Hadron structure from lattice QCD

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Lattice QCD

…is a regularization of Euclidean-space QCD such that the path integral can be done fully non-perturbatively.

- Euclidean spacetime becomes a periodic hypercubic lattice, with spacing a and box size $L_s^3 \times L_t$.
- Path integral over fermion degrees of freedom is done analytically, for each gauge configuration. Solving the Dirac equation with a fixed source yields a source-to-all quark propagator.
- Path integral over gauge degrees of freedom is done numerically using Monte Carlo methods to generate an *ensemble* of *gauge configurations*.
- Various lattice Dirac operators have trade-offs: e.g., domain wall (overlap) fermions have an approximate (exact) chiral symmetry at finite lattice spacing, but are more computationally expensive than Wilson fermions.

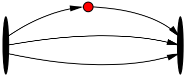
The $a \to 0$ and $L_s, L_t \to \infty$ extrapolations need to be taken by using multiple ensembles.

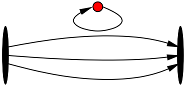
Hadron matrix elements using lattice QCD

To find (forward) matrix elements, compute $C_{2\text{pt}}(t) = \langle N(t)\bar{N}(0)\rangle$ and

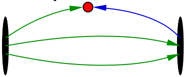
$$C_{3\text{pt}}(T,\tau) = \langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle$$
$$\xrightarrow{L_t\to\infty} \sum_{n,n'} e^{-E_n\tau} e^{-E_{n'}(T-\tau)} \langle 0|N|n'\rangle\langle n'|\mathcal{O}|n\rangle\langle n|\bar{N}|0\rangle$$
$$\xrightarrow[T-\tau\to\infty]{\tau\to\infty} e^{-m_N T} |\langle N|\bar{N}|0\rangle|^2 \langle N|\mathcal{O}|N\rangle$$

For $\mathcal{O}$ a quark bilinear, there are two kinds of quark contractions for $C_{3\text{pt}}$:





Most of the focus has been on nucleon isovector observables, which have no contribution from disconnected diagrams. These are typically computed using the sequential propagator method:



1. Fix source; compute forward propagator.
2. Fix sink and T; compute backward (sequential) propagator.
3. Combine the two to compute many different $\mathcal{O}$, for all $\tau \in [0, T]$.

Quark disconnected diagrams

$$T(\vec{q}, t, \Gamma) \equiv \sum_{\vec{x}} e^{i\vec{q}\cdot\vec{x}} \langle \bar{q}\Gamma q(x) \rangle = -\sum_{\vec{x}} e^{i\vec{q}\cdot\vec{x}} \,\mathrm{Tr}[\Gamma D^{-1}(x,x)]$$

Estimate the all-to-all propagator stochastically using noise sources η that satisfy $E(\eta\eta^\dagger) = I$. By solving $\psi = D^{-1}\eta$, we get

$$D^{-1}(x,y) = E(\psi(x)\eta^\dagger(y)).$$

Various improvements on the basic scheme:

- dilution
- hopping parameter expansion
- truncated solver
- exact low-mode deflation

Contribution to 3-point function is the *correlation* between the disconnected loop T and the 2-point function.
→ high statistics also needed for 2-point function.

Systematic errors

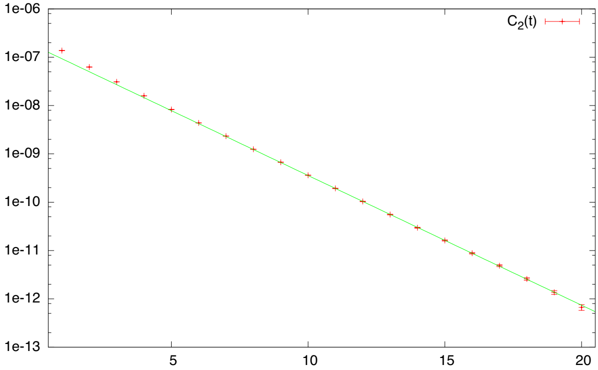
Although calculations with fully-controlled systematics have been done for other observables, this has yet to be done for nucleon structure.

- Quark masses: Most calculations use heavier-than-physical pion masses and rely on an extrapolation to the physical point. Nucleon structure calculations with close-to-physical pion masses have now started to appear.
- Finite volume: $m_\pi L_s \geq 4$ is a typically-aimed-for rule of thumb, but careful $L_s \to \infty$ extrapolations are generally not done.
- Finite temperature: Typically $L_t = 2L_s$ and this issue is not considered separately from finite volume, but the isolation of the ground state could be spoiled if L_t is too small.
- Discretization: Some collaborations have used several lattice spacings and found a negligible effect, but an $a \to 0$ extrapolation is nevertheless necessary.
- Excited states: The problem of correctly isolating the ground state has seen increased attention in recent years; the size of excited-state effects is observable-dependent.

Systematic error: excited states

With interpolating operator $\mathcal{O}$, compute, e.g.,

$$C_{2\text{pt}}(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_n e^{-E_n t} \left|\langle n|\mathcal{O}^\dagger|0\rangle\right|^2$$



For a nucleon, the signal-to-noise asymptotically decays as $e^{-(m_N - \frac{3}{2}m_\pi)t}$.

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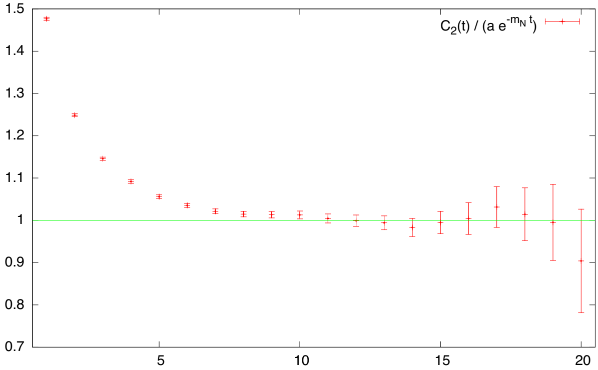

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Removal of excited states

- Standard ratio-plateau method: compute ratio

$$R(T,\tau) = C_{3\text{pt}}(T,\tau)/C_{2\text{pt}}(T)$$
$$= c_{00} + c_{10}e^{-\Delta E\tau} + c_{01}e^{-\Delta E(T-\tau)} + c_{11}e^{-\Delta ET} + \ldots,$$

  then the midpoint $\tau = T/2$ has excited-state contamination that falls off asymptotically as $e^{-\Delta E_{10}T/2}$.

