a^{-1} = 2.176 [*GeV*], 32^3x64 lattice, *L* = 2.902 [*fm*].
  - κ_s = 0.13640 is fixed, κ_{ud} = 0.13700, 0.13727 and 0.13754 are chosen.
- Flat wall source is considered to produce S-wave B-B state.
- The KEK computer system A resources are used.

In unit of MeV	*Esb 1*	*Esb 2*	*Esb 3*
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

u,d quark masses lighter →

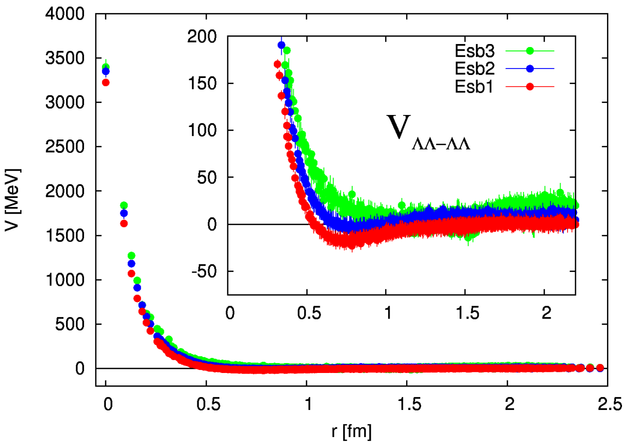


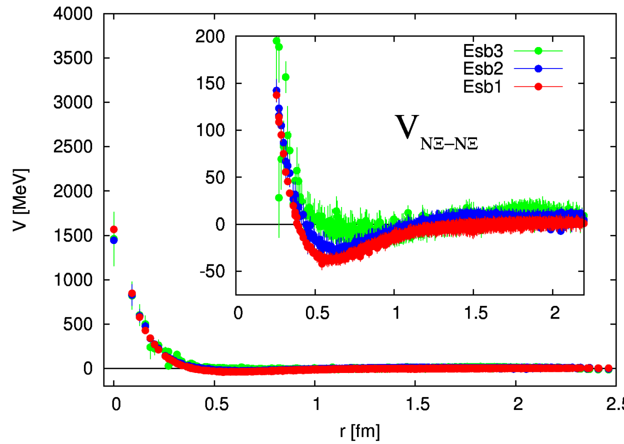
SU(3) breaking effects becomes larger →

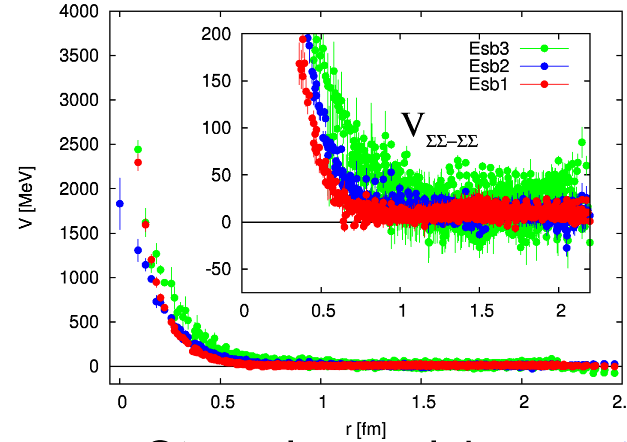
$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel

Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

Diagonal elements



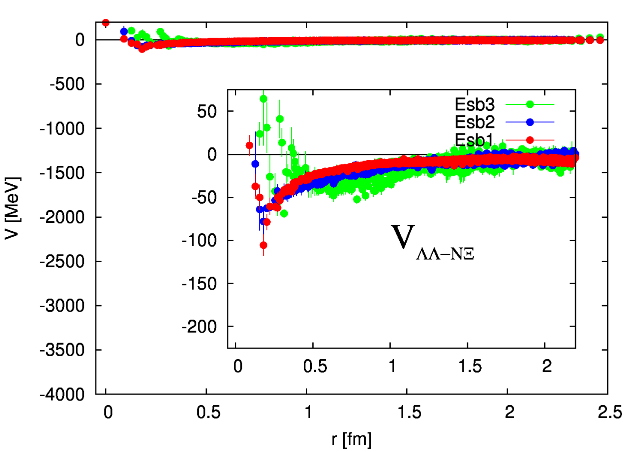


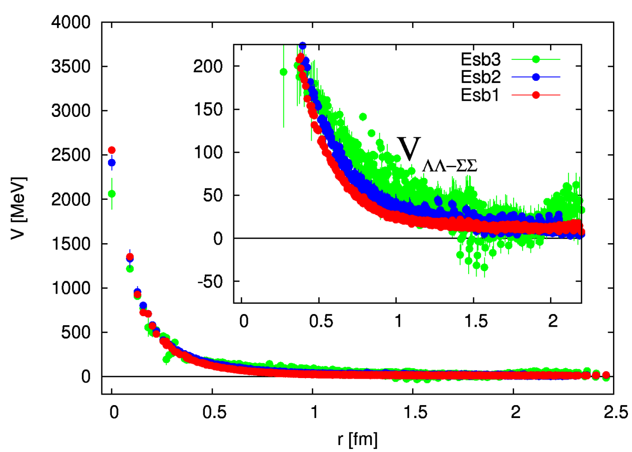


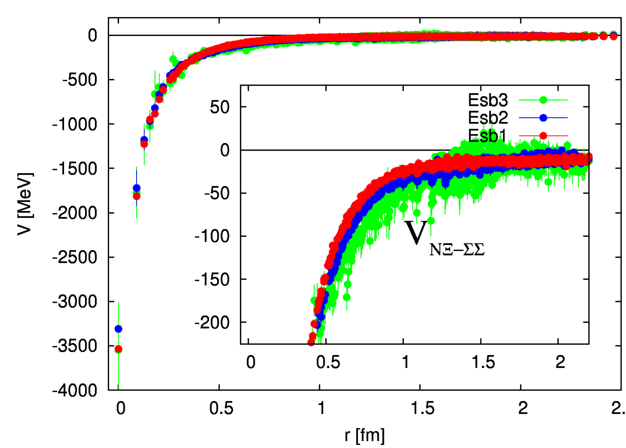
shallow attractive pocket — Deeper attractive pocket — Strongly repulsive

All channels have repulsive core

Off-diagonal elements







In this channel, our group found the "H-dibaryon" in the SU(3) limit.

Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (irrep) basis.

$$\left|\begin{array}{c}\mathbf{1}\\ \mathbf{8}_s\\ \mathbf{27}\end{array}\right\rangle = \begin{pmatrix} -\sqrt{\frac{1}{8}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{3}{8}} \\ -\sqrt{\frac{1}{5}} & \sqrt{\frac{1}{5}} & -\sqrt{\frac{3}{5}} \\ \sqrt{\frac{27}{40}} & \sqrt{\frac{3}{10}} & -\sqrt{\frac{1}{40}} \end{pmatrix} \left|\begin{array}{c}\Lambda\Lambda\\ N\Xi\\ \Sigma\Sigma\end{array}\right\rangle$$

SU(3) Clebsh-Gordan coefficients

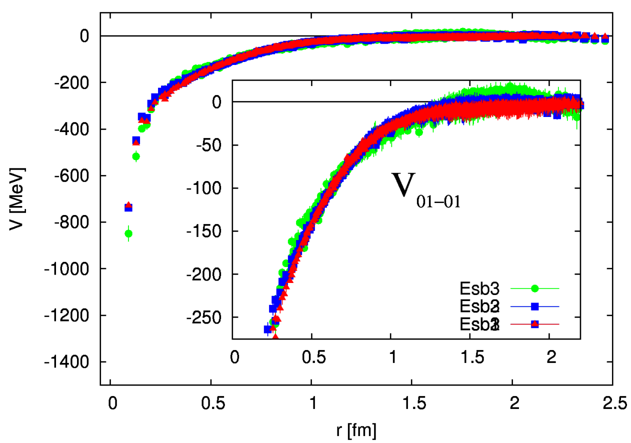
$$U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \rightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

