Mechanisms of chiral symmetry breaking in QCD: a lattice perspective

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QCD action and its (broken) symmetries [Fritzsch, Gell-Mann, Leutwyler 73; Gross, Wilczek 73; Weinberg 73]

- QCD action for $N_F = 2$, $M = \text{diag}(m, m)$

$$S = S_G + \int d^4x \left\{ \bar\psi D\psi + \bar\psi M \psi \right\}, \qquad D = \gamma_\mu(\partial_\mu + iA_\mu)$$

- For $M = 0$ chiral symmetry

$$\psi_{\rm R,L} \to V_{\rm R,L}\,\psi_{\rm R,L} \quad \psi_{\rm R,L} = \left(\frac{1 \pm \gamma_5}{2}\right)\psi$$

Chiral anomaly: measure not invariant

SSB: vacuum not symmetric

$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\rm L} \times \mathrm{SU}(2)_{\rm R} \times \mathrm{U}(1)_{\rm L} \times \mathrm{U}(1)_{\rm R} \times \mathcal{R}_{\rm scale}$

↓ (dim. transm., **chiral anomaly**)

$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\rm L} \times \mathrm{SU}(2)_{\rm R} \times \mathrm{U}(1)_{\rm B=L+R}$

↓ **(Spont. Sym. Break.)**

$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\rm L+R} \times \mathrm{U}(1)_{\rm B}$

↓ (Confinement)

$\mathrm{SU}(2)_{\rm L+R} \times \mathrm{U}(1)_{\rm B}$

- Breaking due to non-perturbative dynamics. Precise quantitative tests are being made on the lattice

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- A comprehensive review of lattice results is provided by the FLAG WG [Aoki et al. 13]

Outline

- Banks–Casher mechanism in QCD:
  - $*$ Renormalizability of the spectral density
  - $*$ The density in QCD Lite
- Witten–Veneziano mechanism in QCD:
  - $*$ Unambiguous definition of the topological susceptibility
  - $*$ Recent numerical results
  - $*$ Beyond the second cumulant
- Conclusions and outlook

Banks–Casher relation [Banks, Casher 80]

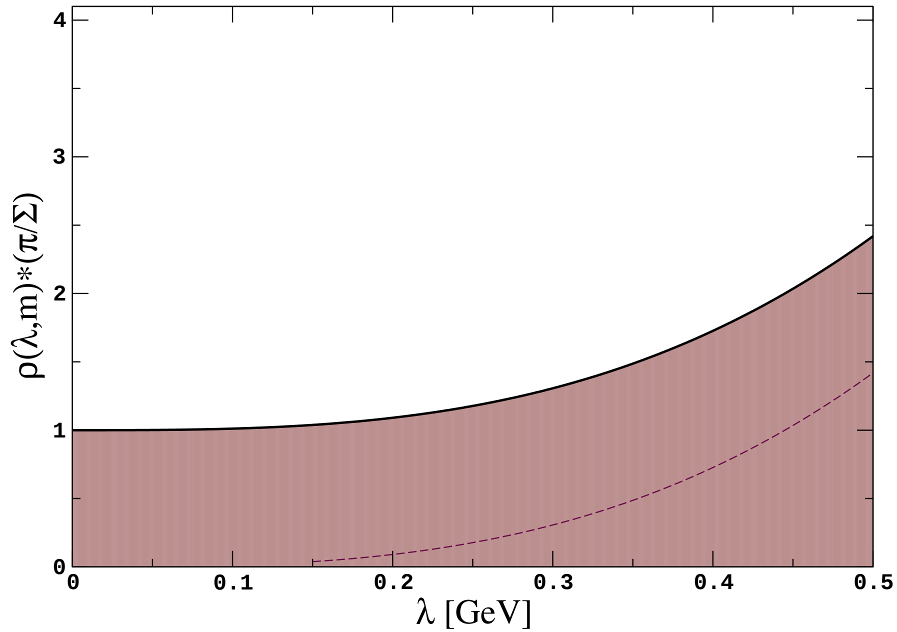
- For each gauge configuration

$$D_m \chi_k = (m + i\lambda_k)\chi_k$$

- The spectral density of D is

$$\rho(\lambda, m) = \frac{1}{V}\sum_k \langle \delta(\lambda - \lambda_k) \rangle$$

where $\langle \ldots \rangle$ indicates path-integral average



- The Banks–Casher relation

$$\lim_{\lambda\to 0}\lim_{m\to 0}\lim_{V\to\infty} \rho(\lambda, m) = \frac{\Sigma}{\pi}$$

provides a link between the condensate and the (non-zero) density at the origin.
To be compared, for instance, with the free case $\rho(\lambda) \propto |\lambda^3|$

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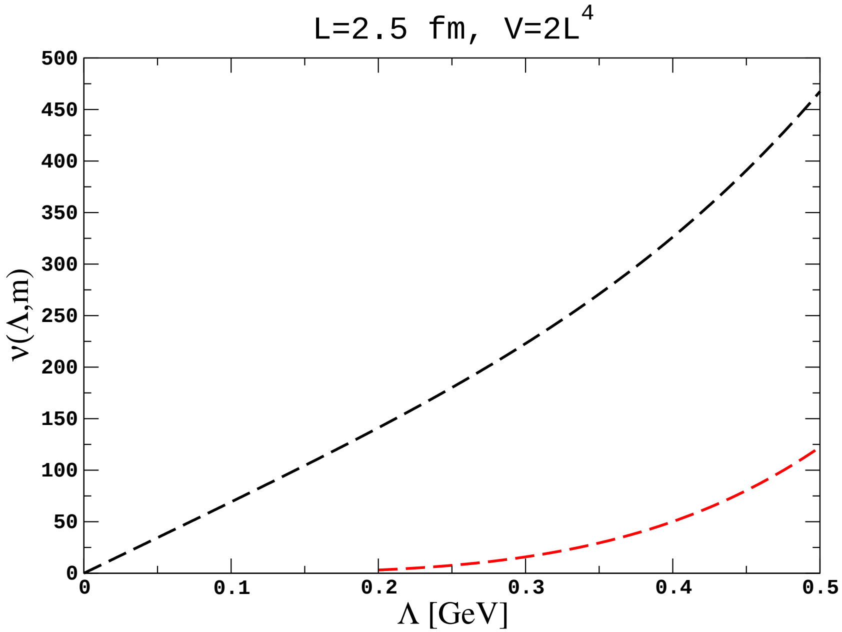