- Summation method (PoS(Lattice 2010) 147 [1011.1358]; *ibid.* 303 [1011.4393]): compute sums

$$S(T) = \sum_{\tau} R(T,\tau) = b + c_{00}T + dTe^{-\Delta ET} + \ldots,$$

  then find their slope, which gives c_{00} with errors that fall off as $Te^{-\Delta E_{10}T}$.

- Alternatives: use the variational method with several interpolating operators; or extrapolate $T \to \infty$ using excited-state fits.

Variance reduction

New technique: all-mode averaging (AMA)
(T. Blum *et al.*, PRD**88** 094503 [1208.4349]; see poster by Eigo Shintani)
Using a set of covariant symmetries G (translations, etc.):

$$\mathcal{O}^{(\text{impr})} = \mathcal{O} - \mathcal{O}^{(\text{appx})} + \frac{1}{N_G} \sum_{g \in G} \mathcal{O}_g^{(\text{appx})},$$

where $\mathcal{O}^{(\text{appx})}$ is a computationally-cheaper approximation to $\mathcal{O}$ that is strongly correlated with it.

Becoming an essential component of calculations at the physical pion mass.

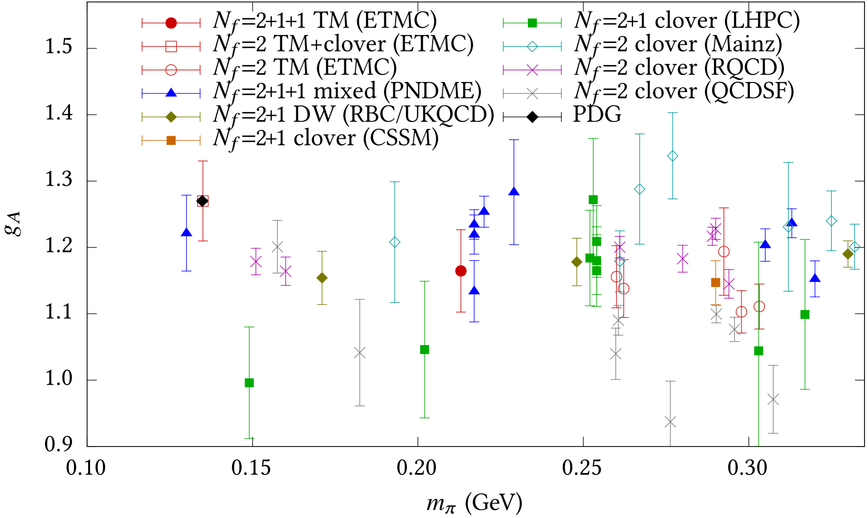
Axial charge g_A

From $n \to p$ transition with zero momentum transfer:

$$\langle p(P,s')|\bar{u}\gamma^\mu\gamma_5 d|n(P,s)\rangle = g_A \bar{u}_p(P,s')\gamma^\mu\gamma_5 u_n(P,s).$$

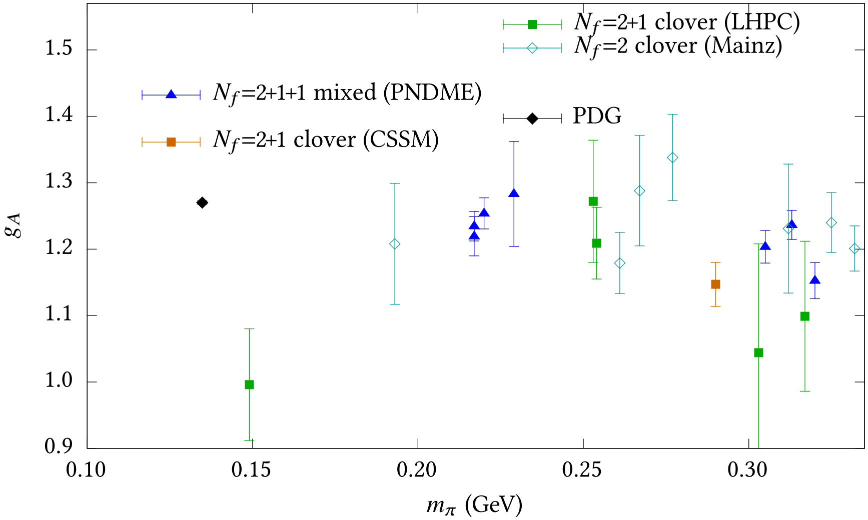
- Well-measured experimentally from beta decay of polarized neutrons: $g_A = 1.2701(25)$.
- Is an isovector quantity, so only connected diagrams are required.
- Is a forward matrix element, which can be determined from a relatively simple analysis.
- $g_A = \langle 1\rangle_{\Delta u - \Delta d} = \Delta\Sigma_{u-d}$ can be understood as the contribution from $(u-d)$ quark spin to nucleon angular momentum.

