Dyons and Confinement at $T \neq 0$

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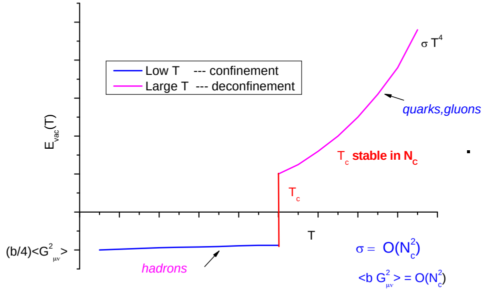
Known facts about pure glue at $T \neq 0$



Order parameter
Polyakov loop

- **Confinement**:
  $< \mathcal{P} > = 0$
- **Deconfinement**:
  $< \mathcal{P} > \neq 0$

Vacuum energy: almost **constant** in **confinement** phase (very few d.o.f), in the **deconfinement** phase approaches σT^4 (Stephen – Boltzman) with $\sigma \sim N_c^2$ d.o.f (gluons). **Strong** phase transition o f **Hagedorne** type at $T_c = 230$ MeV.

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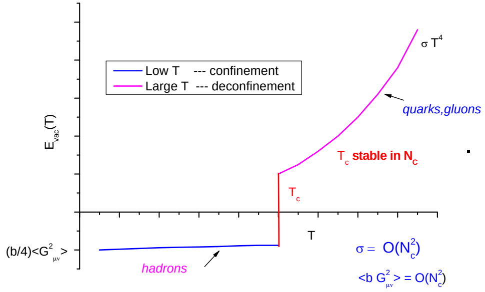

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Description of QCD at non-zero temperature *should* include confinement

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Semiclassical scenario:

True for $N = 1$ SUSY Yang-Mills at all temperatures (no phase transition) $\longrightarrow$ Maybe, true for QCD at temperatures below phase transition (?)

$N = 1$ SUSY Yang Mills at high "temperatures"

Effective potential for holonomy

$$\mathcal{P} = \mathbf{P} \exp i \int_0^T A_4 dt \equiv e^{i\beta v_i Y_i}$$

(Y_i – Cartan generators)

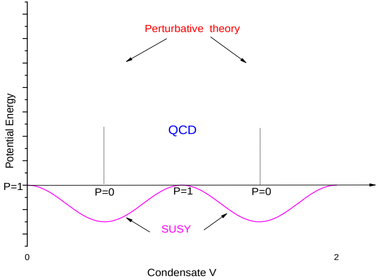
- No perturbative contribution (supersymmetry)
- Non-perturbative contribution: semiclassical solutions (Y.M. eqs of motion with non-trivial holonomy) — **dyons**.

Dyons carry **electric** and **magnetic** charges. N=1 Y.M.theory = Coulomb gas of dyons in $d = 3$ reminds Polyakov's model. Bosonized by complex scalar field (prepotential is **holomorfic**):

$$\mathcal{L} = |\partial_\mu \Phi|^2 + M^2 \left| \exp\left(-\frac{4\pi}{\alpha}\Phi\right) - \exp\left(-4\pi\frac{4\pi}{\alpha}(2\pi\mathbf{T} - \boldsymbol{\Phi})\right)\right|^2 \quad (1)$$

Contribution of $M\bar{M}$ and $L\bar{L}$ dyons, ($SU(2)$-group)

$N = 1$ SUSY Yang Mills at high "temperatures"



- **Minimum of Potential**:

$$v = \pi T, \quad < P >= 0$$

(**Confinement!**) Dual gluon ϕ acquires a mass

$$\mathcal{M}^2 = M_{PV}^2 \exp(-\frac{4\pi^2}{g^2})$$

and accompanied by some (colorless) fermion to fit SUSY.

Everything like in Polyakov's model!

Picture is **valid at all T** while dyons are not necessary dominant at smaller T.

Dyons in QCD

- At $T \neq 0$ constant field A_4 are gauge invariant (global rotation). $A_4 = \vec{v} \cdot \vec{Y}$ (Y_i=Cartans). "Condensates" v_i are gauge invariant and related to eigenvalues of Polyakov's line.
- Dyons are **self-dual** solutuions of Y.M. eqs with non-zero **electric** & **magnetic** charges. There are r (**rank**) elementary monopoles based on **simple roots** with field is **independent on time** (M-monopole) and 1 based on **highest root** $\vec{\alpha}_H$ with potential periodic in time: (L-monopole). At **large** r

$$A_4 = \vec{v} \cdot \vec{Y} + \frac{\vec{m} \cdot \vec{Y}}{r}, \qquad \mathcal{E} = \mathcal{H} = \vec{m} \cdot \vec{Y} \frac{\mathbf{r}}{r^2}$$

Magnetic charge ($\vec{\alpha}^*$ is co-root):

$$\vec{m} = \frac{1}{2} |\alpha_H| \vec{\alpha}^*, \qquad \vec{\alpha}^* = 2 \frac{\vec{\alpha}}{\vec{\alpha} \cdot \vec{\alpha}}$$

Monopole has a **core** with a size $\rho = \vec{\alpha} \cdot \vec{v} / |\alpha_H|$.