- The number of modes in a given energy interval

$$\nu(\Lambda, m) = V \int_{-\Lambda}^{\Lambda} d\lambda \, \rho(\lambda, m) \qquad \nu(\Lambda, m) = \frac{2}{\pi} \Lambda \Sigma V + \ldots$$

grows linearly with Λ, and they condense near the origin with values $\propto 1/V$
In the free case $\nu(\Lambda, m) \propto V\Lambda^4$

Renormalizability and universality of the spectral density [L. G., Lüscher 09]

- Instead of the spectral density, consider the spectral sum

$$\sigma_k(m_v, m) = V \int_{-\infty}^{\infty} d\lambda \frac{\rho(\lambda, m)}{(\lambda^2 + m_v^2)^k}$$

$$= -a^{8k} \sum_{x_1 \dots x_{2k}} \langle P_{12}(x_1) P_{23}(x_2) \dots P_{2k1}(x_{2k}) \rangle$$

$*$ Integral converges if $k \geq 3$

$*$ The relation between $\sigma_k(m_v, m)$ and $\rho(\lambda, m)$ invertible for every k

- Renormalization properties of $\rho(\lambda, m)$ can thus be inferred from those of σ_k

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- Corr. func. of pseudoscalar densities at physical distance renormalized by $(1/Z_m)^{2k}$

- At short distance the flavour structure implies

$$P_{12}(x_1) P_{23}(x_2) \sim C(x_1 - x_2) S_{13}(x_1) \qquad S_{13} = \bar{\psi}_1 \psi_3$$

where $C(x)$ diverges like $|x|^{-3}$ and it is therefore integrable. Analogous argument for all other short-distance singularities. No extra contact terms needed to renormalize σ_k

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$$\begin{aligned}\sigma_k(m_v, m) &= V \int_{-\infty}^{\infty} d\lambda \frac{\rho(\lambda, m)}{(\lambda^2 + m_v^2)^k} \\ &= -a^{8k} \sum_{x_1 \ldots x_{2k}} \langle P_{12}(x_1) P_{23}(x_2) \ldots P_{2k1}(x_{2k}) \rangle\end{aligned}$$

- Once the gauge coupling and the mass(es) are renormalized, the spectral sum

$$\sigma_{k,\mathrm{R}}(m_{v\mathrm{R}}, m_{\mathrm{R}}) = Z_m^{-2k} \sigma_k \left(\frac{m_{v\mathrm{R}}}{Z_m}, \frac{m_{\mathrm{R}}}{Z_m} \right)$$

is ultraviolet finite. Continuum limit universal (if same renormalization cond. are used)

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$$= -a^{8k} \sum_{x_1 \dots x_{2k}} \langle P_{12}(x_1) P_{23}(x_2) \dots P_{2k1}(x_{2k}) \rangle$$

- The spectral density thus renormalizes as

$$\rho_{\mathrm{R}}(\lambda_{\mathrm{R}}, m_{\mathrm{R}}) = Z_m^{-1} \rho \left(\frac{\lambda_{\mathrm{R}}}{Z_m}, \frac{m_{\mathrm{R}}}{Z_m} \right)$$

- For Wilson fermions similar derivation but twisted-mass valence quarks

- The rate of condensation is indeed a renormalizable universal quantity in QCD, and is unambiguously defined once the bare parameters in the action of the theory have been renormalized

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$$= -a^{8k} \sum_{x_1 \dots x_{2k}} \langle P_{12}(x_1) P_{23}(x_2) \dots P_{2k1}(x_{2k}) \rangle$$

- It follows that the mode number is a renormalization-group invariant

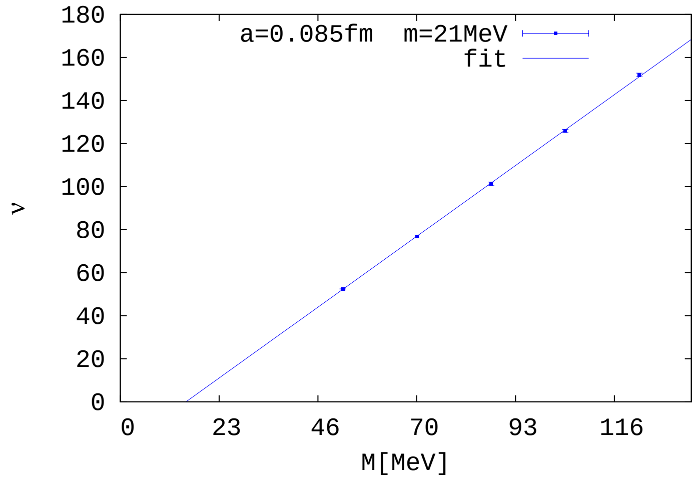
$$\nu_{\mathrm{R}}(\Lambda_{\mathrm{R}}, m_{\mathrm{R}}) = \nu(\Lambda, m)$$

and its continuum limit is universal for any value of Λ and m

Numerical computation for $N_F = 2$ [L. G., Lüscher 09; Cichy et al. 13; Engel et al. 14]

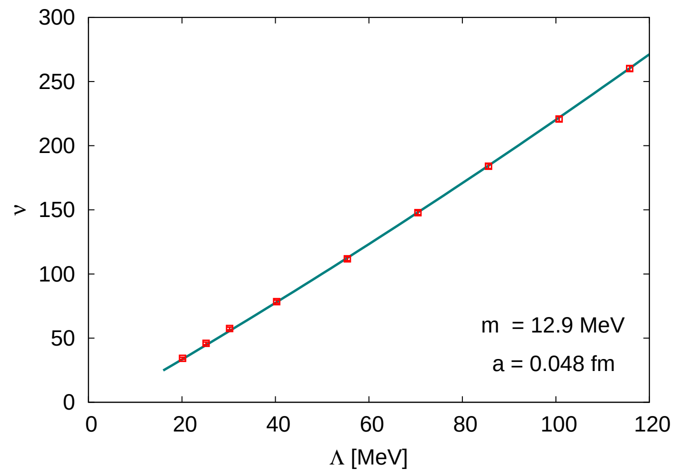
- Twisted-mass QCD [Cichy et al. 13]:

  * $a = 0.054$–0.085 fm
  * $m = 16$–47 MeV
  * $\Lambda = 45$–110 MeV
  * $M = \sqrt{\Lambda^2 + m^2}$



- $O(a)$–improved Wilson fermions [Engel et al. 14]:

  * $a = 0.048$–0.075 fm
  * $m = 6$–37 MeV
  * $\Lambda = 20$–500 MeV
  * $\nu = -9.0(13){+}2.07(7)\Lambda{+}0.0022(4)\Lambda^2$



- The mode number is a nearly linear function in Λ up to approximatively 100–150 MeV. The modes do condense near the origin as predicted by Banks–Casher mechanism.