Axial charge g_A



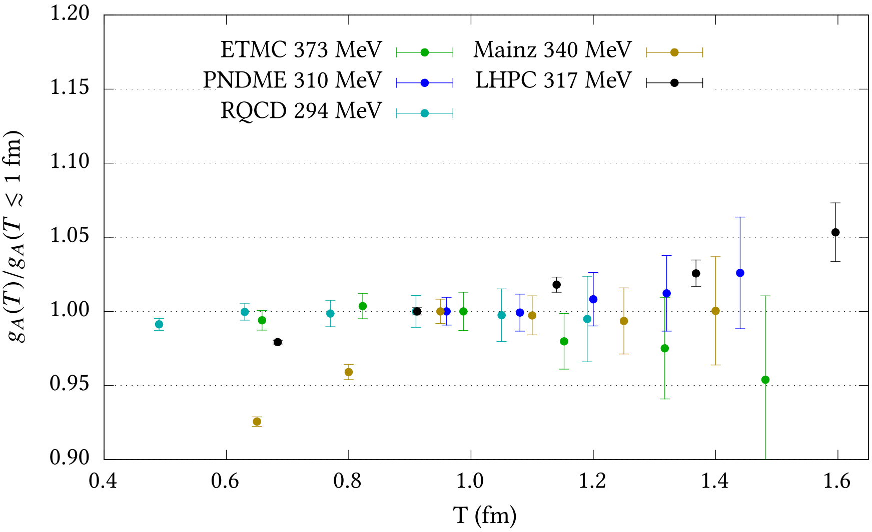
Works that extend below 300 MeV.

Axial charge g_A



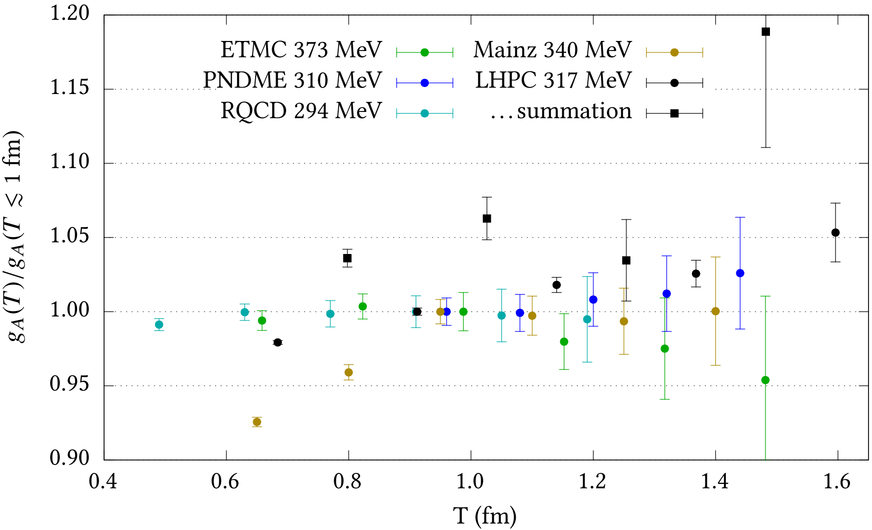
Works that extend below 300 MeV, have $m_\pi L \geq 4$, and control exc. states.

Axial charge g_A systematics: excited states



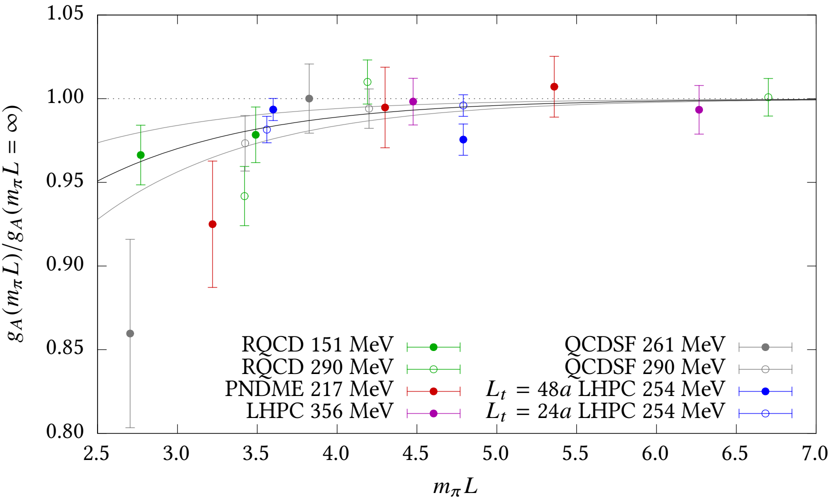
g_A vs. $T_{\text{src-snk}}$, normalized so $g_A(T \lesssim 1 \text{ fm}) = 1$ (ratio method)

Axial charge g_A systematics: excited states



g_A vs. $T_{\text{src-snk}}$, normalized so $g_A(T \lesssim 1\ \text{fm}) = 1$ (ratio, summation methods)

Axial charge g_A systematics: infinite volume extrapolation



Few *fully-controlled* studies; fit them with floating norm
$g_A(m_\pi L, \ldots) = A(\ldots)(1 + Be^{-m_\pi L})$
→ implies −1.1(5)% shift at $m_\pi L = 4$ (χ^2/dof = 20/9).

Electromagnetic form factors

Proton matrix elements of vector current parameterized by Dirac and Pauli form factors:

$$\langle p', s' | J^\mu_{\text{em}} | p, s\rangle = \bar{u}(p', s') \left(\gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2m_p} F_2(Q^2) \right) u(p,s),$$

where $q = p' - p$, $Q^2 = -q^2$. Or alternatively, by the electric and magnetic Sachs form factors,

$$G_E(Q^2) = F_1(Q^2) - \tfrac{Q^2}{(2m_p)^2} F_2(Q^2), \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