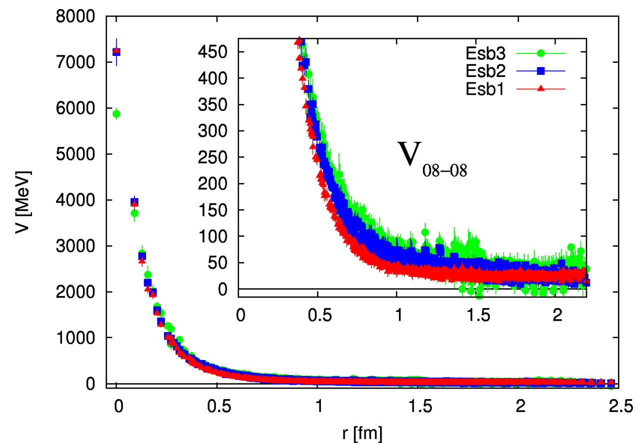
In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

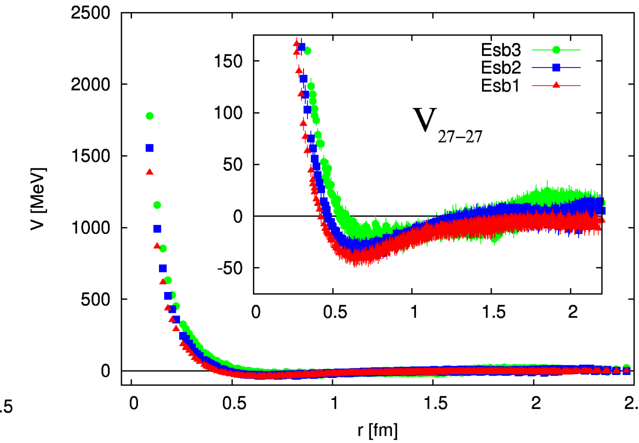
Off-diagonal part of the potential matrix in the SU(3) irrep basis
would be an effectual measure of the SU(3) breaking effect.