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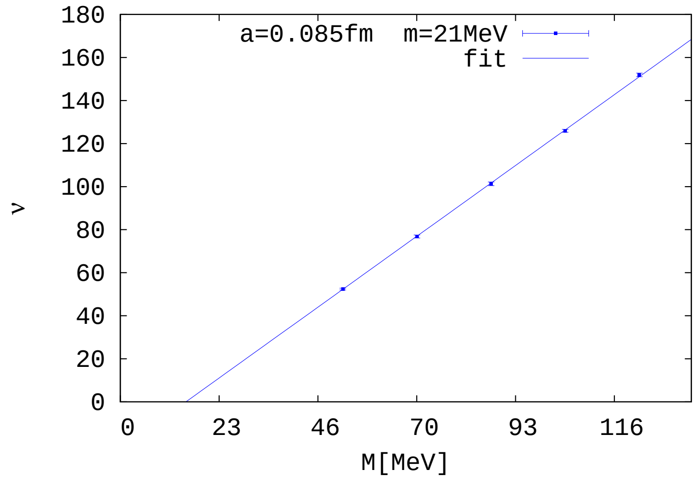

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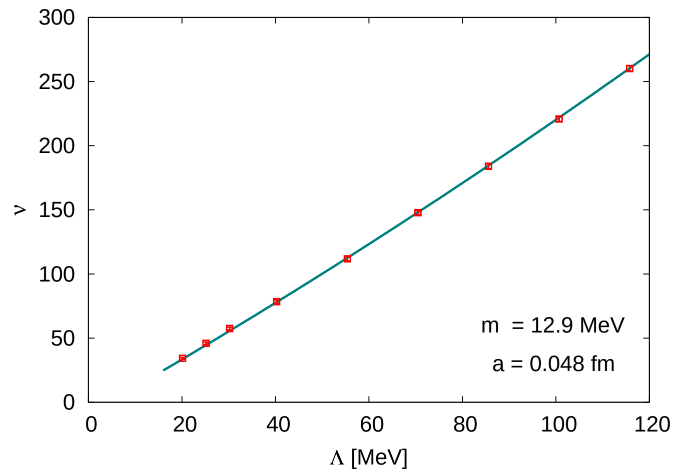


- At fixed lattice spacing and at the percent precision, however, the data show statistically significant corrections from the linear behaviour of $O(10\%)$.

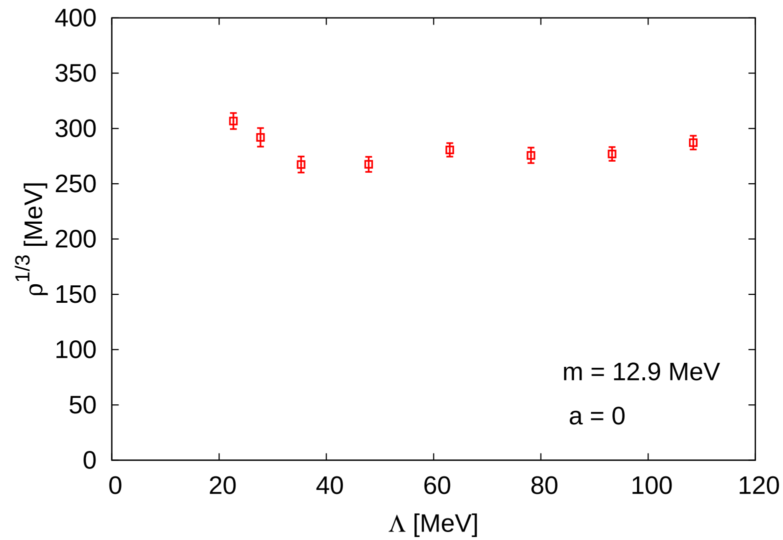
Continuum limit [Engel et al. 14]

- By defining

$$\widetilde{\rho}(\Lambda_1,\Lambda_2,m) = \frac{\pi}{2V}\,\frac{\nu(\Lambda_2)-\nu(\Lambda_1)}{\Lambda_2-\Lambda_1}$$

the continuum limit is taken at fixed m, Λ_1 and Λ_2 [$\Lambda = (\Lambda_1+\Lambda_2)/2$]

- The data are extrapolated linearly in a^2 as dictated by the Symanzik analysis



- The results point to the fact that the spectral density is non-zero and (almost) constant in Λ near the origin and at small quark masses

- This is consistent with the expectations from the Banks–Casher mechanism. In presence of SSB, NLO ChPT indeed predicts

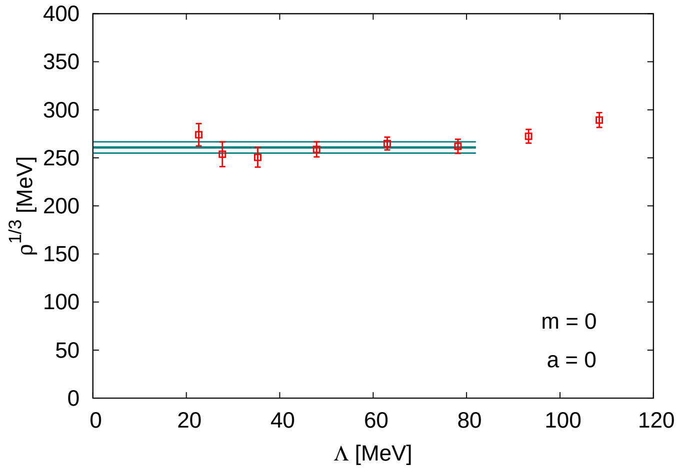
$$\widetilde{\rho}^{\,\rm nlo} = \Sigma\Big\{1+\frac{m\Sigma}{(4\pi)^2F^4}\Big[3\,\bar{l}_6+1-\ln(2)-3\ln\Big(\frac{\Sigma m}{F^2\bar{\mu}^2}\Big)+\tilde{g}_\nu\Big(\frac{\Lambda_1}{m},\frac{\Lambda_2}{m}\Big)\Big]\Big\}$$

where $\tilde{g}_\nu$ is a parameter-free known function (almost) flat for (small) Λ

