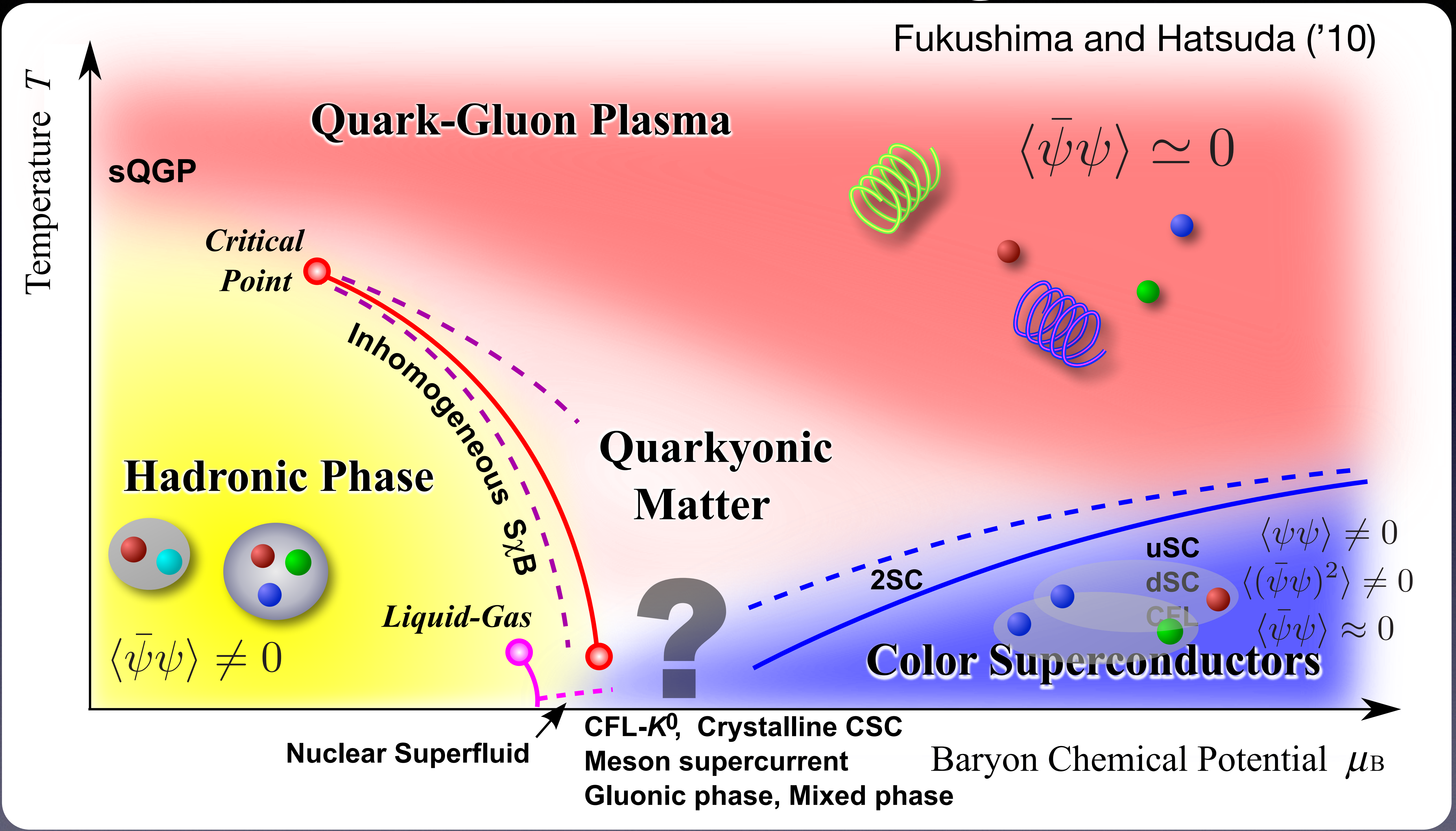
Spontaneous symmetry breaking and Nambu-Goldstone modes in QCD matter

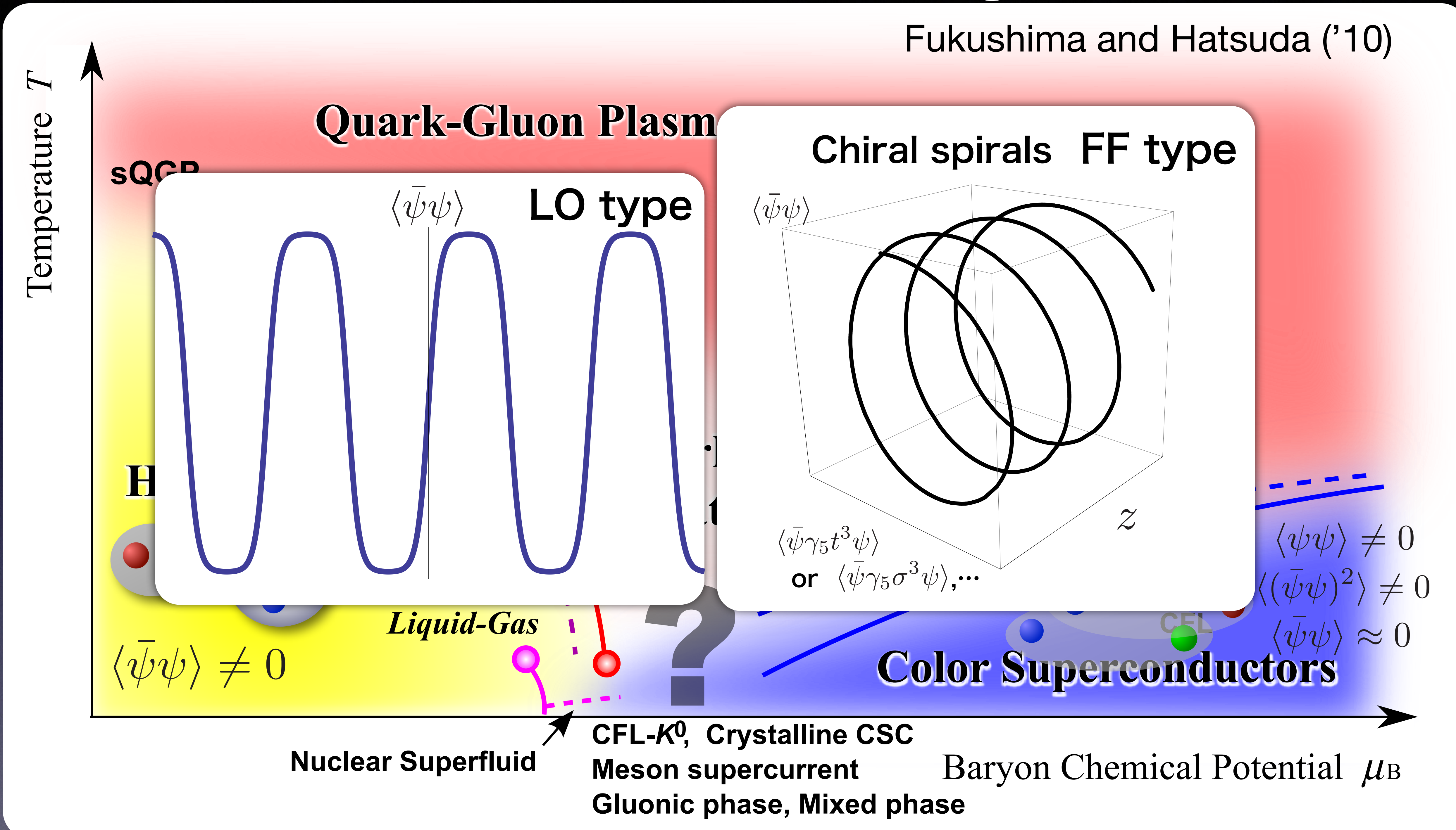
Yoshimasa Hidaka
(Nishina Center, RIKEN)

QCD phase diagram



What are the low-energy excitations?

QCD phase diagram



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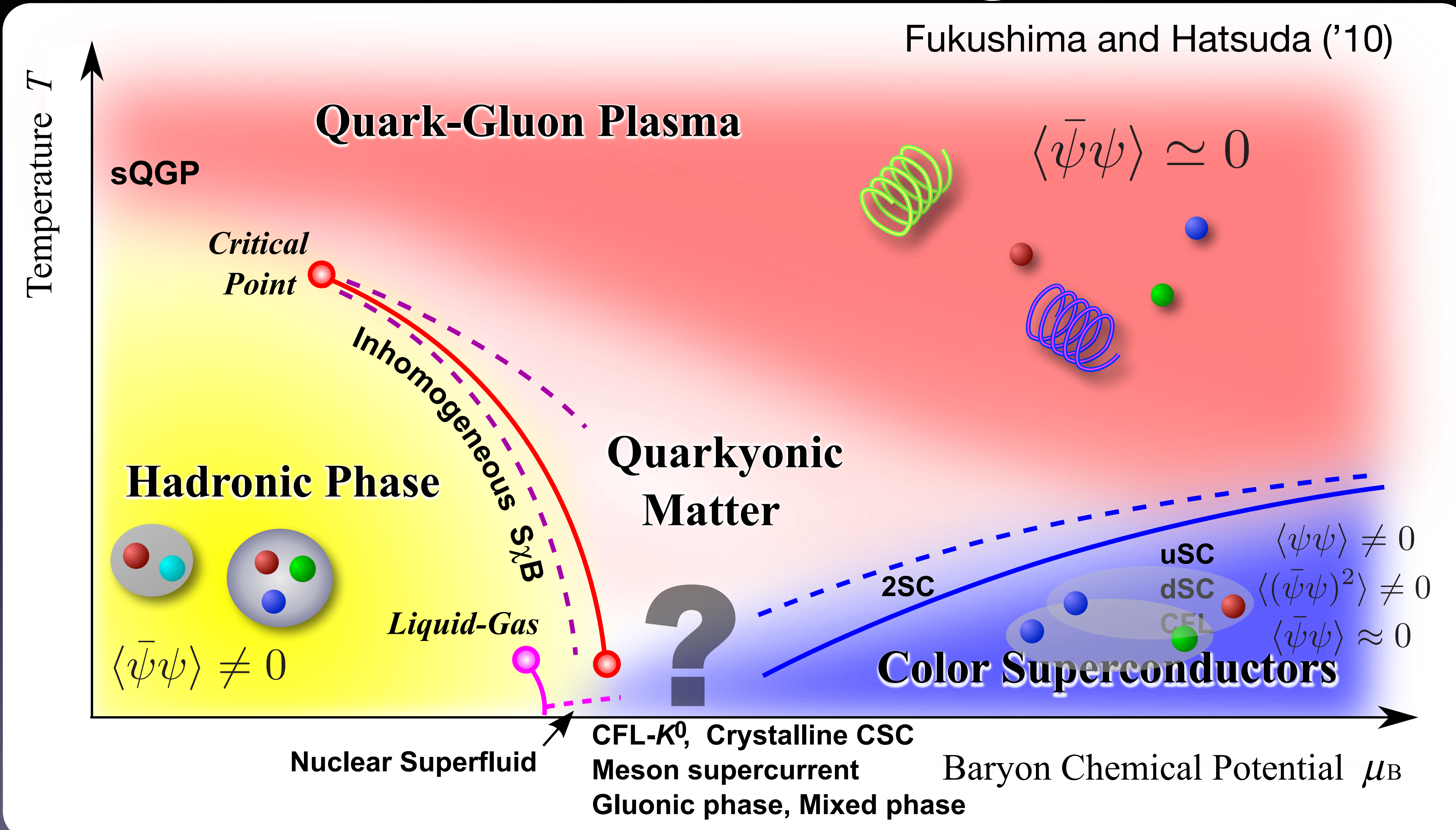