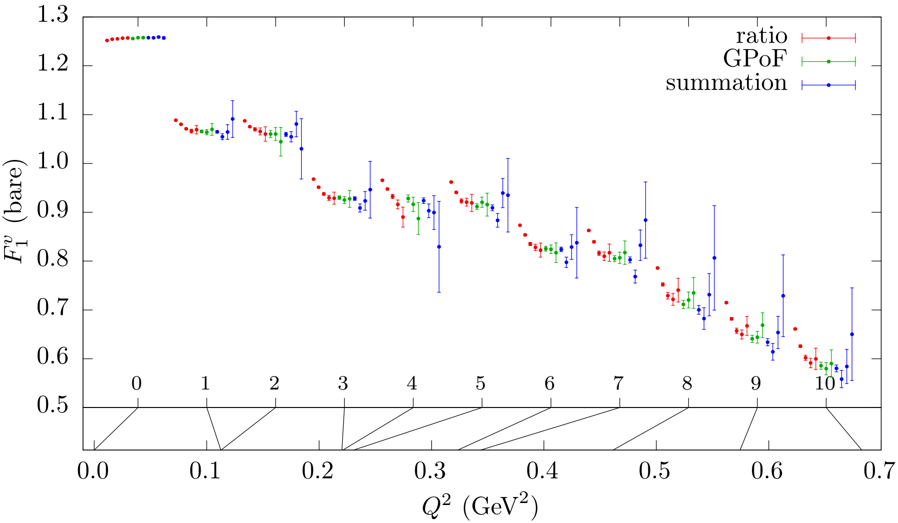
At $Q^2 = 0$, these give the charge and magnetic moment of the proton, and their derivatives define the mean-squared electric and magnetic radii:

$$G_E^p(Q^2) = 1 - \tfrac{1}{6} (r_E^2)^p Q^2 + O(Q^4), \qquad G_M^p(Q^2) = \mu^p \left(1 - \tfrac{1}{6} (r_M^2)^p Q^2 + O(Q^4) \right).$$

To eliminate disconnected diagrams, we take the isovector combination,

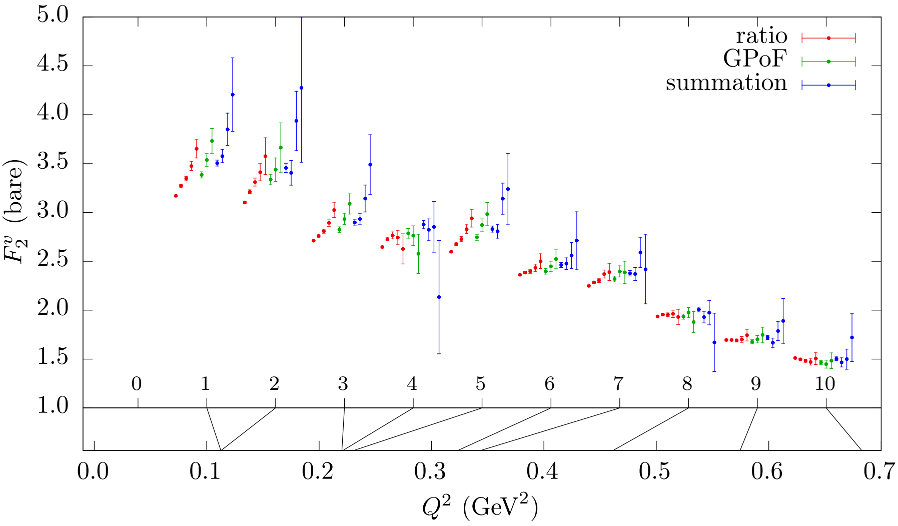
$$G^v_{E,M} = G^p_{E,M} - G^n_{E,M}, \qquad F^v_{1,2} = F^p_{1,2} - F^n_{1,2}.$$

Isovector $F_1(Q^2)$: excited states



LHPC, $m_\pi = 317$ MeV (JG *et al.*, PoS **Lattice 2013**, 276 [1310.7043])

Isovector $F_2(Q^2)$: excited states



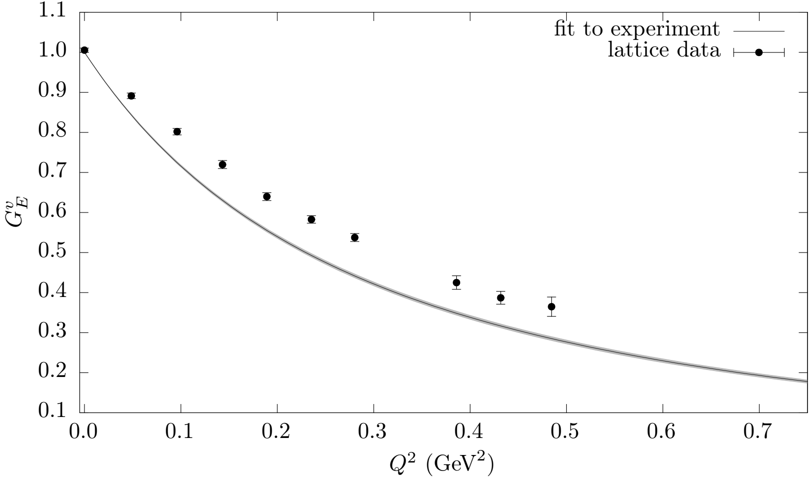
LHPC, m_π = 317 MeV (JG *et al.*, PoS **Lattice 2013**, 276 [1310.7043])

Electromagnetic form factors: excited states

Importance of good control over excited states generally reported whenever they have been studied:

- LHPC: $m_\pi \in [149, 356]$ MeV (JG *et al.*, PLB **734**, 290 [1209.1687]; 1404.4029)
- PNDME: $m_\pi = 220, 310$ MeV (T. Bhattacharya *et al.*, PRD **89**, 094502 [1306.5435])
- ETMC: $m_\pi = 135, 375$ MeV (G. Koutsou, Lattice 2014)
- Mainz: $m_\pi \in [195, 473]$ MeV (G. von Hippel, Lattice 2014)

Isovector $G_E(Q^2)$: comparison with experiment



(JG *et al.*, PoS **Lattice 2013**, 276 [1310.7043]; 1404.4029)

LHPC: $m_\pi = 149$ MeV, $m_\pi L = 4.2$, ratio method, $T = 1.16$ fm

Comparison with Kelly-style fit from W. M. Alberico *et al.*, PRC **79**, 065204 [0812.3539].