Chiral limit [Engel et al. 14]

- In the chiral limit NLO ChPT predicts $\widetilde{\rho}$ to be Λ-independent. By extrapolating with NLO ChPT

$$[\widetilde{\rho}^{\,\overline{\rm MS}}]^{1/3} = [\Sigma_{\rm BK}^{\overline{\rm MS}}(2\,\text{GeV})]^{1/3} = 261(6)(8)\,\text{MeV}$$



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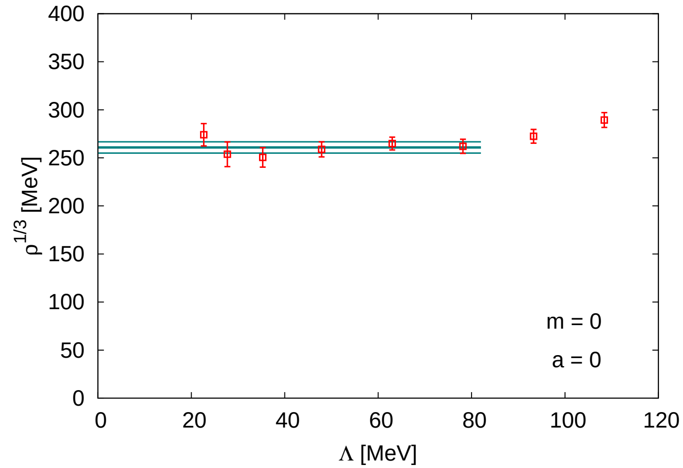
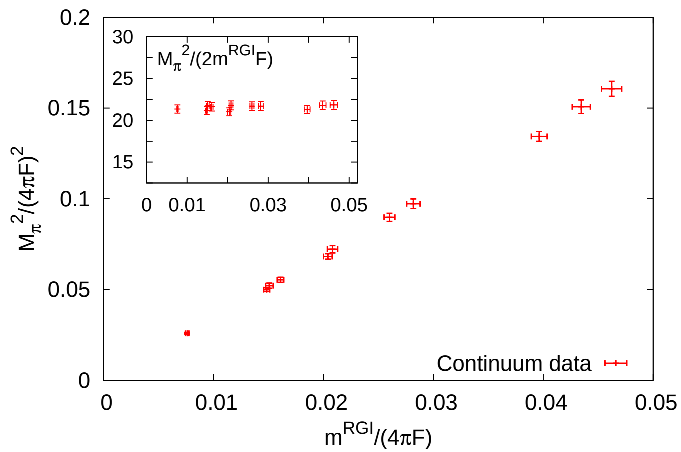


- The distinctive signature of SSB is the agreement between $\widetilde{\rho}$ and the slope of $M_\pi^2 F_\pi^2/2$ with respect to m in the chiral limit



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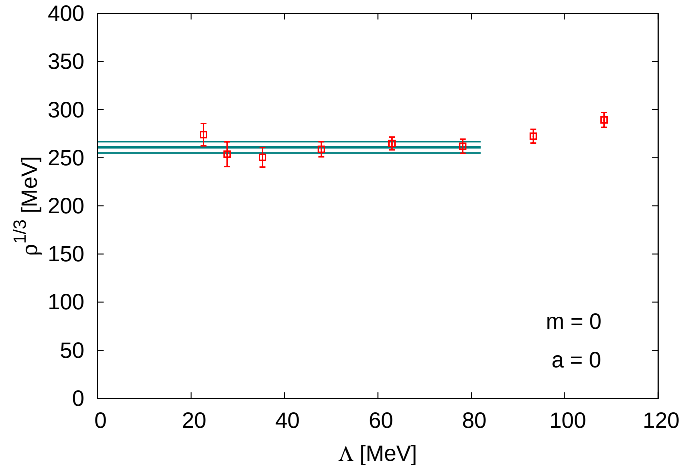

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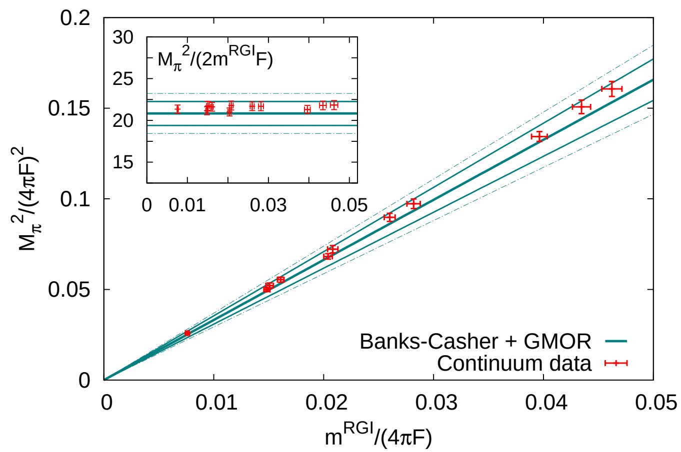
- On the same set of configurations

$$[\Sigma_{\rm GMOR}^{\overline{\rm MS}}(2\,\text{GeV})]^{1/3} = 263(3)(4)\,\text{MeV}$$



- The low-modes of the Dirac operator do condense in the continuum, and the rate of condensation explains the bulk of the pion mass up to $M_\pi \leq 500$ MeV

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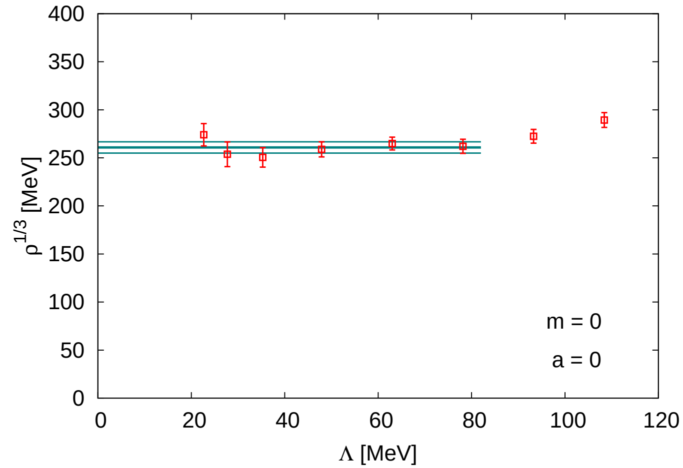