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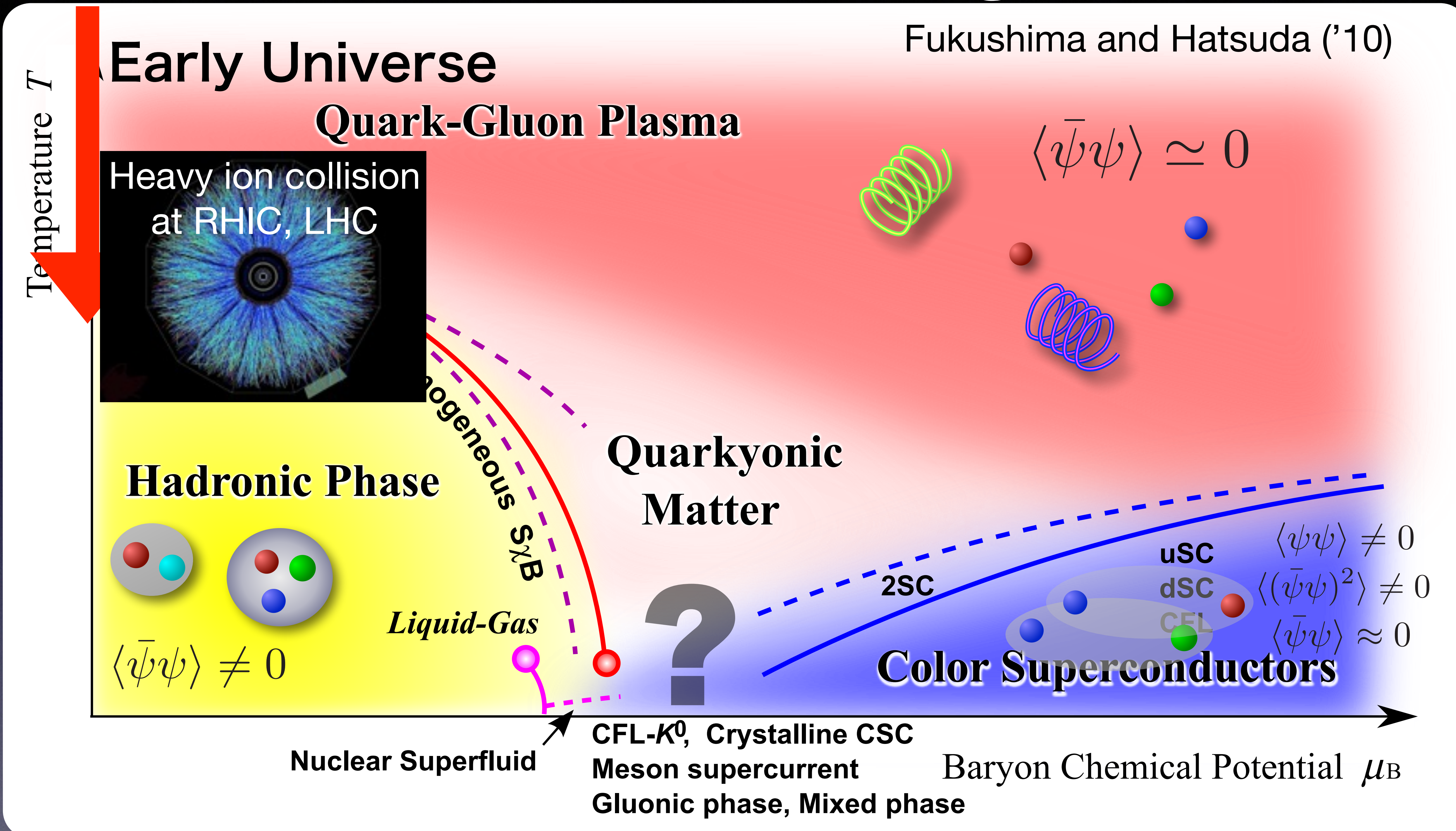

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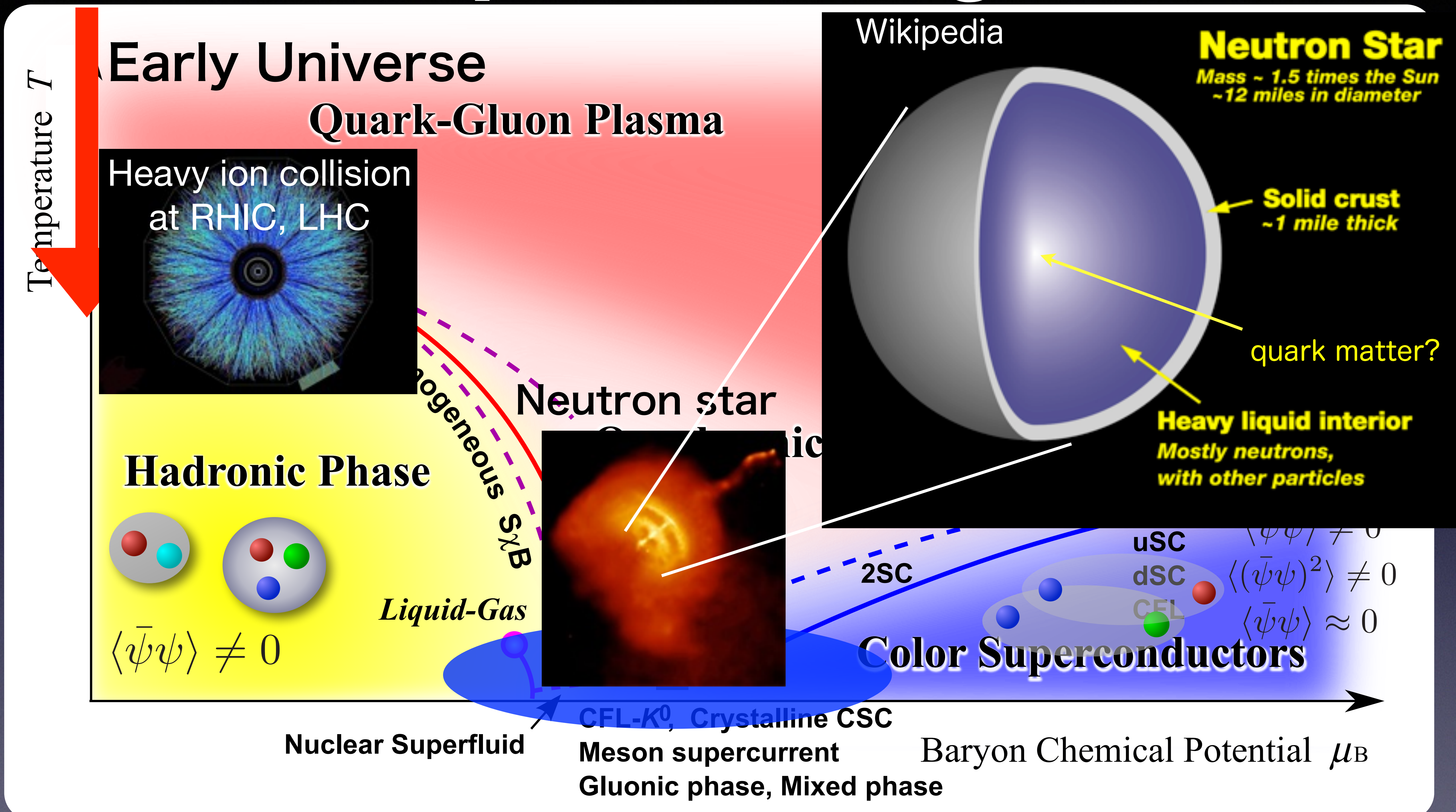


What are the low-energy excitations?

Spontaneous symmetry breaking

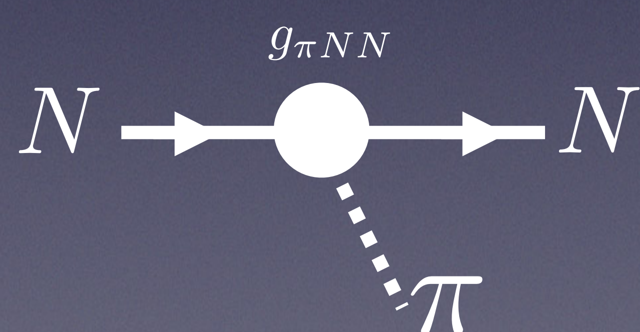
What is important?

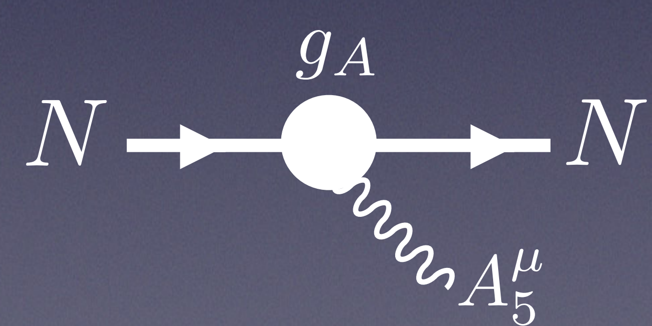
Independence of details of theory

Low-energy theorem

Ex.) Goldberger-Treiman relation

$$g_{\pi NN} = 2m_N g_A / f_\pi$$





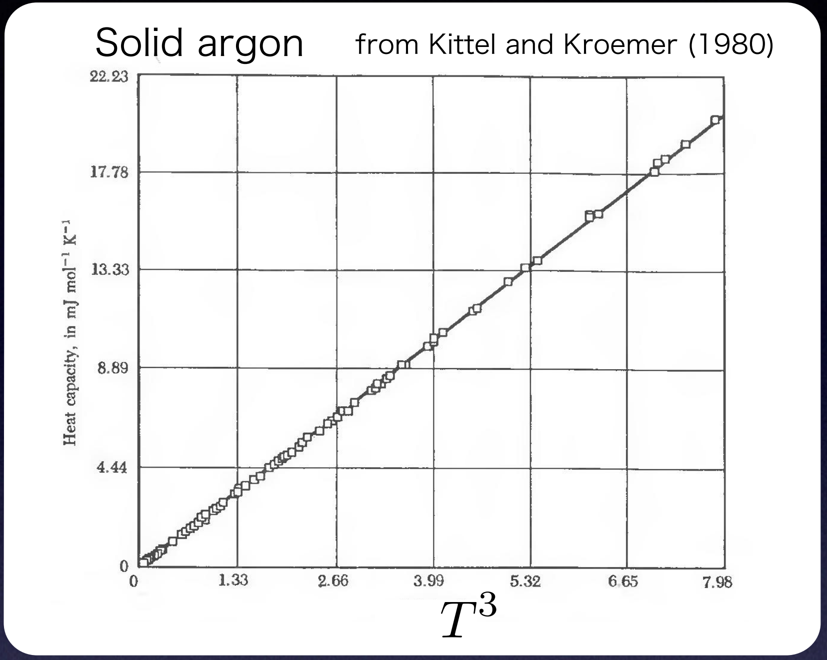
Relation between different vertices.