1, 8_s, 27 (I=0) $^1S_0$ channel

Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

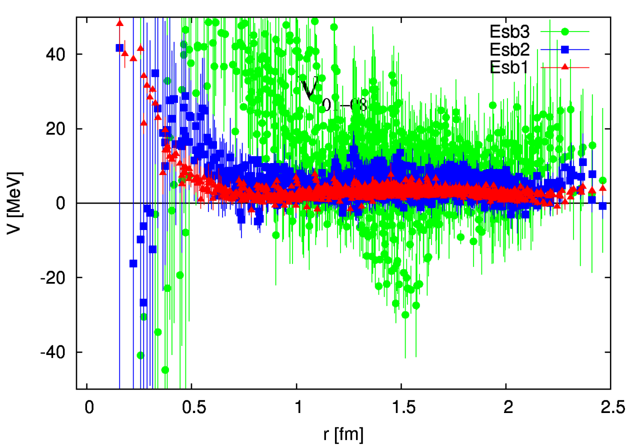


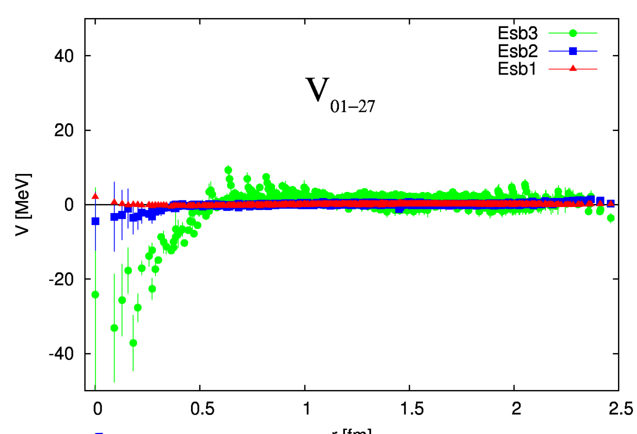


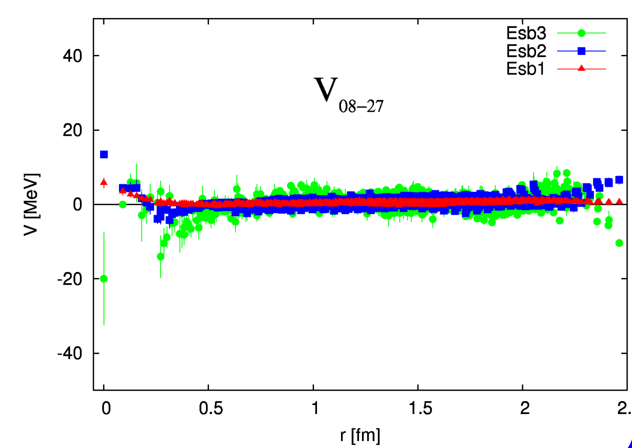


Strongly attractive
H-dibaryon channel

Pauli blocking effect





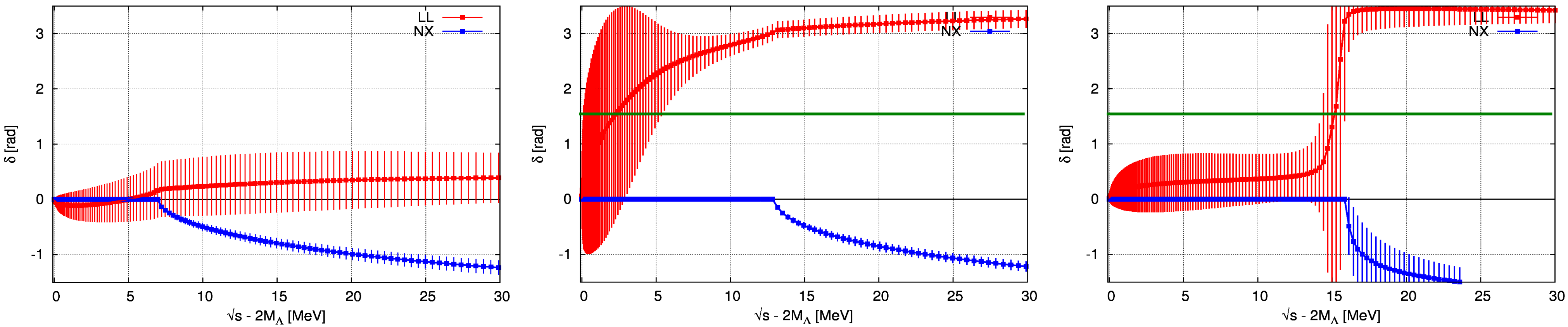


Mixture of singlet and octet
Is relatively larger than the others

27 plet does not mix so much to the other representations

$\Lambda\Lambda$ and $N\Xi$ phase shifts

Esb1 : m_π= 701 MeV | **Esb2 : m_π= 570 MeV** | **Esb3 : m_π= 411 MeV**



Preliminary!

- Esb1:
  - Bound H-dibaryon similar to the SU(3) symmetric case.
- Esb2:
  - H-dibaryon is near the $\Lambda\Lambda$ threshold.
- Esb3:
  - The H-dibaryon resonance energy is close to $N\Xi$ threshold..

- We can see the clear resonance shape in $\Lambda\Lambda$ phase shifts for Esb2 and 3.
- The energy of H-dibaryon becomes closer to the $N\Xi$ threshold as decreasing of quark masses.

Summary and outlook

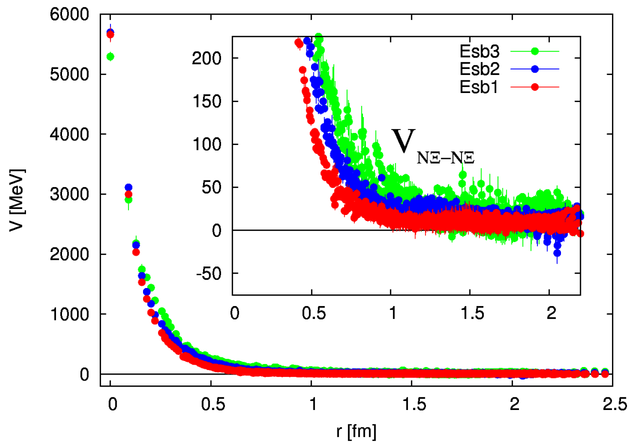
- We have investigated S=-2 BB interactions focusing on the H-dibaryon channel
- Quark mass dependence of potentials can be seen not in long range region but in shot distances as an enhancement of repulsive core.
- Small mixture between different SU(3) irreps can be seen as the flavor SU(3) breaking effect.
- We found that the energy of H-dibaryon becomes closer to the $N\Xi$ threshold as decreasing ud-quark masses.
- SU(3) breaking effects are still small even in m_π/m_K=0.65 situation but it would be change drastically at physical situation m_π/m_K=0.28.