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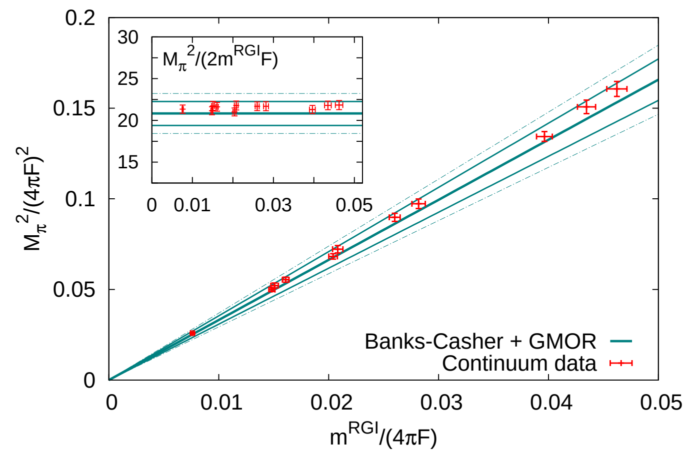

- The dimensionless ratios

$$[\Sigma^{\rm RGI}]^{1/3}/F = 2.77(2)(4)\;, \quad \Lambda^{\overline{\rm MS}}/F = 3.6(2)$$

are “geometrical” properties of the theory. They can be directly compared with your preferred approximation/model

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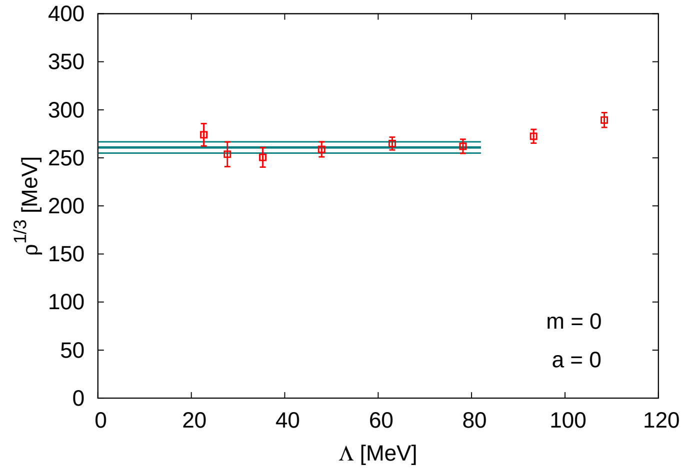

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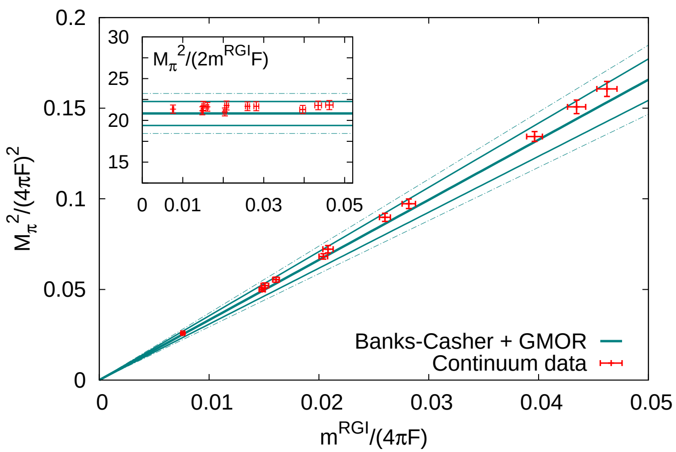

- For instance a large N_c computation gives [Armoni et al. 06]

$$\frac{[\Sigma^{\rm RGI}]^{1/3}}{\Lambda^{\overline{\rm MS}}} = 1.43\left[\frac{N_c}{2\pi^2}\tilde{K}\right]^{1/3} \qquad \tilde{K} = 1 + O(1/N_c)$$

The above lattice measures give $[\Sigma^{\rm RGI}]^{1/3}/\Lambda^{\overline{\rm MS}} = 0.77(4) \implies \tilde{K}^{1/3} = 1.01(5)$

The Witten–Veneziano mechanism [Witten 79; Veneziano 79]

- An axial Ward identity of the chiral group is

$$\int d^4x \, \langle Q(x)Q(0)\rangle = -\frac{m}{N_F}\int d^4x \, \langle P^0(x)Q(0)\rangle \; , \quad Q(x) = -\frac{1}{32\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\Big[F_{\mu\nu}(x)F_{\rho\sigma}(x)\Big]$$

- In the chiral and $N_c \to \infty$ limits

$$\frac{F^2 M^2_{\eta'}}{2N_F} = \chi^{\mathrm{YM}}_\infty \qquad\qquad \chi^{\mathrm{YM}}_\infty = \lim_{N_c\to\infty}\int d^4x \, \langle Q(x)Q(0)\rangle^{\mathrm{YM}} \neq 0$$

- Note that for $N_c \to \infty$:

  $*$ $\mathrm{U}(1)_\mathrm{A}$ is restored

  $*$ η' becomes a Nambu–Goldstone boson $\Longrightarrow M_{\eta'} = 0$

  $*$ At first order in $1/N_c$

$$M^2_{\eta'} = 2N_F\,\frac{\chi^{\mathrm{YM}}}{F^2} + \mathcal{O}(1/N_c^2)$$

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- An unambiguous definition of the topological susceptibility is required. Naive definition would diverge as

$$\chi = \int d^4x \, \langle Q(x)Q(0)\rangle \propto \frac{1}{a^4}$$

- Models for extracting the topological content by "cooling" the gauge configurations developed in the past, for an updated review see [Vicari, Panagopoulos 09]

Unambiguous definitions of the topological susceptibility on the lattice

- At least three families of definitions of χ ultraviolet finite and unambiguous

- From Ginsparg–Wilson fermions
  [LG, Rossi, Testa, Veneziano 01; LG, Rossi, Testa 04; Lüscher 04]

$$\chi_N = \sum_x a^4 \langle Q(x)Q(0)\rangle \qquad Q(x) = -\frac{1}{2a^3}\mathrm{Tr}\Big[\gamma_5 D(x,x)\Big]$$

- From spectral projectors of Dirac operator [LG, Lüscher 09]

$$\chi_P = \frac{\langle \mathrm{Tr}\{\mathrm{P}_M\}\rangle}{V}\,\frac{\langle \mathrm{Tr}\{\gamma_5 \mathrm{P}_M\}\mathrm{Tr}\{\gamma_5 \mathrm{P}_M\}\rangle}{\langle \mathrm{Tr}\{\gamma_5 \mathrm{P}_M \gamma_5 \mathrm{P}_M\}\rangle}$$