Dyons in QCD

Highest co-root can be expanded in a simple co-roots

$$\alpha_H^* = \varkappa_1 \vec{\alpha}_1^* + \ldots + \varkappa_r \vec{\alpha}_r^*$$

$\varkappa_i$ - positive integers, their sum is dual **Coxeter number** $n_C = 1 + \varkappa_1 + \ldots + \varkappa_r$.

- configuration — **KVBLL instanton** with zero electric and magnetic charge consists of n_C dyons: one L-monopole, $\varkappa_1$ M-monopoles based on root $\vec{\alpha}_1$, ... Example: $SU(N_c)$ group — instanton= N_c dyons (1 L-monopole and $N_c - 1$ M-monopoles), G_2 group: instanton = 4 dyons (1 L-monopole and 3 M-monopoles of 2 types).
- **Action** and **topological charge** (self-dual configuration)

$$Q_t = \frac{\vec{m} \cdot \vec{v}}{2\pi T}, \qquad S = \frac{8\pi^2}{g^2} \frac{\vec{m} \cdot \vec{v}}{2\pi T}$$

QCD is NOT SUSY: Interaction of Dyons in QCD

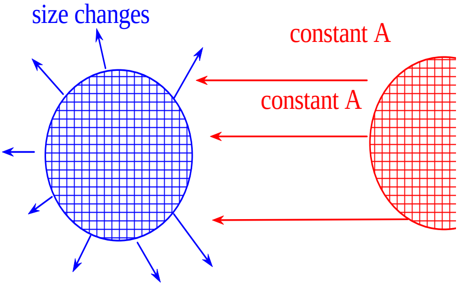
- Total action of n_C dyons in the instanton $\sum S_i = S_{inst} = 8\pi^2/g^2$. **Classical** interaction of dyons — **absent** (KvBLL - coloron is a classical solution interpolating between instanton and dyons)
- **Quantum** interaction consists of:
  - Quantum determinant on **non-zero** modes — numerically small ($2/3N_c$ as compared to $4N_c$
  - Jacobian of transition to zero modes (**metrics of the moduli space**).
- Every dyon – **4 zero modes** -3 translations and 1 $U(1)$-**rotation** ($4n_C$ modes for KvBLL instanton)

$$J^2 = \mathrm{Det}\left(\int d^3x \psi_\mu^{(i)} \psi_\mu^{(j)}\right)$$

-- determinant does not **factorize**.

QCD is NOT SUSY: Interaction of Dyons in QCD

Main effect at large distances – renormalization of the size of one dyon under the field of another



—**determinant** quantum interaction

Size **renormalized**

$$\rho \longrightarrow \rho + \frac{\mathbf{1}}{\mathbf{r}}$$

Metrics of moduli space is **hyper-Kähler** (Ricci tensor is self-dual). **Enough** to restore...

Statistical sum of Dyons in QCD

Statistical sum of dyon gas ($SU(2)$ color)

$$\mathcal{Z} = \sum_{K_L,K_M} \frac{1}{K_L! K_M!} \left(\frac{\Lambda^4 \beta^2}{8\pi^3} \left(\frac{8\pi^2}{g^2} \right)^2 \right)^{K_L+K_M} \int d^3 r_1 \ldots \times$$

$$\mathrm{Det} \begin{pmatrix} v + \sum \frac{1}{r_{1i}} & \ldots & \pm\frac{1}{r_{1i}} & \ldots \\ \pm\frac{1}{r_{21}} & \ldots & \ldots & \ldots \\ \ldots & 2\pi T - v + \sum \frac{1}{r_{ij}} & \pm\frac{1}{r_{1i}} & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

Determinant interaction **cannot be reduced** to the pair dyon interaction, as in classical statistics. Nevertheless, in the **thermodynamic limit** it is equivalent to the **field theory**