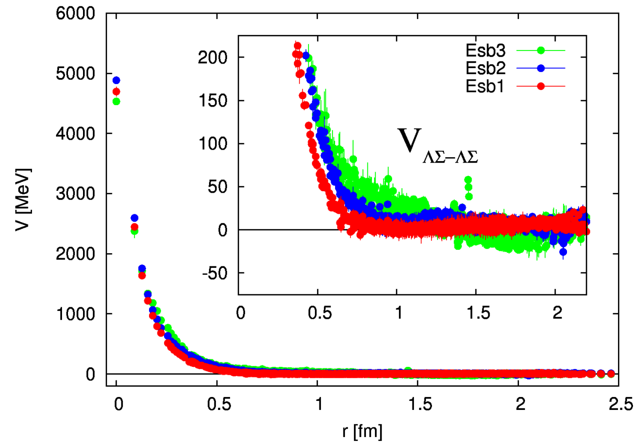
Backup slides

$N\Xi, \Lambda\Sigma$ (I=1) $^1S_0$ channel

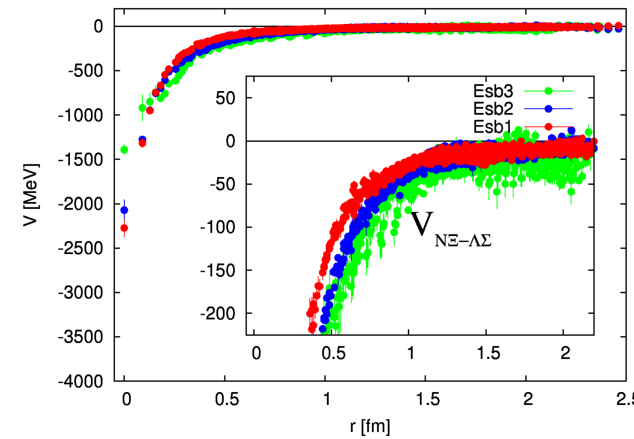
Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