Spontaneous symmetry breaking

Heat capacity (chiral limit):

$$C_{\rm V} = \frac{2}{5}\pi^2 T^3 + \cdots$$

Debye T^3 law



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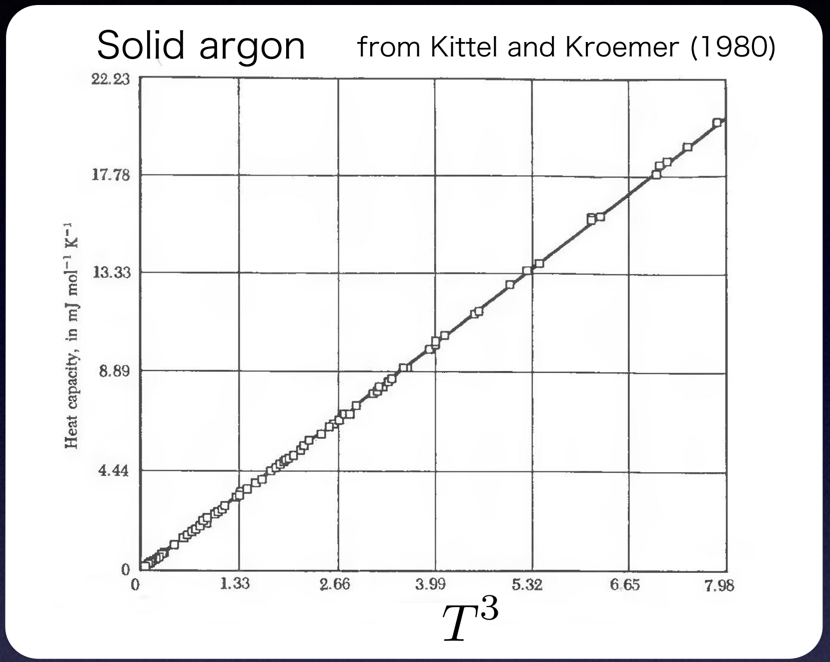

Condensate:

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{8}\frac{T^2}{f_\pi^2} + \cdots$$

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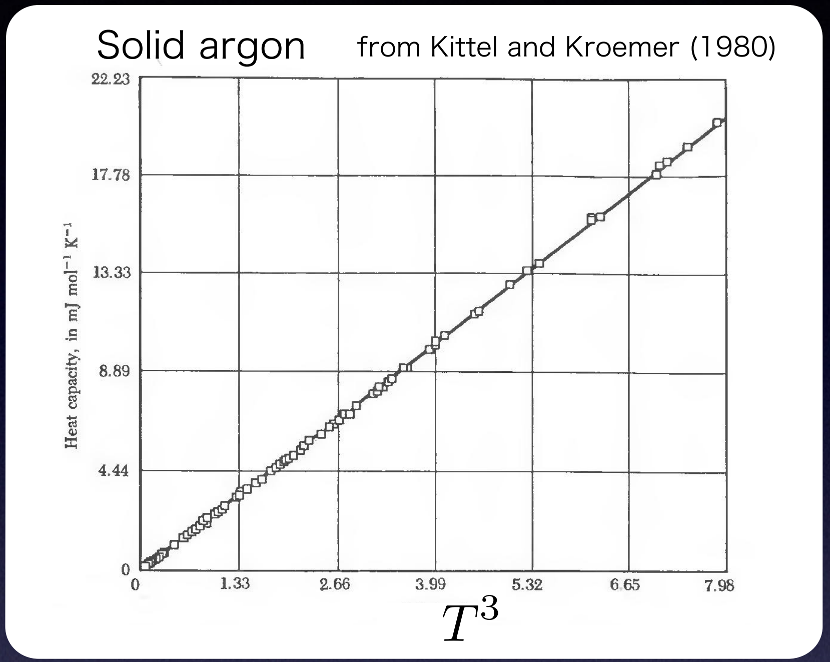
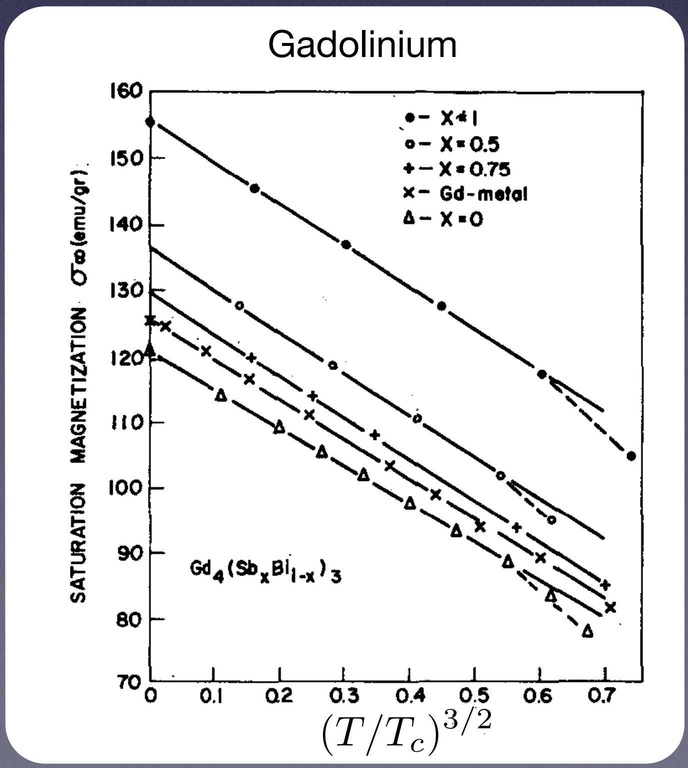


Condensate:

$$\frac{\langle \bar{q}q\rangle_T}{\langle \bar{q}q\rangle_0} = 1 - \frac{1}{8}\frac{T^2}{f_\pi^2} + \cdots$$

$$\frac{\langle M\rangle_T}{\langle M\rangle_0} = 1 - \left(\frac{T}{T_c}\right)^{3/2}$$

cf. Bloch $T^{3/2}$ law



Holtzberg, McGuire, M'ethfessel, Suits, J. Appl. Phys. 35,1033 (1964)

Nambu-Goldstone theorem

Nambu ('60), Goldstone (61), Nambu, Jona-Lasinio ('61), Goldstone, Salam, Weinberg ('62)

For Lorentz invariant vacuum

Spontaneous Symmetry Breaking

⬇

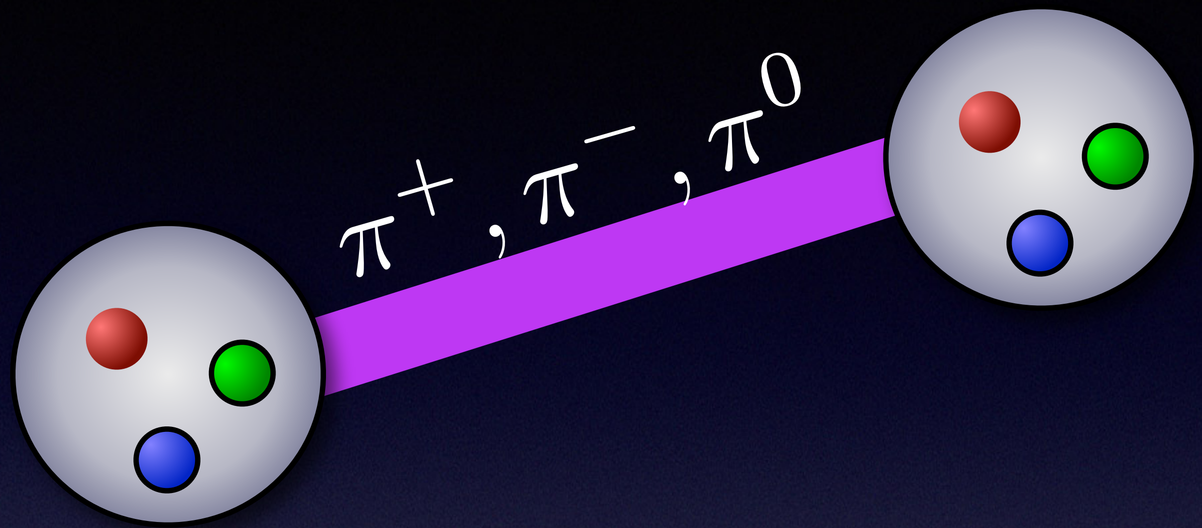
$$N_{\mathrm{NG}} = N_{\mathrm{BS}}$$

# of NG modes **# of broken symmetry**

Dispersion relation: $\omega = c|\boldsymbol{k}|$

QCD in vacuum

Three NG modes: Pions



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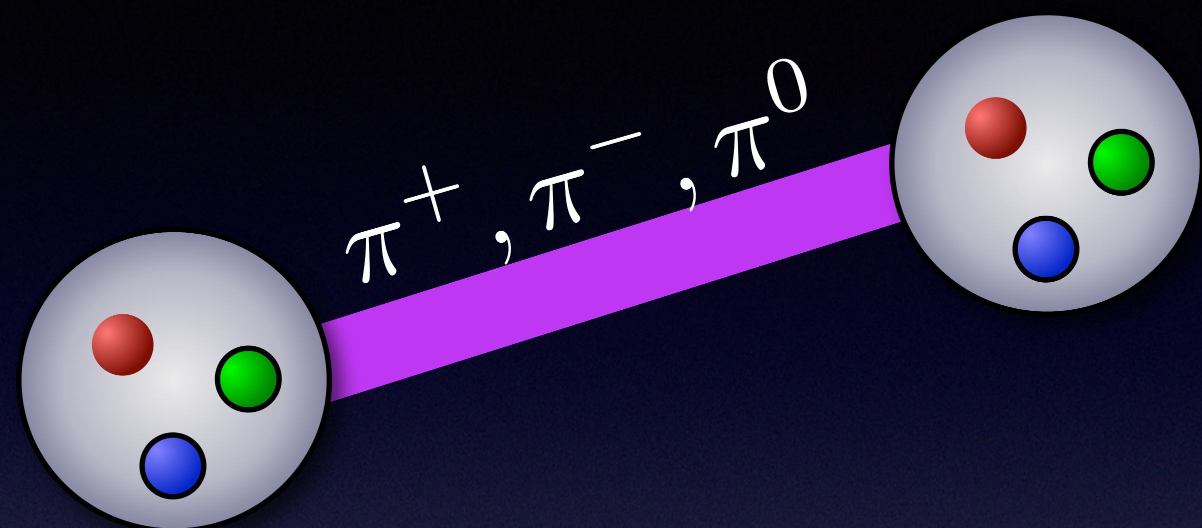
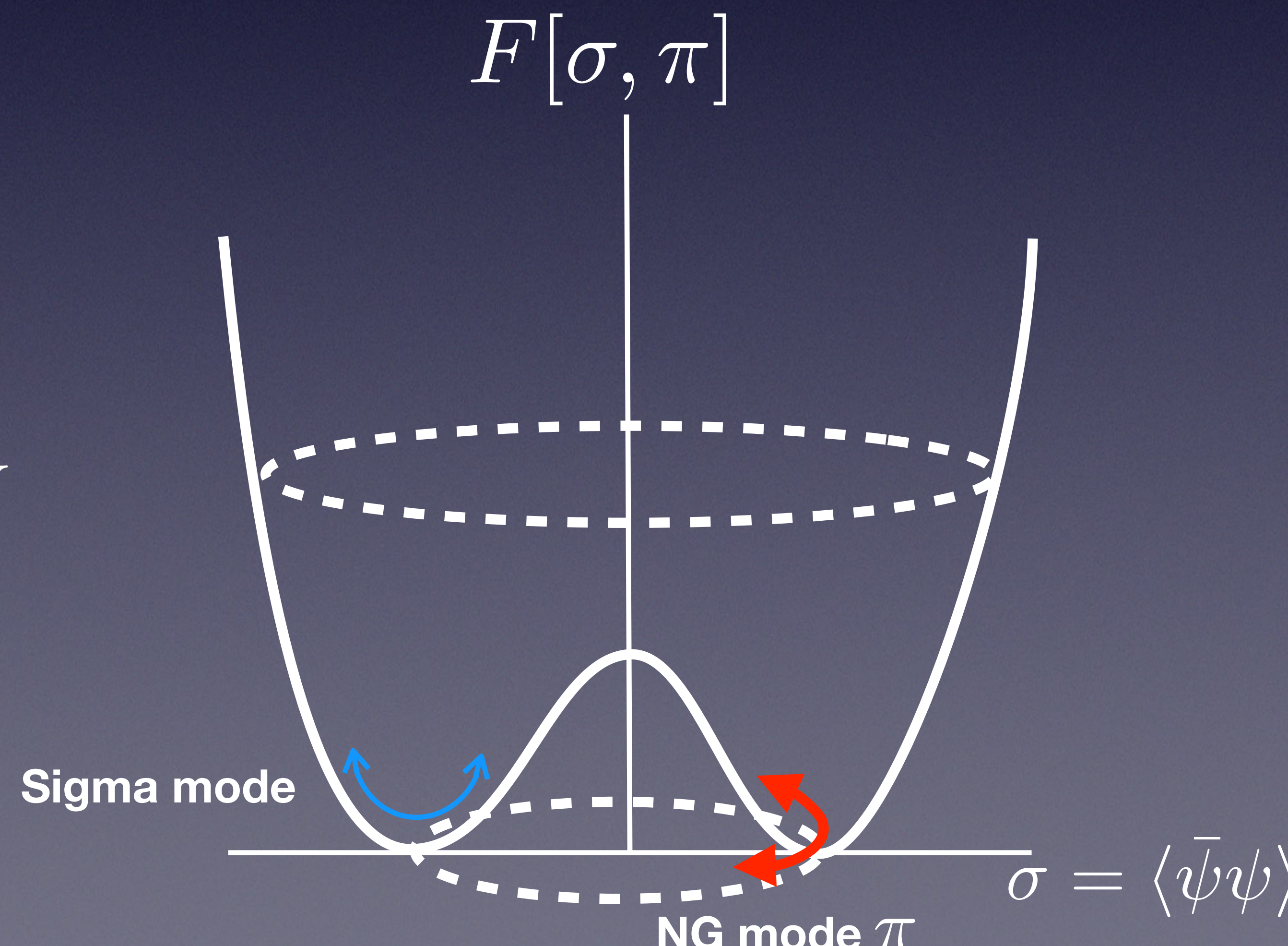