Statistical sum of Dyons in QCD

For every dyon — **one fermion** and **f**ield

$$\mathcal{Z} = \int D\vec{\chi} D\vec{\chi}^{+} \mathbf{Dw_k} \exp\left(\frac{T}{4\pi}\int d^3x[\partial_i\chi_k\partial_i\chi_k^{+} + \xi\chi_k^{+}\frac{\partial^2\mathcal{F}}{\partial w_k \partial w_n}\chi_n]\right)$$

$$\exp\left(-\mathbf{4}\pi\mathbf{m_k}\xi\frac{\partial\mathcal{F}}{\partial\mathbf{w_k}}\right) \times \delta\left(\frac{\mathbf{T}}{\mathbf{4}\pi}\partial^{\mathbf{2}}\mathbf{w_k} - \xi\frac{\partial\mathcal{F}}{\partial\mathbf{w_k}}\right)$$

where k — indices numerating Cartans $(1 \ldots rank$, they label one of $U(1)^{N_c-1}$ subgroups.

$$\mathcal{F} = \sum_{k}^{n_C} \varkappa_k e^{w_k - w_{k+1}}$$

is **affine Toda potential**. ξ — **fugacity of one dyon**

Effective potential: non-perturbative contribution of dyons

Logarithm of statistical sum is **effective potential** for **condensate** $\vec{v}$:

$$V_{eff}(\vec{v}) = -\zeta n_C \left(\frac{S_1(\vec{v})}{\varkappa_1}\right)^{\frac{\varkappa_1}{n_C}} \dots \left(\frac{S_{n_C}(\vec{v})}{\varkappa_{n_C}}\right)^{\frac{\varkappa_{n_C}}{n_C}}$$

where $S_1 \dots S_{n_C}$ are **actions** of the n_C dyons which are present in **KvBLL coloron**. The minimum is in the point of **maximal holonomy**

$$\vec{v} = 4\pi T \frac{\vec{\rho}}{n_C|\alpha_H|}, \qquad \mathcal{P}_{fundamental}(\vec{v}) = 0$$

where $\vec{\rho}$ is **Weyl vector** (which is the half of the sum of positive roots) Polyakov's line (fundamental) is **zero** in this point.

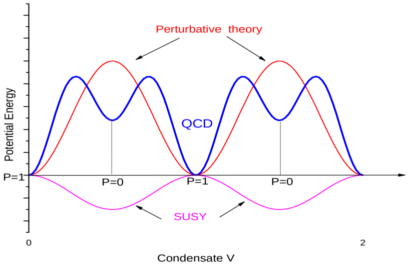
Effective potentia: perturbative contribution of gluons

Quantum corrections in perturbation theory induce **perturbative** effective potential. In 1 loop:

$$V_{eff}(\vec{v}) = \frac{\pi^2 \mathbf{N_{gl}}}{45}\mathbf{T^3} + \frac{2\pi^2}{3}\mathbf{T^3} \sum_{\textbf{all roots}} \left(\frac{\vec{\alpha}_i \vec{v}}{2\pi T|\alpha_H|}\right)^2 \left(1 - \frac{\vec{\alpha}_i \vec{v}}{2\pi T|\alpha_H|}\right)^2$$

First term is Stephen-Boltzman energy, N_{gl} is number of gluons, second – effective potential.
Minimum is in point of **trivial holonomy** $\mathcal{P} = 0$. Effective potential falls with temperature as T^3.

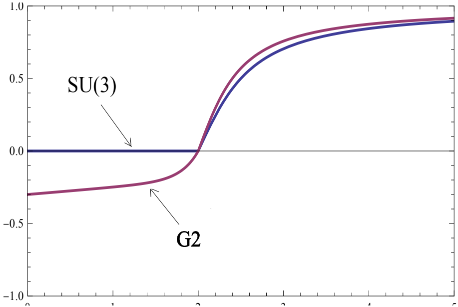
Phase transition: confinement-deconfinement



- Perturbative potential – falls with T
- Dyon Potential **is stable**

At $T = T_c$ minimum with $\mathcal{P} = 0$ becomes more deep — **1st order phase transition to confinement**.

Phase transition: confinement-deconfinement



Below T_c zero Polyakov's loop is supported by **center of the group** (G_2 has no center – **no confinement**).

Physical properties of dyon vacuum

- T_c is **stable** in N_c
- Below T_c free energy **is constant**
- Phase transition is **first order** in $N_c \geq 3$ but **second order** in $SU(2)$.
- Numerically T_c is correct.