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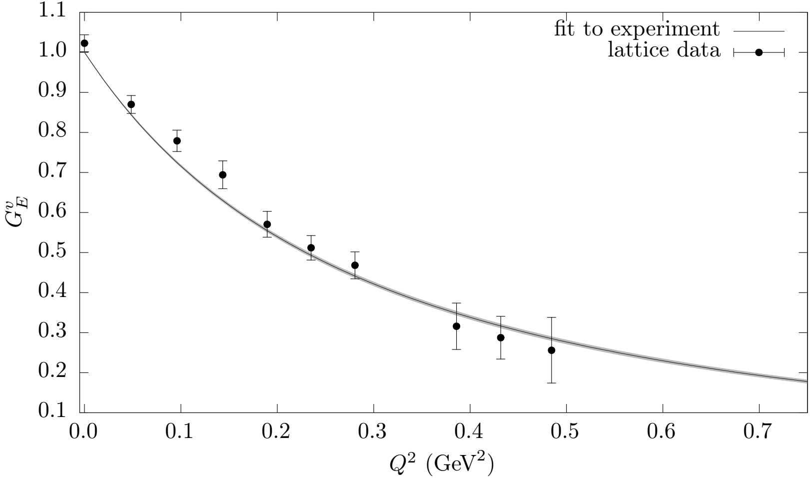

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LHPC: $m_\pi = 149$ MeV, $m_\pi L = 4.2$, summation method

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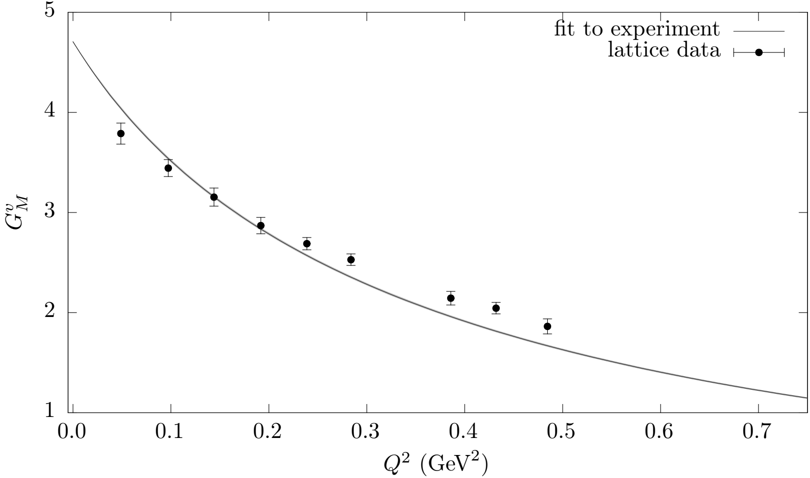
Isovector $G_M(Q^2)$: comparison with experiment



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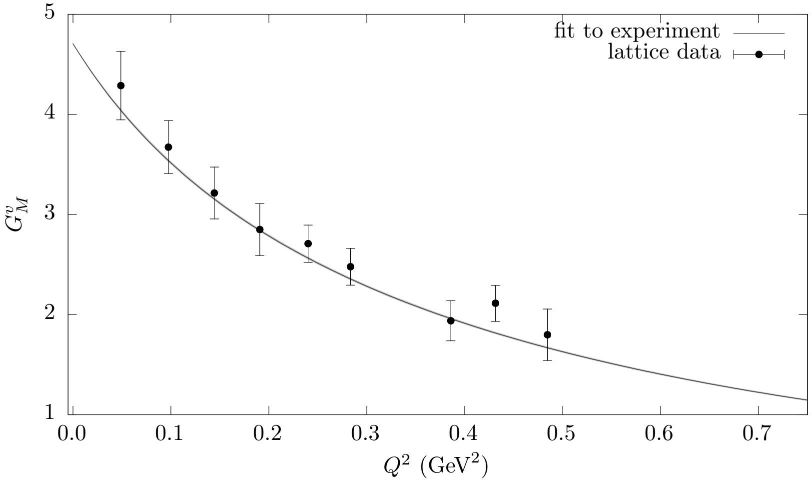

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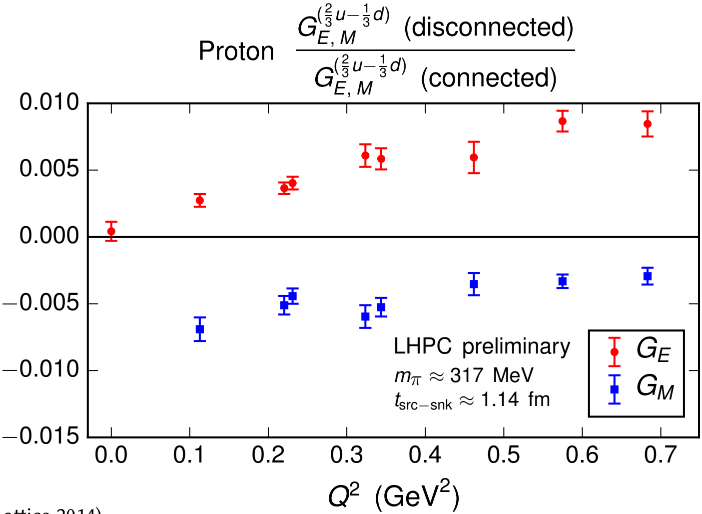
Electromagnetic form factors: disconnected diagrams



(S. Meinel, Lattice 2014)

ETMC also found $< 1\%$ contribution at $m_\pi = 372$ MeV (A. Vaquero, Lattice 2014)

Radii and magnetic moment

Lattice momenta are restricted by periodic BCs:

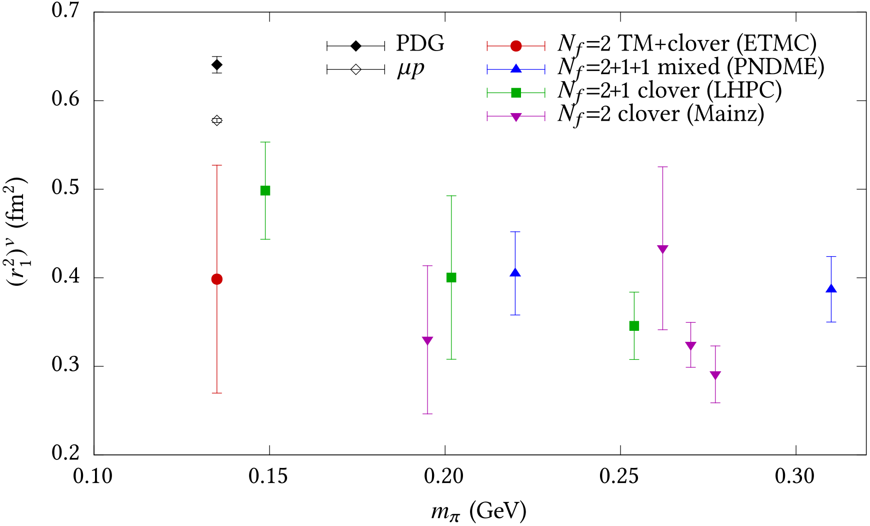
$$Q^2_{\text{min}} \approx \left(\frac{2\pi}{L}\right)^2,$$

which is $\sim 0.05\ \text{GeV}^2$ on the largest lattices used for hadron structure.
To find $r^2_{1,2}$ and κ, we fit a simple function to $F_{1,2}(Q^2)$, often a dipole,

$$F(Q^2) = \frac{F(0)}{\left(1 + \frac{Q^2}{m_D^2}\right)^2}.$$

Given that experimental form-factor data are much more precise and reach smaller Q^2, yet extracting radii from fits is still non-trivial, it is likely that uncertainties from fitting in lattice calculations are typically underestimated.
Nevertheless, this is useful for condensing form factors to fewer observables and comparing different lattice calculations.

Isovector Dirac radius $(r_1^2)^v$



Selecting calculations with: (1) small m_π and
(2) summation or fitting to control excited states.

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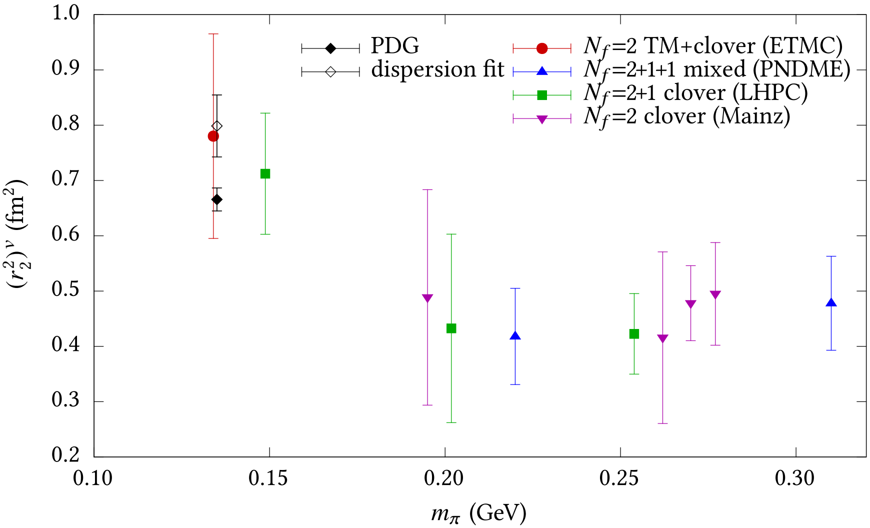


Selecting calculations with: (1) small m_π and
(2) summation or fitting to control excited states.

Targeting the proton radius problem

7σ discrepancy between r_E^p from

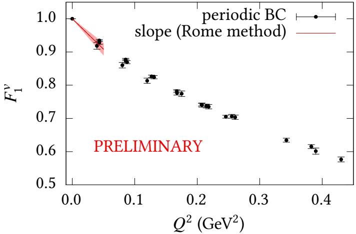
ep scattering and spectroscopy vs. μp spectroscopy.

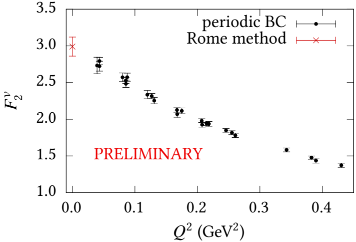
Competing with experimental precision is currently beyond reach for lattice QCD, but the discrepancy in $(r_1^2)^v$ is 10%, so discriminating between the two values may be within reach. But finding r_1^2 from $F_1(Q^2)$ needs better control. There are ways of doing this:

- Larger volumes (possibly in just 1 dimension) to reduce Q^2_{min}.
- Twisted boundary conditions to access arbitrary Q^2.
- Rome method for momentum-derivatives of matrix elements.
  (G. M. de Divitiis *et al.*, PLB **718**, 589 [1208.5914])
  - Recently studied for pion FFs in χPT: gives r^2 with finite-volume effects $\sim L^{1/2}e^{-m_\pi L}$
    (B. Tiburzi, Lattice 2014; 1407.4059).

(Although the latter two do not work for disconnected diagrams.)

Preliminary Rome-method results from LHPC:





Scalar and tensor charges

(T. Bhattacharya *et al.*, PRD **85**, 054512 [1110.6448])

Precision neutron β-decay experiments may be sensitive to BSM physics; leading contributions are controlled by the (not measured experimentally) scalar and tensor charges:

$$\langle p(P,s')|\bar{u}d|n(P,s)\rangle = g_S \bar{u}_p(P,s')u_n(P,s),$$
$$\langle p(P,s')|\bar{u}\sigma^{\mu\nu}d|n(P,s)\rangle = g_T \bar{u}_p(P,s')\sigma^{\mu\nu}u_n(P,s).$$