SSB of approximate symmetry of QCD

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$



Dispersion relation

$$\omega = \sqrt{\boldsymbol{k}^2 + m_\pi^2}$$

“Abnormal Number of NG bosons”

Schafer, Son, Stephanov, Toublan, and Verbaarschot (’01), Miransky, Shovkovy (’02), Blaschke, Ebert, Klimenko, Volkov, Yudichev (’04), Ebert, Klimenko, Yudichev (’05), He, Jin, Zhuang (’06) ,Buchel, Jia, Miransky (’07), ..

$$N_{\mathrm{NG}} \neq N_{\mathrm{BS}}$$

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Kaon-Condensed CFL phase

SSB pattern: $SU(2)_I \times U(1)_Y \to U(1)_{I_3+Y}$

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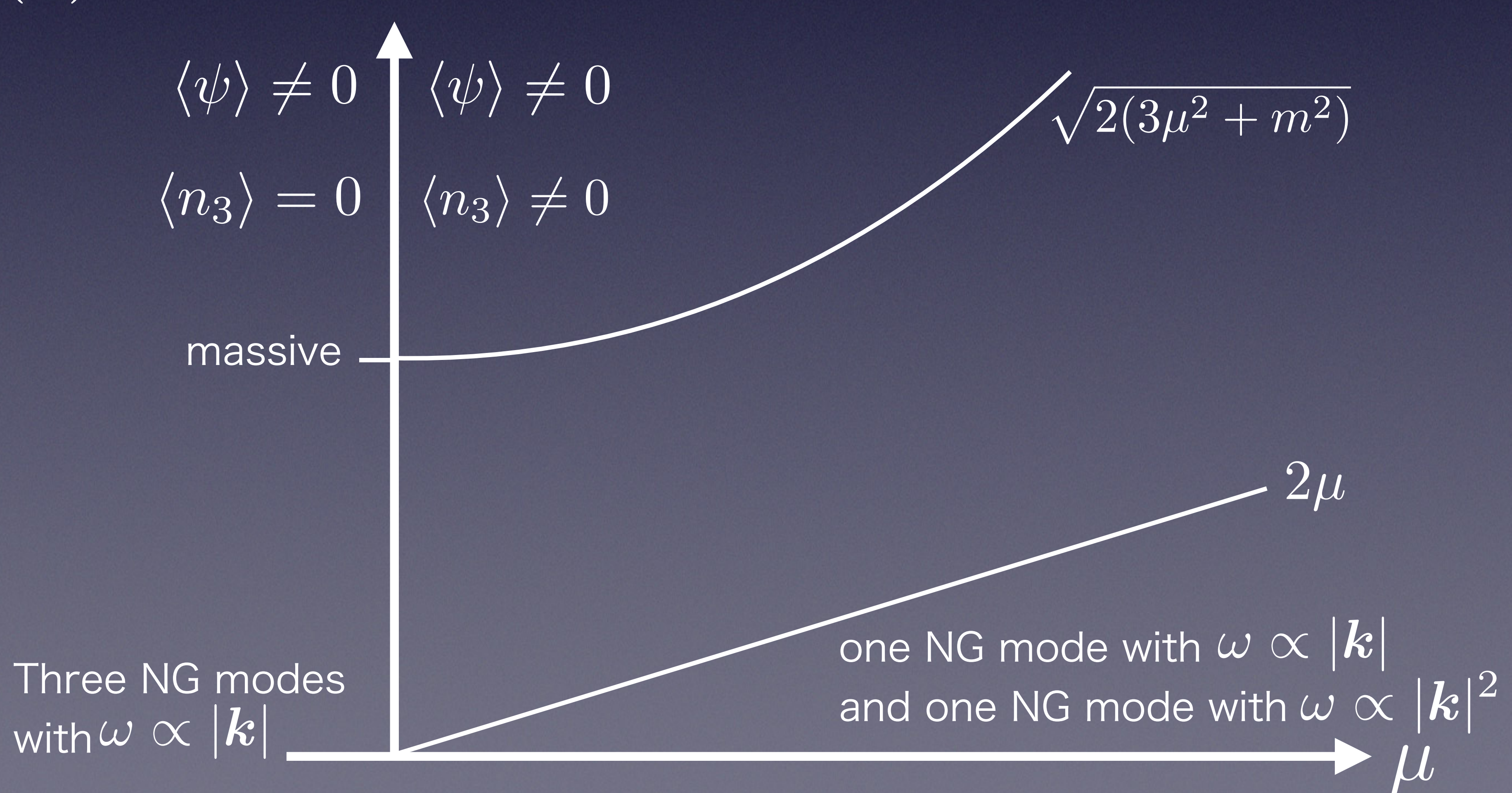
$SU(2) \times U(1)$ **model with chemical potential**



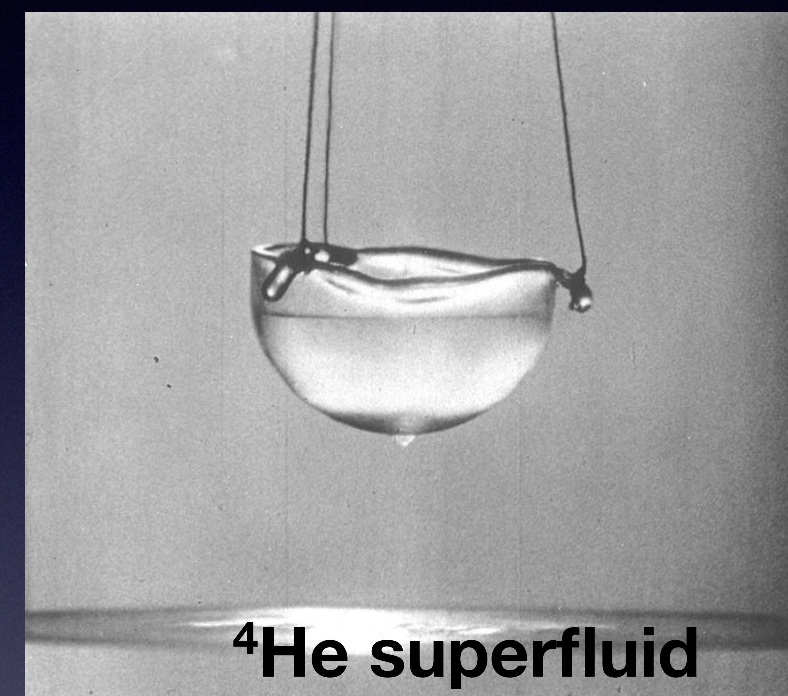
Example of NG modes in condensed matter Phys.

Superfluid phonon

broken of number

Broken generator Q

One phonon $\omega \sim |\boldsymbol{k}|$

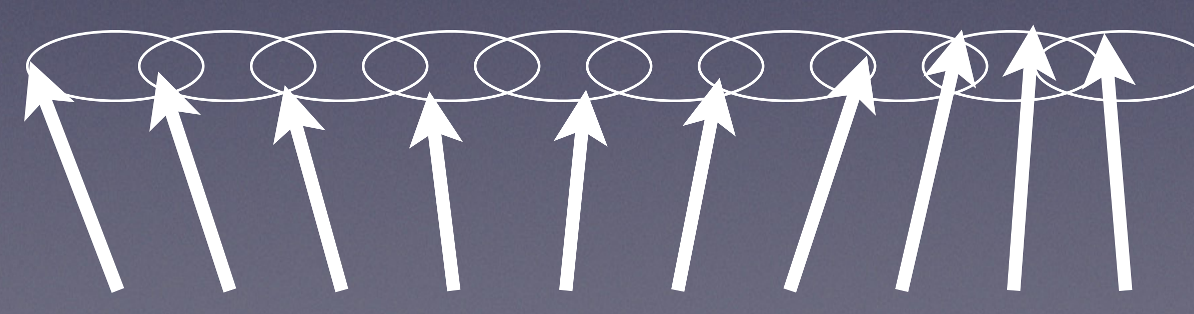


Magnon

Broken of rotation

Two broken generators S_x, S_y

one magnon $\omega \sim \boldsymbol{k}^2$



General theory of
Spontaneous symmetry breaking
(internal symmetry)

Generalization of NG theorem

Nielsen - Chadha ('76)

$$N_{\text{type-I}} + 2N_{\text{type-II}} \geq N_{\text{BS}}$$

Type-I: $\omega \propto k^{2n+1}$ **Type-II:** $\omega \propto k^{2n}$

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$$\langle [iQ_a, Q_b] \rangle = 0 \Rightarrow N_{\text{NG}} = N_{\text{BS}}$$

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$$\langle [iQ_a, Q_b] \rangle \neq 0 \Rightarrow (Q_a, Q_b)$$

canonical conjugate

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canonical conjugate

Watanabe - Brauner ('11)

$$N_{\text{BS}} - N_{\text{NG}} \leq \frac{1}{2} \text{rank} \langle [iQ_a, Q_b] \rangle$$

Recent progress

Effective Lagrangian method **Watanabe, Murayama ('12)**

Mori's projection operator method **YH ('12)**

- $N_{\mathrm{BS}} - N_{\mathrm{NG}} = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$
- $N_{\text{type-I}} + 2N_{\text{type-II}} = N_{\mathrm{BS}}$

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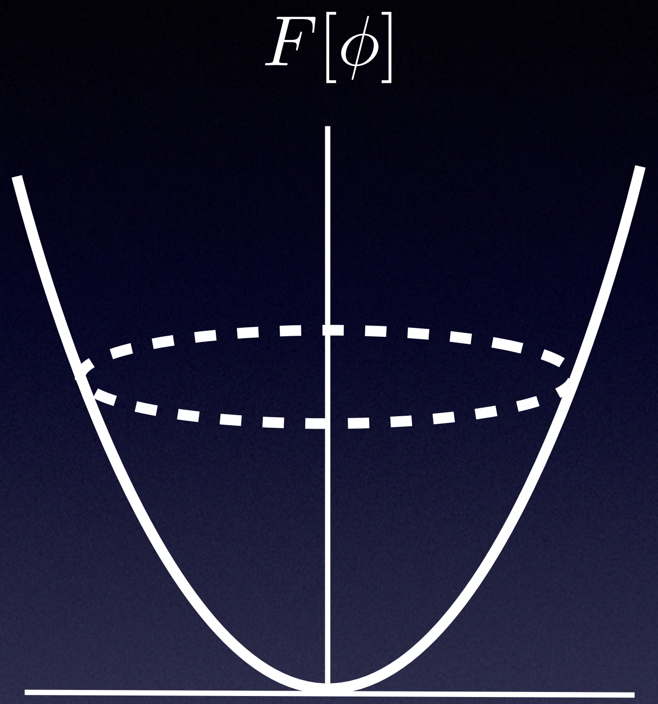
- $N_{\mathrm{BS}} - N_{\mathrm{NG}} = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$
- $N_{\text{type-A}} + 2N_{\text{type-B}} = N_{\mathrm{BS}}$
- $N_{\text{type-B}} = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$