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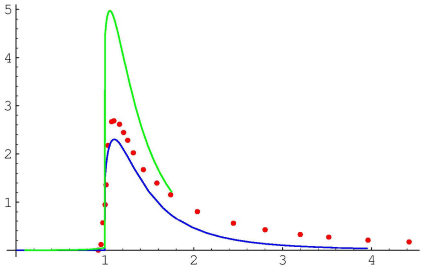
One can **add(????)** anti-dyons and obtain more...

- Free energy below behaves as $O(N_c^2)$ (should be: $\frac{b}{4} < G_{\mu\nu}^2 >$ Gluon condensate is $\sim (255)^4$ **Mev**^{4}.
- Topological susceptibility stable in N_c — different from instantons! (action$=O(1/N_c)$) Numerically $< Q_t^2 >= (187 Mev)^4$.

Thermodynamic properties of dyon vacuum

Thermodynamic properties are more or less OK. Example $(E_{vac} - 3P)/T^4$ (P-pressure) at $T > T_c$. This quantity should be zero if all excitation are massless (gluons)



- Green – G_2-**group**
- Blue – $SU(3)$-**group**
- Red points $SU(3)$ **lattice date** (Karsch)

String tension

At temperature $T \neq 0$ there are **two types of strings**

- Heavy quark potential is defined from **correlator of two Polyakov's lines**.

$$< P(0)P(\vec{r}) >= e^{-\beta V(r)}, \qquad V(r) = \sigma_{el} r$$

(correlator is determined by mass of lowest excitation)

- **Spatial strings** which are related to **area behavior** spatial Wilson loop

$$W[C] = P \exp i \int A_i dx_i \approx \mathbf{e}^{-\sigma_{\mathrm{mag}} \mathbf{Area}}$$

(one has to consider Polyakov's double layer of monopoles and find corresponding soliton.

String tension

Affine Toda chain is **completely integrable** model, all solitons are known and it has a remarkable property

$$\text{Mass of lowest excitation} \quad = \quad \text{Mass of the lowest soliton}$$

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Electrical string tension = **Magnetic string tension**

Restoration of Lorentz symmetry at small temperatures.

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Electrical string tension = Magnetic string tension

Restoration of Lorentz symmetry at small temperatures. **String tension is stable in N_c and numerically OK.**

k-strings

If $N_c \neq 2$ there different **nonminimal strings** corresponding to different representations of the group.

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Not a Casimir scaling!!!. **NOW** lattice people are thinking this is **correct!**

Conclusions

Model **based on dyons**

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Model **based on dyons**

- **Reproduces** all known from the lattice **qualitative** features of pure glue theory. Dependence on N_c and temperature are all correct.
- Ideologically completely follows **t'Hooft-Polyakov scenario of confinement**
- In spite of the fact that the model is quite crude, it appears to be **numerically successful**

Direct check

Presented ideas can **be checked directly** on the lattice. One has to measure **effective potential for Polyakov's line**

- Fix analogue of $A_4(x) = 0$ gauge. We fix all $U(\mathbf{x}, i)$ on all *time slices* $i = 0, \ldots N_t - 2$. The last time slice

$$\mathcal{P}(\mathbf{x}) = U_4(\mathbf{x}, N_t - 1) = diag\{e^{i\varphi_1} \ldots e^{-i\sum \varphi_i}\}$$

- Statistical sum is:

$$Z = \int D\mathcal{P} \int \mathbf{DU_i e^{-\beta S[U_i, \mathcal{P}]}}$$

- The inner integral in space links is **effective potential for Polyakov' line**

Direct measurement was done in *Diakonov,Gattringer, Schaldah, 2012* for $SU(2)$ and $SU(3)$ color groups

Direct check

Unfortunately they met **new type of perturbative ultraviolet divergence**. In paper of 2013 we described the data in the **mean field approximation**. Free energy:

$$f = \varepsilon_{\rm vac} + \frac{\log\left(\frac{N_t}{2\pi\beta}\right) - 2c_2}{2N_t} + \frac{\log(\mathbf{4}\sin^{\mathbf{2}}\varphi)}{\mathbf{N_t}} +$$

$$+\frac{\pi^2}{3N_t^4}\left[-\frac{1}{5} + \mathbf{4}\left(\frac{\varphi}{\pi}\right)^{\mathbf{2}}\left(\mathbf{1} - \frac{\varphi}{\pi}\right)^{\mathbf{2}}\right].$$

This U.V. divergence is related to the known **linear (Coulomb) divergency** of Polyakov's line in perturbation theory. Polyakov's line should be first **renormalized**. Divergency **obscures** the lattice measurements.