- From the Wilson flow [Lüscher 10]

$$\chi_{WF} = \sum_x a^4 \langle Q(x)Q(0)\rangle\,, \qquad Q(x) = -\frac{1}{32\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\Big[G_{\mu\nu}(x)G_{\rho\sigma}(x)\Big]$$

A proof that χ_{WF} is in the same universality class of the other two is still missing

Non-perturbative computation for $N_c = 3$ [Del Debbio et al. 04, Lüscher Palombi 10; Cè et al. 14]

- With the GW definition a fit of the form

$$r_0^4 \chi^{\rm YM}(a,s) = r_0^4 \chi^{\rm YM} + c_1(s)\frac{a^2}{r_0^2}$$

gives

$$r_0^4 \chi^{\rm YM} = 0.059 \pm 0.003$$

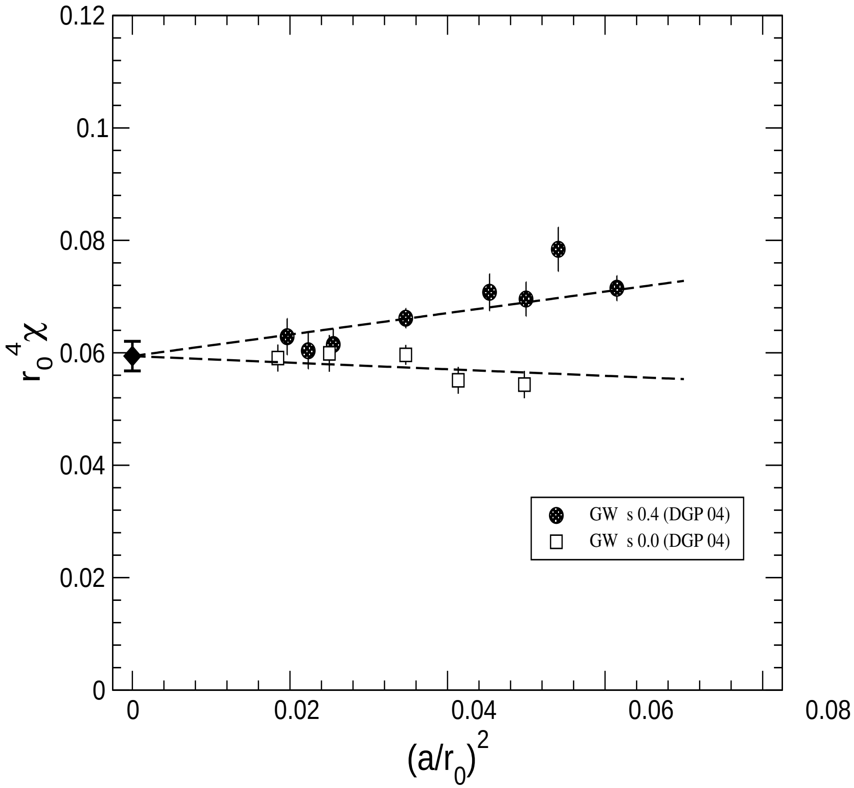
- By setting the scale $F_{\rm K} = 0.113(1)$ GeV

$$\chi^{\rm YM} = (0.191 \pm 0.005 \text{ GeV})^4$$

to be compared with

$$\frac{F^2}{2N_F}(M_\eta^2 + M_{\eta'}^2 - 2M_K^2) \underset{\rm exp}{\approx} (0.175 \text{ GeV})^4$$

- The (leading) QCD anomalous contribution to $M_{\eta'}^2$ supports the Witten–Veneziano explanation for its large experimental value



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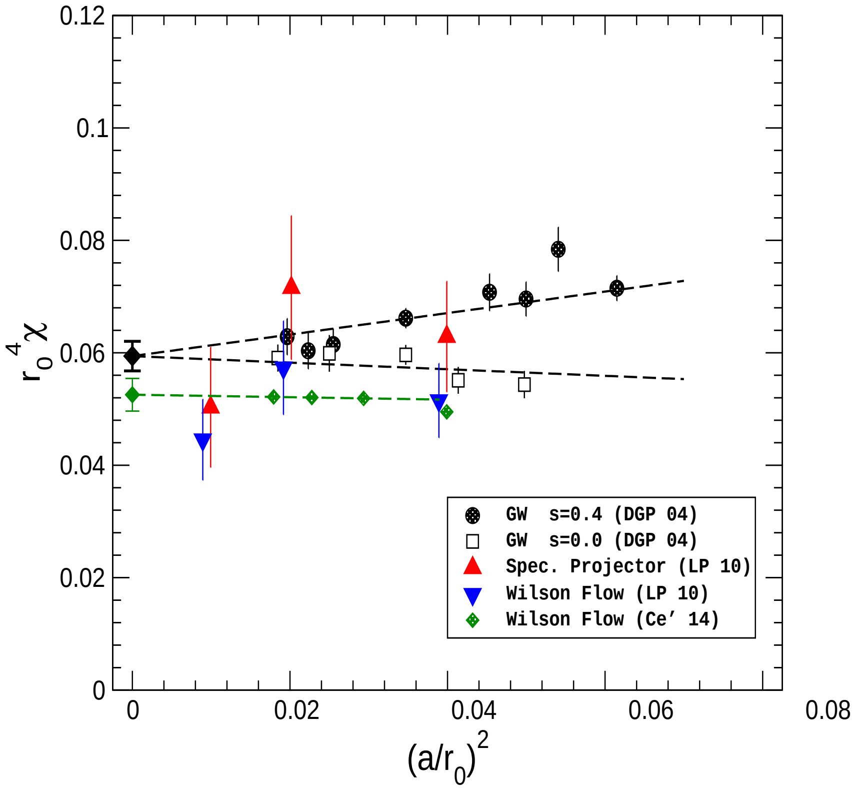
- With the Wilson flow definition (preliminary)

$$r_0^4 \chi^{\rm YM} = 0.053 \pm 0.003$$

which corresponds to

$$\chi^{\rm YM} = (0.185 \pm 0.005 \text{ GeV})^4$$



- From an unsolved problem to a universality test of lattice gauge theory!

How the Witten–Veneziano mechanism works ?