Spontaneous symmetry breaking

For fields



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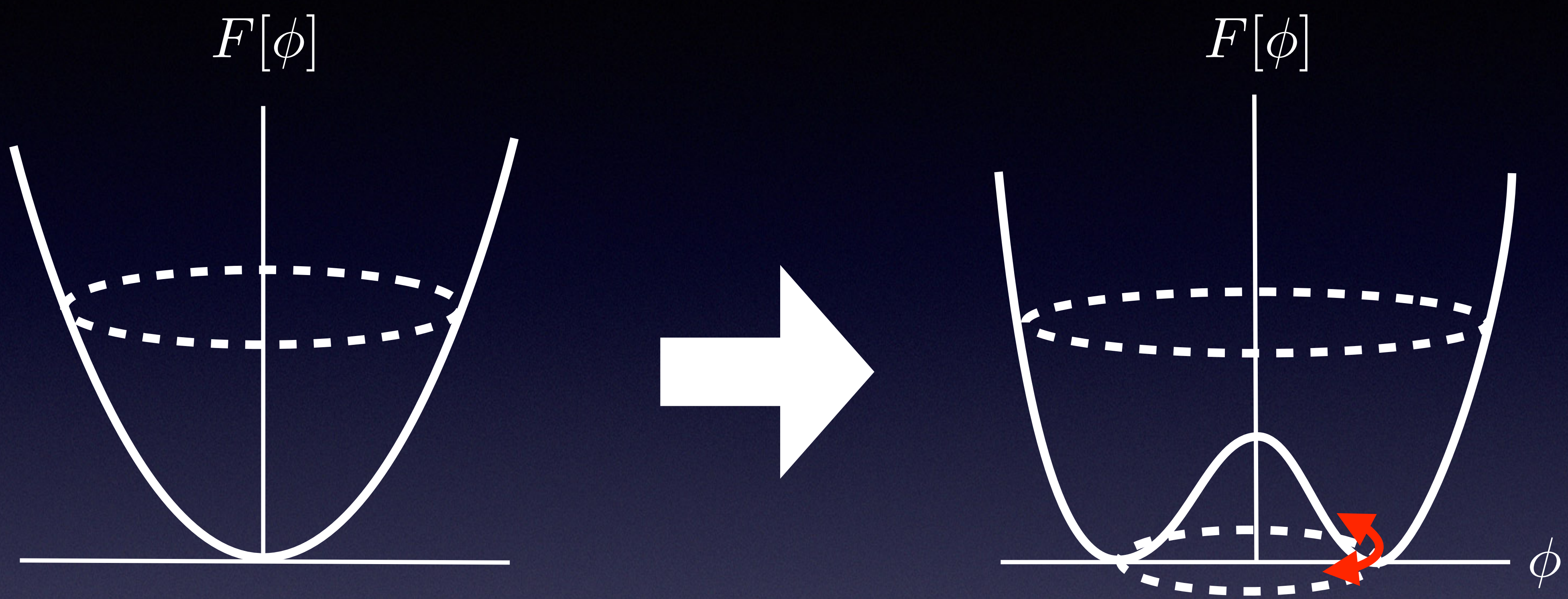
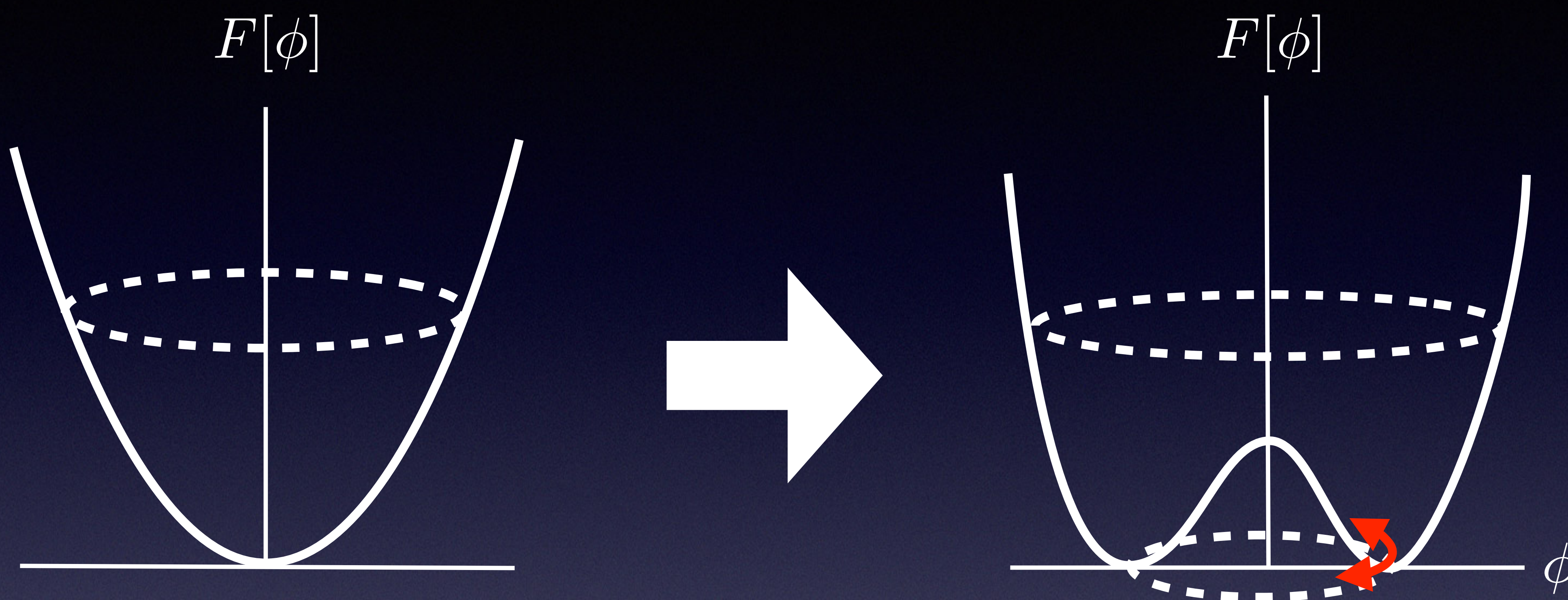