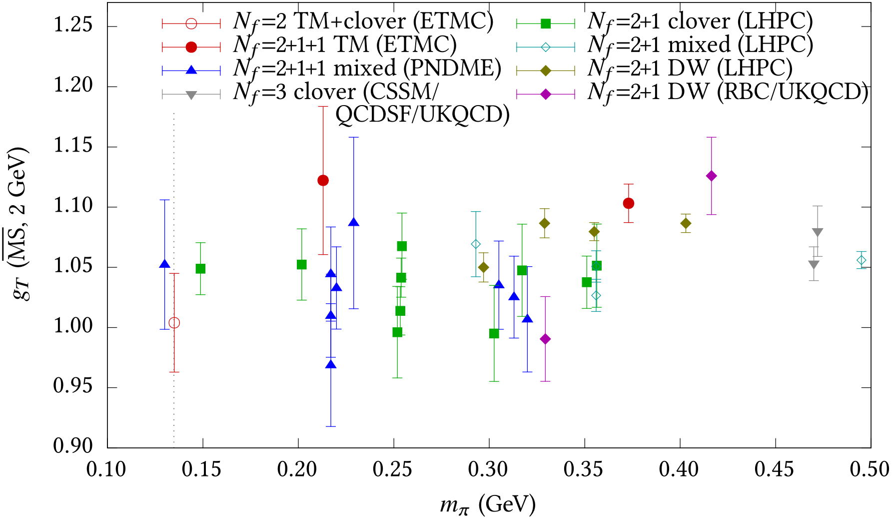
- Tensor charge is also the isovector first moment of transversity:

$$g_T = \langle 1 \rangle_{\delta u - \delta d}.$$

- Scalar charge is related via Feynman-Hellmann theorem to neutron-proton mass splitting:

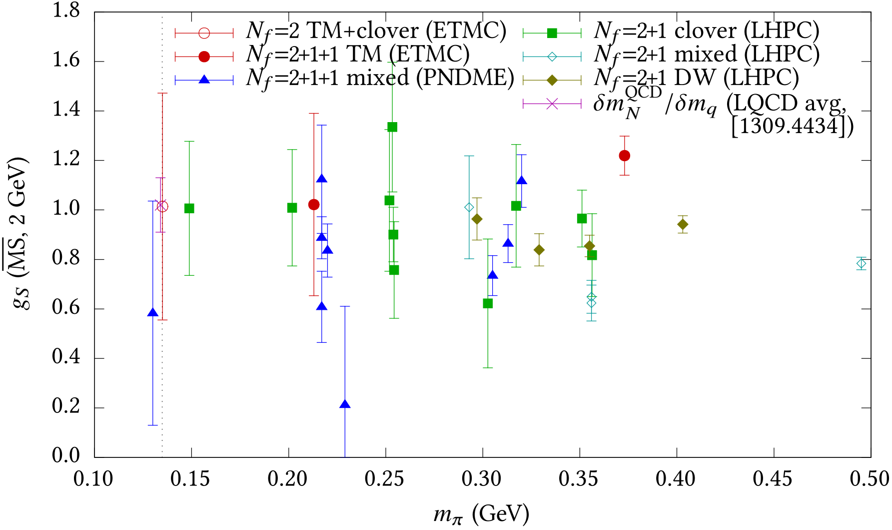
$$g_S = \frac{\partial(m_n - m_p)_{\text{QCD}}}{\partial(m_d - m_u)}.$$

Tensor charge g_T



Systematics seem to be under reasonable control.

Scalar charge g_S



Large statistical error makes systematics hard to resolve.

Momentum fraction

Forward matrix element of traceless quark/gluon energy-momentum tensor:

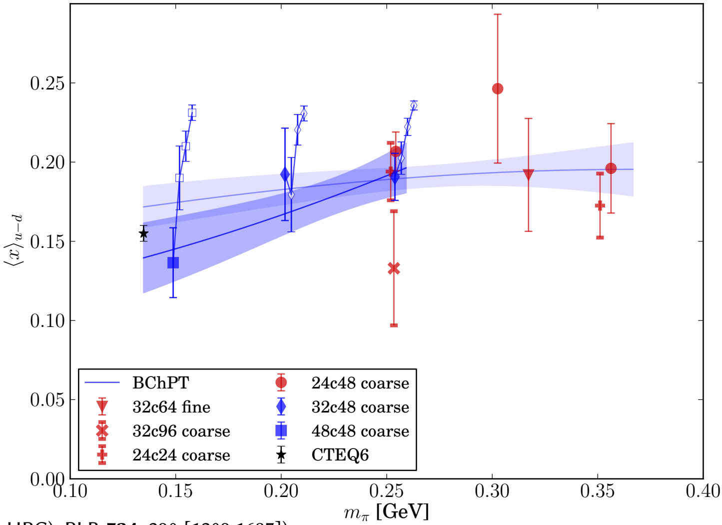
$$\langle p,\lambda'|T^{\mu\nu}_{q,g}|p,\lambda\rangle = \langle x\rangle_{q,g}\bar{u}(p,\lambda')\bar{p}^{\{\mu}\gamma^{\nu\}}u(p,\lambda),$$

where $T^{\mu\nu}_q = \bar{q}\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}q$ and $T^{\mu\nu}_g = G^{\{\mu\alpha a}G_\alpha{}^{\nu\}a}$.

- $\langle x\rangle_{q,g}$ is the average momentum fraction carried by quarks q and $\bar{q}$, or gluons; focus has been on isovector combination $\langle x\rangle_{u-d}$.
- Sum rule:

$$\langle x\rangle_g + \sum_q \langle x\rangle_q = 1$$

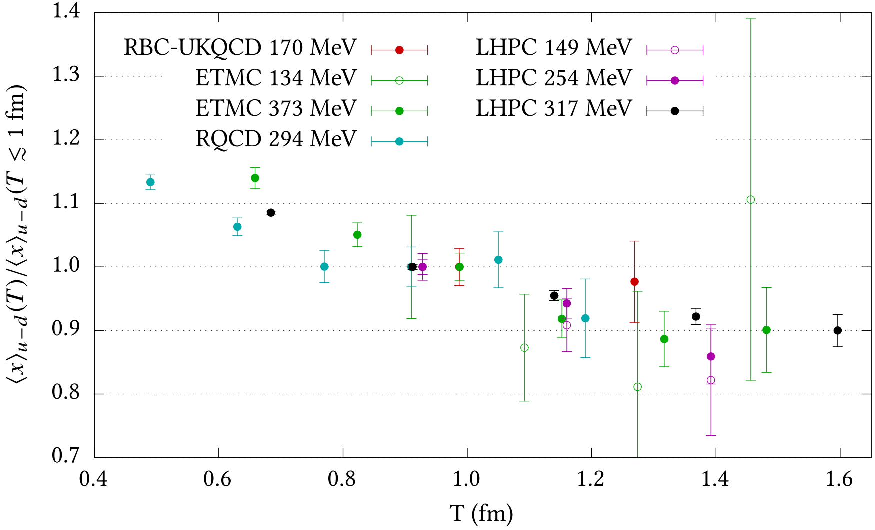
Isovector quark momentum fraction $\langle x \rangle_{u-d}$