Diagonal elements

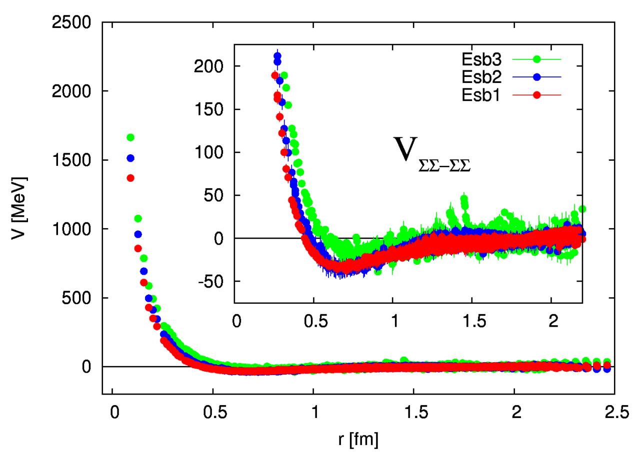




Off-diagonal elements

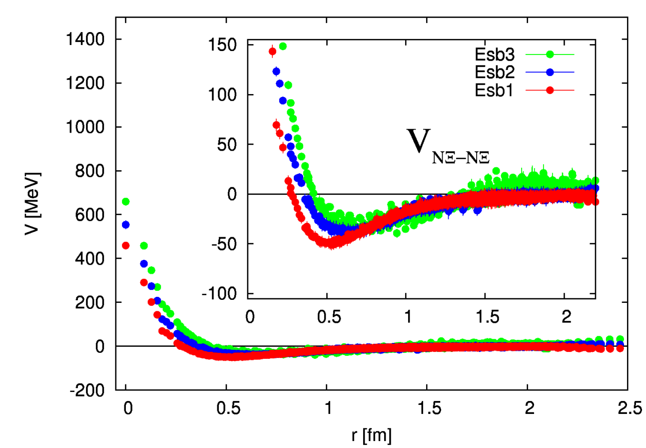


$\Sigma\Sigma$ (I=2) $^1S_0$ channel



Potential shape is similar to NN potential

$N\Xi$ (I=0) $^3S_1$ channel

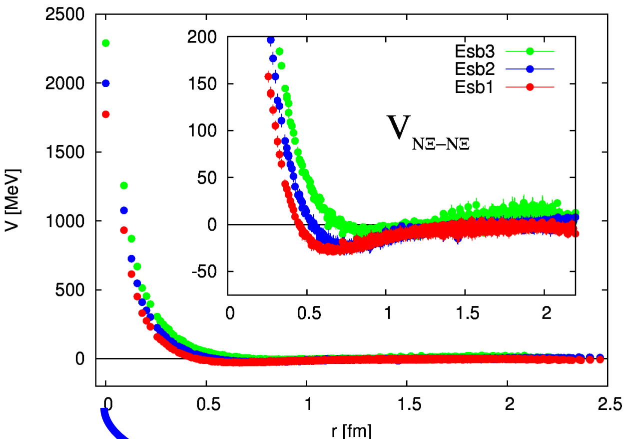


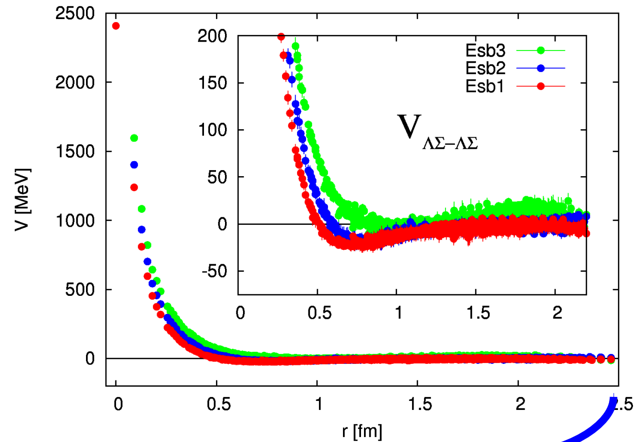
Small repulsive core
Deep attractive pocket

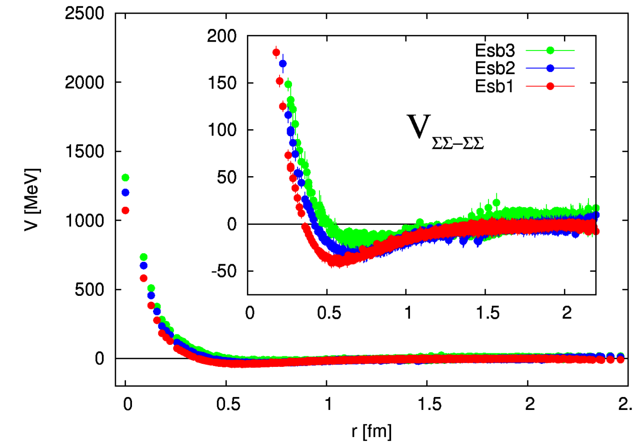
$N\Xi$, $\Sigma\Sigma$, $\Lambda\Sigma$ (I=1) $^3S_1$ channel

Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

Diagonal elements





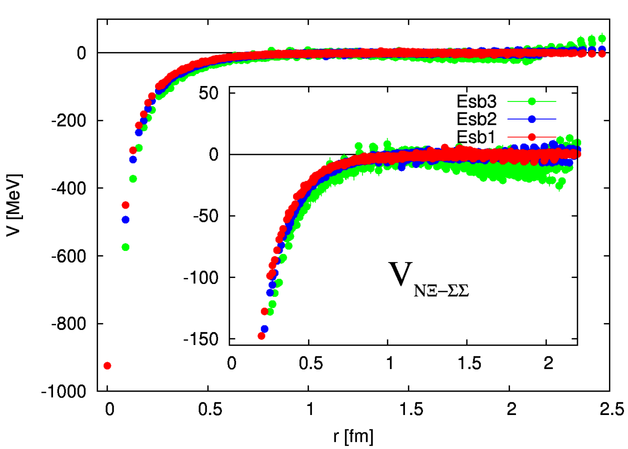


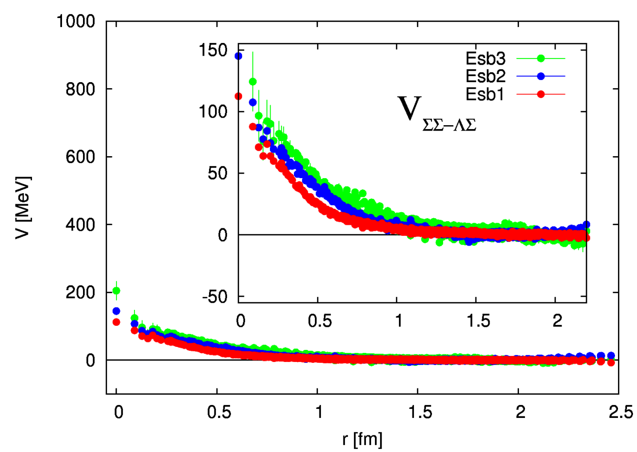
Most attractive

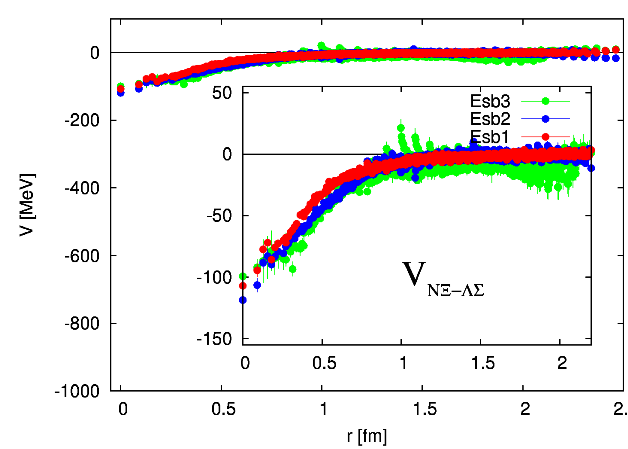
Attractive pocket becomes shallower as a lighter quark mass

All channels have repulsive core

Off-diagonal elements







Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi(E,\vec{r})e^{-E(t-t_0)} = \sum_{\vec{x}} \langle 0|B_i(t,\vec{x}+\vec{r})B_j(t,\vec{x})|E,\nu,t_0\rangle$$

E : Total energy of system

Four point correlator

$$F_{B_1B_2}(\vec{r},t)=\langle 0|T\left[B_1(\vec{r},t)B_2(0,t)(\bar{B}_2\bar{B}_1)_{t_0}\right]|0\rangle=\sum_n A_n\Psi(E_n,\vec{r})e^{-E_n t}$$

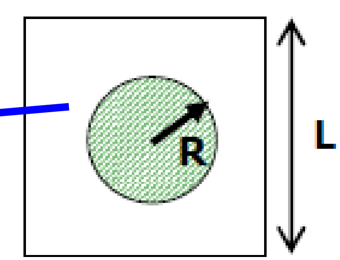
- It satisfies the Helmholtz eq in asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- In interaction region : $(p^2+\nabla^2)\Psi(E,\vec{r})=K(E,\vec{r})$

Local composite interpolating operators

$$B=\epsilon^{abc}(q_a^T C\gamma_5 q_b)q_c \quad \begin{cases} p=udu \quad n=udd \quad \Xi^0=sus \quad \Xi^-=sds \\ \Lambda=\sqrt{\frac{1}{6}}[dsu+sud-2uds] \\ \Sigma^+=-usu \quad \Sigma^0=-\sqrt{\frac{1}{2}}[dsu+usd] \quad \Sigma^-=-dsd \end{cases}$$

NBS wave function has a same asymptotic form with quantum mechanics.
(NBS wave function is characterized from phase shift)