Degeneracy of ground states

Spontaneous symmetry breaking

For fields

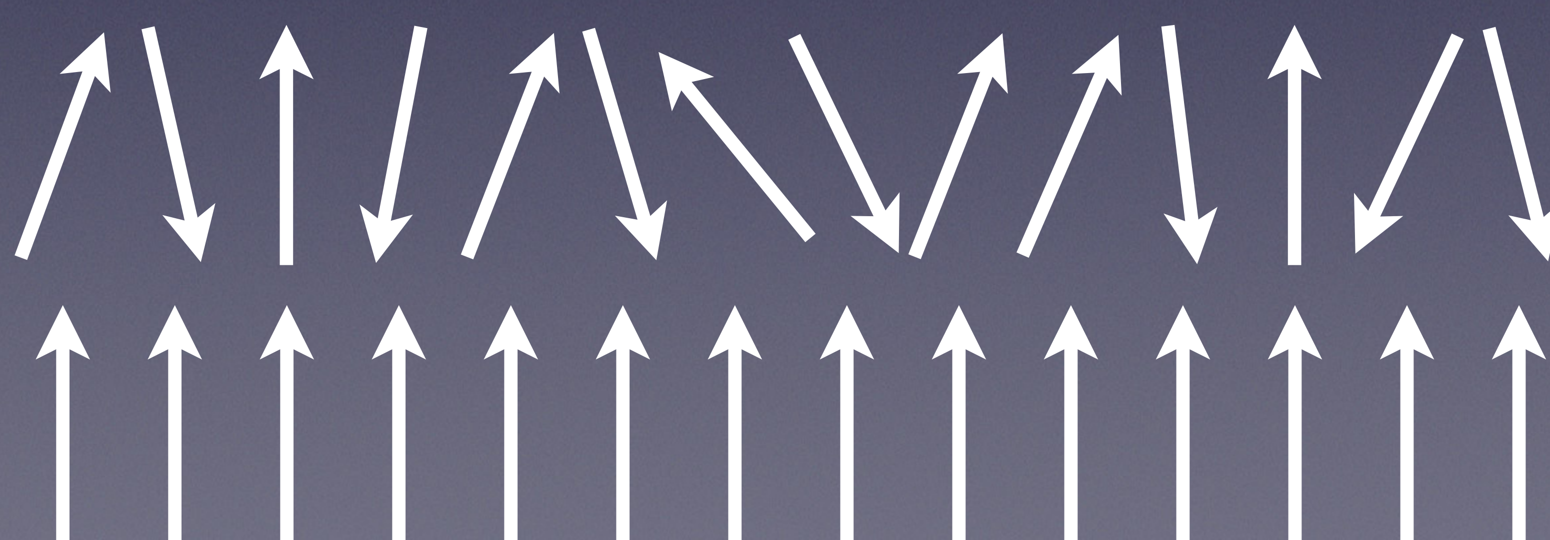


Degeneracy of ground states

For spins

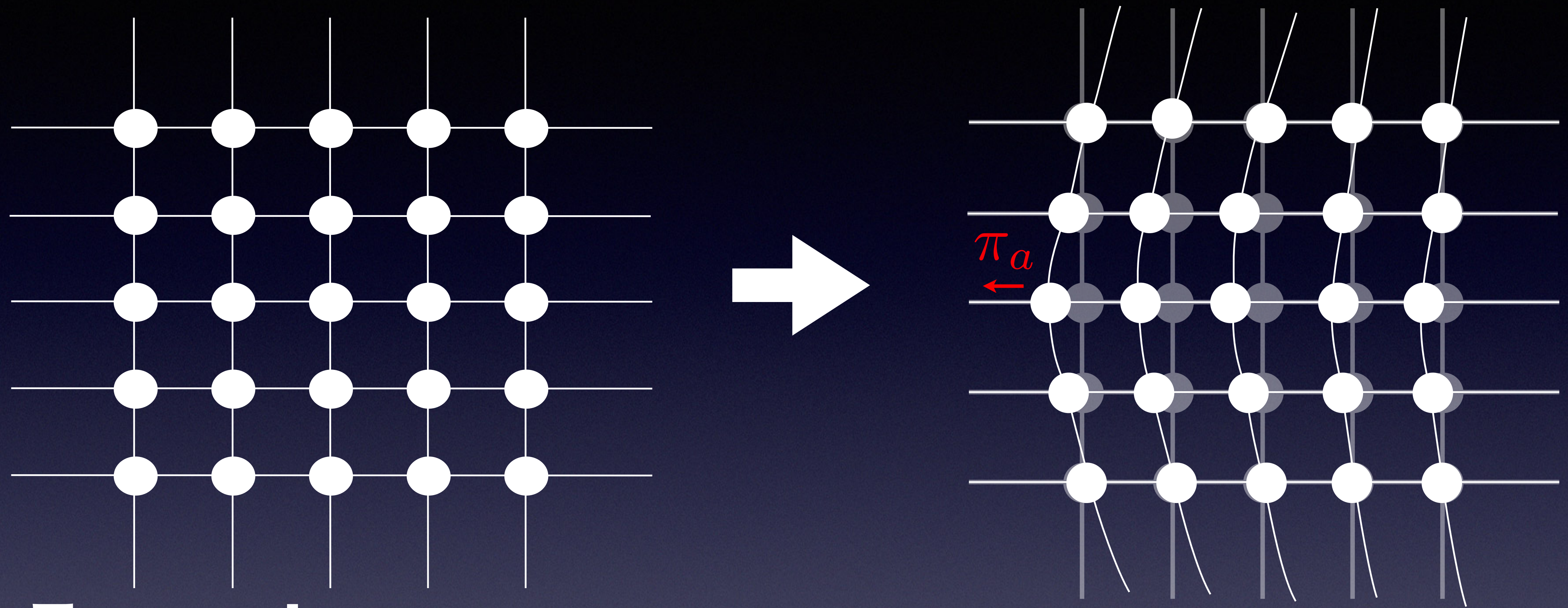
random

aligned

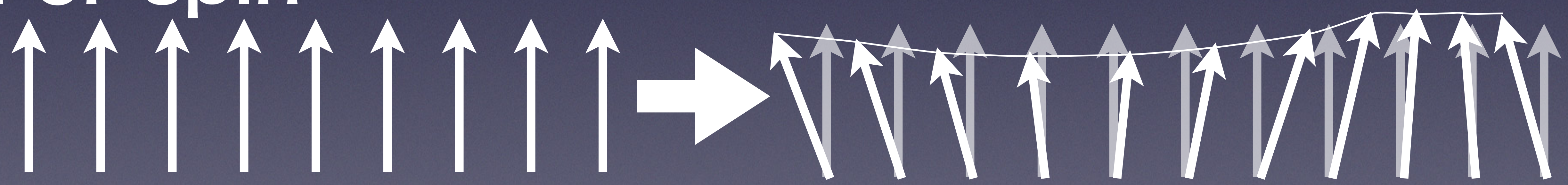


Elasticity

For crystal



For spin



Elasticity

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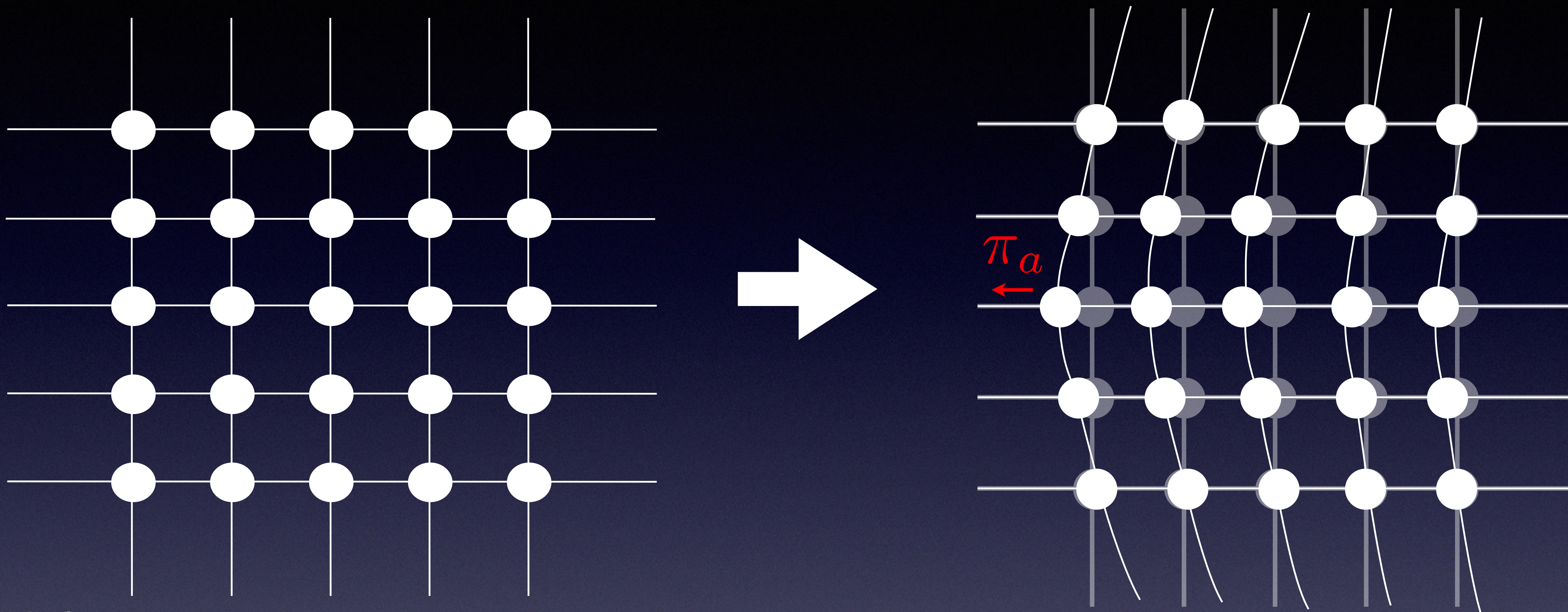
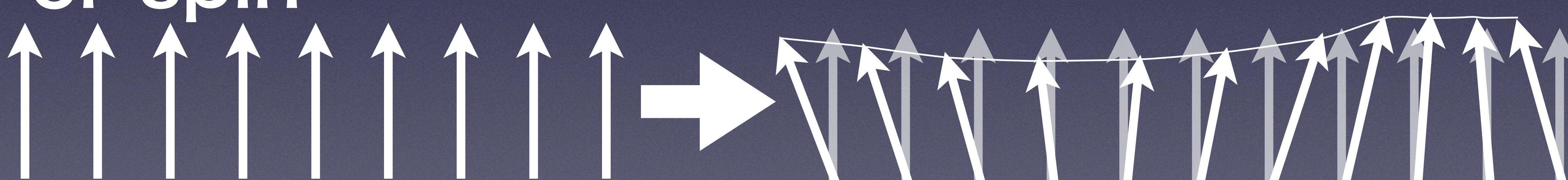


For spin



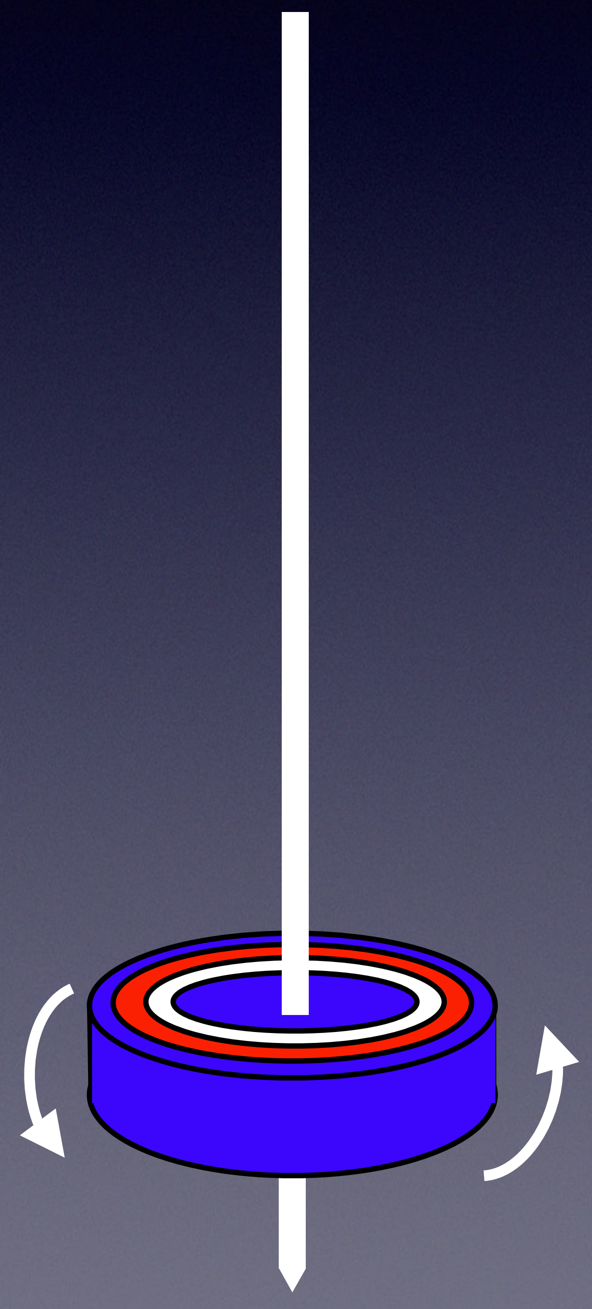
Free energy: $F = \frac{1}{2}(\partial_i \pi^a)^2 + \cdots$

# of elasticity = # of broken symm.

Intuitive example for type-B NG modes

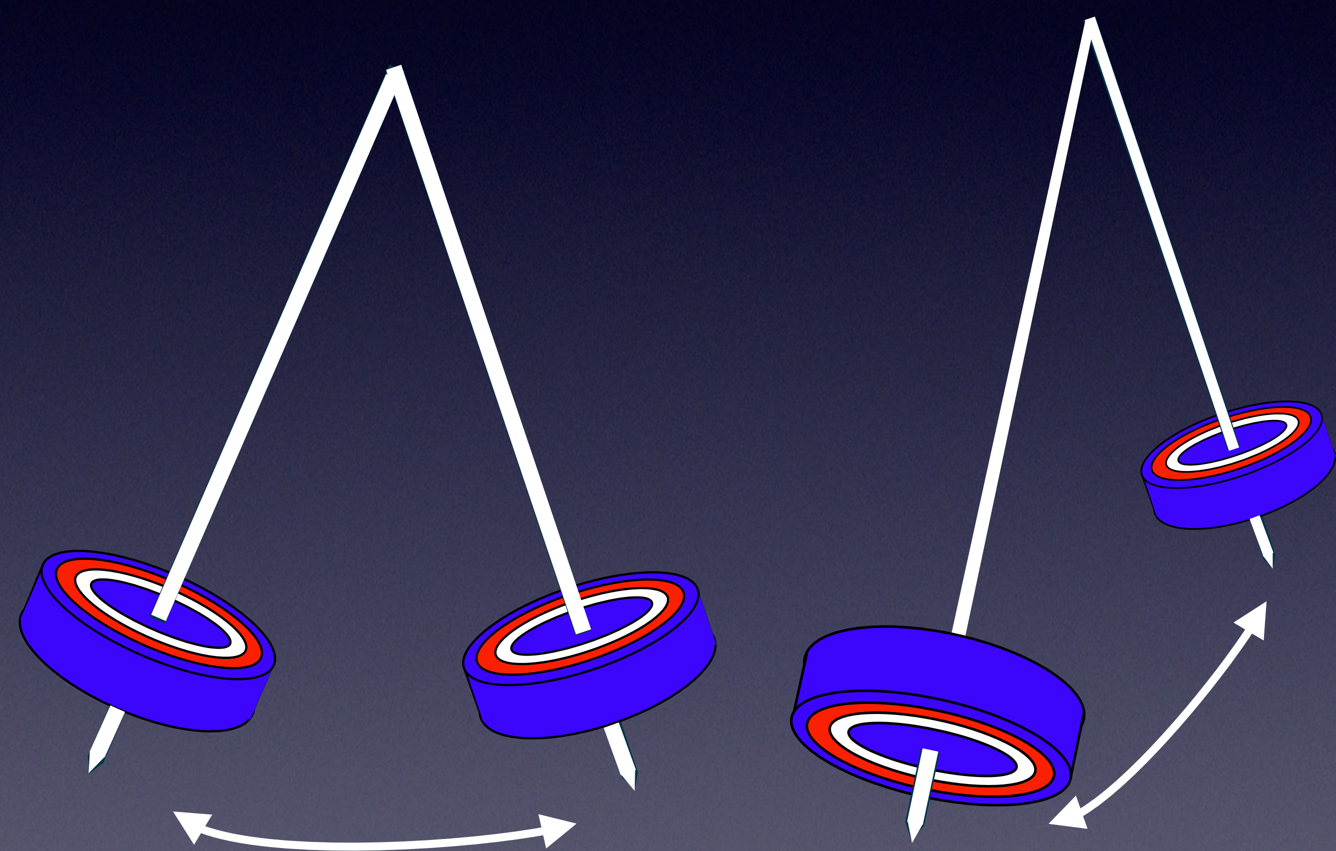
Pendulum with a spinning top

- Rotation symmetry is explicitly broken by a weak gravity
- Rotation along with z axis is unbroken.
- Rotation along with x or y is broken.
- The number of broken symmetry is two.