(JG *et al.* (LHPC), PLB **734**, 290 [1209.1687])

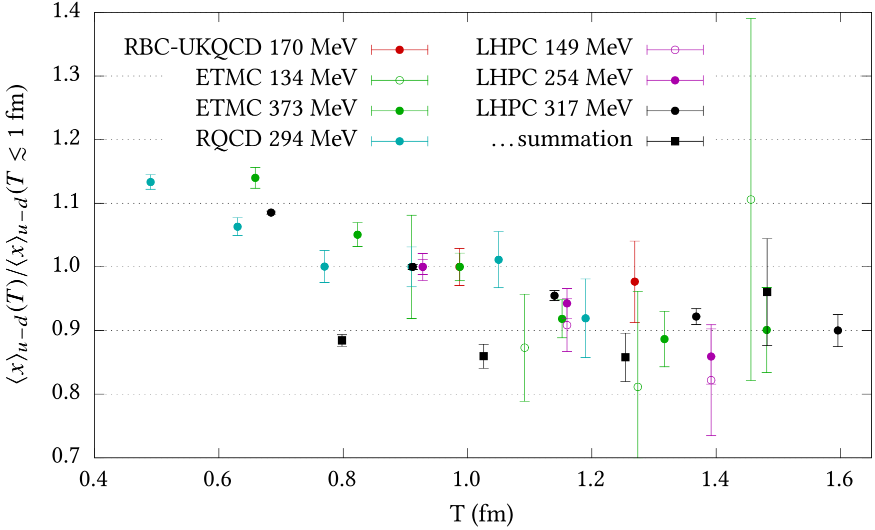
Broad agreement: large excited-state effects (which may grow at small m_π).

Isovector quark momentum fraction $\langle x\rangle_{u-d}$: excited states



$\langle x\rangle_{u-d}$ vs. $T_{\text{src-snk}}$, normalized so $\langle x\rangle_{u-d}(T \lesssim 1\text{ fm}) = 1$ (ratio method)

Isovector quark momentum fraction $\langle x \rangle_{u-d}$: excited states



$\langle x \rangle_{u-d}$ vs. $T_{\text{src-snk}}$, normalized so $\langle x \rangle_{u-d}(T \lesssim 1\text{ fm}) = 1$ (ratio, summation)

Three calculations of disconnected contributions

ETMC (A. Abdel-Rehim *et al.*, PRD **89**, 034501 [1310.6339]): $N_f = 2+1+1$ twisted mass, $a = 0.082$ fm, $m_\pi = 372$ MeV, $m_\pi L = 4.97$, truncated solver + one-end trick, 147k samples of C_{2pt}.

LHPC (S. Meinel, Lattice 2014): $N_f = 2+1$ clover, $a = 0.114$ fm, $m_\pi = 317$ MeV, $m_\pi L = 5.87$, hierarchical probing, 99k samples of C_{2pt}.

LANL (B. Yoon, Lattice 2014): $N_f = 2+1+1$ mixed (clover on staggered sea), $a = 0.12$ fm, $m_\pi = 305$ MeV, $m_\pi L = 4.54$, truncated solver + hopping parameter expansion, 61k samples of C_{2pt} using AMA.

Ratios of disconnected/connected:

Obs	ETMC	LHPC	LANL
g_A^{u+d}	−0.12(2)	−0.12(2)	−0.19(2)
g_T^{u+d}	−0.002(2)	−0.005(10)	−0.039(8)
g_S^{u+d}	0.101(15)	1.756(94)	0.328(25)
$\langle x\rangle_{u+d}$	0.05(13)	0.24(4)	—

Sufficient statistics are achievable; study of systematics is still needed.

Other activities

Far too much for a 25 minute talk, e.g.:

- Sigma terms
- Axial form factors (Mainz, ETMC, ...)
- Form factors of energy-momentum tensor (ETMC, χQCD, ...)
- Polarized, and transversity generalized FFs (ETMC, ...)
- Other hadrons:
  - Pion scalar form factor (Mainz, HPQCD, JLQCD/TWQCD)
  - Electromagnetic form factors of:
    - Pions (various collaborations)
    - Excited nucleons (CSSM)
    - Octet baryons (CSSM/QCDSF/UKQCD)
    - Charmed baryons (K. Utku Can *et al.*)
  - Axial charges (ETMC, CSSM/QCDSF/UKQCD)
- Polarizibilities (George Washington U; see A. Alexandru, Tues. 15:30)
- Light cone operators
  - Direct calculation of PDFs (X. Ji; H.-W. Lin *et al.*; ETMC)
  - Transverse momentum-dependent PDFs (M. Engelhardt *et al.*)

Summary

Quark-connected nucleon matrix elements and form factors are approaching maturity:

- Calculations are now ongoing at the physical pion mass.
- Trade-off between exponential decay of excited states and exponential growth of noise remains a challenge.
- Control over other systematics is within sight.

Disconnected contributions are now being taken seriously:

- Prototype calculations done and ongoing for quark-disconnected and gluonic observables.
- Some efforts for renormalization and mixing have been done.

Calculations of other observables are in a more exploratory stage:

- PDFs and TMDs.
- Structure of other baryons.
- Structure of excited hadrons.