PHYSICAL REVIEW D VOLUME 14, NUMBER 12 15 DECEMBER 1976

Computation of the quantum effects due to a four-dimensional pseudoparticle*

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(Received 28 June 1976)

A detailed quantitative calculation is carried out of the tunneling process described by the Belavin-Polyakov-Schwarz-Tyupkin field configuration. A certain chiral symmetry is violated as a consequence of the Adler-Bell-Jackiw anomaly. The collective motions of the pseudoparticle and all contributions from single loops of scalar, spinor, and vector fields are taken into account. The result is an effective interaction Lagrangian for the spinors.

I. INTRODUCTION

When one attempts to construct a realistic gauge theory for the observed weak, electromagnetic, and strong interactions, one is often confronted with the difficulty that most simple models have

rections, and it determines the scale of the amplitude. Clearly, then, to understand the main features of such an amplitude, complete understanding of all one-loop quantum effects is de-

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CURRENT ALGEBRA THEOREMS FOR THE U(1) "GOLDSTONE BOSON"

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The U(1) problem is reconsidered from the point of view of the $1/N$ expansion. It is argued that various heuristic ideas about the η' are valid from this point of view. Current algebra theorems, similar to soft π theorems, are derived for the η'. They are valid to lowest order in $1/N$.

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U(1) WITHOUT INSTANTONS

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Witten's recent proposal that the U(1) problem might be solved in $1/N$ expanded QCD, is shown to be automatically consistent with expected θ dependences and anomalous Ward identities, if a (modified) Kogut–Susskind mechanism is used. Ward identities are algebraically saturated for large N. A sort of "partial conservation of the U(1) current" is found to hold for the "η" field.

1. Introduction

The history of the U(1) problem* is a particularly frustrating one: although a number of solutions were proposed at various moments, each time they were claimed not to be viable at some deeper level.

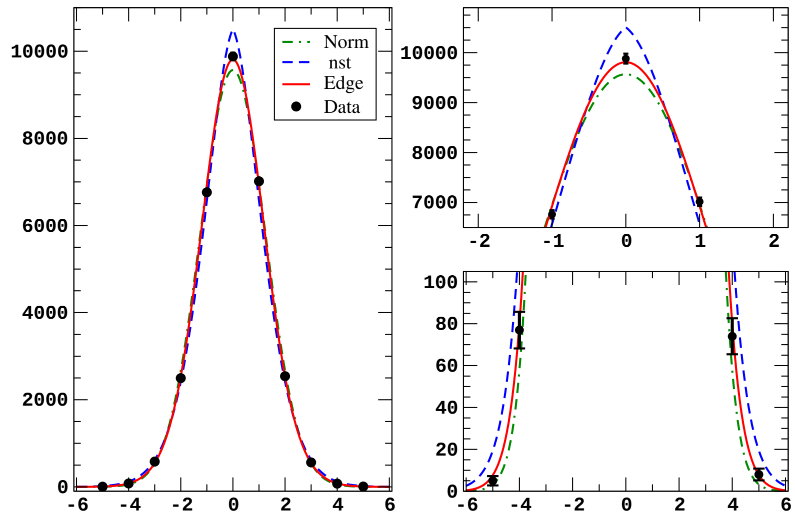
The most recent example of this pattern is 't Hooft's solution [2], i.e., his

How the WV mechanism works ? [LG, Petrarca, Taglienti 07; Cè et al. 14]

- Vacuum energy and charge distribution are

$$e^{-F(\theta)} = \langle e^{i\theta Q} \rangle, \; P_Q = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} e^{-F(\theta)}$$

Their behaviour is a distinctive feature of the configurations that dominate the path integr.



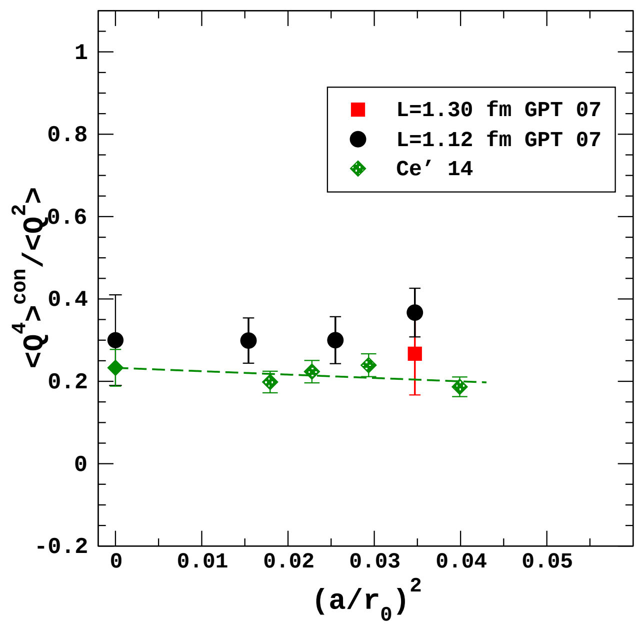
- Large N_c predicts ['t Hooft 74; Witten 79; Veneziano 79]

$$\frac{\langle Q^{2n} \rangle^{\text{con}}}{\langle Q^2 \rangle} \propto \frac{1}{N_c^{2n-2}}$$

- Various conjectures. For example, dilute-gas instanton model gives ['t Hooft 76; Callan et al. 76; . . .]