Intuitive example for type-B NG modes

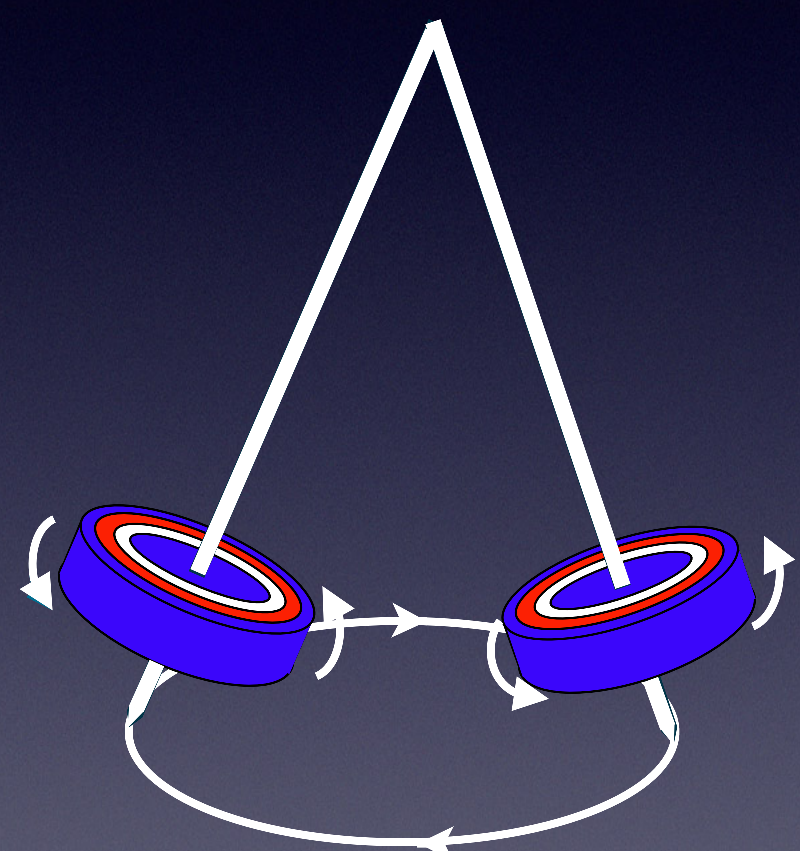
Pendulum has two oscillation motions



if the top is not spinning.

Intuitive example for type-B NG modes

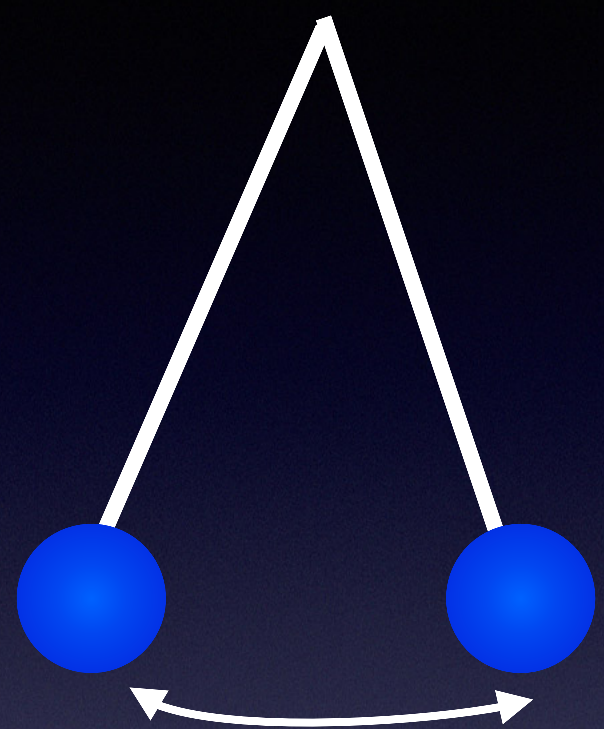
If the top is spinning,

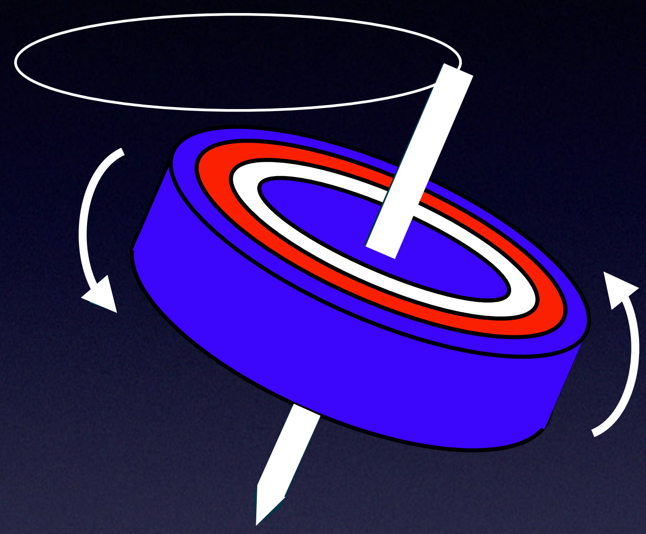


the only one rotation motion (Precession) exists.

In this case, $\{L_x, L_y\}_P = L_z \neq 0$

Two types of excitations





Type-A

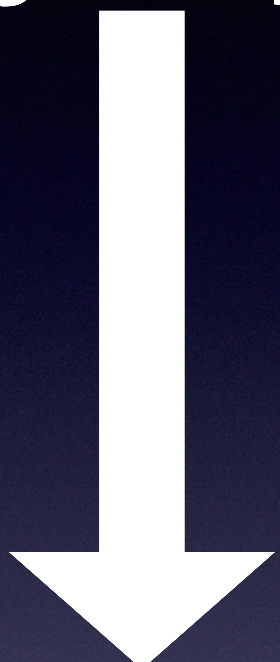
Harmonic oscillation

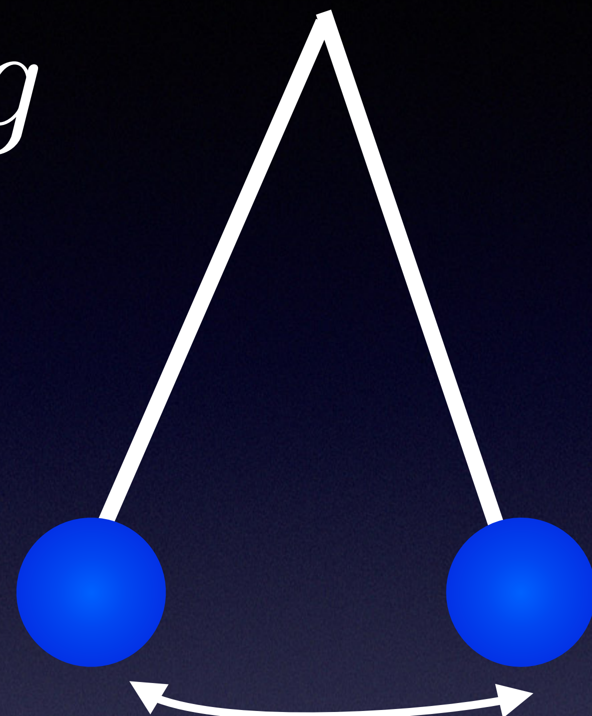
Type-B

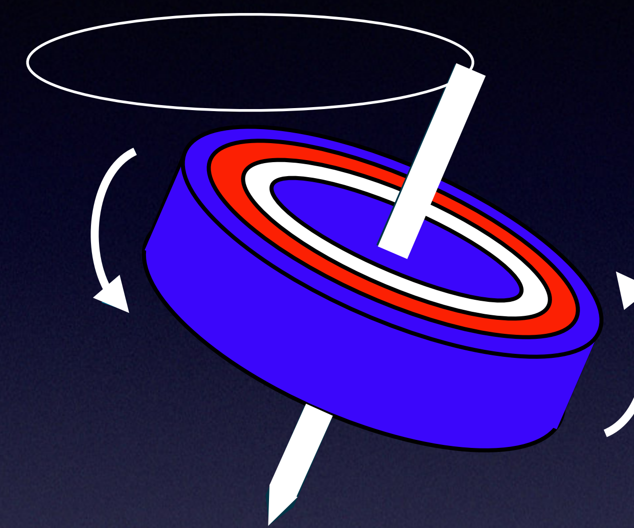
Precession motion

Two types of excitations

gravity g







Type-A

Harmonic oscillation

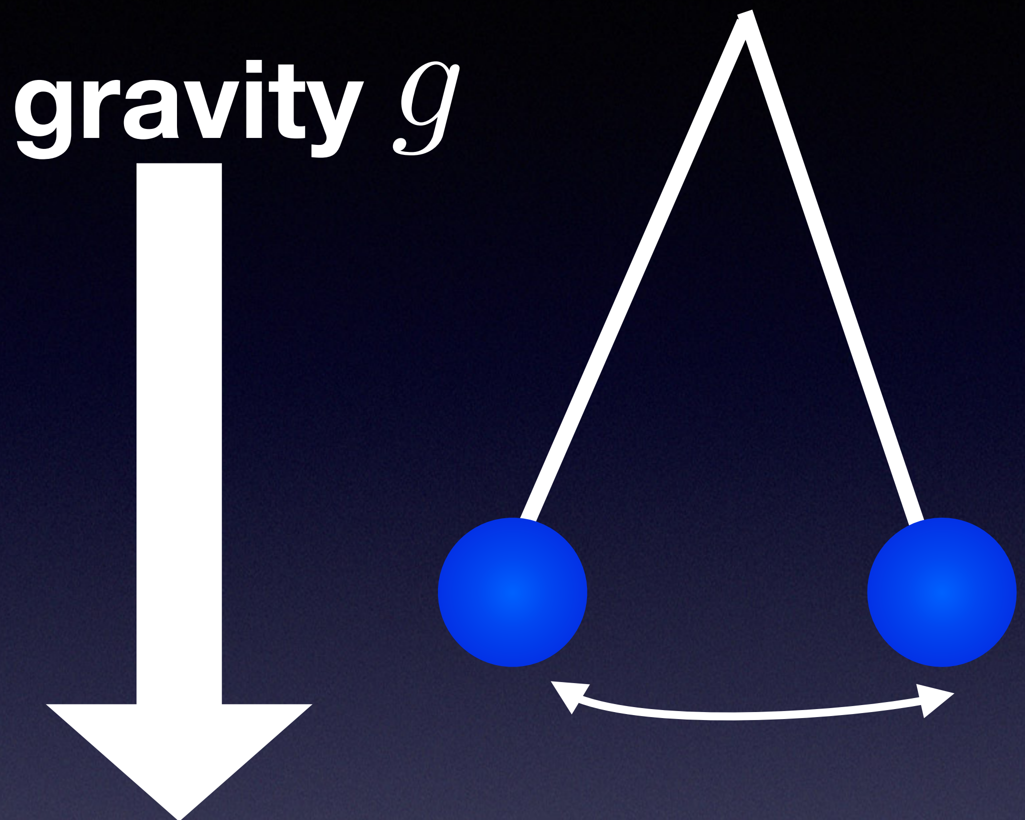
$$\omega \sim \sqrt{g}$$

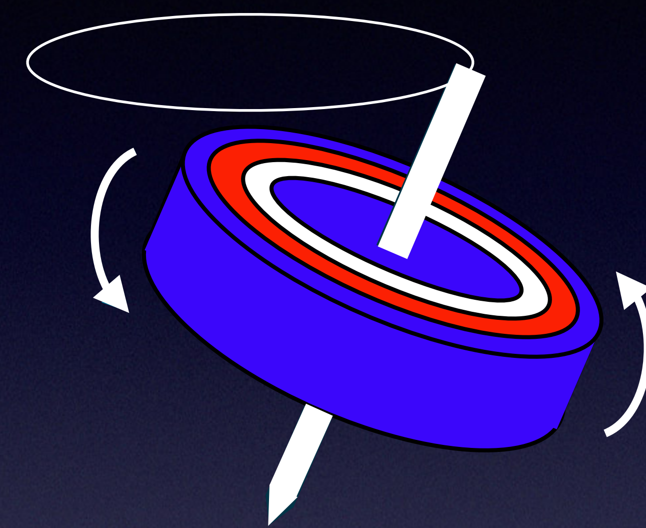
Type-B

Precession motion

$$\omega \sim g$$

Two types of excitations





Type-A

Harmonic oscillation

$$\omega \sim \sqrt{g} \sim \sqrt{k^2}$$

Type-I

Type-B

Precession motion

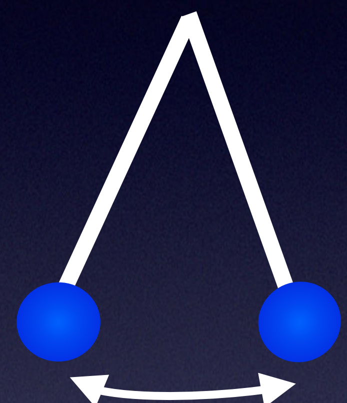
$$\omega \sim g \sim k^2$$

Type-II

Classification

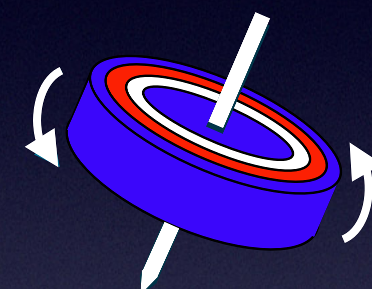
Watanabe, Murayama ('12), YH ('12)

NG modes associated with spontaneous breaking of internal symmetry can be classified by two types:



Type-A

harmonic oscillation



Type-B

precession motion

$$N_{\text{type-A}} = N_{\text{BS}} - 2N_{\text{type-B}} \qquad N_{\text{type-B}} = \frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$$

- $$N_{\text{BS}} - N_{\text{NG}} = \frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$$

Effective Lagrangian approach

Leutwyler('94), Watanabe, Murayama ('12)

Write down all possible terms

Leutwyler('94)

$$\mathcal{L} = \frac{1}{2}\rho_{ab}\pi^a\dot{\pi}^b + \frac{\bar{g}_{ab}}{2}\dot{\pi}^a\dot{\pi}^b - \frac{g_{ab}}{2}\partial_i\pi^a\partial_i\pi^b$$

+higher orders

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+higher orders

No Lorentz symmetry:

The first derivative term may appear.