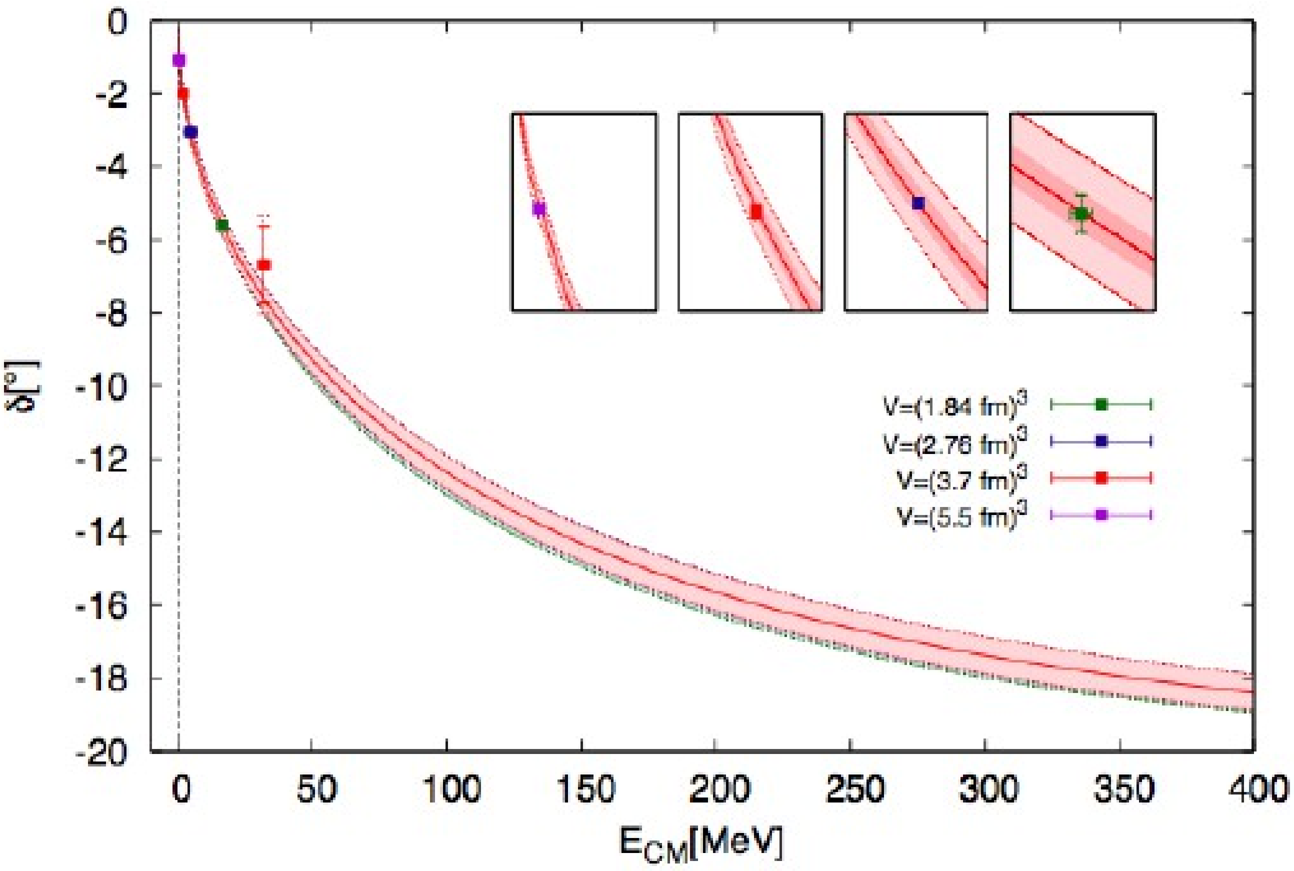
$$\Psi(E,\vec{r})\simeq A\frac{\sin(pr+\delta(E))}{pr}$$



Nuclear Force from Lattice QCD

Comparison between the potential method and Lueshcer's method.

ππ scattering in I = 2 channel



Ns=16,24,32,48, Nt=128, a=0.115

m_{pi} = 940 MeV by Quenched QCD

Kurth et al., arXiv:1305.4462[hep-lat]