$$F^{\text{Inst}}(\theta) = -VA\{\cos(\theta) - 1\}$$

$$\frac{\langle Q^{2n} \rangle^{\text{con}}}{\langle Q^2 \rangle} = 1$$



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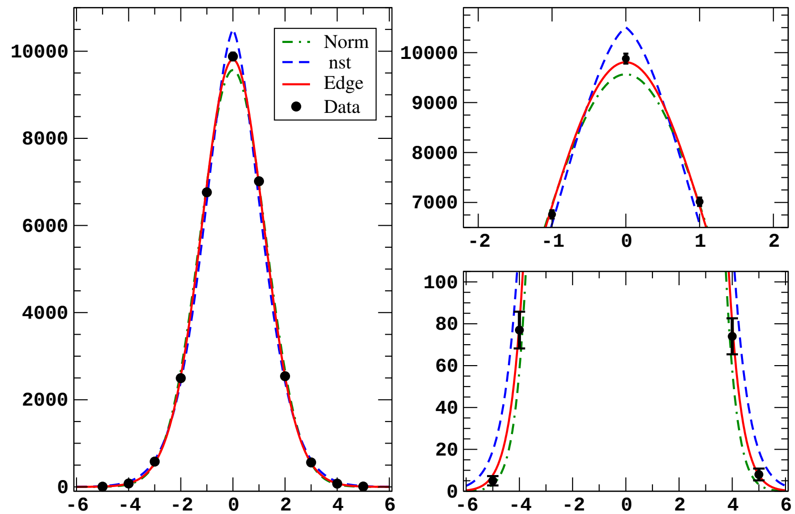
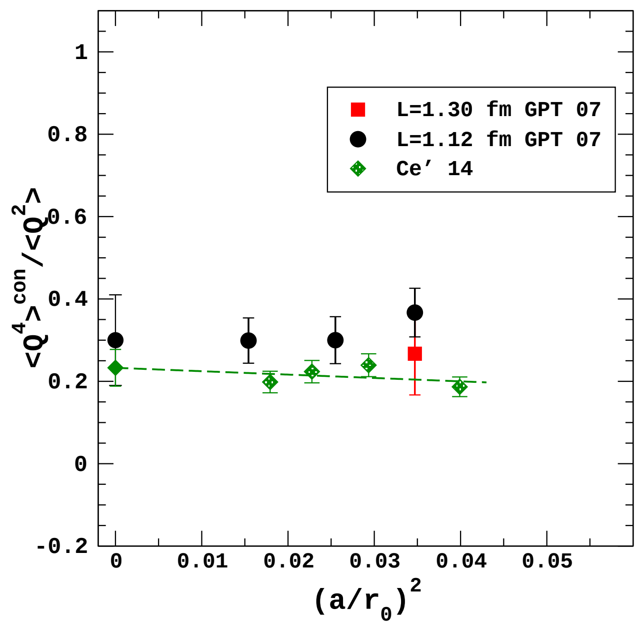


- Lattice computations give

$$\frac{\langle Q^4\rangle^{\rm con}}{\langle Q^2\rangle} = 0.30 \pm 0.11 \quad \text{Ginsparg–Wilson}$$

$$= 0.23 \pm 0.05 \quad \text{Wilson-Flow (Prel.)}$$

i.e. supports large N_c and disfavours a dilute gas of instantons

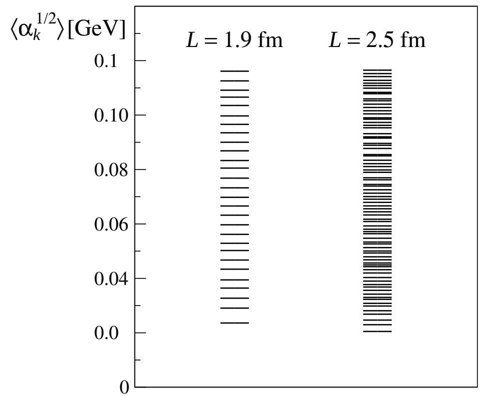


- The anomaly gives a mass to the η' thanks to the NP quantum fluctuations of Q

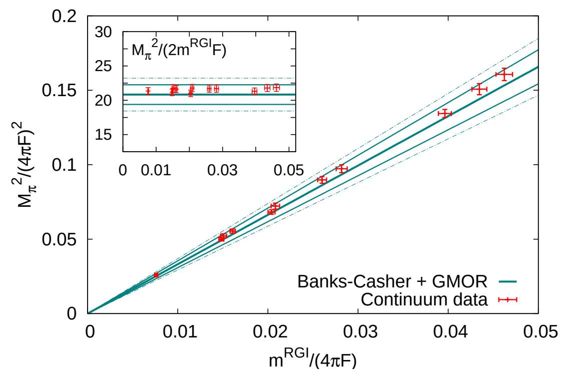
Conclusions and Outlook

- In QCD Lite the low-modes of the Dirac operator do condense in the continuum due to the Banks-Casher mechanism

- The condensation is the most direct piece of theoretical evidence for SSB



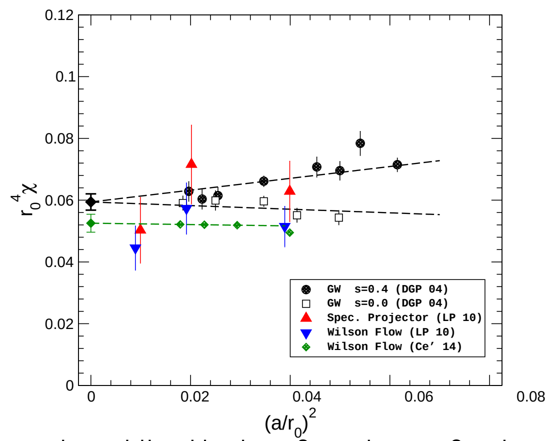
- The rate of condensation explains the bulk of the pion mass up to 0.5 GeV



- Quantum fluctuations of the topological charge in Yang–Mills theory generate a non-zero value of $\chi^{\rm YM}$

- Its value supports the Witten–Veneziano explanation for the large mass of the η'

- Wilson flow is a very powerful tool to study the topological charge distribution on the lattice



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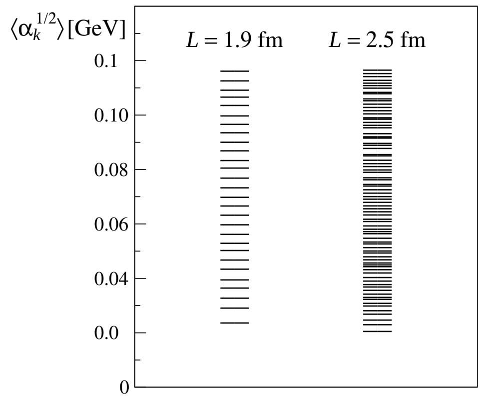

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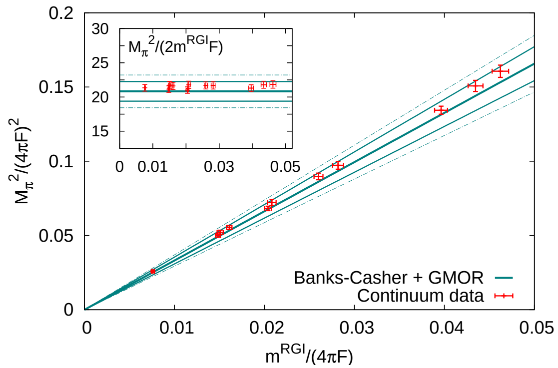

- First attempts to compute χ in full QCD
  [Cichy et al. 14; Bruno et al. 14]

- To arrive at $O(5\%)$ statistical error requires much longer Monte Carlo runs than typically generated in the present full QCD simulations (direct way to measure N_F)