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+higher orders

No Lorentz symmetry:

The first derivative term may appear.

Lagrangian is invariant under symmetry transformation up to surface term.

$$\Rightarrow \rho_{ab} \propto -i\langle[Q_a, j^0_b(x)]\rangle$$

Watanabe, Murayama ('12)

SSB with a small breaking term

$$H = H_0 + hV$$

Symmetric small explicit breaking term

Pseudo NG modes

YH ('12), Hayata, YH(14)

Type-A: $\omega \sim \sqrt{h}$

Ex) pions

Type-B: $\omega \sim h$

Ex) magnon in an external magnetic field

No higher corrections if the explicit breaking term is a charge.

Nicolis, Piazza ('12), ('13)
Watanabe, Brauner, Murayama ('13)

Examples of Type-B NG modes

	N_{BS}	$N_{\text{type-A}}$	$N_{\text{type-B}}$	$\frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$	$N_{\text{type-A}} + 2N_{\text{type-B}}$
Spin wave in ferromanget $SO(3) \to SO(2)$	2	0	1	1	2
NG modes in Kaon condensed CFL $SU(2)_I \times U(1)_Y \to U(1)_{\text{em}}$	3	1	1	1	3
Kelvin waves in vortex Translation $\mathbf{R}^3 \to \mathbf{R}^1$	2	0	1	1	2
nonrelativistic massive CP1 model $U(1) \times \mathbf{R}^3 \to \mathbf{R}^2$	2	0	1	1	2

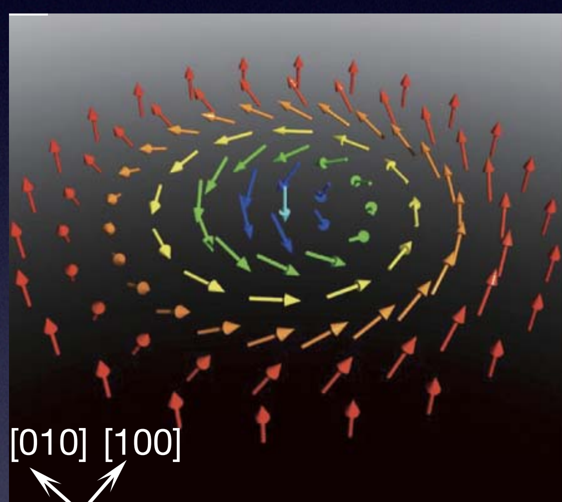
$$N_{\text{type-A}} + 2N_{\text{type-B}} = N_{\text{BS}} \qquad N_{\text{BS}} - N_{\text{NG}} = \frac{1}{2}\text{rank}\langle[iQ_a, Q_b]\rangle$$

Topological soliton and central extensions

Translation-translation

Ex.) 2+1D skyrmion, Kelvin waves

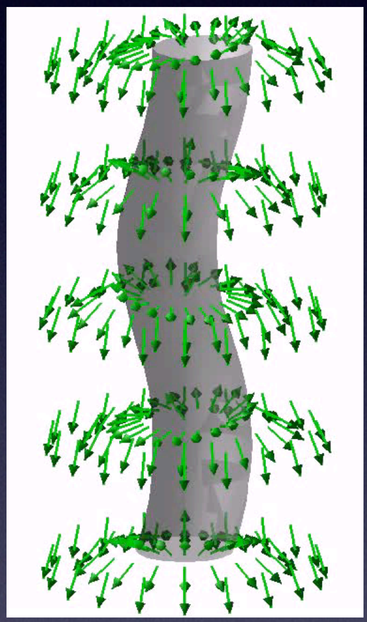
Watanabe, Murayama ('14)   Kobayashi, Nitta ('12,'14)



Yu, et al Nature 465, 901 (2010)

$$[P_x, P_y] \propto N$$

x trans.   y trans.   topological number



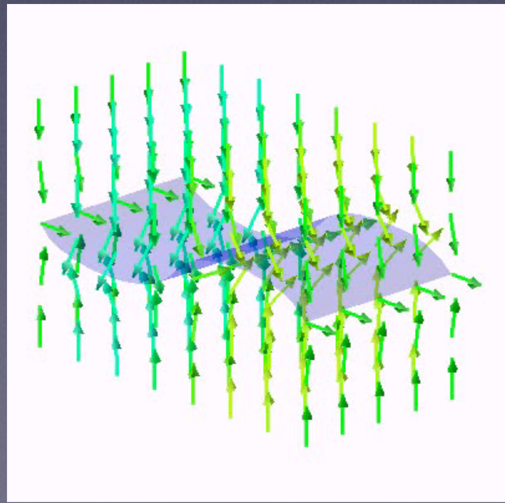
Translation-Internal symm.

Ex.) domain wall in nonrelativistic massive CP^1 model

Kobayashi, Nitta ('14)

$$[P_z, Q] \propto N$$

z trans   U(1) charge   topological number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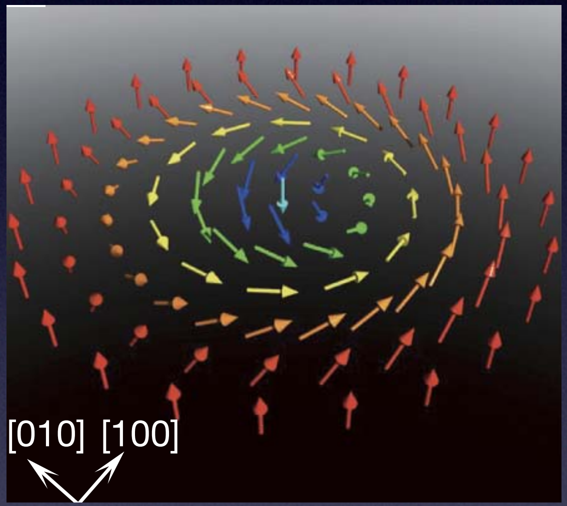
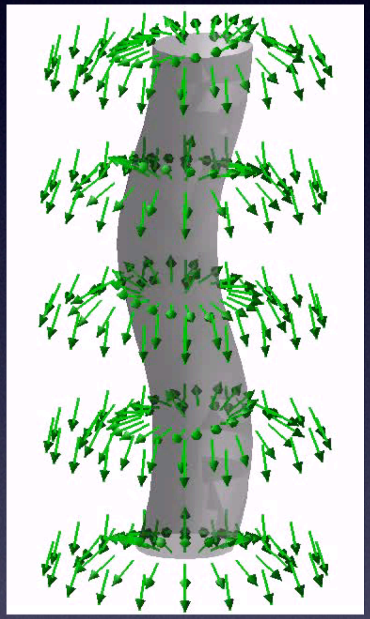


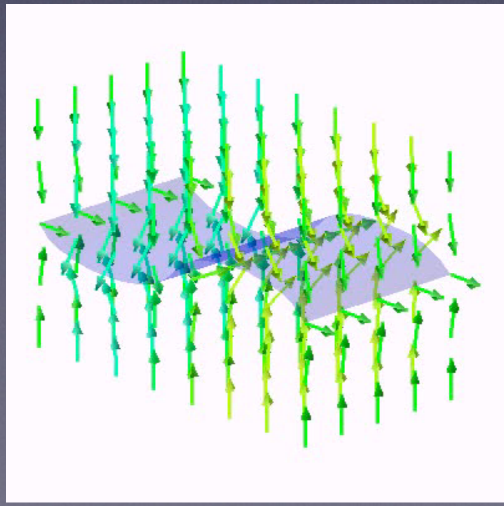
Yu, et al Nature 465, 901 (2010)

$$[P_x, P_y] \propto N$$

x trans. y trans. topological number



Translation-Internal symm.



Kobayashi, Nitta ('14)

Ex.) domain wall in nonrelativistic massive CP^1 model

Kobayashi, Nitta ('14)

$$[P_z, Q] \propto N$$

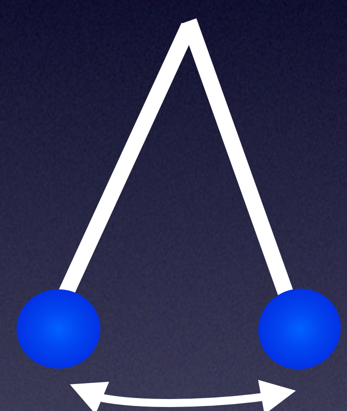
z trans U(1) charge topological number

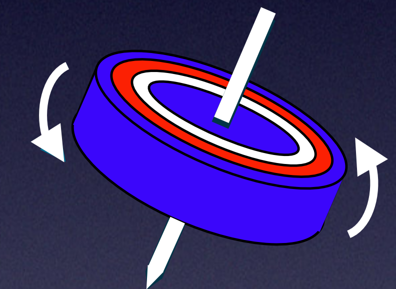
Summary

For SSB of internal symmetries,

classification is completed!

$$N_{\mathrm{BS}} - N_{\mathrm{NG}} = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$$





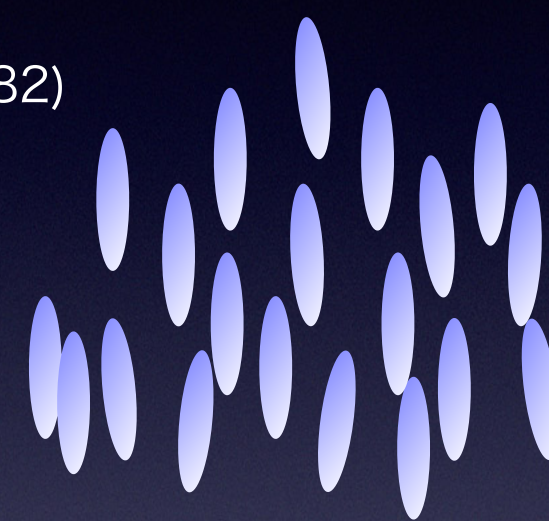
Type-A	**Type-B**
harmonic oscillation	**precession motion**
$N_{\text{type-A}} = N_{\mathrm{BS}} - 2N_{\text{type-B}}$	$N_{\text{type-B}} = \frac{1}{2}\mathrm{rank}\langle[iQ_a, Q_b]\rangle$
$\omega = ak - ibk^2$	$\omega = a'k^2 - ib'k^4$

Spacetime breaking is more complicated

Ex) Liquid crystal (type-A)

Nematic phase: rotation O(3)→O(2)

Dispersion relation: $\omega = ak^2 - ibk^2$ Hosino, Nakano('82)



Real and imaginary parts are the same order (damped oscillation)

In case $a = 0$, (overdamping)

Ex) Capillary wave (Type-B)

cf. Takeuchi, Kasamatsu ('13)   Effective Lagrangian: Watanabe, Murayama ('14)

$\frac{1}{V}\langle [P_z, N] \rangle \neq 0$ $\omega \sim k^{3/2}$